

Δ isobar masses, large N_c relations, and the quark model

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Motivated by recent remarks on the Δ^+ mass and comparisons between the quark model and relations based on large- N_c with perturbative flavor breaking, two sets of Δ masses consistent with these constraints are constructed. These two sets, based either on an experimentally determined mass splitting or a quark model of isospin symmetry breaking, are shown to be inconsistent. The model dependence of this inconsistency is examined, and suggestions for improved experiments are made. An explicit quark model calculation and mass relations based on the large- N_c limit with perturbative flavor breaking are compared. The expected level of accuracy of such relations is realized in the quark model, except for mass relations derived by combining others which correspond to different SU(6) representations. It is shown that the Δ^0 and Δ^{++} pole masses and $\Delta^0 - \Delta^+ = (\Delta^- - \Delta^{++})/3 \approx 1.5$ MeV are more consistent with model expectations than the analogous Breit-Wigner masses and their splittings. [S0556-2821(99)02301-2]

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I. INTRODUCTION

The standard Δ masses [1] have been used in a number of comparisons with predictions based on large- N_c with perturbative flavor breaking [2,3] and the quark model [4,5]. The agreement generally has been poor. While the $\Delta(1232)$ resonance has been extensively studied in both strong and electromagnetic reactions, only the Δ^0 and Δ^{++} masses have precise values, and the Δ^- mass has never been determined. Values for the Δ^0 and Δ^{++} masses come mainly from analyses of elastic pion-nucleon scattering [6–8], and the Δ^+ mass has been extracted from analyses of pion photoproduction data [1,9].

In this paper, we first note that the agreement with theory is much improved when the Δ^+ mass of Ref. [9] is removed. The justification for doing so has recently been clarified [10]. Having done this, we require a pair of additional constraints to determine the full set of Δ isobar masses. The first constraint is the most reliable relation based on large- N_c and perturbative flavor breaking given by Jenkins and Lebed [2], and involves only Δ masses. We will consider the different sets of Δ isobar masses which arise from the choice of a second constraint. One possibility is to use a linear combination of Δ masses determined from an analysis of elastic π^\pm scattering from the deuteron. A value for this linear combination

$$D = \Delta^- - \Delta^{++} + \frac{1}{3}(\Delta^0 - \Delta^+) \quad (1.1)$$

(a particle's name is used for its mass here and in what follows) has been extracted by Pedroni *et al.* [11].

Another possibility is to use a theoretically reliable relation between the Δ and Σ^* masses from Ref. [2], together with a quark model estimate of the *difference* between the Σ^* and Σ mass splittings. As justification for this latter approach, in Secs. II and III we carefully examine the predic-

tions of our quark model for isospin splittings and compare with relations based on large- N_c and perturbative flavor breaking, and with the “experimental” masses, to see where they differ. Comparisons of a similar nature have recently been completed by Rosner [5]. The present study extends this work through the use of a dynamical quark model which allows for SU(6) symmetry breaking in the baryon wave functions, and also for non-spectator effects [12,4], where the interactions of a pair of quarks with a given flavor and total spin are allowed to depend on the flavor and spin of the remaining quark. Our study also differs numerically through the use of different experimental input.

We will show that these two different approaches give quite different results for the Δ masses, which implies an inconsistency between the measurements of the Δ^0 and Δ^{++} Breit-Wigner masses, the extracted value of D , and our prediction based on large N_c and the quark model. In Sec. IV, we will show that the $\Delta^0 - \Delta^{++}$ mass splitting based on pole mass values is more consistent with quark model and large N_c expectations.

