# *CP*-violating asymmetries in charmless nonleptonic decays $B\to PP, PV, VV$ in the factorization approach

A. Ali
*Deutsches Elektronen-Synchrotron DESY, 22607 Hamburg, Germany*

G. Kramer and Cai-Dian Lü
*II. Institut für Theoretische Physik, Universität Hamburg, 22761 Hamburg, Germany*

(Received 22 May 1998; published 17 November 1998)

We present estimates of the direct (in decay amplitudes) and indirect (mixing-induced) *CP*-violating asymmetries in the nonleptonic charmless two-body decay rates for $B\to PP$, $B\to PV$, and $B\to VV$ decays and their charged conjugates, where $P(V)$ is a light pseudoscalar (vector) meson. These estimates are based on a generalized factorization approach making use of next-to-leading order perturbative QCD contributions which generate the required strong phases. No soft final state interactions are included. We study the dependence of the asymmetries on a number of input parameters and show that there are at least two (possibly three) classes of decays in which the asymmetries are parametrically stable in this approach. The decay modes of particular interest are $\overset{(-)}{B^0}\to\pi^+\pi^-$, $\overset{(-)}{B^0}\to K_S^0\pi^0$, $\overset{(-)}{B^0}\to K_S^0\eta'$, $\overset{(-)}{B^0}\to K_S^0\eta$, and $\overset{(-)}{B^0}\to\rho^+\rho^-$. Likewise, the *CP*-violating asymmetry in the decays $\overset{(-)}{B^0}\to K_S^0 h^0$ with $h^0=\pi^0$, $K_S^0$, $\eta$, $\eta'$ is found to be parametrically stable and large. Measurements of these asymmetries will lead to a determination of the phases $\sin 2\alpha$ and $\sin 2\beta$ and we work out the relationships in these modes in the present theoretical framework. We also show the extent of the so-called ''penguin pollution'' in the rate asymmetry $A_{CP}(\pi^+\pi^-)$ and of the ''tree shadow'' in the asymmetry $A_{CP}(K_S^0\eta')$ which will effect the determination of $\sin 2\alpha$ and $\sin 2\beta$ from the respective measurements. *CP*-violating asymmetries in $B^\pm$ decays depend on a model parameter in the penguin amplitudes and theoretical predictions require further experimental or theoretical input. Of these, *CP*-violating asymmetries in $B^\pm\to\pi^\pm\eta'$, $B^\pm\to K^{*\pm}\eta$, $B^\pm\to K^{*\pm}\eta'$, and $B^\pm\to K^{*\pm}\rho^0$ are potentially interesting and are studied here.
[S0556-2821(98)01821-9]

PACS number(s): 13.25.Hw, 12.15.Hh, 12.38.Bx

## I. INTRODUCTION

Recent measurements by the CLEO Collaboration [1,2] of a number of decays $B\to h_1h_2$, where $h_1$ and $h_2$ are light hadrons such as $h_1h_2=\pi\pi$, $\pi K$, $\eta' K$, $\omega K$, have lead to renewed theoretical interest in understanding hadronic $B$ decays [3].

In a recent work [4] we have calculated the branching fractions of two-body nonleptonic decays $B\to PP$, $PV$, $VV$, where $P$ and $V$ are the lowest lying light pseudoscalar and vector mesons, respectively. The theoretical framework used was based on the next-to-leading logarithmic improved effective Hamiltonian and a factorization ansatz for the hadronic matrix elements of the four-quark operators [5]. We worked out the parametric dependence of the decay rates using currently available information on the weak mixing matrix elements, form factors, decay constants, and quark masses. In total we considered seventy six decay channels with a large fraction of them having branching ratios of order $10^{-6}$ or higher which hopefully will be measured in the next round of experiments on $B$ decays. The recently measured decay modes $B^0\to K^+\pi^-$, $B^+\to K^+\eta'$, $B^0\to K^0\eta'$, $B^+\to\pi^+K^0$, and $B^+\to\omega K^+$ are shown to be largely in agreement with the estimates based on factorization [4–6]. This encourages us to further pursue this framework and calculate quantities of experimental interest in two-body nonleptonic $B$ decays.

