Nonspectator effects and B meson lifetimes from a field-theoretic calculation

Hai-Yang Cheng* and Kwei-Chou Yang†
Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China
(Received 5 May 1998; published 23 November 1998)

The B meson lifetime ratios are calculated to the order of $1/m_b^3$ in heavy quark expansion. The predictions of those ratios are dependent on four unknown hadronic parameters B_1, B_2, ε_1, and ε_2, where B_1 and B_2 parametrize the matrix elements of color singlet-singlet four-quark operators and ε_1 and ε_2 the matrix elements of color octet-octet operators. We derive the renormalization-group improved QCD sum rules for these parameters within the framework of heavy quark effective theory. The results are $B_1(m_b)=0.96\pm0.04$, $B_2(m_b)=0.95\pm0.02$, $\varepsilon_1(m_b)=-0.14\pm0.01$, and $\varepsilon_2(m_b)=-0.08\pm0.01$ to zeroth order in $1/m_b$. The resultant B meson lifetime ratios are $\tau(B^-)/\tau(B_d)=1.11\pm0.02$ and $\tau(B_s)/\tau(B_d)\approx1$ in SU(3) symmetry limit. [S0556-2821(98)05223-0]

PACS number(s): 13.25.Hw, 11.55.Hx, 12.38.Lg, 12.39.Hg

I. INTRODUCTION

A QCD-based formulation for treatment of inclusive heavy hadron decays has been developed in past years [1–3]. According to the optical theorem, the inclusive decay rates are related to the imaginary part of certain forward scattering amplitudes along the physical cut. Since the cut is dominated by physical intermediate hadron states such as resonances which are nonperturbative in nature, *a priori* the operator product expansion (OPE) or heavy quark expansion cannot be carried out on the physical cut. Nevertheless, for inclusive semileptonic decays, OPE can be employed for some smeared or averaged physical quantities. For example, by integrating out the neutrino energy, one can apply the OPE to the double differential cross section $d^2\Gamma/(dq^2dE_l)$ by deforming the contour of integration into the unphysical region far away from the physical cut [4]. Therefore, global quark-hadron duality, namely the matching between the hadronic and OPE-based expressions for decay widths and smeared spectra in semileptonic B or bottom baryon decays, follows from the OPE and is justified except for a small portion of the contour near the physical cut which is of order $\Lambda_{\rm QCD}/m_Q$. Unfortunately, there is no analogous variable to be integrated out in inclusive nonleptonic decays, allowing an analytic continuation into the complex plane. As a result, one has to invoke the assumption of local quark-hadron duality in order to appply the OPE in the physical region [5]. It is obvious that local duality is theoretically less firm and secure than global duality. In order to test the validity of local quark-hadron duality, it is thus very important to have a reliable estimate of the heavy hadron lifetimes within the OPE framework and compare them with experiment.

In the heavy quark limit, all bottom hadrons have the same lifetimes, a well-known result in the parton picture. With the advent of heavy quark effective theory and the OPE approach for the analysis of inclusive weak decays, it is realized that the first nonperturbative correction to bottom hadron lifetimes starts at order $1/m_b^2$ and it is model independent (for a review, see [6]). However, the $1/m_b^2$ corrections are small and essentially canceled out in the lifetime ratios. The nonspectator effects such as W-exchange and Pauli interference due to four-quark interactions are of order $1/m_Q^3$, but their contributions can be potentially significant due to a phase-space enhancement by a factor of $16\pi^2$. As a result, the lifetime differences of heavy hadrons come mainly from the above-mentioned nonspectator effects.

The world average values for the lifetime ratios of bottom hadrons are [7]

$$\frac{\tau(B^-)}{\tau(B_d^0)}=1.07\pm0.03,$$
$$\frac{\tau(B_s^0)}{\tau(B_d^0)}=0.94\pm0.04, \qquad (1.1)$$
$$\frac{\tau(\Lambda_b)}{\tau(B_d^0)}=0.79\pm0.05.$$

Since the model-independent prediction of $\tau(\Lambda_b)/\tau(B_d)$ to order $1/m_b^2$ is very close to unity [see Eq. (2.7) below], the conflict between theory and experiment for this lifetime ratio is quite striking and has received a lot of attention [8–15]. One possible reason for the discrepancy is that local quark-hadron duality may not work in the study of nonleptonic inclusive decay widths. Another possibility is that some hadronic matrix elements of four-quark operators are probably larger than what naively expected so that the nonspectator effects of order $16\pi^2/m_b^3$ may be large enough to explain the observed lifetime difference between the Λ_b and B_d. Therefore, as stressed by Neubert and Sachrajda [11], one cannot conclude that local duality truly fails before a reliable field-theoretical calculation of the four-quark matrix elements is obtained. Contrary to the $1/m_b^2$ corrections, the estimate of the nonspectator effects is, unfortunately, quite model dependent.

Conventionally, the hadronic matrix elements of four-quark operators are evaluated using the factorization approximation for mesons and the quark model for baryons. However, as we shall see, nonfactorizable effects absent in

*Email address: phcheng@ccvax.sinica.edu.tw
†Email address: kcyang@phys.sinica.edu.tw

the factorization hypothesis can affect the B meson lifetime ratios significantly. In order to have a reliable estimate of the hadronic parameters B_1, B_2, ε_1, and ε_2 in the meson sector, to be introduced below, we will apply the QCD sum rule to calculate these unknown parameters. After a brief review on the status of the OPE approach for the B hadron lifetime ratios in Sec. II, we proceed to derive in Sec. III the renormalization-group improved QCD sum rules for the parameters B_i and ε_i and present a detailed analysis. Section IV gives discussions and conclusions.

II. A BRIEF OVERVIEW

Within the heavy quark expansion framework, we will focus in this paper on the study of the four-quark matrix elements of the B meson to understand the problem with B meson lifetime ratios. Before proceeding, let us briefly review the content of the theory. Applying the optical theorem, the inclusive decay width of the bottom hadron H_b containing a b quark can be expressed in the form

$$\Gamma(H_b\to X)=\frac{1}{m_{H_b}}\ \mathrm{Im}\ \int \mathrm{d}^4x\langle H_b|T\{i\mathcal{L}_{\rm eff}(x),\mathcal{L}_{\rm eff}(0)\}|H_b\rangle, \tag{2.1}$$

where $\mathcal{L}_{\rm eff}$ is the relevant effective weak Lagrangian that contributes to the particular final state X. When the energy release in a b quark decay is sufficiently large, it is possible to express the nonlocal operator product in Eq. (2.1) as a series of local operators in powers of $1/m_b$ by using the OPE technique. In the OPE series, the only locally gauge invariant operator with dimension four, $\bar{b}i\not{D}b$, can be reduced to $m_b\bar{b}b$ by using the equation of motion. Therefore, the first nonperturbative correction to the inclusive B hadron decay width starts at order $1/m_b^2$.[1] As a result, the inclusive decay width of a hadron H_b can be expressed as [1,2]

$$\begin{aligned}\Gamma(H_b\to X)=\frac{G_F^2m_b^5|V_{\rm CKM}|^2}{192\pi^3}\,\frac{1}{2m_{H_b}}\Bigg\{ & c_3^X\langle H_b|\bar{b}b|H_b\rangle \\ &+c_5^X\frac{\left\langle H_b\left|\bar{b}\frac{1}{2}g_s\sigma_{\mu\nu}G^{\mu\nu}b\right|H_b\right\rangle}{m_b^2} \\ &+\sum_n c_6^{X(n)}\frac{\langle H_b|O_6^{(n)}|H_b\rangle}{m_b^3}+O(1/m_b^4)\Bigg\},\end{aligned} \tag{2.2}$$

where $V_{\rm CKM}$ denotes some combination of the Cabibbo-Kobayashi-Maskawa parameters and c_i^X reflect short-distance dynamics and phase-space corrections. The matrix elements in Eq. (2.2) can be systematically expanded in powers of $1/m_b$ in heavy quark effective theory (HQET) [17], in which the b-quark field is represented by a four-velocity-dependent field denoted by $h_v^{(b)}(x)$.