II. Δ MASSES

In the work of Jenkins and Lebed [2], relations between the masses of octet and decuplet baryons are estimated at various orders of a perturbative expansion in flavor breaking and in powers of $1/N_c$. The first constraint that we will use to determine a set of Δ masses is

$$\Delta_3 \equiv \Delta^{++} - 3\Delta^+ + 3\Delta^0 - \Delta^- = 0. \quad (2.1)$$

This relation is predicted [2] to have an accuracy [13] of order $\epsilon'' \epsilon' / N_c^3$, where ϵ' is an isospin-symmetry violating parameter for the strong interaction mass splittings, and $\epsilon'' \approx \epsilon'$ is an isospin symmetry breaking parameter for electromagnetic mass splittings. With the parameters of Ref. [2], this means that Δ_3 is expected to be of order 10^{-3} MeV or

smaller. We will show below that our quark model, which breaks flavor and isospin symmetry explicitly, satisfies Eq. (2.1) to a similar degree of accuracy.

A set of Δ masses can be constructed with minimal theoretical input by using the value of D extracted by Pedroni *et al.* [11] and the accurate relation Eq. (2.1),

$$\Delta^0 - \Delta^+ = \frac{3D}{10} = 1.38 \pm 0.06 \text{ MeV},$$

$$\Delta^- - \Delta^{++} = \frac{9D}{10} = 4.14 \pm 0.18 \text{ MeV}. \quad (2.2)$$

If these relations are combined with Breit-Wigner masses [1,6,14] for the Δ^0 (1233.6 ± 0.5 MeV) and Δ^{++} (1230.9 ± 0.3 MeV), we have

$$\Delta^+ = 1232.2 \pm 0.5 \text{ MeV},$$

$$\Delta^- = 1235.0 \pm 0.35 \text{ MeV}. \quad (2.3)$$

Although the Δ^+ mass is poorly known, we note that the current range of values, given in the Review of Particle Properties [1] (1231.5 ± 0.3 MeV, *excluding* the value of Ref. [9]), is consistent with this value. This exercise has also been performed by Lebed [15], with slightly different results. However, we will show in what follows that this prescription leads to a set of masses which is in conflict with a result based on a combination of relations derived from the large N_c limit with perturbative flavor breaking and the quark model.

A relation given by Jenkins and Lebed [2],

$$\Delta_2 = 2\Sigma_2^*, \quad (2.4)$$

between the quantities

$$\Delta_2 \equiv (\Delta^{++} + \Delta^-) - (\Delta^+ + \Delta^0) \quad (2.5)$$

and

$$\Sigma_2^* \equiv \Sigma^{*+} + \Sigma^{*-} - 2\Sigma^{*0} \quad (2.6)$$

[with $\Sigma^* \equiv \Sigma(1385)$], can also be used to constrain the Δ masses. This relation is expected to be accurate to $\epsilon''\epsilon/N_c^3 \approx 3 \times 10^{-5}$, where ϵ is an $SU(3)_f$ symmetry violating parameter (with $\epsilon \gg \epsilon' \approx \epsilon''$), so that [2] corrections to this relation should be of order 0.15 MeV. Figure 1 illustrates the pattern of Δ and Σ^* splittings which results from imposing Eqs. (2.1) and (2.4).

Since the current value $\Sigma_2^* = 2.6 \pm 2.1$ MeV extracted from data [1] is quite uncertain, our approach is to estimate Σ_2^* by using the value $\Sigma_2 \equiv \Sigma^+ + \Sigma^- - 2\Sigma^0 = 1.71 \pm 0.18$ MeV, also extracted from the data [1], and a dynamical quark model prediction of the difference $\Sigma_2^* - \Sigma_2$.

