$\mathbf{SU}(3)$ breaking in hyperon beta decays: A prediction for $\mathbf{\Xi}^0 \rightarrow \Sigma^+ e \, \overline{\nu}$

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On the basis of a previous analysis of hyperon semileptonic decay data, a prediction is presented for g_1/f_1 in the $\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$ β -decay. The analysis takes into account SU(3) breaking in this sector via the inclusion of mass-driven corrections. A rather precise measurement of the above channel by the KTeV experiment at Fermilab will shortly be available. Since the dependence on the SU(3) parameters, *F* and *D*, is identical to that of the neutron β -decay, such a measurement will provide a rather stringent test of SU(3) and the models used to describe its violation in these decays. The prediction given here for the above decay is $g_1 / f_1 = 1.17$, which leads to a rate of 0.80×10^6 s⁻¹ and thus a branching fraction of 2.3×10^{-4} . [S0556-2821(99)00701-8]

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I. INTRODUCTION

In recent years the precision of experimental hyperon semileptonic decay data has improved steadily $[1-4]$ with parameters and rates now known to within a few percent. Indeed, the present accuracy demands an approach for applying corrections due to the breaking of $SU(3)$. However, there are several methods proposed in the literature; all describe the data with varying degrees of success, from different starting points and with differing output values for the parameters involved $(e.g., F \text{ and } D)$.

The imminent release of an entirely new branching ratio, that of $\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$, will permit testing the various approaches. This channel is being studied by the KTeV experiment at Fermilab $[5]$. It is important to note that the angular correlations will also be measured $[6]$. It turns out that this particular channel has an axial decay constant given by $g_1 / f_1 = F + D$, which in the absence of SU(3) breaking would therefore be identical to that of neutron β -decay (by far the most precisely known). Thus, provided both Γ and g_1 / f_1 are measured independently, such a channel can provide a sensitive test of the methods used to describe $SU(3)$ symmetry breaking in this sector.

II. HYPERON SEMILEPTONIC DECAY DATA

The present situation with regard to the hyperon semileptonic decay (HSD) data is shown in Table I, which represents the useful available knowledge. As discussed in $[7]$, the disagreement between the neutron lifetime and the value of g_1 / f_1 extracted from β -decay angular correlations [8] requires some care, to avoid clouding the issue of $SU(3)$ breaking. The present value of the neutron lifetime is 887.0 \pm 2.0 s and *g*₁/*f*₁ (from angular correlations) is 1.2601(25)¹ $[4]$; i.e., both are known independently to approximately 0.2%. The value of the relevant Cabibbo-KobayashiMaskawa matrix element extracted from the *ft* values of the eight super-allowed nuclear β -decay Fermi transitions is V_{ud} =0.9740(5) [9]. This is to be compared with the values V_{ud} =0.9795(20), from the neutron lifetime and g_1 / f_1 , and V_{ud} =0.9758(4), from the so-called K_{l3} decays (V_{us} $=0.2188(16)$ [10]).

The displacements from the central values are all very small, < 0.2 %. Thus, to neutralize the contribution of the neutron discrepancy to the global χ^2 , a mean value for V_{ud} is first extracted from the nuclear ft and K_{13} data. Then using this value, a combined fit to the $\Gamma_{n\to p}$ and g_1/f_1 is made. Finally, the errors of the $\Gamma_{n\to p}$, g_1/f_1 , and mean V_{ud} values are multiplied by the $\sqrt{\chi^2}$ so obtained; these are used in all fits:

$$
\Gamma(n \to p\overline{1}\overline{\nu}) = (1.1274 \pm 0.0055) \times 10^{-3} \text{ s}^{-1}, \quad (1)
$$

$$
g_1/f_1 = 1.2601 \pm 0.0055,\tag{2}
$$

$$
V_{ud} = 0.9752 \pm 0.0007. \tag{3}
$$

III. SU(3) ANALYSES

In Table II the results of a series of fits to the hyperon semileptonic decay-data are displayed: the symmetric fit uses three parameters $(F, D \text{ and } V_{ud})$, and the SU(3) breaking is described by one further parameter (described in the following). We use the mean value obtained from the combined nuclear ft analysis and K_{13} decays just described, and impose the unitarity constraint $V_{ud}^2 + V_{us}^2 = 1$ (neglecting V_{ub} 50.0033 ± 0.0008 [4]). The parametrizations of the SU(3) breaking used are the so-called center-of-mass correction $[11]$ (fit A), which is described in detail in $[7]$, and an alternative breaking scheme (fit B), using an $SU(3)$ motivated mass dependence for the axial couplings $[12]$.