In Eq. (2.2) c_i^X are functions of c_1 and c_2, the Wilson coefficients in the effective Hamiltonian

$$\begin{aligned}\mathcal{H}_{\rm eff}^{\Delta B=1}=\frac{G_F}{\sqrt{2}}[&V_{cb}V_{uq}^*(c_1(\mu)O_1^u(\mu)\\ &+c_2(\mu)O_2^u(\mu))+V_{cb}V_{cq}^*(c_1(\mu)O_1^c(\mu)\\ &+c_2(\mu)O_2^c(\mu))+\cdots]+\mathrm{H.c.},\end{aligned} \tag{2.3}$$

where $q=d,s$, and

$$\begin{aligned}O_1^u&=\bar{c}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1-\gamma_5)u,\\ O_2^u&=\bar{q}\gamma_\mu(1-\gamma_5)b\bar{c}\gamma^\mu(1-\gamma_5)u.\end{aligned} \tag{2.4}$$

The scale and scheme dependence of the Wilson coefficients $c_{1,2}(\mu)$ are canceled out by the corresponding dependence in the matrix element of the four-quark operators $O_{1,2}$. That is, the four-quark operators in the effective theory have to be renormalized at the same scale μ and evaluated using the same renormalization scheme as that for the Wilson coefficients. Schematically, we can write $\langle\mathcal{H}_{\rm eff}\rangle=c(\mu)\langle O(\mu)\rangle=c(\mu)g(\mu)\langle O\rangle_{\rm tree}=c^{\rm eff}\langle O\rangle_{\rm tree}$, where the effective Wilson coefficients $c_i^{\rm eff}$ are renormalization scale and scheme independent. Then the factorization approximation or the quark model is applied to evaluate the hadronic matrix elements of the operator O at tree level. The explicit expression for $g(\mu)$, the perturbative corrections to the four-quark operators renormalized at the scale μ, has been calculated in the literature [18,19]. To the next-to-leading order (NLO) precision, we have [20][2]

$$c_1^{\rm eff}=1.149,\quad c_2^{\rm eff}=-0.325. \tag{2.5}$$

Replacing c_i by $c_i^{\rm eff}$ and using $m_b=(4.85\pm0.25)$ GeV, $(m_c/m_b)^2=0.089$, $|V_{cb}|=0.039$, $G_{B_u,B_d}=0.366$ GeV^2, $G_{B_s}=0.381$ GeV^2 [21], $G_{\Lambda_b}=0$, $K_{B_u,B_d}\approx K_{\Lambda_b}\approx0.4$ GeV^2, $K_{B_s}=1.02\times K_{B_u,B_d}$ [22] together with the nonleptonic inclusive results to the next-to-leading order [23], we find numerically

[1]It is emphasized in [16] that the cancellation of the $1/m_Q$ corrections to the inclusive decay width occurs when it is expressed in terms of the running short-distance quark mass, e.g., the $\overline{\rm MS}$ mass, rather than the pole quark mass.

[2]The effective Wilson coefficients given in Eq. (2.5) are derived from $c_i(\mu)$ at $\mu=m_b$ to the NLO [12]. Nevertheless, it is not difficult to explicitly check the scale and scheme independence of $c_i^{\rm eff}$. For example, the authors of [19] obtain $c_1^{\rm eff}=1.160$ and $c_2^{\rm eff}=-0.334$ at $\mu=2.5$ GeV. Therefore, $c_i^{\rm eff}$ are very insensitive to the chosen μ scale, as it should be. It is known that the Wilson coefficient c_2 at the NLO: $c_2=-0.185$ in the naive dimension regularization scheme and $c_2=-0.228$ in the 't Hooft–Veltman scheme [18], deviates substantially from the leading-order value $c_2-0.308$ at $\mu=m_b(m_b)$. However, the resultant $c_2^{\rm eff}$ is scheme independent and its value is close to the leading-order one.

$$\Gamma_{\rm SL}(B\to e\bar{\nu}X)=(4.18^{+1.20}_{-0.99})\times 10^{-14}\ \ \text{GeV},$$

$$\Gamma_{\rm SL}(\Lambda_b\to e\bar{\nu}X)=(4.32^{+1.24}_{-1.01})\times 10^{-14}\ \ \text{GeV},$$

$$\Gamma(B)=\Gamma_{\rm NL}(B)+2.24\Gamma_{\rm SL}(B\to e\bar{\nu}X)=(3.61^{+1.04}_{-0.84})\times 10^{-13}\ \ \text{GeV},$$

$$\Gamma(\Lambda_b)=\Gamma_{\rm NL}(\Lambda_b)+2.24\Gamma_{\rm SL}(\Lambda_b\to e\bar{\nu}X)=(3.65^{+1.04}_{-0.85})\times 10^{-13}\ \ \text{GeV}. \tag{2.6}$$

It follows that the lifetime ratios of the H_b hadrons are

$$\frac{\tau(B^-)}{\tau(B_d)}=1+O(1/m_b^3),$$
$$\frac{\tau(B_s)}{\tau(B_d)}=1.0005+O(1/m_b^3), \tag{2.7}$$
$$\frac{\tau(\Lambda_b)}{\tau(B_d)}=0.99+O(1/m_b^3).$$

Note that $\tau(B_s)$ here refers to the average lifetime of the two CP eigenstates of the B_s meson. It is evident that the $1/m_b^2$ corrections are too small to explain the shorter lifetime of the Λ_b relative to that of the B_d. To the order of $1/m_b^3$, the nonspectator effects due to Pauli interference and W-exchange parametrized in terms of the hadronic parameters [11]: B_1, B_2, ε_1, ε_2, $\tilde{B}$, and r (see below), may contribute significantly to lifetime ratios due to a phase-space enhancement by a factor of $16\pi^2$. The four-quark operators relevant to inclusive nonleptonic B decays are

$$\begin{aligned}
O^q_{V-A}&=\bar{b}_L\gamma_\mu q_L\bar{q}_L\gamma^\mu b_L,\\
O^q_{S-P}&=\bar{b}_R q_L\bar{q}_L b_R,\\
T^q_{V-A}&=\bar{b}_L\gamma_\mu t^a q_L\bar{q}_L\gamma^\mu t^a b_L,\\
T^q_{S-P}&=\bar{b}_R t^a q_L\bar{q}_L t^a b_R,
\end{aligned} \tag{2.8}$$

where $q_{R,L}=[(1\pm\gamma_5)/2]q$ and $t^a=\lambda^a/2$ with λ^a being the Gell-Mann matrices. For the matrix elements of these four-quark operators between B hadron states, we follow [11] to adopt the following definitions:

$$\frac{1}{2m_{B_q}}\langle\bar{B}_q|O^q_{V-A}|\bar{B}_q\rangle\equiv\frac{f^2_{B_q}m_{B_q}}{8}B_1,$$

$$\frac{1}{2m_{B_q}}\langle\bar{B}_q|O^q_{S-P}|\bar{B}_q\rangle\equiv\frac{f^2_{B_q}m_{B_q}}{8}B_2,$$

$$\frac{1}{2m_{B_q}}\langle\bar{B}_q|T^q_{V-A}|\bar{B}_q\rangle\equiv\frac{f^2_{B_q}m_{B_q}}{8}\varepsilon_1,$$

$$\frac{1}{2m_{B_q}}\langle\bar{B}_q|T^q_{S-P}|\bar{B}_q\rangle\equiv\frac{f^2_{B_q}m_{B_q}}{8}\varepsilon_2,$$

$$\frac{1}{2m_{\Lambda_b}}\langle\Lambda_b|O^q_{V-A}|\Lambda_b\rangle\equiv-\frac{f^2_{B_q}m_{B_q}}{48}r,$$

$$\frac{1}{2m_{\Lambda_b}}\langle\Lambda_b|T^q_{V-A}|\Lambda_b\rangle\equiv-\frac{1}{2}\left(\tilde{B}+\frac{1}{3}\right)\frac{1}{2m_{\Lambda_b}}\times\langle\Lambda_b|O^q_{V-A}|\Lambda_b\rangle, \tag{2.9}$$

where f_{B_q} is the B_q meson decay constant defined by

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}_q(p)\rangle=if_{B_q}p_\mu. \tag{2.10}$$

Under the factorization approximation, $B_i=1$ and $\varepsilon_i=0$, and under the valence quark approximation $\tilde{B}=1$ [11].

