Exclusive hadronic D decays to η' and η

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The hadronic decay modes $D^0\to(\bar{K}^0,\bar{K}^{*0})(\eta,\eta')$ and $(D^+,D_s^+)\to(\pi^+,\rho^+)(\eta,\eta')$ are studied in the generalized factorization approach. The form factors for $(D,D_s^+)\to(\eta,\eta')$ transitions are carefully evaluated by taking into account the wave function normalization of the η and η'. The predicted branching ratios are generally in agreement with experiment except for $D^0\to\bar{K}^0\eta'$, $D^+\to\pi^+\eta$ and $D_s^+\to\rho^+\eta'$; the calculated decay rates for the first two decay modes are too small by an order of magnitude. We show that the weak decays $D^0\to K^-\pi^+$ and $D^+\to K^+\bar{K}^0$ followed by resonance-induced final-state interactions (FSI), which are amenable technically, are able to enhance the branching ratios of $D^0\to\bar{K}^0\eta'$ and $D^+\to\pi^+\eta$ dramatically without affecting the agreement between theory and experiment for $D^0\to\bar{K}^0\eta$ and $D^+\to\pi^+\eta'$. We argue that it is difficult to understand the observed large decay rates of $D_s^+\to\rho^+\eta'$ and $\rho^+\eta$ simultaneously; FSI, W annihilation, and the production of excess η' from gluons are not helpful in this regard. The large discrepancy between the factorization hypothesis and experiment for the ratio of $D_s^+\to\rho^+\eta'$ and $D_s^+\to\eta' e^+\nu$ remains an enigma. [S0556-2821(99)01001-2]

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I. INTRODUCTION

The exclusive rare B decays to the η' have recently received a great deal of attention since the observed large branching ratio of $B^-\to\eta' K^-$ by CLEO is substantially higher than the naive theoretical estimates (for a review, see [1]). It has stimulated many theoretical studies and speculation. It is natural to reexamine the hadronic decays of the charmed mesons into the final states containing an η or η'. Experimentally, CLEO has recently remeasured the decay modes $(D_s^+,D^+)\to(\pi^+,\rho^+)\eta^{(\prime)}$ [2]. Combined with the previous measurements of $D^0\to\bar{K}^{0(*)}\eta^{(\prime)}$, we see an η' enhancement for $(D_s^+,D^+)\to\pi^+\eta'$ over $(D_s^+,D^+)\to\pi^+\eta$ and for $D^0\to\bar{K}^0\eta'$ over $D^0\to\bar{K}^0\eta$ (see Table I). Also, very large branching ratios for $D_s^+\to\rho^+\eta'$ and $D_s^+\to\rho^+\eta$ are confirmed by the new data [2]. Theoretically, the factorization approach of Bauer, Stech and Wirbel (BSW) [3] predicts less η' production than η in $D^0\to\bar{K}^0\eta^{(\prime)}$ and $D_s^+\to\pi^+\eta^{(\prime)}$ decays, in disagreement with experiment (see Table I). Moreover, the decay $D^+\to\pi^+\eta$ is severely suppressed in the BSW approach, about two orders of magnitude smaller than the experimental measurement. Likewise, the predicted branching ratios for $D^0\to\bar{K}^{*0}\eta$ and $D_s^+\to\rho^+\eta'$ are also too small. Many different theoretical attempts have been made in the past to explain the data [4–7].

Care must be taken when applying the BSW form factors for $(D,D_s)\to(\eta,\eta')$ transitions as the wave function normalizations of the η and η' are not taken into account in the original BSW analysis [9]. In this paper we will evaluate these form factors in a consistent way and present an updated analysis in the generalized factorization approach. Then we proceed to propose that final-state interactions (FSI) in resonance formation are responsible for the discrepancy between theory and experiment for the above-mentioned η'/η ratios and for the decay rate of $D^+\to\pi^+\eta$. Since some resonances are known to exist in the charm mass region and since the charm decay is not very energetic, FSI are expected to play an essential role in the nonleptonic charm decays. We shall show in the present paper that $D^+\to\pi^+\eta$ and $D^0\to\bar{K}^0\eta'$ are essentially generated from FSI. Finally, we will comment on the observed large branching ratio for the decay $D_s^+\to\rho^+\eta'$.

II. GENERALIZED FACTORIZATION

The effective weak Hamiltonian for nonleptonic charm decay relevant to the present paper is

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\Big\{V_{cs}^*V_{ud}(c_1(\mu)(\bar{u}d)(\bar{s}c)+c_2(\mu)(\bar{u}c)(\bar{s}d))+\sum_{q=d,s}V_{cq}^*V_{uq}(c_1(\mu)O_1^q(\mu)+c_2(\mu)O_2^q(\mu))\Big\}, \tag{2.1}$$

with $O_1^q=(\bar{u}q)(\bar{q}c)$ and $O_2^q=(\bar{u}c)(\bar{q}q)$, where $(\bar{q}_1q_2)\equiv\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ and $c_{1,2}(\mu)$ are the Wilson coefficient functions. The mesonic matrix elements of four-quark operators are customarily evaluated under the factorization approximation. It is known that naive factorization fails to describe color-suppressed charm decays. Therefore, it is necessary and mandatory to take into account nonfactorizable contributions to the weak decay amplitudes. For $D\to PP,VP$ decays (P: pseudoscalar meson, V: vector meson), the effects of nonfactorization characterized by the parameters χ_1 and χ_2 can be lumped into the effective parameters a_1 and a_2 [8]:

$$a_1 = c_1 + c_2\left(\frac{1}{N_c} + \chi_1\right), \quad a_2 = c_2 + c_1\left(\frac{1}{N_c} + \chi_2\right), \tag{2.2}$$

where N_c is the number of colors. If $\chi_{1,2}$ are universal (i.e., process independent) in charm decays, then we still have a new factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters $a_{1,2}$. By treating $a_{1,2}$ as free parameters, they can be determined from experiment. For example, neglecting the W-exchange contribution and assuming that final-state interactions can be described by isospin phase shifts, we find that

$$a_1(D \to \bar{K}\pi) = 1.25, \quad a_2(D \to \bar{K}\pi) = -0.51 \tag{2.3}$$

from the data of $D^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$ and $D^+ \to \bar{K}^0\pi^+$ decays.