Besides branching fractions, other observables which will help to test the factorization approach and give information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix [7] are *CP*-violating rate asymmetries in partial decay rates. In the past a large variety of ways has been proposed to observe *CP* violation in $B$ decays [8]. One method is to study *CP*-violating asymmetries in the time-dependence of the neutral $B$ meson decay rates in specific modes, which involve an interference between two weak amplitudes. Asymmetries in charged $B$ decays require an interference between two amplitudes involving both a CKM phase and a final state strong interaction phase-difference. Such asymmetries occur also in decays of neutral $B$ mesons in which $B^0$ and $\overline{B^0}$ do not decay into common final states or where these states are not *CP*-eigenstates. In these decays the weak phase difference arises from the superposition of various penguin contributions and the usual tree diagrams in case they are present. The strong-phase differences arise through the absorptive parts of perturbative penguin diagrams (hard final state interaction) [9] and nonperturbatively (soft final state interaction).

When a $B^0$ and $\overline{B^0}$ decay to a common final state $f$, $B^0-\overline{B^0}$ mixing plays a crucial role in determining the *CP*-violating asymmetries, requiring time-dependent measurements. For the final states which are both *CP*-eigenstates and involve only one weak phase in the decays, the *CP*-violating asymmetry is independent of the hadronic matrix elements. This occurs in the well studied $\overset{(-)}{B^0}\to J/\psi K_S$ decays making it possible to extract the value of $\sin 2\beta$ with no hadronic uncertainties. For neutral $B$ decays into two light mesons such as direct translation of the $CP$-violating asymmetries in terms of $CP$-violating phases $\alpha$, $\beta$ and $\gamma$ will not be possible, in general. Hence, the predicted asymmetries are subject to hadronic uncertainties. In principle, these uncertainties can be removed by resorting to a set of time-dependent and time-independent measurements as suggested in the literature [10–13]. In practice, this program requires a number of difficult measurements. We pursue here the other alternative, namely we estimate these uncertainties in a specific model, which can be tested experimentally in a variety of decay modes.

$CP$-violating asymmetries are expected in a large number of $B$ decays; in particular the partial rate asymmetries in all the $B\to h_1 h_2$ decay modes and their charge conjugates studied in [4] are potentially interesting for studying $CP$ violation. We recall that $CP$-violating asymmetries in $B\to h_1 h_2$ decays have been studied earlier in the factorization framework [8,14–16]. With the measurement of some of the $B\to h_1 h_2$ decays [1,2], some selected modes have received renewed interest in this approach [17–19]. These papers, however, make specific assumptions about $\xi\equiv 1/N_c$ (here $N_c$ is the number of effective colors) and certain other input parameters; in particular, the earlier ones used CKM-parameter values which are now strongly disfavored by recent unitarity fits [20,21] and/or they do not include the anomaly contributions (or not quite correctly) and the latter ones make specific assumptions about $\xi$, which may or may not be consistent with data on $B\to h_1 h_2$ decays. We think it is worthwhile to study again these $CP$-violating asymmetries by including theoretical improvements [5,6] and determine their $N_c$-and other parametric dependences.

Following our previous work [4] we study this on the basis of the factorization approach. We consider the same seventy six decay channels as in [4] and calculate the $CP$-violating asymmetries for charged and neutral $B$ decays with the classification $I$ to $V$ as in [4] to distinguish those channels which can be predicted with some certainty in the factorization approach. These are the class-I and class-IV (and possibly some class-III) decays, whose decay amplitudes are $N_c$-stable and which do not involve delicate cancellations among components of the amplitudes. In our study here, we invoke two models to estimate the form factor dependence of the asymmetries, study their dependence on the effective coefficients of the QCD and electroweak penguin operators in term of $N_c$, the dependence on $k^2$, the virtuality of the gluon, photon or $Z$ in the penguin amplitudes decaying into the quark-antiquark pair $q\overline{q'}$ in $b\to qq'\overline{q'}$ and, of course, the CKM parameters. The last of these is the principal interest in measuring the $CP$-violating asymmetries. Our goal, therefore, is to identify, by explicit calculations, those decay modes whose $CP$-violating asymmetries are relatively insensitive to the variations of the rest of the parameters.