Approach A is to apply center-of-mass or recoil corrections to the axial coupling constant for the process $A \rightarrow B l \nu$ according to the following formula $\lfloor 13 \rfloor$:

$$
g_1 = g_1^{\text{SU(3)}} \left\{ 1 - \frac{\langle p^2 \rangle}{3m_A m_B} \left[\frac{1}{4} + \frac{3m_B}{8m_A} + \frac{3m_A}{8m_B} \right] \right\}.
$$
 (4)

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The slight change in the value since the publication of $[7]$ has no visible effect on any of the fits.

	Γ (10 ⁶ s ⁻¹)		g_1/f_1	
Decay	$l = e^{-}$	$l = \mu^-$	$l = e^{-}$	SU(3)
$n \rightarrow pl\bar{\nu}$	$1.1274 \pm 0.0025^{\text{a}}$		1.2601 ± 0.0025	$F+D$
$\Lambda^0 \rightarrow pl\bar{\nu}$	3.161 ± 0.058	0.60 ± 0.13	0.718 ± 0.015	$F + D/3$
$\Sigma^- \rightarrow n l \overline{\nu}$	6.88 ± 0.23	3.04 ± 0.27	-0.340 ± 0.017	$F-D$
$\Sigma^- \rightarrow \Lambda^0 l \overline{\nu}$	0.387 ± 0.018			$-\sqrt{\frac{2}{3}}D^{\mathrm{b}}$
$\Sigma^+ \rightarrow \Lambda^0 \overline{l} \nu$	0.250 ± 0.063			$-\sqrt{\frac{2}{3}}D^{\mathrm{b}}$
Ξ ⁻ \rightarrow $\Lambda^0 l\bar{\nu}$	3.35 ± 0.37^c	2.1 ± 2.1 ^d	0.25 ± 0.05	$F-D/3$
Ξ ⁻ \rightarrow Σ ⁰ $l\bar{\nu}$	0.53 ± 0.10			$F+D$

TABLE I. The hyperon semi-leptonic data used in this analysis $[4]$, g_1/f_1 indicates the value as extracted from angular correlations. The last column shows the SU(3) formula for g_1 / f_1 .

^aThe rate is given in 10^{-3} s⁻¹.

 b As $f_1 = 0$, the absolute expression for is g_1 given.

 c A scale factor of 2 is included, following the PDG practice for discrepant data.

^dThese data are not used in the fits.

A similar correction to the vector piece is entirely negligible $(in$ accordance with the Ademollo-Gatto theorem $[14]$ and thus here f_1 is taken to have its naive SU(3) CVC value. The mean momentum squared, $\langle p^2 \rangle$, is calculated by Donoghue, Holstein and Klimt [13] using a bag model to be 0.43 GeV^2 , here it is left as a free parameter and is determined in fit A to be 0.43 ± 0.11 GeV². The results for approach B are necessarily rather similar as it effectively corresponds to a linearization of Eq. (4) .

As can be seen from Table II, the data clearly indicate the presence of $SU(3)$ breaking, which is well described by the correction schemes adopted. Note also that the value of the ratio F/D is largely unaffected by the breaking, changing by less than 2%, and should thus not be considered as an indicator of the importance of $SU(3)$ breaking. In both schemes a possible additional breaking in the $|\Delta S=1|$ decays has been neglected; in previous fits this was found to be at most about 2%; in any case, it is essentially absorbed into the extracted value of $\sin^2 \theta_c$ and has negligible effect on *F* and *D*.

IV. THE PREDICTION FOR $\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$

At this point a prediction is possible for any of the remaining HSD's: in particular, the $\Xi^0 \rightarrow \Sigma^+ e\bar{\nu}$ β -decay. Since independent measurements of both the decay rate and the angular correlations should be obtained, this measurement will, in principle, simultaneously allow separate control over the smallness of the corrections associated with the $|\Delta S = 1|$ decays (assumed here) and of the validity of corrections applied in the above analysis.

The values obtained are shown in Table III. Included in the error for the branching fraction is the contribution from the error on the total decay width of the Ξ^0 , which is about 3%, the others are those returned by the global fit. The difference between the two breaking fits, A and B, is an indication of the expected systematic uncertainty arising from this type of description, and which we thus estimate to be less than 3%. The spread is also small compared to the shift from the naïve values.