We next consider the two-body decays of charmed mesons into the η or η'. Neglecting W-exchange or W-annihilation, it is easily seen that $D_s^+ \to (\pi^+, \rho^+)\eta^{(\prime)}$ proceed through the color-allowed external W-emission, $D^0 \to \bar{K}^{0(*)}\eta^{(\prime)}$ via the color-suppressed internal W-emission, and $D^+ \to (\pi^+, \rho^+)\eta^{(\prime)}$ receive contributions from both external and internal W-emission diagrams. Under the generalized factorization hypothesis, it is straightforward to write down the decay amplitudes of the charmed meson decays to the final state containing an η or η':

$$A(D^0 \to \bar{K}^0\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_2 (X^{(D\eta^{(\prime)},K)} + 2X^{(D,K\eta^{(\prime)})}),$$

$$A(D^0 \to \bar{K}^{*0}\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_2 (X^{(D\eta^{(\prime)},K^*)} + 2X^{(D,K^*\eta^{(\prime)})}),$$

$$A(D^+ \to \pi^+\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} [a_1 X^{(D\eta^{(\prime)},\pi)} + a_2 (X_d^{(D\pi,\eta^{(\prime)})} - X_s^{(D\pi,\eta^{(\prime)})}) + 2a_1 X^{(D,\eta^{(\prime)}\pi)}],$$

$$A(D^+ \to \rho^+\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} [a_1 X^{(D\eta^{(\prime)},\rho)} + a_2 (X_d^{(D\rho,\eta^{(\prime)})} - X_s^{(D\rho,\eta^{(\prime)})}) + 2a_1 X^{(D,\eta^{(\prime)}\rho)}],$$

$$A(D_s^+ \to \pi^+\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_1 (X^{(D_s\eta^{(\prime)},\pi)} + 2X^{(D_s,\eta^{(\prime)}\pi)}),$$

$$A(D_s^+ \to \rho^+\eta^{(\prime)}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_1 (X^{(D_s\eta^{(\prime)},\rho)} + 2X^{(D_s,\eta^{(\prime)}\rho)}), \tag{2.4}$$

where use of the approximation $V_{cs}^* V_{us} \approx -V_{cd}^* V_{ud}$ has been made and $X^{(DP_1,P_2)}$ denotes the factorizable amplitude with the meson P_2 being factored out:

$$X^{(DP_1,P_2)} = \langle P_2|(\bar{q}_1 q_2)|0\rangle\langle P_1|(\bar{q}_3 c)|D\rangle. \tag{2.5}$$

Explicitly,

$$X^{(D\eta^{(\prime)},K)} = i f_K (m_D^2 - m_{\eta^{(\prime)}}^2) F_0^{D\eta^{(\prime)}}(m_K^2),$$

$$X^{(D\eta^{(\prime)},\pi)} = i f_\pi (m_D^2 - m_{\eta^{(\prime)}}^2) F_0^{D\eta^{(\prime)}}(m_\pi^2),$$

$$X_q^{(D\pi,\eta^{(\prime)})} = i f_{\eta^{(\prime)}}^q (m_D^2 - m_\pi^2) F_0^{D\pi^\pm}(m_{\eta^{(\prime)}}^2),$$

$$X^{(D_s\eta^{(\prime)},\pi)} = i f_\pi (m_{D_s}^2 - m_{\eta^{(\prime)}}^2) F_0^{D_s\eta^{(\prime)}}(m_\pi^2),$$

$$X^{(D\eta^{(\prime)},K^*)} = 2 f_{K^*} m_{K^*} F_1^{D\eta^{(\prime)}}(m_{K^*}^2)(\varepsilon \cdot p_D),$$

$$X^{(D\eta^{(\prime)},\rho)} = 2 f_\rho m_\rho F_1^{D\eta^{(\prime)}}(m_\rho^2)(\varepsilon \cdot p_D),$$

$$X^{(D_s\eta^{(\prime)},\rho)} = 2 f_\rho m_\rho F_1^{D_s\eta^{(\prime)}}(m_\rho^2)(\varepsilon \cdot p_{D_s}),$$

$$X_q^{(D\rho,\eta^{(\prime)})} = 2 f_{\eta^{(\prime)}}^q m_\rho A_0^{D\rho}(m_{\eta^{(\prime)}}^2)(\varepsilon \cdot p_D), \tag{2.6}$$

where $\langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}\rangle = i f^q_{\eta^{(\prime)}} p_\mu$, and form factors F_0, F_1 and A_0 are those defined in [9]. The amplitude $X^{(D,\eta^{(\prime)}P)}$ in Eq. (2.4) denotes W-exchange or W-annihilation, for example,

$$X^{(D,\eta\pi)} = \langle \eta\pi^+|(\bar{u}d)|0\rangle\langle 0|(\bar{d}c)|D^+\rangle.$$

To determine the decay constant $f^q_{\eta'}$, we need to know the wave functions of the physical η' and η states which are related to that of the SU(3) singlet state η_0 and octet state η_8 by

$$\eta' = \eta_8 \sin\theta + \eta_0\cos\theta, \qquad \eta = \eta_8\cos\theta - \eta_0\sin\theta, \tag{2.7}$$

with $\theta \approx -20°$. When the $\eta-\eta'$ mixing angle is $-19.5°$, the η' and η wave functions have simple expressions [10]:

$$|\eta'\rangle = \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d + 2\bar{s}s\rangle, \qquad |\eta\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d - \bar{s}s\rangle, \tag{2.8}$$

recalling that

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle, \qquad |\eta_8\rangle = \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle. \tag{2.9}$$

At this specific mixing angle, $f^u_{\eta'} = \frac{1}{2} f^s_{\eta'}$ in the SU(3) limit. Introducing the decay constants f_8 and f_0 by $\langle 0|A^0_\mu|\eta_0\rangle = i f_0 p_\mu$ and $\langle 0|A^8_\mu|\eta_8\rangle = i f_8 p_\mu$, then $f^u_{\eta'}$ and $f^s_{\eta'}$ are related to f_8 and f_0 by

$$f^u_{\eta'} = \frac{f_8}{\sqrt{6}}\sin\theta + \frac{f_0}{\sqrt{3}}\cos\theta, \qquad f^s_{\eta'} = -2\frac{f_8}{\sqrt{6}}\sin\theta + \frac{f_0}{\sqrt{3}}\cos\theta. \tag{2.10}$$

Likewise, for the η meson

$$f^u_{\eta} = \frac{f_8}{\sqrt{6}}\cos\theta - \frac{f_0}{\sqrt{3}}\sin\theta, \qquad f^s_{\eta} = -2\frac{f_8}{\sqrt{6}}\cos\theta - \frac{f_0}{\sqrt{3}}\sin\theta. \tag{2.11}$$