To the order of $1/m_b^3$, we find that the B-hadron lifetime ratios are given by

$$\frac{\tau(B^-)}{\tau(B_d^0)}=1+\left(\frac{f_B}{185\ \ \text{MeV}}\right)^2(0.043B_1+0.0006B_2-0.61\varepsilon_1+0.17\varepsilon_2),$$

$$\frac{\tau(B_s^0)}{\tau(B_d^0)}=1+\left(\frac{f_B}{185\ \ \text{MeV}}\right)^2(-1.7\times10^{-5}B_1+1.9\times10^{-5}B_2-0.0044\varepsilon_1+0.0050\varepsilon_2),$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d^0)}=0.99+\left(\frac{f_B}{185\ \ \text{MeV}}\right)^2[-0.0006B_1+0.0006B_2-0.15\varepsilon_1+0.17\varepsilon_2-(0.014+0.019\tilde{B})r]. \tag{2.11}$$

The above results are similar to that given in [11]. We see that the coefficients of the color singlet-singlet operators are one to two orders of magnitude smaller than those of the color octet-octet operators. This implies that even a small deviation from the factorization approximation $\varepsilon_i=0$ can have a sizable impact on the lifetime ratios. It was argued in [11] that the unknown nonfactorizable contributions render it impossible to make reliable estimates on the magnitude of the lifetime ratios and even the sign of corrections. That is, the theoretical prediction for $\tau(B^-)/\tau(B_d)$ is not necessarily larger than unity. In the next section we will apply the QCD sum rule method to estimate the aforementioned hadronic parameters, especially ε_i.

III. THE QCD SUM RULE CALCULATION

In this section we will employ the method of QCD sum rules within the framework of HQET. Since the b quark is treated as a static quark with an infinite quark mass in HQET and since HQET is a low-energy effective theory, it is natural to regard the matrix elements of Eq. (2.9) as defined at the scale m_b and to evaluate the corresponding hadronic matrix elements in HQET at a scale $\mu < m_b$. Indeed, it has been argued that the estimate of hadronic matrix elements of four-quark operators using the factorization hypothesis for mesons and the quark model for baryons becomes more reliable if the operators are renormalized at a typical hadronic scale [24]. In the sum rule approach, the correlation function (or the so-called Green function), within the QCD framework, can be expanded as a series of operators $O_n(\mu)$ multiplied by the Wilson coefficients $C_n(-2\omega/\mu, g_s)/\omega^n$, where ω is an external momentum flowing in (or out) the correlation function and μ is the factorization scale that separates the long-distance part O_n from the short-distance one C_n. The quality of the convergence of the OPE series is controlled by the value of the external momentum ω. The factorization scale μ cannot be chosen too small, otherwise the strong coupling constant α_s would be so large that Wilson coefficients cannot be perturbatively calculated. Four-quark operators are sometimes renormalized at a typical scale $\mu_h \approx 0.67$ GeV, corresponding to the coupling constant $\alpha_s^{\overline{\rm MS}}(\mu_h) \sim \mathcal{O}(1)$. However, such a scale is not quite suitable for the sum rule calculation. Instead we choose $\mu = 1$ GeV as the lowest possible factorization scale in the ensuing study. After summing over the logarithmic dependence $\alpha_s^m \ln^m(-2\omega/\mu)$ by the renormalization-group method, one obtains the nonperturbative quantity $X(\mu)$ which can be extracted from the correlation function in the following form:

$$X(\mu) \sim \sum_n \frac{C_n(1, g_s(-2\omega))}{\omega^n} \left(\frac{\alpha_s(\mu)}{\alpha_s(-2\omega)} \right)^{\gamma_n - \Sigma_j \gamma_j} O_n(\mu), \tag{3.1}$$

where γ_n are the anomalous dimensions of O_n and γ_j the anomalous dimensions of the currents appearing in the correlation function. As noted above, the four-quark operators in Eq. (2.9) are defined at the scale $\mu = m_b$. In HQET where the b quark is treated as a static quark, we can use the renormalization group equation to express them in terms of the operators renormalized at a scale $\Lambda_{\rm QCD} \ll \mu \ll m_b$. These operators have the hybrid anomalous dimensions [25–27] and their renormalization-group evolution is determined by the anomalous dimensions in HQET. The operators O^q_{V-A} and T^q_{V-A}, and similarly O^q_{S-P} and T^q_{S-P}, mix under renormalization. In the leading logarithmic approximation, the renormalization-group equation of the operator pair (O,T) governed by the hybrid anomalous dimension matrix reads[3]

$$\frac{d}{dt}\begin{pmatrix} O \\ T \end{pmatrix} = \frac{3\alpha_s}{2\pi} \begin{pmatrix} C_F & -1 \\ -\frac{C_F}{2N_c} & \frac{1}{2N_c} \end{pmatrix} \begin{pmatrix} O \\ T \end{pmatrix}, \tag{3.2}$$

where $t = \frac{1}{2}\ln(Q^2/\mu^2)$, $C_F = (N_c^2 - 1)/2N_c$, and effects of penguin operators induced from evolution have been neglected.

The solution to the evolution equation Eq. (3.2) has the form

$$\begin{pmatrix} O \\ T \end{pmatrix}_Q = \begin{pmatrix} \frac{8}{9} & \frac{2}{3} \\ -\frac{4}{27} & \frac{8}{9} \end{pmatrix} \begin{pmatrix} L_Q^{9/(2\beta_0)} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{D}_{\boldsymbol{\mu}}, \tag{3.3}$$

where

$$\mathbf{D}_{\boldsymbol{\mu}} = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}_\mu = \begin{pmatrix} O - \frac{3}{4}T \\ \frac{1}{6}O + T \end{pmatrix}_\mu, \tag{3.4}$$

$$L_Q = \frac{\alpha_s(\mu)}{\alpha_s(Q)}, \tag{3.5}$$

and $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$ is the leading-order expression of the β-function with n_f being the number of light quark flavors. The subscript μ in Eq. (3.4) and in what follows denotes the renormalization point of the operators. Given the evolution equation (3.3) for the four-quark operators, we see that the hadronic parameters B_i and ε_i normalized at the scale m_b are related to that at $\mu = 1$ GeV by

$$\begin{pmatrix} B_i \\ \varepsilon_i \end{pmatrix}_{m_b} = \begin{pmatrix} \frac{8}{9} & \frac{2}{3} \\ -\frac{4}{27} & \frac{8}{9} \end{pmatrix} \begin{pmatrix} L_{m_b}^{9/(2\beta_0)} & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} B_i - \frac{3}{4}\varepsilon_i \\ \frac{1}{6}B_i + \varepsilon_i \end{pmatrix}_{\mu = 1\,\text{GeV}}, \tag{3.6}$$

and hence

$$B_i(m_b) \simeq 1.54 B_i(\mu) - 0.41 \varepsilon_i(\mu),$$
$$\varepsilon_i(m_b) \simeq -0.090 B_i(\mu) + 1.07 \varepsilon_i(\mu), \tag{3.7}$$

with $\mu = 1$ GeV, where uses have been made of $\alpha_s(m_Z) = 0.118$, $\Lambda^{(4)}_{\overline{MS}} = 333$ MeV, $m_b = 4.85$ GeV, $m_c = 1.45$ GeV and

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln\frac{Q^2}{\Lambda^2}} \left(1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln\left(\ln\frac{Q^2}{\Lambda^2}\right)}{\ln\frac{Q^2}{\Lambda^2}} \right)$$

---

[3] One of the off-diagonal anomalous dimension matrix elements in Eq. (3.2) has a sign opposite to that obtained in [11], but the final result in Eq. (3.6) is in full agreement with the results derived there.

to the next-to-leading order with $\beta_1=51-\frac{19}{3}n_f$. The above results (3.7) indicate that renormalization effects are quite significant.