V. CONCLUSIONS

First of all, as has been demonstrated in detail elsewhere $[12]$, the axial couplings extracted from the hyperon decays are well described by a parametrization motivated by the mass differences in the baryon octet. The results discussed above permit a precise prediction for the $\Xi^0 \rightarrow \Sigma^+ e \overline{\nu}$ β -decay: here both the g_1/f_1 and the expected decay rate have been presented. The values given may be compared to another prediction in the literature due to Flores-Mendieta, Jenkins, and Manohar $[15]$. In a breaking scheme based on the $1/N_c$ expansion, the authors cited find a value for g_1/f_1 considerably smaller than that predicted here: $f_1 = 1.12$ and $g_1 = 1.02$, or $g_1 / f_1 = 0.91$ (their fit B), which leads to a rate of 0.68×10^6 s⁻¹. Their prediction for the SU(3) parameters is $F/D = 0.46$, to be compared with 0.57 above. In an alternative fit, where f_1 is left at its SU(3) value, they obtain $g_1 / f_1 = 1.03$ (their fit A) and 0.65×10^6 s⁻¹. In either case

TABLE II. SU(3) symmetric and breaking fits to the modified data, including the external V_{ud} from nuclear *ft* and K_{13} analyses (see the text for details).

Fit	V_{ud}	Parameters F	D	χ^2 /DOF	F/D
Sym.	0.9749 ± 0.0003	0.465 ± 0.006	0.798 ± 0.006	2.3	0.582
A	0.9743 ± 0.0004	0.460 ± 0.006	0.806 ± 0.006	1.2	0.571
B	0.9744 ± 0.0004	0.459 ± 0.006	0.807 ± 0.006	1.2	0.571

TABLE III. The values obtained for the axial coupling (g_1/f_1) , rate (Γ) and branching fraction (*B*) for the $\Xi^0 \rightarrow \Sigma^+ e \overline{\nu}$ *β*-decay. The errors quoted are purely those returned by the fitting routine.

Fit	g_1/f_1	Γ (10 ⁶ s ⁻¹)	$B(10^{-4})$
Sym.	1.26^a	0.89 ± 0.01	2.58 ± 0.05
A	1.17 ± 0.03	0.80 ± 0.03	2.32 ± 0.10
B	1.14 ± 0.03	0.78 ± 0.03	2.26 ± 0.12

^aNo serious error can be associated with the value of g_1/f_1 in the symmetric fit as it should be precisely that of the neutron β -decay.

both g_1/f_1 and the rate are considerably smaller than the results of the present analysis, which in turn are considerably smaller than a naive fit. Thus, the various possibilities should be distinguishable in an experiment with good statistics, such as KTeV.

It should perhaps be mentioned that the Flores-Mendieta *et al.* fit also includes data on the baryon decuplet nonleptonic decays, which in fact dominate the final results. The overall fit, according to the value of χ^2 returned, is rather poor. Moreover, their approach applied to the hyperon semileptonic decay data alone produces results similar to those reported in this paper $[16]$.

Secondly, in this analysis, as too in $[15]$, the possibility of a weak electric (g_2) contribution has been neglected. It is therefore worth remarking that experimental data on the $\Sigma^- \rightarrow ne\bar{\nu}$ β -decay [2] indicate that such a second-class current contribution may be non-negligible. Indeed, the data marginally prefer a sizable g_2 and thus a much reduced value for g_1 there. If such were the case, then the question would also arise as to the relevance of g_2 in other decays, where the experimental analysis has typically assumed it zero.

Thirdly, a paper often quoted in the literature as providing evidence for large breaking effects, similar to those found in [15], is that of Ehrnsperger and Schater $[17]$. There the authors apply an *ad hoc* one-parameter (*a* below) correction to the angular correlation data alone:

$$
\frac{F}{D} = \left(\frac{F}{D}\right)^{\text{SU(3)}} \left[1 + a\frac{(m_A + m_B) - (m_n + m_p)}{(m_A + m_B) + (m_n + m_p)}\right],\tag{5}
$$

where *a* is found to be \sim 2.7; thus, the limiting value of *F*/*D* is 0.49 ± 0.08 (note the large error). However, since the breaking is treated as affecting only the ratio *F*/*D* and not the sum, such a solution implies that the $\Xi^0 \rightarrow \Sigma^+ l \bar{\nu}$ decay has g_1/f_1 identical to that of the neutron despite the enormous mass shift.

Finally, before closing, let us recall another decay for which the rates and angular correlations are also expected to have very large corrections and thus to be highly sensitive to SU(3) breaking: namely, $\Xi^- \rightarrow \Sigma^0 e \nu$. Here too, the fact that $g_1 / f_1 = F + D$ makes it highly desirable to improve on the present limited experimental knowledge for this process.

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