In this pursuit, the sensitivity of the asymmetries on $k^2$ is a stumbling block. As the branching ratios are relatively insensitive to the parameter $k^2$, this dependence can be removed only by the measurement of at least one of the $CP$-violating asymmetries sensitive to it (examples of which are abundant), enabling us to predict quite a few others. A mean value of $k^2$ can also be estimated in specific wave function models [14]—an alternative we do not consider here. However, quite interestingly, we show that a number of class-I and class-IV (hence $N_c$-stable) decays involving $B^0/\overline{B^0}$ mesons have $CP$-violating asymmetries which are also stable against variation in $k^2$. Hence, in this limited number of decays, the asymmetries can be reliably calculated within the factorization framework. We find that the $CP$-violating asymmetries in the following decays are particularly interesting and relatively stable: $\overset{(-)}{B^0}\to\pi^+\pi^-$, $\overset{(-)}{B^0}\to K_S^0\pi^0$, $\overset{(-)}{B^0}\to K_S^0\eta$, $\overset{(-)}{B^0}\to K_S^0\eta'$ and $\overset{(-)}{B^0}\to\rho^+\rho^-$. Likewise, the $CP$-violating asymmetry in the decays $\overset{(-)}{B^0}\to K_S^0 h^0$ with $h^0=\pi^0$, $K_S^0$, $\eta$, $\eta'$ is large as the individual decay modes have the same intrinsic $CP$ parity. The $k^2$ dependences in the individual asymmetries in this sum, which are small to start with but not negligible, compensate each other resulting in a $CP$-violating asymmetry which is practically independent of $k^2$. Ideally, i.e., when only one decay amplitude dominates, the asymmetries in the mentioned decays measure one of the $CP$-violating phases $\alpha$ and $\beta$. In actual decays, many amplitudes are present and we estimate their contribution in the asymmetries. To quantify this more pointedly, we work out the dependence of the time-integrated partial rate asymmetry $A_{CP}(\pi^+\pi^-)$ in the decays $\overset{(-)}{B^0}\to\pi^+\pi^-$ on $\sin 2\alpha$ and show the extent of the so-called ''penguin pollution.'' Likewise, we work out the dependence of $A_{CP}(K_S^0\eta')$, $A_{CP}(K_S^0\pi^0)$, $A_{CP}(K_S^0\eta)$ and $A_{CP}(K_S^0 h^0)$ on $\sin 2\beta$. We also study the effect of the tree contribution, which we call a ''tree shadow'' of the penguin-dominated amplitude, on $A_{CP}(K_S^0\eta')$. The $CP$-violating asymmetries in $B^\pm$ decays are in general $k^2$-dependent. Supposing that this can be eventually fixed, as discussed above, the interesting asymmetries in $B^\pm\to h_1 h_2$ decays in our approach are $B^\pm\to\pi^\pm\eta'$, $B^\pm\to K^{*\pm}\eta$, $B^\pm\to K^{*\pm}\eta'$ and $B^\pm\to K^{*\pm}\rho^0$. We study the asymmetries in the mentioned decays and also in $\overset{(-)}{B^0}\to\rho^\pm\pi^\pm$ in detail in this paper.

The effects of soft final state interactions (SFI) may influence some (or all) of the estimates presented here for the asymmetries. By the same token, decay rates are also susceptible to such nonperturbative effects [22–27], which are, however, notoriously difficult to quantify. We think that the role of SFI in $B\to h_1 h_2$ decays will be clarified already as the measurements of the branching ratios become more precise and some more decays are measured. Based on the ''color transparency'' argument [28], we subscribe to the point of view that the effects of SFI are subdominant in decays whose amplitudes are not (color)-suppressed. However, it should be noted that the effects of the so-called nonperturbative ''charm penguins'' [29] are included here in the factorization approach in terms of the leading power $(1/m_c^2)$ corrections which contribute only to the decays $B\to h_1 h_2$ involving an $\eta$ or $\eta'$ [6], as explained in the next section.