It is easily seen from Eqs. (3.3) and (3.4) that the normalized operator D_1 (or D_2) is simply multiplied by $L_Q^{9/(2\beta_0)}$ (or 1) when it evolves from a renormalization point μ to another point Q. In what follows, we will apply this property to derive the renormalization-group improved QCD sum rules for D_j at the typical scale $\mu=1$ GeV. We define the new four-quark matrix elements as follows:

$$\frac{1}{2m_{B_q}}\langle\overline{B}_q|D_j^{(i)}(\mu)|\overline{B}_q\rangle\equiv\frac{f_{B_q}^2 m_{B_q}}{8}d_j^{(i)}(\mu), \tag{3.8}$$

where the superscript (i) denotes $(V-A)$ four-quark operators for $i=1$ and $(S-P)$ operators for $i=2$, and $d_j^{(i)}$ satisfy

$$\begin{pmatrix} d_1^{(i)} \\ d_2^{(i)} \end{pmatrix}_\mu=\begin{pmatrix} B_i-\frac{3}{4}\varepsilon_i \\ \frac{1}{6}B_i+\varepsilon_i \end{pmatrix}_\mu. \tag{3.9}$$

Since the terms linear in four-quark matrix elements are already of order $1/m_b^3$, we only need the relation between the full QCD field $b(x)$ and the HQET field $h_v^{(b)}(x)$ to the zeroth order in $1/m_b$: $b(x)=e^{-im_b v\cdot x}\{h_v^{(b)}(x)+\mathcal{O}(1/m_b)\}$. Therefore, in analogue to Eq. (2.8), we define the relevant four-quark operators in HQET as

$$O_{V-A}^v=\overline{h}_{vL}^{(b)}\gamma_\mu q_L\overline{q}_L\gamma^\mu h_{vL}^{(b)},$$
$$O_{S-P}^v=\overline{h}_{vR}^{(b)}q_L\overline{q}_L h_{vR}^{(b)},$$
$$T_{V-A}^v=\overline{h}_{vL}^{(b)}\gamma_\mu t^a q_L\overline{q}_L\gamma^\mu t^a h_{vL}^{(b)},$$
$$T_{S-P}^v=\overline{h}_{vR}^{(b)}t^a q_L\overline{q}_L t^a h_{vR}^{(b)}. \tag{3.10}$$

The corresponding hadronic matrix elements of these four-quark operators are parametrized by

$$\frac{1}{2}\langle\overline{B}(v)|O_{V-A}^v|\overline{B}(v)\rangle\equiv\frac{F^2(m_b)}{8}B_1^v(\mu),$$
$$\frac{1}{2}\langle\overline{B}(v)|O_{S-P}^v|\overline{B}(v)\rangle\equiv\frac{F^2(m_b)}{8}B_2^v(\mu),$$
$$\frac{1}{2}\langle\overline{B}(v)|T_{V-A}^v|\overline{B}(v)\rangle\equiv\frac{F^2(m_b)}{8}\varepsilon_1^v(\mu),$$
$$\frac{1}{2}\langle\overline{B}(v)|T_{S-P}^v|\overline{B}(v)\rangle\equiv\frac{F^2(m_b)}{8}\varepsilon_2^v(\mu), \tag{3.11}$$

where the heavy-flavor-independent decay constant F defined in the heavy quark limit is given by

$$\langle 0|\overline{q}\gamma^\mu\gamma_5 h_v^{(b)}|\overline{B}(v)\rangle=iF(\mu)v^\mu. \tag{3.12}$$

The decay constant $F(\mu)$ depends on the scale μ at which the effective current operator is renormalized and it is related to the scale-independent decay constant f_B of the B meson by

$$F(m_b)=f_B\sqrt{m_B}. \tag{3.13}$$

Notice that F in Eq. (3.11) is chosen to be normalized at the scale m_b.

To complete the aim of obtaining the matrix elements of four-quark operators, we apply the method of QCD sum rules [28]. We consider the following three-point correlation function:

$$\Pi_{\alpha,\beta}^{D_j^{v(i)}}(\omega,\omega')=i^2\int dxdy e^{i\omega v\cdot x-i\omega' v\cdot y}$$
$$\times\langle 0|T\{[\overline{q}(x)\Gamma_\alpha h_v^{(b)}(x)]D_j^{v(i)}(0)$$
$$\times[\overline{q}(y)\Gamma_\beta h_v^{(b)}(y)]^\dagger\}|0\rangle, \tag{3.14}$$

of the operator $D_j^{(i)}$ defined in Eq. (3.4), where $\Gamma_\alpha=\gamma_\alpha\gamma_5$. However, this current interpolates not only the heavy mesons with quantum number $J^P=0^-$ but also that with quantum number $J^P=1^+$. Therefore, we need to decompose Γ_α into $\Gamma_\alpha=\Gamma_\alpha^{AV}-v_\alpha\Gamma^{PS}$, with $\Gamma_\alpha^{AV}=(\gamma+v)_\alpha\gamma_5$ for $J^P=1^+$ and $\Gamma^{PS}=\gamma_5$ for $J^P=0^-$. As a consequence, $\Pi_{\alpha\beta}^{D_j^{v(i)}}$ is recast to

$$\Pi_{\alpha\beta}^{D_j^{v(i)}}=(-g_{\alpha\beta}+v_\alpha v_\beta)\Pi_{D_j^{v(i)}}^{AV}+v_\alpha v_\beta\Pi_{D_j^{v(i)}}^{PS}, \tag{3.15}$$

where

$$(-g_{\alpha\beta}+v_\alpha v_\beta)\Pi_{D_j^{v(i)}}^{AV}$$
$$=i^2\int dxdy e^{i\omega v\cdot x-i\omega' v\cdot y}$$
$$\times\langle 0|T\{[\overline{q}(x)\Gamma_\alpha^{AV}h_v^{(b)}(x)]D_j^{v(i)}(0)$$
$$\times[\overline{q}(y)\Gamma_\beta^{AV}h_v^{(b)}(y)]^\dagger\}|0\rangle,$$

$$\Pi_{D_j^{v(i)}}^{PS}=i^2\int dxdy e^{i\omega v\cdot x-i\omega' v\cdot y}$$
$$\times\langle 0|T\{[\overline{q}(x)\Gamma^{PS}h_v^{(b)}(x)]D_j^{v(i)}(0)$$
$$\times[\overline{q}(y)\Gamma^{PS}h_v^{(b)}(y)]^\dagger\}|0\rangle. \tag{3.16}$$

In deriving Eq. (3.15) we have applied the relations

$$i^2\int dxdy e^{i\omega v\cdot x-i\omega' v\cdot y}\langle 0|T\{[\overline{q}(x)\Gamma_\alpha^{AV}h_v^{(b)}(x)]D_j^{v(i)}(0)$$
$$\times[\overline{q}(y)\Gamma^{PS}h_v^{(b)}(y)]^\dagger\}|0\rangle=0 \tag{3.17}$$

and

$$i^2\int dxdy e^{i\omega v\cdot x-i\omega' v\cdot y}\langle 0|T\{[\overline{q}(x)\Gamma^{PS}h_v^{(b)}(x)]D_j^{v(i)}(0)\times[\overline{q}(y)\Gamma_\beta^{AV}h_v^{(b)}(y)]^\dagger\}|0\rangle=0. \quad (3.18)$$

Note that only the correlation function Π^{PS} is relevant to our purpose. It can be written in the double dispersion relation form

$$\Pi^{PS}_{D_j^{v(i)}}(\omega,\omega')=\int\int\frac{ds}{s-\omega}\frac{ds'}{s'-\omega'}\rho^{D_j^{v(i)}}. \quad (3.19)$$