The pairing models of baryon isospin splittings of Cutkosky [4] and Rosner [5] assume the universality of splittings

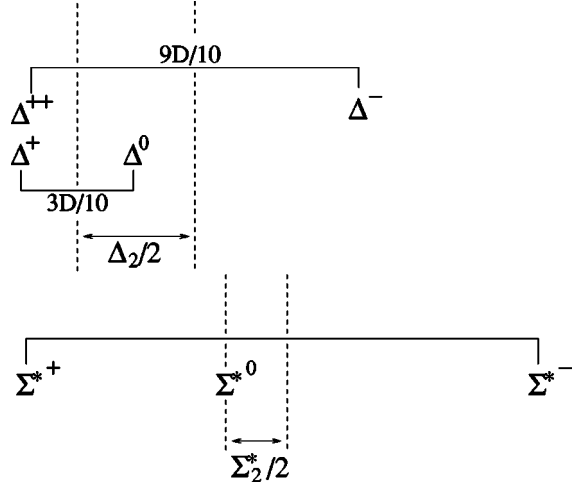


FIG. 1. Δ and Σ^* mass splittings, incorporating the relations $\Delta_3 = 0$ and $\Sigma_2^* = \Delta_2/2$.

of a given type within the ground state octet and decuplet baryons. This amounts to the assumption of $SU(6)$ flavor-spin symmetry in the wave functions, although the interactions must be allowed to depend on the light-strange quark mass difference and so break $SU(3)_f$. This is the spectator approximation of Franklin [12], where the strong and electromagnetic interactions between a given pair of quarks do not depend on the flavor or spin of the remaining quark in the baryon. Models of this kind give $\Sigma_2^* = \Sigma_2$. Dynamical models such as that used here [16] and by Isgur [17] allow for breaking of $SU(6)$ symmetry in the wave functions and so allow for and calculate non-spectator effects. However, these models neglect potentially important additional effects due to electromagnetic box and penguin graphs, as shown by Stephenson, Maltman, and Goldman [18], and these may also contribute to non-spectator effects.

It is pointed out in Ref. [4] that isospin splittings of the N and Δ baryons are much smaller than those of the hyperons because of pair terms which cancel in equal mass systems, but not in the hyperons. As a consequence, the $n-p$ and Δ splittings are more sensitive to non-spectator effects which cannot be written as pair terms, and which may not necessarily show the same cancellations. It is, therefore, important to include these effects when examining the Δ mass splittings in a model (such as ours) constrained to fit $n-p$ and other isospin splittings. Note that certain of the mass relations based on large- N_c and perturbative flavor breaking mentioned here are either satisfied by construction or by virtue of the assumption of $SU(6)$ symmetry in the wave functions of the model of Refs. [4] and [5], and that our model allows for explicit breaking of these relations.

As an example of such an effect, any model which consistently treats the hyperfine contact interaction and the isospin splittings will predict that the $\Delta^0 - \Delta^+$ splitting is slightly larger than $n-p$, because the effect of $m_d - m_u > 0$ on the quark kinetic energy is less diluted by relativistic effects in the Δ . Explicitly, $(m_d^2 + p^2)^{1/2} - (m_u^2 + p^2)^{1/2}$ is smaller than $m_d - m_u$ for a finite quark momentum p . The nucleon has a net attractive contact interaction which gives the quarks a larger mean momentum, whereas the Δ has a

repulsive contact interaction. Slight differences in the magnitude of the electrostatic and magnetic interactions introduce almost no difference between these two splittings. Although they are somewhat reduced in magnitude in the larger Δ state with slower moving quarks, these two terms come in with opposite sign and so the differences largely cancel. The relation $\Delta^0 - \Delta^+ = n - p$ from Jenkins and Lebed [2] and Rosner [5] has an expected accuracy of $\epsilon'/N_c^3 \approx \epsilon'/N_c^2$, which is lower order than, say, Eq. (2.4).