This paper is organized as follows: In Sec. II we review the salient features of the generalized factorization framework used in estimating the $B\to h_1 h_2$ decay rates in [4]. In Sec. III we give the formulas from which the various $CP$-violating asymmetries for the charged and neutral $B$ decays are calculated. Section IV contains the numerical results for the $CP$-violating coefficients, required for time-dependent measurements of the $CP$-violating asymmetries in $B^0$ and $\overline{B^0}$ decays, and time-integrated $CP$-violating asymmetries. The numerical results are tabulated for three specific values of the effective number of colors $N_c=2,3,\infty$, varying $k^2$ in the range $k^2=m_b^2/2\pm 2\ \mathrm{GeV}^2$, and two sets of the CKM parameters. We show the CKM-parametric dependence of the $CP$-violating asymmetries for some representative decays belonging to the class-I, class-III and class-IV decays, which have stable asymmetries and are estimated to be measurably large in forthcoming experiments at $B$ factories and hadron machines. Finally, in this section we study some decay modes which have measurable but $k^2$-dependent $CP$-violating asymmetries, mostly involving $B^\pm$ decays but also a couple of $B^0/\overline{B^0}$ decays. Section V contains a summary of our results and conclusions.

## II. GENERALIZED FACTORIZATION APPROACH AND CLASSIFICATION OF $B\to h_1 h_2$ DECAYS

The calculation of the $CP$-violating asymmetries reported here is based on our work described in [4]. There, we started from the short-distance effective weak Hamiltonian $H_{\mathrm{eff}}$ for $b\to s$ and $b\to d$ transitions. We write below $H_{\mathrm{eff}}$ for the $\Delta B=1$ transitions with five active quark flavors by integrating out the top quark and the $W^\pm$ bosons:

$$\mathcal{H}_{\mathrm{eff}}=\frac{G_F}{\sqrt{2}}\left[V_{ub}V_{uq}^*(C_1O_1^u+C_2O_2^u)+V_{cb}V_{cq}^*(C_1O_1^c+C_2O_2^c)-V_{tb}V_{tq}^*\left(\sum_{i=3}^{10}C_iO_i+C_gO_g\right)\right], \tag{1}$$

where $q=d,s$; $C_i$ are the Wilson coefficients evaluated at the renormalization scale $\mu$ and $V_{ij}$ are the CKM matrix elements for which we shall use the Wolfenstein parameterization [30]. The operators $O_i^u$ and $O_i^c$ with $i=1,2$ are the current-current four-quark operators inducing the $b\to uq\overline{q}$ and $b\to cq\overline{q}$ transitions, respectively. The rest of the operators are the QCD penguin operators $(O_3,\ldots,O_6)$, electroweak penguin operators $(O_7,\ldots,O_{10})$, and $O_g$ represents the chromomagnetic penguin operator. The operator basis for $H_{\mathrm{eff}}$ is given in [4] together with the coefficients $C_1,\ldots,C_6$, evaluated in NLL precision, and $C_7,\ldots,C_{10}$, and $C_g$, evaluated in LL precision. Effects of weak annihilation and $W$-exchange diagrams have been neglected.

Working in NLL precision, the quark level matrix elements of $H_{\mathrm{eff}}$ are treated at the one-loop level. They can be rewritten in terms of the tree-level matrix elements of the effective operators with new coefficients $C_1^{\mathrm{eff}},\ldots,C_{10}^{\mathrm{eff}}$ (for details see [4] and the references quoted therein). The effective coefficients $C_1^{\mathrm{eff}},C_2^{\mathrm{eff}},C_8^{\mathrm{eff}}=C_8$, and $C_{10}^{\mathrm{eff}}=C_{10}$ have no absorptive parts to the order we are working. The effective coefficient $C_3^{\mathrm{eff}}$, $C_4^{\mathrm{eff}}$, $C_5^{\mathrm{eff}}$, $C_6^{\mathrm{eff}}$, $C_7^{\mathrm{eff}}$ and $C_9^{\mathrm{eff}}$ contain contributions of penguin diagrams with insertions of tree operator $O_{1,2}$, denoted by $C_t$ and $C_e$ in [4] and with insertions of the QCD penguin operators $O_3$, $O_4$ and $O_6$ (denoted by $C_p$ in [4]). These penguinlike matrix elements have absorptive parts which generate the required strong phases in the quark-level matrix elements. The contributions $C_t$ and $C_e$ depend on the CKM matrix elements. All three functions $C_t$, $C_p$ and $C_e$ depend on quark masses, the scale $\mu$, and $k^2$, and are given explicitly in Eqs. (10), (11) and (14), respectively, of Ref. [4].