The results of the QCD sum rules can be obtained in the following way. On the phenomenological side, which is the sum of the relevant hadron states, this correlation function can be written as

$$\Pi^{PS}_{D_j^{v(i)}}(\omega,\omega')=\frac{F^2(m_b)F^2(\mu)d_j^{(i)}}{16(\overline{\Lambda}-\omega)(\overline{\Lambda}-\omega')}+\cdots, \quad (3.20)$$

where $\overline{\Lambda}$ is the binding energy of the heavy meson in the heavy quark limit and ellipses denote resonance contributions. On the theoretical side, the correlation function Π^{PS} can be alternatively calculated in terms of quarks and gluons using the standard OPE technique. Then we equate the results on the phenomenological side with that on the theoretical side. However, since we are only interested in the properties of the ground state at hand, e.g., the B meson, we shall assume that contributions from excited states (on the phenomenological side) are approximated by the spectral density on the theoretical side of the sum rule, which starts from some thresholds (say, $\omega_{i,j}$ in this study). To further improve the final result under consideration, we apply the Borel transform to both external variables ω and ω'. After the Borel transform [28],

$$\mathbf{B}[\Pi^{PS}_{D_j^{v(i)}}(\omega,\omega')]=\lim_{\substack{m\to\infty\\ -\omega'\to\infty\\ -\frac{\omega'}{mt'}\ \text{fixed}}}\lim_{\substack{n\to\infty\\ -\omega\to\infty\\ -\frac{\omega}{nt}\ \text{fixed}}}\frac{1}{n!m!}(-\omega')^{m+1}\times\left[\frac{d}{d\omega'}\right]^m(-\omega)^{n+1}\left[\frac{d}{d\omega}\right]^n\Pi^{PS}_{D_j^{v(i)}}(\omega,\omega'), \quad (3.21)$$

the sum rule gives

$$\frac{F^2(m_b)F^2(\mu)}{16}e^{-\overline{\Lambda}/t_1}e^{-\overline{\Lambda}/t_2}d_j^{(i)}=\int_0^{\omega_{i,j}}ds\int_0^{\omega_{i,j}}ds' e^{-(s/t_1+s'/t_2)}\rho^{\rm QCD}, \quad (3.22)$$

where $\omega_{i,j}$ is the threshold of the excited states and $\rho^{\rm QCD}$ is the spectral density on the theoretical side of the sum rule. Because the Borel windows are symmetric in variables t_1 and t_2, it is natural to choose $t_1=t_2$. However, unlike the case of the normalization of the Isgur-Wise function at zero recoil, where the Borel mass is approximately twice as large as that in the corresponding two-point sum rule [29], in the present case of the three-point sum rule at hand, we find that the working Borel windows can be chosen as the same as that in the two-point sum rule since in our analysis the output results depend weakly on the Borel mass. Therefore, we choose $t_1=t_2=t$. By the renormalization group technique, the logarithmic dependence $\alpha_s\ln(2t/\mu)$ can be summed over to produce a factor like $[\alpha_s(\mu)/\alpha_s(2t)]^\gamma$. After some manipulation we obtain the sum rule results:

$$\frac{F^2(m_b)F^2(\mu)}{16}e^{-2\overline{\Lambda}/t}\begin{pmatrix}d_1^{v(i)}\\ d_2^{v(i)}\end{pmatrix}_\mu=\left(\frac{\alpha_s(2t)}{\alpha_s(\mu)}\right)^{4/\beta_0}\left(\frac{1-2\delta\frac{\alpha_s(2t)}{\pi}}{1-2\delta\frac{\alpha_s(\mu)}{\pi}}\right)\begin{pmatrix}L_t^{-9/(2\beta_0)} & 0\\ 0 & 1\end{pmatrix}\times\begin{pmatrix}{\rm OPE}_{B_{i,1}}-\frac{3}{4}{\rm OPE}_{\varepsilon_{i,1}}\\ \frac{1}{6}{\rm OPE}_{B_{i,2}}+{\rm OPE}_{\varepsilon_{i,2}}\end{pmatrix}_t, \quad (3.23)$$

where

$$\begin{aligned}{\rm OPE}_{B_{i,j}}&\simeq\frac{1}{4}({\rm OPE})^2_{2pt;i,j},\\ {\rm OPE}_{\varepsilon_{1,j}}&\simeq-\frac{1}{16}\left[-\frac{\langle\overline{q}g_s\sigma\cdot Gq\rangle}{8\pi^2}t(1-e^{-\omega_{1,j}/t})+\frac{\langle\alpha_sG^2\rangle}{16\pi^3}t^2(1-e^{-\omega_{1,j}/t})^2\right],\\ {\rm OPE}_{\varepsilon_{2,j}}&\simeq\mathcal{O}(\alpha_s),\end{aligned} \quad (3.24)$$

with

$$({\rm OPE})_{2pt;i,j}=\frac{1}{2}\left\{\int_0^{\omega_{i,j}}ds s^2e^{-s/t}\frac{3}{\pi^2}\times\left[1+\frac{\alpha_s}{\pi}\left(\frac{17}{3}+\frac{4\pi^2}{9}-2\ln\frac{s}{t}\right)\right]-\left(1+\frac{2\alpha_s}{\pi}\right)\langle\overline{q}q\rangle+\frac{\langle\overline{q}g_s\sigma\cdot Gq\rangle}{16t^2}\right\}. \quad (3.25)$$

For reason of consistency, in the following numerical analysis we will neglect the finite part of radiative one loop corrections in ${\rm OPE}_{B_{i,j}}$ and ${\rm OPE}_{\varepsilon_{i,j}}$ [and in Eq. (3.28)]. The parameter δ in Eq. (3.23) is some combination of the β functions and anomalous dimensions [see Eq. (4.2) of [30]] and is numerically equal to -0.23. The relevant parameters normalized at the scale t are related to those at μ by [31,29,32]

$$F(2t)=F(\mu)\left(\frac{\alpha_s(2t)}{\alpha_s(\mu)}\right)^{-2/\beta_0}\frac{1-\delta\frac{\alpha_s(\mu)}{\pi}}{1-\delta\frac{\alpha_s(2t)}{\pi}},$$

$$\langle\bar{q}q\rangle_{2t}=\langle\bar{q}q\rangle_{\mu}\cdot\left(\frac{\alpha_s(2t)}{\alpha_s(\mu)}\right)^{-4/\beta_0},$$

$$\langle g_s\bar{q}\sigma\cdot Gq\rangle_{2t}=\langle g_s\bar{q}\sigma\cdot Gq\rangle_{\mu}\left(\frac{\alpha_s(2t)}{\alpha_s(\mu)}\right)^{2/(3\beta_0)},$$

$$\langle\alpha_s G^2\rangle_{2t}=\langle\alpha_s G^2\rangle_{\mu}, \tag{3.26}$$

where $\langle\cdots\rangle$ stands for $\langle 0|\cdots|0\rangle$. In the calculation of the correlation function, we have also used the fixed-point gauge (the Fock-Schwinger gauge) $x^{\mu}A_{\mu}(x)=0$ with A_{μ} being an external gluon field. Under this gauge, the generalized quark propagator in the external gluon field reads

$$\begin{aligned}S^{ab}_{q(ij)}(0,x)=\int\frac{d^4p}{(2\pi)^4}e^{ip\cdot x}\Bigg[&\frac{i\delta^{ab}}{\not{p}-m_q}+\frac{i}{4}\frac{\lambda^n_{ab}}{2}g_sG^n_{\mu\nu}(0)\frac{\sigma^{\mu\nu}(\not{p}+m_q)+(\not{p}+m_q)\sigma^{\mu\nu}}{(p^2-m_q^2)^2}\\&-\frac{iG^n_{\mu\nu}(0)\lambda^n_{ab}}{4}g_sx^{\nu}\left(\frac{1}{\not{p}-m_q}\gamma^{\mu}\frac{1}{\not{p}-m_q}\right)\Bigg]_{ij}\\&+:q^a_i(0)\bar{q}^b_j(0):+x_{\mu}:q^a_i(0)(D^{\mu}\bar{q}^b_j(0)):+\frac{x_{\mu}x_{\nu}}{2!}:q^a_i(0)(D^{\mu}D^{\nu}\bar{q}^b_j(0)):+\cdots,\end{aligned} \tag{3.27}$$

where a and b are the color indices, i and j the Lorentz indices.

Let us explain the results obtained in Eqs. (3.23) and (3.24). OPE_{B_i} is obtained by substituting $D_j^{v(i)}$ by O^v in $\Pi^{PS}_{D_j^{v(i)}}$ [cf. Eq. (3.16)] and it can be approximately factorized as the product of $(\mathrm{OPE})_{2pt;i,j}$ with itself, which is the same as the theoretical part in the two-point $F(\mu)$ sum rule [29–31]. In the series of $(\mathrm{OPE})_{2pt;i,j}$, we have neglected the contribution proportional to $\langle\bar{q}q\rangle^2$. (More precisely, it is equal to $\alpha_s\langle\bar{q}q\rangle^2\pi/324$; see Ref. [29].) Nevertheless, the result of $(\mathrm{OPE})_{B_i}$ in Eq. (3.24) is reliable up to dimension six, as the contributions from the $\langle\bar{q}q\rangle^2$ terms in $(\mathrm{OPE})_{2pt;i,j}$ are much smaller than the term $(1+\alpha_s/\pi)^2\langle\bar{q}q\rangle^2/16$ that we have kept [see Eq. (3.25)]. Note that in $(\mathrm{OPE})_{B_i}$ the contribution involving the gluon condensate is proportional to the light quark mass and hence can be neglected. Likewise, $\mathrm{OPE}_{\varepsilon_i}$ is the theoretical side of the sum rule, and it is obtained by substituting $D_j^{v(i)}$ by T^v in Eq. (3.16). To the order of dimension-five, the main contributions to $\mathrm{OPE}_{\varepsilon_i}$ are depicted in Fig. 1. Here we have neglected the dimension-6 four-quark condensate of the type $\langle\bar{q}\Gamma\lambda^a q\bar{q}\Gamma\lambda^a q$. Its contribution is much less than that from dimension-five or dimension-four condensates and hence unimportant (see [33] for similar discussions). It should be emphasized that nonfactorizable contributions to the parameters B_i arise mainly from the O^v-T^v operator mixing.