Applying the results

$$\frac{f_8}{f_\pi} = 1.38 \pm 0.22, \quad \frac{f_0}{f_\pi} = 1.06 \pm 0.03, \quad \theta = -22.0° \pm 3.3°, \tag{2.12}$$

extracted from a recent analysis of the data of $\eta,\eta' \to \gamma\gamma$ and $\eta,\eta' \to \pi\pi\gamma$ [11] yields

$$f^u_\eta = 99 \text{ MeV}, \qquad f^s_\eta = -108 \text{ MeV},$$
$$f^u_{\eta'} = 47 \text{ MeV}, \qquad f^s_{\eta'} = 131 \text{ MeV}. \tag{2.13}$$

To compute the form factors $F_0^{D\eta^{(\prime)}}$ and $F_0^{D_s\eta^{(\prime)}}$, we will first apply the nonet symmetry relations

$$\sqrt{6}F_0^{D\eta_8}(0) = \sqrt{3}F_0^{D\eta_0}(0) = F_0^{D\pi^\pm}(0),$$
$$-\sqrt{\frac{6}{2}}F_0^{D_s\eta_8}(0) = \sqrt{3}F_0^{D_s\eta_0}(0) = F_0^{D_sK}(0), \tag{2.14}$$

to determine $F_0^{D\eta_{0,8}}(0)$ and $F_0^{D_s\eta_{0,8}}(0)$, and then relate them to the form factors $F_0^{D\eta^{(\prime)}}$ and $F_0^{D_s\eta^{(\prime)}}$ via

$$F_0^{D\eta} = F_0^{D\eta_8}\cos\theta - F_0^{D\eta_0}\sin\theta,$$
$$F_0^{D\eta'} = F_0^{D\eta_8}\sin\theta + F_0^{D\eta_0}\cos\theta. \tag{2.15}$$

Using $F_0^{D\pi^\pm}(0) \approx F_0^{DK}(0) \approx 0.75$ as inferred from experiment [12], and taking $F_0^{D_sK}(0) \approx 0.76$ extracted from the data of $D_s^+ \to K^+\bar{K}^0$ and $K^{*+}\bar{K}^0$ for $a_2 = -0.51$, we obtain

$$F_0^{D\eta}(0) = 0.446, \qquad F_0^{D\eta'}(0) = 0.287,$$
$$F_0^{D_s\eta}(0) = -0.411, \qquad F_0^{D_s\eta'}(0) = 0.639. \tag{2.16}$$

Note that the form factor $F_0^{D_s\eta}$ has a sign opposite to $F_0^{D_s\eta'}$ due to the sign difference of the strange quark content in the η and η' [see Eq. (2.8)]. Using the above form factors for $D_s^+ \to \eta^{(\prime)}$ transition, we have computed the semileptonic decay rates of $D_s^+ \to \eta^{(\prime)}e^+\nu$ and found an agreement with experiment.

The form factors for $D \to \eta^{(\prime)}$ and $D_s \to \eta^{(\prime)}$ transitions also have been calculated by BSW [9] in a relativistic quark model. However, form factors obtained there did not include the wave function normalizations and mixing angles.[1] For example, for $D\to\eta$ transition, BSW put in the $u\bar{u}$ constitutent quark mass only, and for $D_s\to\eta$ the $s\bar{s}$ quark masses. In this way, BSW obtained [3]

$$F_0^{D\eta_{u\bar{u}}}(0) = 0.681, \qquad F_0^{D\eta'_{u\bar{u}}}(0) = 0.655,$$
$$F_0^{D_s\eta_{s\bar{s}}}(0) = 0.723, \qquad F_0^{D_s\eta'_{s\bar{s}}}(0) = 0.704. \tag{2.17}$$

To compute the physical form factors one has to take into account the wave function normalizations of η and η':

$$F_0^{D\eta} = \left(\frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\right)F_0^{D\eta_{u\bar{u}}}, \tag{2.18}$$

$$F_0^{D\eta'} = \left(\frac{1}{\sqrt{6}}\sin\theta + \frac{1}{\sqrt{3}}\cos\theta\right)F_0^{D\eta'_{u\bar{u}}},$$

[1] We are grateful to A. N. Kamal for pointing this out to us.

TABLE I. Branching ratios (in units of %) of the charmed meson decays to an η or η'. The BSW predictions [3] are for the η-η' mixing angle $\theta=-10°$, while ours are for $\theta=-22°$.

Decay	BSW [3]	This work without FSI	This work with resonant FSI	Expt. [2,14]
$D^0\to\overline{K}^0\eta$	0.31	0.50	$0.54^{+0.01}_{-0.02}$	0.71 ± 0.10
$D^0\to\overline{K}^0\eta'$	0.12	0.10	$0.90^{+0.27}_{-0.45}$	1.72 ± 0.26
$D^0\to\overline{K}^{*0}\eta$	0.28	0.76	0.74	1.9 ± 0.5
$D^0\to\overline{K}^{*0}\eta'$	0.002	0.004	0.02	<0.11
$D^+\to\pi^+\eta$	0.002	0.011	0.12	0.30 ± 0.06
$D^+\to\pi^+\eta'$	0.15	0.25	0.59	0.50 ± 0.10
$D^+\to\rho^+\eta$	0.06	0.20	0.20	<0.68
$D^+\to\rho^+\eta'$	0.03	0.07	0.07	<0.52
$D_s^+\to\pi^+\eta$	3.66	2.43	1.30	1.73 ± 0.47
$D_s^+\to\pi^+\eta'$	2.14	3.32	4.37	3.71 ± 0.98
$D_s^+\to\rho^+\eta$	6.87	5.92	5.92	10.7 ± 3.1
$D_s^+\to\rho^+\eta'$	1.94	3.86	3.86	10.0 ± 2.9

$$F_0^{D_s\eta}=-\left(\frac{2}{\sqrt{6}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)F_0^{D_s\eta_{s\bar{s}}},$$

$$F_0^{D_s\eta'}=\left(-\frac{2}{\sqrt{6}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)F_0^{D_s\eta'_{s\bar{s}}}.$$

Then the mixing angle $\theta=-10°$ leads to

$$\text{BSW:}\quad F_0^{D\eta}(0)=0.342,\quad F_0^{D\eta'}(0)=0.326,\quad F_0^{D_s\eta}(0)=-0.509,\quad F_0^{D_s\eta'}(0)=0.500. \tag{2.19}$$

The above are the form factors used in the original BSW analysis for $(D,D_s)\to(\eta,\eta')$ transitions [3]. One can check that if $\theta=-22°$ is used, the BSW form factors will be close to ours as given in Eq. (2.16).