Having defined $H_{\mathrm{eff}}$ in terms of the four-quark operators $O_i$ and their effective coefficients $C_i^{\mathrm{eff}}$ the calculation of the hadronic matrix elements of the type $\langle h_1h_2|O_i|B\rangle$ proceeds with the generalized factorization assumption [31]. The result of this calculation for the various $B\to PP$, $PV$ and $VV$ decays are written down in detail in [4]. The hadronic matrix elements depend on the CKM matrix elements, which contain the weak phases, the form factors and decay constants of current matrix elements, various quark masses and other parameters. The quantities $a_i$, given in terms of the effective short-distance coefficient $C_i^{\mathrm{eff}}$,

$$a_i=C_i^{\mathrm{eff}}+\frac{1}{N_c}C_{i+1}^{\mathrm{eff}}\quad(i=\mathrm{odd});$$

$$a_i=C_i^{\mathrm{eff}}+\frac{1}{N_c}C_{i-1}^{\mathrm{eff}}\quad(i=\mathrm{even}), \tag{2}$$

where $i$ runs from $i=1,\ldots,10$, are of central phenomenological importance. The terms in Eq. (2) proportional to $\xi=1/N_c$ originate from fierzing the operators $O_i$ to produce quark currents to match the quark content of the hadrons in the initial and final state after adopting the factorization assumption. This well-known procedure results in general in matrix elements with the right flavor quantum number but involves both color singlet-singlet and color octet-octet operators. In the naive factorization approximation, one discards the color octet-octet operators. This amounts to having $N_c=3$ in Eq. (2). To compensate for these neglected octet-octet and other nonfactorizing contribution one treats $\xi\equiv 1/N_c$ in Eq. (2) as a phenomenological parameter. In theory, $\xi$ can be obtained only by fully calculating the octet-octet and other nonfactorizing contributions and can, in principle, be different for each of the ten $a_i$.

Starting from the numerical values of the ten perturbative short distance coefficients $C_i^{\mathrm{eff}}(i=1,\ldots,10)$ we investigated in [4] the $N_c$ dependence of the ten effective coefficients $a_i$ for the four types of current-current and penguin induced decays, namely $b\to s(\overline{b}\to\overline{s})$ and $b\to d(\overline{b}\to\overline{d})$. It was found, that $a_1$, $a_4$, $a_6$, $a_8$ and $a_9$ are rather stable with respect to variations of $\xi$ in the usually adopted interval $\xi\in[0,1/2]$ (or $2<N_c<\infty$) for all four types of transitions, whereas $a_2$, $a_3$, $a_5$, $a_7$ and $a_{10}$ depend very much on $\xi$.