The quark model predictions given here are made within a model similar to that of Ref. [16], with some important differences noted below. The strong contact interaction used in the baryon spectrum calculation of Ref. [19] was convoluted with a Gaussian smearing function with the form $\exp(-\sigma_{ij}^2 r_{ij}^2)$, where r_{ij} is the separation of quarks i and j , and the smearing parameter σ_{ij} was 1.83 GeV for a light-quark pair (the smearing parameter is taken to depend on the quark mass; see Ref. [19] for details). This can be interpreted as a strong form factor for the light constituent quarks, and this smearing parameter implies a relatively small strong size for the constituent quark. On the other hand, relativistic calculations of the electromagnetic form factors of the nucleon carried out with light-cone techniques require a substantially larger electromagnetic size for the constituent quark in order to fit the nucleon form factors using the resulting wave functions [20–22]. The magnetic component of the electromagnetic interaction *between* quarks, which is one source of isospin-violating mass splittings, was smeared in Ref. [16], also with a substantially smaller smearing parameter γ_{em} than that used for the strong contact interaction.

This implies that smaller smearing parameters should be used in the strong contact interaction, coupled with a larger strong coupling $\alpha_s(Q^2=0)$ to preserve the size of the contact splittings. This reduces the level of high-momentum components in the nucleon, and therefore reduces the electromagnetic size of the quarks required to fit the nucleon moments, bringing the strong and electromagnetic constituent quark sizes into rough agreement.

Wave functions have been generated for the ground state octet and decuplet baryons with a strong contact interaction which is smeared with $\sigma_{ij}=0.9$ GeV for light quark pairs [with similar reductions for the s -(u,d) and s - s quark pairs], and with an increased $\alpha_s(0)$, which result in a fit to the ground state baryon and the entire light-quark baryon spectra of similar quality to that of Ref. [19]. The resulting wave functions for the nucleons have been shown to give an adequate fit to the nucleon elastic form factors within a light-cone model [21,22]. The parameters of the isospin splitting model of Ref. [16] have been readjusted to fit the measured splittings, yielding $\delta m = m_d - m_u = 3.6$ MeV and $\gamma_{em} = 1.0$ GeV, with an unchanged magnetic relativistic suppression factor $\epsilon_{magn} = -0.297$. The results for light-quark baryons are $n - p = 1.3$ MeV, $D \equiv \Delta^- - \Delta^{++} + (\Delta^0 - \Delta^+)/3 = 4.9$ MeV, and $\Delta_2 = 3.5$ MeV. Our results confirm within a dynamical model (constrained by the baryon spectrum, nucleon form factors, and the measured isospin splittings) the expected accuracy of the best of the relations based on large- N_c and perturbative flavor breaking of Ref. [2]; we find

$\Delta_3 = 0.002$ MeV and $\Delta_2 - 2\Sigma_2^* = -0.026$ MeV for this fit. Our quark model explicitly breaks SU(6) symmetry, which allows a slight difference $\Sigma_2^* - \Sigma_2 = 0.046$ MeV, with $\Sigma_2 = 1.70$ MeV, consistent with the measured value of 1.71 ± 0.18 MeV. These results suggest that it should be a good approximation to constrain Δ_2 using Eq. (2.4) and our quark model prediction that Σ_2^* should be only slightly larger than Σ_2 . As a result, we will adopt the value

$$\Delta_2 = 2(1.71 \pm 0.18 + 0.046) \text{ MeV} = 3.51 \pm 0.36 \text{ MeV}. \quad (2.7)$$

This value of Δ_2 is quite different from the value implied by our first set of masses, which are based on the Breit-Wigner Δ^0 and Δ^{++} masses, $\Delta_3 = 0$, and the extracted value of D . To illustrate this point, we eliminate either Δ^- or Δ^+ from Eqs. (2.1) and (2.5) to obtain

$$\begin{aligned} \Delta^+ &= \frac{\Delta^0 + \Delta^{++}}{2} - \frac{\Delta_2}{4}, \\ \Delta^- &= \frac{3\Delta^0 - \Delta^{++}}{2} + \frac{3\Delta_2}{4}, \end{aligned} \quad (2.8)$$