For the q^2 dependence of form factors in the region where q^2 is not too large, we shall use the pole dominance ansatz, namely, $f(q^2)=f(0)/[1-(q^2/m_*^2)]^n$, where m_* is the pole mass given in [3]. A direct calculation of $D\to P$ and $D\to V$ form factors at timelike momentum transfer is available in the relativistic light-front quark model [13] with the results that the q^2 dependence of the form factors A_0,F_1 is a dipole behavior (i.e., $n=2$), while F_0 exhibits a monopole dependence ($n=1$). Note that in the BSW model, the q^2 dependence of A_0,F_1 is assumed to be the same as F_0, namely a monopole behavior.

Applying Eqs. (2.3), (2.13), (2.16) and the form factor $A_0^{D\rho}(0)=0.63$ [13], we have calculated the branching ratios for $(D^+,D_s^+)\to(\pi^+,\rho^+)\eta^{(\prime)}$ and $D^0\to(\overline{K}^0,\overline{K}^0*)\eta^{(\prime)}$ decays, as summarized in Table I (see the third column), where use has been made of the charmed meson lifetimes [14]

$$\tau(D^0)=4.15\times10^{-13}\ \text{s},$$
$$\tau(D^+)=1.057\times10^{-12}\ \text{s},$$
$$\tau(D_s^+)=4.67\times10^{-13}\ \text{s}. \tag{2.20}$$

For comparison, the experimental measurements and the BSW predictions [3] based on $a_1=1.25$, $a_2=-0.51$, Eq. (2.19) for form factors $F_0^{D\eta^{(\prime)}}$ and $F_0^{D_s\eta^{(\prime)}}$ and a monopole q^2 dependence for all the form factors are also exhibited in Table I. It is clear that our results differ from the BSW predictions mainly for the decay modes $D^0\to\overline{K}^{*0}\eta$, $D^+\to\pi^+\eta$ and for the η'/η ratio in $D_s^+\to\pi^+\eta^{(\prime)}$ due to the form factor differences in Eqs. (2.16) and (2.19) and the q^2 dependence for form factors A_0 and F_1. We see from Table I that the mixing angle $\theta=-22°$ agrees better with experiment than the angle $-10°$ and that our predictions are in general consistent with experiment except for the decays: $D^0\to\overline{K}^0\eta'$, $D^+\to\pi^+\eta$ and $D_s^+\to\rho^+\eta'$; the branching ratios of the first two decay modes are too small by an order of magnitude. Hence, there are three difficulties with the factorization approach in describing the hadronic D decays to an η and η'. First, it is naively expected that $\mathcal{B}(D^0\to\overline{K}^0\eta')\ll\mathcal{B}(D^0\to\overline{K}^0\eta)$ due to the form factor suppression $F_0^{D\eta'}(0)/F_0^{D\eta}(0)=0.64$ and the less phase space available to the former. However, experimentally it is the other way around: $\mathcal{B}(D^0\to\overline{K}^0\eta')\sim2.4\mathcal{B}(D^0\to\overline{K}^0\eta)$. Second, the predicted branching ratio for $D^+\to\pi^+\eta$ is too small by one order of magnitude. This is attributed to the fact that the sign of $X_s^{(D\pi,\eta')}$ is opposite to $X_d^{(D\pi,\eta')}$ and that there is a large cancellation between the external W-emission amplitude $a_1X^{(D\eta,\pi)}$ and the internal W-emission one $a_2(X_d^{(D\pi,\eta)}-X_s^{(D\pi,\eta)})$. Third, while the generalized factorization is successful in predicting $\mathcal{B}(D_s^+\to\pi^+\eta)$ and $\mathcal{B}(D_s^+\to\pi^+\eta')$ and marginally for $D_s^+\to\rho^+\eta$, its prediction for $\mathcal{B}(D_s^+\to\rho^+\eta')$ is too small by about 2σ compared to experiment. This has motivated some authors [15] (see also [16]) to advocate an enhancement mechanism in which two gluons are produced in the $c\bar{s}$ annihilation process and then hadronized into an η'.

III. FINAL-STATE INTERACTIONS

In the previous section we have pointed out three problems with the factorization approach for dealing with the two-body D decays to an η or η'. One issue is that final-state interactions (FSI) and nonspectator W-exchange or W-annihilation effects are not taken into account thus far. It is customary to argue that the W-exchange contribution is neg-
ligible due to helicity and color suppression.[2] Therefore, it is very unlikely that the nonspectator effects due to W-

[2]In the factorization approach, the W-exchange amplitude in $D\to P_1P_2$ decay is suppressed by a factor of $[(m_1^2-m_2^2)/m_D^2](F_0^{P_1P_2}(m_D^2)/F_0^{DP_1}(m_2^2))$ relative to the external W-emission (assuming that P_2 is factored out). The form factor $F_0^{P_1P_2}(q^2)$, which is antisymmetric in P_1 and P_2, is suppressed at large momentum transfer $q^2=m_D^2$.

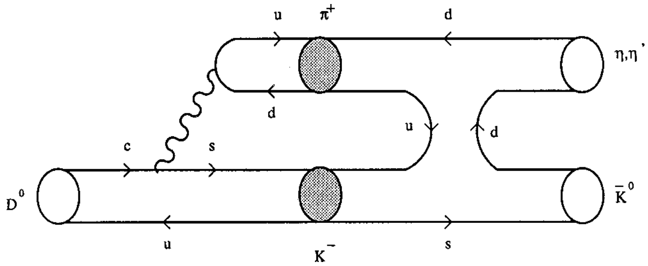

FIG. 1. Contributions to $D^0\to\overline{K^0}\eta(\eta')$ from the weak decay $D^0\to K^-\pi^+$ followed by a resonant rescattering.

exchange or W-annihilation can account for the large discrepancy between theory and experiment for $D^0\to\overline{K^0}\eta'$ and $D^+\to\pi^+\eta$. It remains to be seen if FSI could be the underlying mechanism responsible for the large enhancement of the above-mentioned decay modes. The importance of FSI has long been realized in charm decay since some resonances are known to exist at energies close to the mass of the charmed meson. Consequently, the inelastic scattering effects are crucial for understanding the pattern of charm weak decays. For example, the ratio $R=\Gamma(D^0\to\overline{K^0}\pi^0)/\Gamma(D^0\to K^-\pi^+)$ is predicted to be only of order 3×10^{-4} in the naive factorization approach, while experimentally it is measured to be 0.55 ± 0.06 [14]. It is known that the weak decay $D^0\to K^-\pi^+$ followed by the inelastic rescattering $K^-\pi^+\to\overline{K^0}\pi^0$ can raise $\mathcal{B}(D^0\to\overline{K^0}\pi^0)$ dramatically and lower $\mathcal{B}(D^0\to K^-\pi^+)$ slightly.