At this point, it is useful to compare our analysis with the similar QCD sum rule studies in [33] and [15]. First, Chernyak [33] used the chiral interpolating current for the B meson, so that all light quark fields in his correlators are purely left-handed. As a result, there are no quark-gluon mixed condensates as these require the presence of both left- and right-handed light quark fields. Indeed, the gluon condensate contribution enters into the ε_1 sum rule with an additional factor of 4 in comparison with ours. Second, our results for $\mathrm{OPE}_{\varepsilon_i}$ are very different from that obtained by Baek *et al.* [15]. The reason is that they calculated the full $\Pi^{\varepsilon_i,\alpha}_{\alpha}$ [obtained by replacing $D_j^{v(i)}$ by T^v in Eq. (3.14)] rather than the pseudoscalar part of $\Pi^{\varepsilon_i,\alpha}_{\alpha}$. Therefore, their results are mixed with the 1^+ to 1^+ transitions. Also a subtraction of the contribution from excited states is not carried out in [15] for the three-point correlation function, though it is justified to do so for two-point correlation functions. Indeed, in the following analysis, one will find that after subtracting the contribution from excited states, the contributions of $\mathrm{OPE}_{\varepsilon_i}$ are largely suppressed. Furthermore, as in the study of the B meson decay constant [29], we find that the renormalization-group effects are very important in the sum

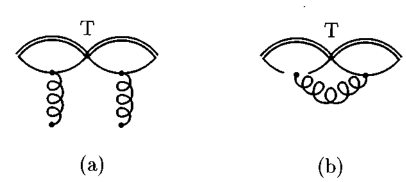

FIG. 1. The main diagrams contributing to $\mathrm{OPE}_{\varepsilon_i}$ [cf. Eq. (3.24)]: (a) the contribution from the gluon condensate, and (b) the contribution from the quark-gluon mixed condensate. In (b) the mirror-symmetric diagram is included in the calculation. The double lines denote heavy quarks in HQET.

rule analysis. Consequently, there is not much difference between the resulting values of ε_1 and ε_2. Moreover, ε_i at $\mu=m_b$ are largely enhanced by renormalization-group effects.

The value of F in Eq. (3.23) can be substituted by

$$F^2(\mu)e^{-\bar{\Lambda}/t}=\left[\frac{\alpha_s(2t)}{\alpha_s(\mu)}\right]^{4/\beta}\left[\frac{1-2\delta\frac{\alpha_s(2t)}{\pi}}{1-2\delta\frac{\alpha_s(\mu)}{\pi}}\right]\times\left\{\int_0^{\omega_0}ds s^2 e^{-s/t}\frac{3}{\pi^2}\times\left[1+\frac{\alpha_s(2t)}{\pi}\left(\frac{17}{3}+\frac{4\pi^2}{9}-2\ln\frac{s}{t}\right)\right]-\left(1+\frac{2\alpha_s(2t)}{\pi}\right)\langle\bar{q}q\rangle_{2t}+\frac{\langle\bar{q}g_s\sigma\cdot Gq\rangle_{2t}}{16t^2}\right\}, \tag{3.28}$$

which can be obtained from the two-point sum rule approach [30,31,29]. For the numerical analysis, we use the following values of parameters [32,34]

$$\langle\bar{q}q\rangle_{\mu=1\text{ GeV}}=-(240\ \text{MeV})^3,$$

$$\langle\alpha_s G^2\rangle_{\mu=1\text{ GeV}}=0.0377\ \text{GeV}^4,$$

$$\langle\bar{q}g_s\sigma_{\mu\nu}G^{\mu\nu}q\rangle_{\mu=1\text{ GeV}}=(0.8\ \text{GeV}^2)\times\langle\bar{q}q\rangle_{\mu=1\text{ GeV}}, \tag{3.29}$$

as input and neglect the finite part of radiative one loop corrections. Since in our convention $D_\mu=\partial_\mu-ig_sA_\mu$, we have $\langle g_s\bar{q}\sigma\cdot Gq\rangle=m_0^2\langle\bar{q}q\rangle$. Next, in order to determine the thresholds $\omega_{i,j}$ we employ the B meson decay constant $f_B=(185\pm25\pm17)$ MeV obtained from a recent lattice-QCD calculation [35] and the relation [36]

$$f_B=\frac{F(m_b)}{\sqrt{m_B}}\left(1-\frac{2}{3}\frac{\alpha_s(m_b)}{\pi}\right)\left(1-\frac{(0.8\sim1.1)\ \text{GeV}}{m_b}\right), \tag{3.30}$$

that takes into account QCD and $1/m_b$ corrections. Using the relation between $F(m_b)$ and $F(\mu)$ given by Eq. (3.26) and $m_b=(4.85\pm0.25)$ GeV, we obtain

$$F(\mu=1\ \text{GeV})\cong(0.34\sim0.52)\ \text{GeV}^{3/2}. \tag{3.31}$$

Since the $\bar{\Lambda}$ parameter in Eq. (3.28) can be replaced by the $\bar{\Lambda}$ sum rule obtained by applying the differential operator $t^2\partial\ln/\partial t$ to both sides of Eq. (3.28), the $F(\mu)$ sum rule can be rewritten as

$$F^2(\mu)=[\text{right hand side of Eq. (3.28)}]\times\exp\left[t\frac{\partial}{\partial t}\ln[\text{right hand side of Eq. (3.28)}]\right], \tag{3.32}$$

which is $\bar{\Lambda}$-free. Then using the result (3.31) as input, the threshold ω_0 in the $F(\mu)$ sum rule, Eq. (3.32), is determined. The result for ω_0 is $1.25-1.65$ GeV. A larger $F(\mu=1\ \text{GeV})$ corresponds to a larger ω_0. The working Borel window lies in the region 0.6 GeV$<t<$1 GeV, which turns out to be a reasonable choice [31]. Substituting the value of ω_0 back into the $\bar{\Lambda}$ sum rule, we obtain $\bar{\Lambda}=0.48-0.76$ GeV in the Borel window 0.6 GeV$<t<$1 GeV. This result is consistent with the choice $m_b=(4.85\pm0.25)$ GeV, recalling that in the heavy quark limit, $\bar{\Lambda}=m_B-m_b$. To extract the $d_j^{v(i)}$ sum rules, one can take the ratio of Eq. (3.28) and Eq. (3.23) to eliminate the contribution of $F^2/\exp(\bar{\Lambda}/t)$. This means one has chosen the same $\bar{\Lambda}$ both in Eq. (3.28) and Eq. (3.23). Since quark-hadron duality is the basic assumption in the QCD sum rule approach, we expect that the same result of $\bar{\Lambda}$ also can be obtained using the $\bar{\Lambda}$ sum rules derived from Eq. (3.23) (see [37,32] for a further discussion). This property can help us to determine consistently the threshold in 3-point sum rule, Eq. (3.23). Therefore, we can apply the differential operator $t^2\partial\ln/\partial t$ to both sides of Eq. (3.23), the $d^{v(i)}$ sum rule, to obtain new $\bar{\Lambda}$ sum rules. The requirement of producing a reasonable value for $\bar{\Lambda}$, say $0.48-0.76$ GeV, provides severe constraints on the choices of $\omega_{i,j}$. With a careful study, we find that the best choice in our analysis is