which give the expressions

$$\begin{aligned} \Delta^0 - \Delta^+ &= \frac{\Delta^0 - \Delta^{++}}{2} + \frac{\Delta_2}{4}, \\ \Delta^- - \Delta^{++} &= 3(\Delta^0 - \Delta^+), \end{aligned} \quad (2.9)$$

where the last relation follows trivially from $\Delta_3 = 0$. Combining the value $\Delta^0 - \Delta^{++} = 2.7 \pm 0.3$ MeV, which results from the Breit-Wigner masses [14], and Eq. (2.2) for $\Delta^0 - \Delta^+$ which is based on the value of D of Pedroni *et al.*, we see that Eqs. (2.9) require $\Delta_2 \approx 0$ and so $\Sigma_2^* \approx 0$, in conflict with Eq. (2.7).

Equivalently, inserting our value for Δ_2 from Eq. (2.7) and the Breit-Wigner masses for Δ^0 and Δ^{++} into Eq. (2.9) we find

$$\Delta^0 - \Delta^+ = 2.2 \pm 0.2 \text{ MeV}. \quad (2.10)$$

Comparing to Eqs. (2.2) we see that the effect of this approach has been to adopt a value $D = 10(\Delta^0 - \Delta^+)/3 \approx 7.5 \pm 1$ MeV which is significantly larger than that extracted by Pedroni *et al.* [11]. A value of D this large is also disfavored in our quark model.

This suggests that there is an inconsistency between the Breit Wigner values for the Δ^0 and Δ^{++} masses, the value $D = 4.6 \pm 0.2$ MeV, and the analysis combining Eq. (2.4) with our quark model result. Note that this argument is based on the difficulty of accommodating substantially unequal values of Σ_2^* and Σ_2 in the quark model. In our quark model both Σ_2 and Σ_2^* have negligible contributions from the dependence of the kinetic energy and strong interactions on the $m_d - m_u$ mass difference. Their positive values result from a cancellation between a positive Coulomb term (≈ 3.4 MeV) and a negative magnetic term

(≈ -1.7 MeV). Although the Coulomb and magnetic terms are slightly larger in the spatially smaller (from the net negative contact interaction) ground state Σ , the cancellation is more complete in Σ_2 , which results in $\Sigma_2 < \Sigma_2^*$. In Cutkosky's pairing model [4], the $m_d - m_u$ terms are exactly zero, and the same partial cancellation between electric and magnetic terms occurs, but by fiat $\Sigma_2^* = \Sigma_2$. A value for Σ_2^* close to zero while Σ_2 is close to the value extracted from experiment is, therefore, inconsistent with such quark models. We will return to this point in Sec. IV.

III. ACCURACY OF MASS RELATIONS IN THE QUARK MODEL

As our analysis of the Δ isobar masses depends crucially on the relations in Eqs. (2.1) and (2.4), it is of some interest to test the predicted accuracy of these and other relations based [2] on large- N_c and perturbative flavor breaking with a dynamic quark model which includes $SU(3)_f$ breaking effects, as well as $SU(6)$ symmetry breaking and effects higher order in the isospin-symmetry violating quantities such as $\delta m/m \equiv 2(m_d - m_u)/(m_u + m_d)$. Certain of these relations cannot be compared to experiment due to large experimental uncertainties, particularly in the splittings of the Δ states. Our quark model can provide estimates for the level of accuracy of such relations.