There are several different forms of FSI: elastic scattering and inelastic scattering such as quark exchange, resonance formation, . . . , etc. As emphasized in [17], the resonance formation of FSI via $q\bar{q}$ resonances is probably the most important one. Since FSI are nonperturbative in nature, in general it is notoriously difficult to calculate their effects. Nevertheless, as we shall see below, the effect of resonance-induced FSI can be estimated provided that the mass and the width of the nearby resonances are known. Before embarking on a detailed analysis, it is instructive to elucidate qualitatively how resonant FSI work for the decay $D^0\to\overline{K^0}\eta'$ as an example. Consider the weak decay $D^0\to K^-\pi^+$ followed by the strong-interaction process: $K^-\pi^+\to$scalar resonances$\to\overline{K^0}\eta'$ (see Fig 1). Note that Fig. 1 has the same topology as the W-exchange diagram, a point we will come back to later. Denote the amplitude by $r_d(r_s)$ when the $d\bar{d}$ $(s\bar{s})$ pair is created and combined with the $s\bar{d}$ quarks to form the final state $\overline{K^0}\eta'$. Assuming SU(3) symmetry for the $d\bar{d}$ and $s\bar{s}$ creation and taking the $\eta-\eta'$ mxing angle θ to be $-19.5°$, it is easily seen that

$$A(D^0\to\overline{K^0}\eta')_{\rm FSI}=r_d+2r_s=3r_d,$$
$$A(D^0\to\overline{K^0}\eta)_{\rm FSI}=r_d-r_s=0,$$
$$\text{for}\quad \theta=-19.5°. \tag{3.1}$$

Since the branching ratio of $D^0\to K^-\pi^+$ is large enough, $\mathcal{B}(D^0\to K^-\pi^+)=(3.83\pm0.12)\%$ [14], it is quite plausible that resonance-induced FSI could enhance $\mathcal{B}(D^0\to\overline{K^0}\eta')$ by an order of magnitude without affecting the original good agreement between theory and experiment for $D^0\to\overline{K^0}\eta$. Therefore, this mechanism enables us to understand why the decay rate of $D^0\to\overline{K^0}\eta'$ is larger than $D^0\to\overline{K^0}\eta$, even though the factorizable contribution to the former is smaller than the latter.

We will repeat the analysis of [17] to study the effects of resonant FSI for the decays $D^0\to\overline{K^0}\eta^{(\prime)}$. It turns out that the quark-diagram approach put forward in [18,19] is quite suitable for this purpose. In this approach, all two-body nonleptonic weak decays of charmed mesons can be expressed in terms of six distinct quark diagrams: $\mathcal{A}$, the external W-emission diagram; $\mathcal{B}$, the internal W-emission diagram; $\mathcal{C}$, the W-exchange diagram; $\mathcal{D}$, the W-annihilation diagram; $\mathcal{E}$, the horizontal W-loop diagram; and $\mathcal{F}$, the vertical W-loop diagram. It should be stressed that these quark diagrams are classified according to their topologies and hence they are *not* Feynman graphs. The quark-diagram amplitudes for $D^0\to K^-\pi^+$, $\overline{K^0}\pi^0$, $\overline{K^0}\eta_{ns}$ and $\overline{K^0}\eta_s$, where $\eta_{ns}=(1/\sqrt{2})(u\bar{u}+d\bar{d})$ and $\eta_s=s\bar{s}$, are given by (see Table III of [18]):

$$A(D^0\to(\bar{K}\pi)_{3/2})=\frac{1}{\sqrt{3}}(\mathcal{A}+\mathcal{B}),$$
$$A(D^0\to(\bar{K}\pi)_{1/2})=\frac{1}{\sqrt{6}}(2\mathcal{A}-\mathcal{B}+3\mathcal{C}),$$
$$A(D^0\to\overline{K^0}\eta_{ns})=\frac{1}{\sqrt{2}}(\mathcal{B}+\mathcal{C}),$$
$$A(D^0\to\overline{K^0}\eta_s)=\mathcal{C}. \tag{3.2}$$

For FSI through $q\bar{q}$ resonances, we consider the D-type coupling for the strong interaction $P_1P_2\to P'$ (P': scalar meson), namely $\kappa{\rm Tr}(P'\{P_1,P_2\})$ with κ being a flavor-symmetric strong coupling [17]. Noting that $(\bar{K}\pi)_{3/2}$ does not couple to $(\bar{K}\pi)_{1/2}$, $\overline{K^0}\eta_{ns}$, and $\overline{K^0}\eta_s$ via FSI, the strong reaction matrix K_0, which is related to the S matrix by $S_0=(1+iK_0)/(1-iK_0)$, for the $I=\frac{1}{2}$ sector has the form:

$$K_0=\kappa^2\begin{pmatrix}\frac{3}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{\sqrt{2}}\\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\\ \frac{\sqrt{3}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1\end{pmatrix} \tag{3.3}$$

in the basis of $(\overline{K}\pi)_{1/2}$, $\overline{K}^0\eta_{ns}$, $\overline{K}^0\eta_s$. The eigenvalues and eigenvectors of the K_0 matrix are

$$\lambda_1=3\kappa^2,\quad (PP)_1=\frac{1}{\sqrt{6}}[\sqrt{3}(\overline{K}\pi)_{1/2}+\overline{K}^0\eta_{ns}+\sqrt{2}(\overline{K}^0\eta_s)],$$

$$\lambda_2=0,\quad (PP)_2=\frac{1}{\sqrt{6}}[-\sqrt{3}(\overline{K}\pi)_{1/2}+\overline{K}^0\eta_{ns}+\sqrt{2}(\overline{K}^0\eta_s)],$$

$$\lambda_3=0,\quad (PP)_3=\frac{1}{\sqrt{3}}[\sqrt{2}(\overline{K}^0\eta_{ns})-\overline{K}^0\eta_s]. \tag{3.4}$$