$$\omega_{i,1}=-0.02\ \text{GeV}+\omega_0,$$

$$\omega_{1,2}=-0.5\ \text{GeV}+\omega_0,$$

$$\omega_{2,2}=-0.22\ \text{GeV}+\omega_0. \tag{3.33}$$

Applying the above relations with $\omega_0=(1.25\sim1.65)$ GeV and substituting $F(\mu)$ in Eq. (3.23) by (3.28), we study numerically the $d_j^{v(i)}$ sum rules. In Fig. 2, we plot B_i^v and ε_i^v as a function t, where $B_i^v=8d_1^{v(i)}/9+2d_2^{v(i)}/3$, and $\varepsilon_i^v=-4d_1^{v(i)}/27+8d_2^{v(i)}/9$. The dashed and solid curves stand for B_i^v and ε_i^v, respectively, where we have used $\omega_0=1.4$ GeV [the corresponding decay constant is $f_B=175\sim195$ MeV or $F(\mu)=0.405\pm0.005$ GeV$^{3/2}$]. The final results for the hadronic parameters B_i and ε_i are (see Fig. 2)[4]

$$B_1^v(\mu=1\ \text{GeV})=0.60\pm0.02,$$

$$B_2^v(\mu=1\ \text{GeV})=0.61\pm0.01,$$

$$\varepsilon_1^v(\mu=1\ \text{GeV})=-0.08\pm0.01,$$

$$\varepsilon_2^v(\mu=1\ \text{GeV})=-0.024\pm0.006. \tag{3.34}$$

[4]For comparison, the sum rule results obtained in [15] and [33] are $\varepsilon_1^{\text{BLLS}}(\mu)=-0.041\pm0.022$, $\varepsilon_2^{\text{BLLS}}(\mu)=0.061\pm0.035$ [15] and $\varepsilon_1^{\text{C}}(\mu)\approx-0.15$, $\varepsilon_2^{\text{C}}(\mu)\approx0$ [33], respectively, with μ being a typical hadronic scale $\sim$0.70 GeV. Note that the definition of $B_i(\mu)$ and $\varepsilon_i(\mu)$ in [15,33] is different from ours by a factor of $F^2(m_b)/F^2(\mu)$, that is, $\varepsilon_i^{\text{BLLS,C}}(\mu)=\varepsilon_i(\mu)\times F^2(m_b)/F^2(\mu)$.

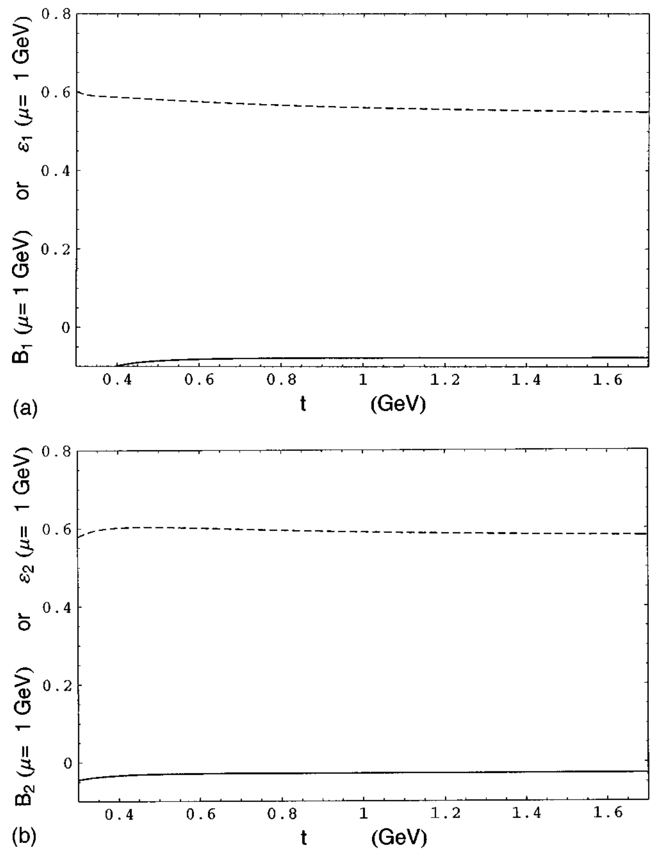

FIG. 2. $B_i^v(\mu)$ and $\varepsilon_i^v(\mu)$ as a function t, where $B_i^v = 8d_1^{v(i)}/9 + 2d_2^{v(i)}/3$, and $\varepsilon_i^v = -4d_1^{v(i)}/27 + 8d_2^{v(i)}/9$. The dashed and solid curves stand for B_i^v and ε_i^v, respectively. Here we have used $\omega_0 = 1.4$ GeV and Eq. (3.33).

The numerical errors come mainly from the uncertainty of $\omega_0 = 1.25 \sim 1.65$ GeV. Some intrinsic errors of the sum rule approach, say quark-hadron duality or α_s corrections, will not be considered here.

Substituting the above results into Eq. (3.7) yields

$$B_1(m_b) = 0.96 \pm 0.04 + O(1/m_b),$$

$$B_2(m_b) = 0.95 \pm 0.02 + O(1/m_b),$$

$$\varepsilon_1(m_b) = -0.14 \pm 0.01 + O(1/m_b),$$

$$\varepsilon_2(m_b) = -0.08 \pm 0.01 + O(1/m_b). \qquad (3.35)$$

It follows from Eq. (2.11) that

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.11 \pm 0.02,$$

$$\frac{\tau(B_s)}{\tau(B_d)} \approx 1,$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.99 - \left(\frac{f_B}{185\ \text{MeV}}\right)^2 (0.007 + 0.020\tilde{B})r, \qquad (3.36)$$

to the order of $1/m_b^3$. Note that we have neglected the corrections of SU(3) symmetry breaking to the nonspectator effects in $\tau(B_s)/\tau(B_d)$. We see that the prediction for $\tau(B^-)/\tau(B_d)$ is in agreement with the current world average: $\tau(B^-)/\tau(B_d) = 1.07 \pm 0.03$ [7], whereas the heavy-quark-expansion-based result for $\tau(B_s)/\tau(B_d)$ deviates somewhat from the central value of the world average:[5] 0.94 ± 0.04. Thus it is urgent to carry out more precise measurements of the B_s lifetime. Using the existing sum rule estimate for the parameter r [10] together with $\tilde{B} = 1$ gives $\tau(\Lambda_b)/\tau(B_d) \geqslant 0.98$. Therefore, the $1/m_b^3$ nonspectator corrections are not responsible for the observed lifetime difference between the Λ_b and B_d.

IV. DISCUSSIONS AND CONCLUSIONS

The prediction of B meson lifetime ratios depends on the nonspectator effects of order $16\pi^2/m_b^3$ in the heavy quark expansion. These effects can be parametrized in terms of the hadronic parameters B_1, B_2, ε_1, and ε_2, where B_1 and B_2 characterize the matrix elements of color singlet-singlet four-quark operators and ε_1 and ε_2 the matrix elements of color octet-octet operators.

As emphasized in [12], one should not be contented with the agreement between theory and experiment for the lifetime ratio $\tau(B^-)/\tau(B_d)$. In order to test the OPE approach for inclusive nonleptonic decay, it is even more important to calculate the absolute decay widths of the B mesons and compare them with the data. From Eqs. (2.6) and (3.35) and considering the contributions of the nonspectator effects, we obtain

$$\Gamma_{\text{tot}}(B_d) = (3.61^{+1.04}_{-0.84}) \times 10^{-13}\ \text{GeV},$$

$$\Gamma_{\text{tot}}(B^-) = (3.34^{+1.04}_{-0.84}) \times 10^{-13}\ \text{GeV}, \qquad (4.1)$$

noting that the next-to-leading QCD radiative correction to the inclusive decay width has been included. The absolute decay widths strongly depend on the value of the b quark mass.