We have already seen above that the most highly suppressed $I=2$ and $I=3$ operators from Ref. [2] yield relations ($\Delta_2 = 2\Sigma_2^*$ and $\Delta_3 = 0$ respectively) with predicted accuracies which are realized in our model. There are also several $I=1$ mass relations. One is $\Delta_1 - 10\Sigma_1^* + 10\Xi_1^* = 0$, with $\Delta_1 \equiv 3(\Delta^{++} - \Delta^-) + \Delta^+ - \Delta^0$, $\Sigma_1 \equiv \Sigma^+ - \Sigma^-$, and $\Xi_1^* \equiv \Xi^{*0} - \Xi^{*-}$. In our quark model we have $\Delta_1 - 10\Sigma_1^* + 10\Xi_1^* = -0.20$ MeV, which corresponds to an accuracy of 6×10^{-6} , which compares favorably with the predicted accuracy of this relation from Ref. [2] of $\epsilon' \epsilon^2/N_c^3 \approx 10^{-5}$. Similarly, the Coleman-Glashow relation $N_1 - \Sigma_1 + \Xi_1 = 0$ is satisfied by our quark model to within 0.03 MeV, which corresponds to an accuracy of 8×10^{-6} , and is predicted to be accurate [2] to $\epsilon' \epsilon/N_c^2 \approx 10^{-4}$ in the large- N_c and $SU(3)_f$ limit.

Two additional $I=1$ relations from Ref. [2] with an expected accuracy of $\epsilon' \epsilon/N_c^2$ are $\Delta_1 - 3\Sigma_1^* - 4\Xi_1^* = 0$ and $\Sigma_1^* - 2\Xi_1^* = 0$. These relations are found [2] by eliminating the unmeasured $\Lambda\Sigma^0$ mixing mass from mass relations which correspond to different $SU(6)$ representations, and are both rather poorly satisfied by our dynamical model, which has $\Delta_1 - 3\Sigma_1^* - 4\Xi_1^* = 6.1$ MeV, which corresponds to an accuracy of 4×10^{-4} , and $\Sigma_1^* - 2\Xi_1^* = -0.91$ MeV, which corresponds to an accuracy of 2×10^{-4} . A similar lack of agreement is obtained for a wide range of parameters. This suggests that some mass relations which span more than one $SU(6)$ representation, and those derived from them, may not be consistent with our dynamical model. This problem could likely be avoided by calculating the mixing mass [17] and using the original relations from Ref. [2] which correspond to a single $SU(6)$ representation.

IV. BREIT-WIGNER VERSUS POLE MASSES

It is interesting to note that our quark model fit *without* imposition of constraints from the Δ masses gives a value $D = 4.9$ MeV, close to the value $D = 4.6 \pm 0.2$ MeV of Pedroni *et al.* This is also true of the fit of Cutkosky [4]. If instead of using the Breit-Wigner masses to determine $\Delta^0 - \Delta^{++}$ we use the pole masses [1,8,10,23], we find a smaller splitting

$$\Delta^0 - \Delta^{++} \approx 1 \text{ MeV.} \quad (4.1)$$

A similar result for this splitting was found by Cutkosky [4] in a fit to the octet and decuplet baryons which excluded the Δ masses. Evaluating Eq. (2.9) with the pole mass difference and our value of Δ_2 gives

$$\begin{aligned} \Delta^0 - \Delta^+ &\approx 1.5 \text{ MeV,} \\ \Delta^- - \Delta^{++} &\approx 4.5 \text{ MeV,} \end{aligned} \quad (4.2)$$

which is at least compatible with our quark model expectation that $\Delta^0 - \Delta^+$ should be slightly larger than $n - p = 1.3$ MeV, though the uncertainty associated with pole mass splittings cannot support any more quantitative conclusions. This naturally leads one to consider whether pole or Breit-Wigner masses should be used in mass relations. As has been pointed out by Höhler [24], the pole position (and not the Breit-Wigner mass) is a quantity which can be most rigorously associated with a resonance.

One obvious way to address this question is to use the best $I=0$ mass relation from Ref. [2] with an expected accuracy of ϵ^3/N_c^3 ,

$$\Delta_0 = 3(\Sigma_0^* - \Xi_0^*) + \Omega_0, \quad (4.3)$$

where B_0 is the average of the isobar masses of baryon B . This leads, unfortunately, to a central value between the Δ Breit-Wigner and pole (1210 MeV) masses. Dillon has noted [25] that the consistency of Eq. (4.3) with the Δ pole masses is improved if pole values [26] are used consistently for each particle. This distinction has no effect on the Ω mass, but does shift the Σ_0^* and Ξ_0^* terms slightly. However, the agreement with Eq. (4.3) is about the same, and within the expected accuracy, if either Breit-Wigner or pole masses are consistently used. Here, again, improved values for the Σ^* (and Ξ^*) Breit-Wigner and pole masses would lead to a corresponding improvement in our understanding of the Δ .