In this new basis, the weak decay amplitudes are unitarized by FSI as [17]

$$A(D^0\to(PP)_i)\to \cos\delta_i e^{i\delta_i}A(D^0\to(PP)_i), \tag{3.5}$$

as required by the unitarity of the S matrix (known as Watson's theorem) with δ_i being the eigenphases of the K matrix. It is then straightforward to show from Eqs. (3.2), (3.4), (3.5) that resonance-induced FSI amount to modifying the W-exchange amplitude by [17]

$$\mathcal{C}\to\mathcal{C}+\left(\mathcal{C}+\frac{1}{3}\mathcal{A}\right)(\cos\delta_1 e^{i\delta_1}-1) \tag{3.6}$$

and leaving the other quark-diagram amplitudes intact, where $\delta_1=3\kappa^2$. This is consistent with what has been expected before: The resonance contribution to FSI, which arises mainly from the external W-emission diagram for the decay $D^0\to(\overline{K}\pi)_{1/2}$ followed by final-state $q\overline{q}$ resonance, has the same topology as the W-exchange quark diagram. We thus see that even if the short-distance W-exchange vanishes, as commonly asserted, a long-distance W-exchange still can be induced via FSI in resonance formation.

Substituting Eq. (3.6) back into Eq. (3.2) and neglecting the short-distance W-exchange contribution, we obtain

$$A(D^0\to\overline{K}^0\eta)=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\Bigg[a_2X^{(D\eta,K)}-a_1X^{(DK,\pi)}\frac{\cos\delta e^{i\delta}-1}{3}\times\left(\frac{\cos\theta}{\sqrt{6}}+\frac{2}{\sqrt{3}}\sin\theta\right)\Bigg],$$

$$A(D^0\to\overline{K}^0\eta')=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\Bigg[a_2X^{(D\eta',K)}-a_1X^{(DK,\pi)}\frac{\cos\delta e^{i\delta}-1}{3}\times\left(\frac{\sin\theta}{\sqrt{6}}-\frac{2}{\sqrt{3}}\cos\theta\right)\Bigg], \tag{3.7}$$

where $a_1X^{(DK,\pi)}$ is the factorizable amplitude for $D^0\to K^-\pi^+$ and $X^{(DK,\pi)}=if_\pi(m_D^2-m_K^2)F_0^{DK}(m_\pi^2)$.

In order to determine the phase shift δ, we shall assume that there exist nearby resonances in the charmed-meson mass region and that the phase is related to the Breit-Wigner resonance by

$$\frac{1}{2i}(e^{2i\delta}-1)=\sin\delta e^{i\delta}=\frac{\Gamma_*}{2(m_*-m_D)-i\Gamma_*}, \tag{3.8}$$

in the rest frame of the charmed meson, where m_* and Γ_* are the mass and width of the resonance, respectively. It follows that

$$\tan\delta=\frac{\Gamma_*}{2(m_*-m_D)}. \tag{3.9}$$

For parity-violating $D\to PP$ decays, there is a 0^+ resonance $K_0^*(1950)$ in $(s\overline{d})$ quark content with mass $1945\pm10\pm20$ MeV and width $210\pm34\pm79$ MeV [14]. It is clear from Table I that the resultant branching ratio of $D^0\to\overline{K}^0\eta'$ is enhanced by resonance-induced FSI by one order of magnitude, whereas $D^0\to\overline{K}^0\eta$ remains essentially unaffected. Therefore, we conclude that it is the final-state interaction that accounts for the bulk of $\mathcal{B}(D^0\to\overline{K}^0\eta')$ and explains its larger decay rate than $D^0\to\overline{K}^0\eta$.

For decays $D^0\to\overline{K}^{*0}\eta^{(\prime)}$, they can proceed through the processes $D^0\to K^{*-}\pi^+, K^-\rho^+\to\overline{K}^{*0}\eta^{(\prime)}$. Following the quark-diagram notation of [18] that primed amplitudes are for the case that the vector meson is produced from the charmed quark decay, we write

$$A(D^0\to\overline{K}^{*0}\eta_{ns})=\frac{1}{\sqrt{2}}(\mathcal{B}'+\mathcal{C}'),\quad A(D^0\to\overline{K}^{*0}\eta_s)=\mathcal{C}. \tag{3.10}$$

Repeating the same analysis as before, one obtains (see [17] for details)

$$\mathcal{C}\to\mathcal{C}+\frac{1}{2}\left[\mathcal{C}+\mathcal{C}'+\frac{1}{3}(\mathcal{A}+\mathcal{A}')\right](\cos\delta e^{i\delta}-1),$$

$$\mathcal{C}'\to\mathcal{C}'+\frac{1}{2}\left[\mathcal{C}+\mathcal{C}'+\frac{1}{3}(\mathcal{A}+\mathcal{A}')\right](\cos\delta e^{i\delta}-1), \tag{3.11}$$

where $\mathcal{A}$ is the external W-emission amplitude for $D^0\to K^-\rho^+$ and $\mathcal{A}'$ for $D^0\to K^{*-}\pi^+$. Neglecting the short-distance W-exchange, we obtain

$$A(D^0\to\overline{K}^{*0}\eta)=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[a_2X^{(D\eta,K^*)}-a_1(X^{(DK^*,\pi)}+X^{(DK,\rho)})\times\frac{\cos\delta e^{i\delta}-1}{6}\left(\frac{\cos\theta}{\sqrt{6}}+\frac{2}{\sqrt{3}}\sin\theta\right)\right],$$

$$A(D^0\to\overline{K}^{*0}\eta')=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[a_2X^{(D\eta',K^*)}-a_1(X^{(DK^*,\pi)}+X^{(DK,\rho)})\times\frac{\cos\delta e^{i\delta}-1}{6}\left(\frac{\sin\theta}{\sqrt{6}}-\frac{2}{\sqrt{3}}\cos\theta\right)\right],\quad(3.12)$$

with $X^{(DK^*,\pi)}=-2f_\pi m_{K^*}A_0^{DK^*}(m_\pi^2)(\varepsilon\cdot p_D)$ and $X^{(DK,\rho)}=-2f_\rho m_\rho F_1^{DK}(m_\rho^2)(\varepsilon\cdot p_D)$. The relevant 0^- resonance for $D\to\overline{K}^*\eta^{(\prime)}$ decays is the $K(1830)$ with mass ~ 1830 MeV and width ~ 250 MeV [14]. As shown in Table I, the resonance effect has almost no impact on $D^0\to\overline{K}^{*0}\eta$. The smallness of $\mathcal{B}(D^0\to\overline{K}^{*0}\eta')$ of order 2×10^{-4} is due mainly to the severe phase-space suppression. Note that our predictions for $D^0\to\overline{K}^0\eta'$ and $\overline{K}^{*0}\eta$ are still slightly smaller than experiment and that so far we have not considered the effects of W-exchange and FSI other than resonance formation.