The problem with the absolute decay width $\Gamma(B)$ is intimately related to the B meson semileptonic branching ratio $\mathcal{B}_{\text{SL}}$. Unlike the semileptonic decays, the heavy quark expansion in inclusive nonleptonic decay is *a priori* not justified due to the absence of an analytic continuation into the complex plane and hence local duality has to be invoked in order to apply the OPE directly in the physical region. If the shorter lifetime of the Λ_b relative to that of the B_d meson is confirmed in the future and/or if the lifetime ratio $\tau(B_s)/\tau(B_d)$ is observed to be different from unity, then it is very likely that local quark-hadron duality is violated in nonleptonic decays. It should be stressed that local duality is *exact* in the heavy quark limit, but its systematic $1/m_Q$ expansion is still lacking. Empirically, it has been suggested in

[5]For example, the neutral B meson lifetimes are measured at CDF to be [38]: $\tau(B_d) = 1.58 \pm 0.09 \pm 0.02$ ps and $\tau(B_s) = 1.34^{+0.23}_{-0.19} \pm 0.05$ ps.

[9] that the presence of linear $1/m_b$ correction, described by the ansatz that the b quark mass m_b is replaced by the decaying bottom hadron mass m_{H_b} in the m_b^5 factor in front of all nonleptonic widths, will account for the observed lifetime difference between the Λ_b and B_d. To be specific, the ansatz $\Gamma_{\mathrm{NL}} \to \Gamma_{\mathrm{NL}}(m_{H_b}/m_b)^5$ will lead to the result [12]:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.76, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.94. \tag{4.2}$$

This simple prescription not only solves the lifetime ratio problem but also provides the correct absolute decay widths for the Λ_b and the B mesons. The predicted lifetime hierarchy

$$\tau(\Lambda_b) > \tau(\Xi_b^-) > \tau(\Xi_b^0) > \tau(\Omega_b) \tag{4.3}$$

for bottom baryons is in sharp contrast to the OPE-based lifetime pattern [12]:

$$\tau(\Omega_b) \simeq \tau(\Xi_b^-) > \tau(\Lambda_b) \simeq \tau(\Xi_b^0). \tag{4.4}$$

Of course, whether this empirical ansatz truly works or whether it can be justified in a more fundamental way (see, for example, [39]) remains to be investigated. Nevertheless, it is worth emphasizing that, although a linear $1/m_Q$ correction to the inclusive nonleptonic decay rate is possible [40,41], the violation of local quark-hadron duality does not necessarily imply the presence of $1/m_Q$ terms in inclusive widths and hence the aforementioned ansatz.

To conclude, we have derived in heavy quark effective theory the renormalization-group improved sum rules for the hadronic parameters B_1, B_2, ε_1, and ε_2 appearing in the matrix element of four-quark operators. The results are $B_1(m_b) = 0.96 \pm 0.04$, $B_2(m_b) = 0.95 \pm 0.02$, $\varepsilon_1(m_b) = -0.14 \pm 0.01$, and $\varepsilon_2(m_b) = -0.08 \pm 0.01$ to the zeroth order in $1/m_b$. The resultant B-meson lifetime ratios are $\tau(B^-)/\tau(B_d) = 1.11 \pm 0.02$ and $\tau(B_s)/\tau(B_d) \approx 1$.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Council of R.O.C. under Grant No. NSC87-2112-M-001-048.

[1] I. I. Bigi, N. G. Uraltsev, and A. I. Vainshtein, Phys. Lett. B **293**, 430 (1992); **297**, 477(E) (1993); I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993).

[2] B. Blok and M. Shifman, Nucl. Phys. **B399**, 441 (1993); **B399**, 459 (1993).

[3] A. Manohar and M. B. Wise, Phys. Rev. D **49**, 1310 (1994); B. Blok, L. Koyrakh, M. Shifman, and A. Vainshtein, *ibid.* **49**, 3356 (1994).

[4] J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B **247**, 399 (1990).

[5] A. F. Falk, M. B. Wise, and I. Dunietz, Phys. Rev. D **51**, 1183 (1995).

[6] G. Bellini, I. I. Bigi, and P. J. Dornan, Phys. Rep. **289**, 1 (1997); I. I. Bigi, Report No. UND-HEP-96-BIG06, hep-ph/9612293.

[7] For updated world averages of B hadron lifetimes, see J. Alcaraz *et al.* (LEP B Lifetime Group), http://wwwcn.cern.ch/~claires/lepblife.html.

[8] N. G. Uraltsev, Phys. Lett. B **376**, 303 (1996); J. L. Rosner, *ibid.* **379**, 267 (1996).

[9] G. Altarelli, G. Martinelli, S. Petrarca, and F. Rapuano, Phys. Lett. B **382**, 409 (1996); G. Martinelli, Nucl. Instrum. Methods Phys. Res. A **384**, 241 (1996).

[10] P. Colangelo and F. De Fazio, Phys. Lett. B **387**, 371 (1996).

[11] M. Neubert and C. T. Sachrajda, Nucl. Phys. **B483**, 339 (1997); M. Neubert, Report No. CERN-TH/97-148, hep-ph/9707217.

[12] H. Y. Cheng, Phys. Rev. D **56**, 2783 (1997).

[13] H. J. Lipkin and P. J. O'Donnell, Phys. Lett. B **409**, 412 (1997).

[14] T. Ito, M. Matsuda, and Y. Matsui, Prog. Theor. Phys. **99**, 271 (1998).

[15] M. S. Baek, J. Lee, C. Liu, and H. S. Song, Phys. Rev. D **57**, 4091 (1998).

[16] A. Sinkovics, R. Akhoury, and V. I. Zakharov, Phys. Rev. D **58**, 114025 (1998).

[17] For a review, see M. Neubert, Phys. Rep. **245**, 259 (1994).

[18] A. J. Buras, M. Jamin, M. E. Lautenbacher, and P. H. Weisz, Nucl. Phys. **B370**, 69 (1992); A. J. Buras, M. Jamin, and M. E. Lautenbacher, *ibid.* **B408**, 209 (1993).

[19] A. Ali and C. Greub, Phys. Rev. D **57**, 2996 (1998).

[20] H. Y. Cheng and B. Tseng, Phys. Rev. D **58**, 094006 (1998).

[21] Particle Data Group, C. Cao *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[22] T. Huang and C.-W. Luo, in *Particle Theory and Phenomenology*, XVII International Karimierz Meeting on Particle Physics, Iowa State University, 1995, edited by K.E. Lassila *et al.* (World Scientific, Singapore, 1996), p. 116.

[23] E. Bagan, P. Ball, V. M. Braun, and P. Gosdzinsky, Phys. Lett. B **342**, 362 (1995); **374**, 363(E) (1996); E. Bagan, P. Ball, B. Fiol, and P. Gosdzinsky, *ibid.* **351**, 546 (1995); M. Lu, M. Luke, M. J. Savage, and B. H. Smith, Phys. Rev. D **55**, 2827 (1997).

[24] See, for example, B. Blok and M. Shifman, in *Proceedings of the Third Workshop on the Physics at a Tau-Charm Factory*, Marbella, Spain, 1993, edited by J. Kirkby and R. Kirkby (Editions Frontières, Gif-sur-Yvette, France, 1994).

[25] M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **41**, 120 (1985); Sov. Phys. JETP **64**, 698 (1986).

[26] M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **45**, 292 (1987).

[27] H. D. Politzer and M. B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988).

[28] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[29] M. Neubert, Phys. Rev. D **45**, 2451 (1992); **46**, 1076 (1992).

[30] P. Ball and V. M. Braun, Phys. Rev. D **49**, 2472 (1994).
[31] E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, Phys. Lett. B **278**, 457 (1992).
[32] K.-C. Yang, Phys. Rev. D **57**, 2983 (1998).
[33] V. Chernyak, Nucl. Phys. **B457**, 96 (1995).
[34] K.-C. Yang, W-Y. P. Hwang, E. M. Henley, and L. S. Kisslinger, Phys. Rev. D **47**, 3001 (1993); W-Y. P. Hwang and K.-C. Yang, *ibid.* **49**, 460 (1994).
[35] C. Bernard *et al.*, hep-ph/9709328.
[36] P. Ball, Nucl. Phys. **B421**, 593 (1994).
[37] B. Blok and M. Lublinsky, Phys. Rev. D **57**, 2676 (1998).
[38] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **57**, 5382 (1998).
[39] C. H. Jin, Phys. Rev. D **56**, 2928 (1997).
[40] P. Colangelo, C. A. Dominguez, and G. Nardulli, Phys. Lett. B **409**, 417 (1997).
[41] B. Grinstein and R. F. Lebed, Phys. Rev. D **57**, 1366 (1998).