While the Δ "mass" has been variously quoted near 1210 MeV, 1232 MeV, and even 1241 MeV [25,27,28], the differences are mainly due to the model dependence in the separation of resonance and background contributions. The pole position remains stable near 1210– $i50$ MeV in all of these works. As an exercise, we have repeated our quark model calculation of the isospin splittings but with the strong contact interaction altered to fit $\Delta - N$ evaluated with the Δ pole mass, leaving all other details of the model unchanged. The resulting values of $n - p$, D , and Δ_2 were largely unchanged at 1.3 MeV, 4.8 MeV, and 3.5 MeV respectively, although the magnitudes of the splittings Σ_1 , Σ_1^* , and Ξ_1 were re-

duced. This simple exercise suggests that it may be possible to accommodate the average pole masses and their difference in a quark model of this kind.

V. CONCLUSIONS

Two different constructions for the Δ isobar masses are compared. Both are based on the well determined Δ^0 and Δ^{++} masses, and a relation based on perturbative flavor breaking and large- N_c , with an expected very high degree of accuracy realized in our dynamical quark model. The additional constraint is taken to be either a combination of Δ masses extracted from π^\pm deuteron elastic scattering data or a second large- N_c relation in combination with a quark model calculation. The expected high degree of accuracy of this second large- N_c relation is also realized in our explicit quark model. If the Breit-Wigner Δ^0 and Δ^{++} masses are adopted, these two different constructions are in conflict. The quark model relation, upon which this conflict is based, is a basic consequence of the type of model used, and does not depend sensitively on parameter values.

The accuracy of certain relations based on perturbative flavor breaking and large- N_c is unknown because of uncertainties in the masses extracted from the data. We have found that in most cases the predicted accuracy is realized in our model. The exceptions are relations derived from combining mass relations which correspond to different SU(6) representations. We have also shown that the relations between masses based on perturbative flavor breaking and large- N_c and those derived from the quark model are more consistent with the Δ pole masses than the corresponding Breit-Wigner values. This suggests that $\Delta^0 - \Delta^{++} = (\Delta^- - \Delta^{++})/3 \approx 1.5$ MeV and the pole mass difference $\Delta^0 - \Delta^{++} \approx 1$ MeV are consistent with theoretical expectations.

We support Rosner's assertion that improved Σ^* and Ξ^* masses are vitally important. These would sharpen comparisons between the large- N_c and quark model predictions and allow more quantitative comparisons between mass relations using Breit-Wigner and pole masses. With one additional Δ mass, we could use Eq. (2.1) to determine the remaining state, bypassing the deuteron data. While the Δ^+ mass determined from pion photoproduction data is the most obvious candidate, we should point out a potential problem. This is most obvious if we rewrite Eq. (2.1) in the form

$$\Delta^- = \Delta^{++} + 3(\Delta^0 - \Delta^+), \quad (5.1)$$

which would be utilized, given the Δ^0 and Δ^{++} masses. Since Δ^0 has a larger uncertainty than Δ^{++} and the "expected" $\Delta^0 - \Delta^+$ splitting is only about 1.5 MeV, we could easily have an experimental $\Delta^0 - \Delta^+$ splitting consistent with zero. This uncertainty would then be magnified in our estimate of the Δ^- mass. While a direct measurement of the Δ^- mass would have the greatest impact, it is unfortunately the least favorable experiment, involving the extraction of $\pi^- n$ scattering from a deuteron target.

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