We next turn to the Cabibbo-suppressed decays $D^+\to(\pi^+,\rho^+)\eta^{(\prime)}$. As noted in passing, in the absence of FSI, the branching ratio of $D^+\to\pi^+\eta$ is very small, of order 10^{-4}, owing to a large cancellation between external and internal W-emission amplitudes. Since $D^+\to K^+\overline{K}^0$ has a relatively large branching ratio, $\mathcal{B}(D^+\to K^+\overline{K}^0)=(7.2\pm1.2)\times10^{-3}$ [14], it is conceivable that $D^+\to\pi^+\eta$ can receive significant contributions from resonant FSI through the process $D^+\to K^+\overline{K}^0\to\pi^+\eta$. (Note that $\pi^+\pi^0$ does not couple to $\pi^+\eta^{(\prime)}$ by strong interactions.) The quark diagram amplitudes for $D^+\to\pi^+\eta^{(\prime)}$ are given by [18]

$$A(D^+\to K^+\overline{K}^0)=-(\mathcal{A}-\mathcal{D}),$$

$$A(D^+\to\pi^+\eta_{ns})=\frac{1}{\sqrt{2}}(\mathcal{A}+\mathcal{B}+2\mathcal{D}),$$

$$A(D^+\to\pi^+\eta_s)=-\mathcal{B}.\quad(3.13)$$

Proceeding as before, resonance-induced coupled-channel effects among the three channels: $K^+\overline{K}^0,\pi^+\eta_{ns}$ and $\pi^+\eta_s$ will only modify the magnitude and phase of the W-annihilation amplitude and leave the other quark-diagram amplitudes unaffected:

$$\mathcal{D}\to\mathcal{D}+\left(\mathcal{D}+\frac{1}{3}\mathcal{A}\right)(\cos\delta e^{i\delta}-1).\quad(3.14)$$

Hence,

$$A(D^+\to\pi^+\eta)=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}\left[a_1X^{(D\eta,\pi)}+a_2(X_d^{(D\pi,\eta)}-X_s^{(D\pi,\eta)})-\frac{\sqrt{2}}{3}a_1X^{(DK,K)}(\cos\delta e^{i\delta}-1)\times\left(\frac{\cos\theta}{\sqrt{3}}-\sqrt{\frac{2}{3}}\sin\theta\right)\right],$$

$$A(D^+\to\pi^+\eta')=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}\left[a_1X^{(D\eta',\pi)}+a_2(X_d^{(D\pi,\eta')}-X_s^{(D\pi,\eta')})-\frac{\sqrt{2}}{3}a_1X^{(DK,K)}(\cos\delta e^{i\delta}-1)\times\left(\frac{\sin\theta}{\sqrt{3}}+\sqrt{\frac{2}{3}}\cos\theta\right)\right],\quad(3.15)$$

where $X^{(DK,K)}=if_K(m_D^2-m_K^2)F_0^{DK}(m_K^2)$.

A nearby 0^+ resonance a_0 in the charm mass region has not been observed. We shall follow [7] to employ $m_{a_0}=1869$ MeV and $\Gamma_{a_0}=300$ MeV, where the mass is estimated from the equispacing formula $m_{a_0}^2=m_{K_0^*}^2-m_K^2-m_\pi^2$. Numerically, both $\mathcal{B}(D^+\to\pi^+\eta)$ and $\mathcal{B}(D^+\to\pi^+\eta')$ are enhanced, in particular the former is increased by an order of magnitude (see Table I).

Contrary to $\pi^+\eta$ and $\pi^+\eta'$ final states, resonant FSI are negligible for $\rho^+(\eta,\eta')$ states for the following reason. The G parity of $\rho\eta$ and $\rho\eta'$ is even, while the $J=0$, $I=1$ meson resonance made from a quark-antiquark pair (i.e., $u\bar{d}$) has odd G parity. This is also true for the W-annihilation process $c\bar{d}\to u\bar{d}$. As stressed in [20], the even-G state $\rho\eta$ or $\rho\eta'$ does not couple to any single meson resonances, nor to the state produced by the W-annihilation diagram with no gluons emitted by the initial state before annihilation. We would like to remark that at the factorizable amplitude level $|A(D^+\to\rho^+\eta')|>|A(D^+\to\rho^+\eta)|$, but $\mathcal{B}(D^+\to\rho^+\eta')<\mathcal{B}(D^+\to\rho^+\eta)$ due to the lack of phase space available to the former.

As for $D_s^+\to\pi^+\eta^{(\prime)}$ decays, the quark diagram amplitudes are

$$A(D_s^+\to K^+\overline{K}^0)=\mathcal{B}+\mathcal{D},\quad A(D_s^+\to\pi^+\eta_{ns})=\sqrt{2}\mathcal{D},$$
$$A(D_s^+\to\pi^+\eta_s)=\mathcal{A}. \tag{3.16}$$

The analysis of resonant coupled-channel effects is the same as $D^+\to\pi^+\eta^{(\prime)}$ and it leads to [17]

$$\mathcal{D}\to\mathcal{D}+\left(\mathcal{D}+\frac{1}{3}\mathcal{B}\right)(\cos\delta e^{i\delta}-1), \tag{3.17}$$

where $\mathcal{B}$ is the internal W-emission amplitude for $D_s^+\to K^+\overline{K}^0$. Neglecting W-annihilation as before, we obtain from Eqs. (3.16) and (3.17) that

$$A(D_s^+\to\pi^+\eta)=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\Bigg[a_1X^{(D_s\eta,\pi)}+\frac{\sqrt{2}}{3}a_2X^{(D_sK,K)}(\cos\delta e^{i\delta}-1)\times\left(\frac{\cos\theta}{\sqrt{3}}-\sqrt{\frac{2}{3}}\sin\theta\right)\Bigg],$$

$$A(D_s^+\to\pi^+\eta')=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\Bigg[a_1X^{(D_s\eta',\pi)}+\frac{\sqrt{2}}{3}a_2X^{(D_sK,K)}(\cos\delta e^{i\delta}-1)\times\left(\frac{\sin\theta}{\sqrt{3}}+\sqrt{\frac{2}{3}}\cos\theta\right)\Bigg], \tag{3.18}$$

with $X^{(D_sK,K)}=if_K(m_{D_s}^2-m_K^2)F_0^{D_sK}(m_K^2)$. It is interesting to remark that $D_s^+\to\pi^+\eta$ is suppressed in the presence of resonant FSI, while $D_s^+\to\pi^+\eta'$ is enhanced (see Table I). This is ascribed to the fact that the external W-emission amplitudes for $D_s^+\to\pi^+\eta$ and $\pi^+\eta'$ are opposite in sign due to a relative sign difference between the form factors $F_0^{D_s\eta}$ and $F_0^{D_s\eta'}$.

The same argument that resonance-induced FSI and W-annihilation without gluon emission in the initial state do not contribute to $D^+\to\rho^+\eta^{(\prime)}$ also applies to $D_s^+\to\rho^+\eta^{(\prime)}$. As a consequence, the large observed branching ratio of $D_s^+\to\rho^+\eta'$ is surprising. Theoretically, it is very difficult to raise the branching ratio of the $\rho\eta'$ mode from 3.9% to the level of 10% without suppressing $D_s^+\to\rho^+\eta$. First, in general the effect of FSI is useful and significant for the weak decay $D\to X$ only if there exists a decay $D\to Y$ with a sufficiently large decay rate, i.e., $\mathcal{B}(D\to Y)\gg\mathcal{B}(D\to X)$, and if X and Y channels couple through FSI. For $D_s^+\to VP$ decays, the branching ratio of $D_s^+\to\phi\pi^+$ is only 3.6% [14], which is even smaller than $D_s^+\to\rho^+\eta$. Hence, FSI in any form are unlikely to raise $\mathcal{B}(D_s^+\to\rho^+\eta',\rho^+\eta)$ substantially. Second, an enhancement mechanism has been suggested in [15] that a $c\bar{s}$ pair annihilates into a W^+ and two gluons, then the two gluons hadronize into a favor-singlet η_0. Since $\eta_0=\eta'\cos\theta-\eta\sin\theta$ and the mixing angle θ is negative, it is evident that if $\mathcal{B}(D_s^+\to\rho^+\eta')$ is enhanced by this mechanism, $\mathcal{B}(D_s^+\to\rho^+\eta)$ will be suppressed due to the destructive interference between the external W-emission and the gluon-mediated process, recalling that the external W-emission amplitudes for $D_s^+\to\rho^+\eta$ and $D_s^+\to\rho^+\eta'$ are opposite in sign. Hence, if $\mathcal{B}(D_s^+\to\rho^+\eta')$ is accommodated by this new mechanism, then we will have a hard time explaining $\mathcal{B}(D_s^+\to\rho^+\eta)$. The W-annihilation diagram, which is not subject to color and helicity suppression in $(D_s^+,D^+)\to\rho^+\eta^{(\prime)}$ decays, is expected to play some role. Even a small contribution from W-annihilation, say $\mathcal{D}/\mathcal{A}\sim0.2$, can easily increase the decay rate by a factor of 2. However, by the same reasoning as shown above, when W-annihilation raises the branching ratio of one of the $D_s^+\to\rho^+\eta^{(\prime)}$ decay modes, it will lower the other one. Third, the phase-space factor relevant to $D_s^+\to\rho^+\eta^{(\prime)}$ is $p_c[(m_{D_s}^2-m_\rho^2-m_{\eta^{(\prime)}}^2)^2-4m_\rho^2m_{\eta^{(\prime)}}^2]$ with p_c being the c.m. momentum. The phase-space suppression of $\rho\eta'$ relative to $\rho\eta$ is found to be 0.27. In order to achieve $\mathcal{B}(D_s^+\to\rho^+\eta')\sim\mathcal{B}(D_s^+\to\rho^+\eta)\sim10\%$, a new mechanism must be introduced to overcome the phase-space suppression for the former and in the meantime it should not lower the decay rate of the latter. To our knowledge, it is difficult to speculate such a mechanism.

Since the decay rates of $D_s^+\to\rho^+\eta^{(\prime)}$ are sensitive to the form factors $F_1^{D_s\eta^{(\prime)}}$, it is advantageous to consider the ratios $R_{\eta^{(\prime)}}\equiv\Gamma(D_s^+\to\rho^+\eta^{(\prime)})/\Gamma(D_s^+\to\eta^{(\prime)}e^+\nu)$ in order to test the generalized factorization hypothesis. Neglecting W-annihilation, factorization leads to the form-factor-independent predictions $R_\eta=2.9$ and $R_{\eta'}=3.5$, while experimentally $R_\eta=4.4\pm1.2$ and $R_{\eta'}=12.0\pm4.3$ [2]. [Our value for $R_{\eta'}$ is slightly different from the result $R_{\eta'}=2.9$ obtained in [5] as we use a dipole q^2 dependence for the form factors $F_1^{D_s\eta^{(\prime)}}(q^2)$.] We have argued that FSI, W-annihilation and the production of excess η' from gluons are not helpful in understanding the very large branching ratios of $D_s^+\to\rho^+\eta^{(\prime)}$. Hence, the very large value of $R_{\eta'}$ remains an enigma.

IV. CONCLUSIONS

With the improved $(D,D_s^+)\to(\eta,\eta')$ form factors and decay constants of the η and η', we have employed the generalized factorization approach to reanalyze the decays of charmed mesons into the final states containing an η or η'. We show that resonant FSI are able to enhance $\mathcal{B}(D^0\to\overline{K}^0\eta')$ and $\mathcal{B}(D^+\to\pi^+\eta)$ by an order of magnitude. Resonance-induced couple-channel effects will suppress $D_s^+\to\pi^+\eta$ and enhance $D_s^+\to\pi^+\eta'$. Contrary to $D\to P\eta^{(\prime)}$ decays, resonant FSI play only a minor role for $D^0\to\overline{K}^{*0}\eta^{(\prime)}$ and do not contribute to $(D^+,D_s^+)\to\rho^+\eta^{(\prime)}$. We argue that it is difficult to understand the observed large decay rates of the $\rho^+\eta'$ and $\rho^+\eta$ decay modes of D_s^+ simultaneously. FSI are not helpful due to the absence of $D_s^+ \to PP$ decays that have much larger decay rates than $D_s^+ \to \rho^+ \eta'$. W-annihilation and a possible production of the η' due to gluon-mediated processes can in principle enhance $\mathcal{B}(D_s^+ \to \rho^+ \eta')$, but, unfortunately, they will also suppress $\mathcal{B}(D_s^+ \to \rho^+ \eta)$.

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