Based on this result we introduced a classification of factorized amplitudes which is an extension of the classification for tree decays in [32] relevant for $B$ decays involving charmed hadrons. These classes I, II, III, IV, and V are fully described in [4] and will be used also in this work. The classes I, II, and III in the decays $B \to h_1 h_2$ are defined as in previous work [32]. They involve dominantly (or only) current-current transitions. Class IV and V involve pure penguin or penguin-dominated decays. The classification is such, that decays in classes I and IV are stable against variations of $N_c$, whereas decays in classes II and V depend strongly on $\xi = 1/N_c$ and decays in class III have an intermediate status, sometimes depending more, sometimes less on $\xi$. We concluded in [4] that decay rates in the classes I and IV decays can be predicted in the factorization approximation. The decays in class II and V have sometimes rather small weak transition matrix elements, depending on the values of the effective $N_c$ and CKM matrix elements. This introduces delicate cancellations which makes their amplitudes rather unstable as a function of $N_c$. Predicting the decay rates in these classes involves a certain amount of theoretical fine-tuning, and hence we are less sure about their estimates in the factorization approach. Depending on the value of $\xi$, it is probable that other contributions not taken into account in the factorization approach used in [4], like annihilation, $W$ exchange or soft final state interactions, are important. We expect that the matrix elements of the decays in class-I and class-IV (and most class-III), being dominantly of $O(1)$ as far as their $N_c$-dependence is concerned will be described, in the first approximation, by a universal value of the parameter $\xi$. We are less sure that this will be the case for class-II and class-V decays. As we show here, this $\xi$-sensitivity of the decay rates reflects itself also in estimates of the *CP*-violating rate asymmetries.

There is also an uncertainty due to the non-perturbative penguin contributions [29], as we do not know how to include their effects in the amplitudes $\langle h_1 h_2 | H_{\rm eff} | B \rangle$ from first principles. However, these effects can be calculated as an expansion in $1/m_c^2$ in the factorization approach. The dominant diagram contributing to the power corrections is the process $b \to s[c\overline{c} \to g(k_1) g(k_2)]$, which was calculated in the full theory (standard model) in [33]. In the operator product language which we are using, this contribution can be expressed as a new induced effective Hamiltonian [6]:

$$H_{\rm eff}^{gg} = -\frac{\alpha_s}{2\pi} a_2 \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \Delta i_5 \left( \frac{q^2}{m_c^2} \right) \frac{1}{k_1 \cdot k_2} O^{gg}, \quad (3)$$

where the operator $O^{gg}$ is defined as

$$O^{gg} \equiv G_a^{\alpha\beta} (D_\beta \widetilde{G}_{\alpha\mu})_a \overline{s} \gamma^\mu (1 - \gamma_5) b, \quad (4)$$

with $\widetilde{G}_{\mu\nu,a} = 1/2 \epsilon_{\alpha\beta\mu\nu} G_a^{\alpha\beta}$, and $G_a^{\alpha\beta}$ being the QCD field strength tensor. This formula holds for on-shell gluons $q^2 = (k_1 + k_2)^2 = 2 k_1 \cdot k_2$, and the sum over the color indices is understood. The function $\Delta i_5(z)$ is defined as [6]

$$\Delta i_5(z) = -1 + \frac{1}{z} \left[ \pi - 2 \arctan\left( \frac{4}{z} - 1 \right)^{1/2} \right]^2 \quad \text{for } 0 < z < 4. \quad (5)$$

The $H_{\rm eff}^{gg}$ gives a nonlocal contribution but one can expand the function $\Delta i_5(z)$ in $z$ for $z < 1$ and the leading term in this expansion can be represented as a higher dimensional local operator. In fact, it is just the chromomagnetic analogue of the operator considered by Voloshin [34] to calculate the power $(1/m_c^2)$ corrections in the radiative decay $B \to X_s + \gamma$. Now comes the observation made in [6] that in the assumption of factorization, only the states which have nonzero matrix elements $\langle M | \alpha_s G_a^{\alpha\beta} (D_\beta \widetilde{G}_{\alpha\mu})_a | 0 \rangle$ contribute to the $1/m_c^2$ corrections in the decay rates for $B \to M h$. For $M = \eta, \eta'$, this matrix element is determined by the QCD anomaly, and $q^2$ also gets fixed with $q^2 = m_{\eta^{(\prime)}}^2$ which justifies the expansion. For the decays $B \to \eta^{(\prime)} K^{(*)}$, the $1/m_c^2$ effects were calculated in [6] in the decay rates. For the two-body $B \to h_1 h_2$ decays, these are the only $1/m_c^2$ contributions in the factorization approach. They are included here in the estimates of the rates and the asymmetries. Note that as the function $\Delta i_5(m_{\eta^{(\prime)}}^2 / m_c^2)$ has no absorptive part, there is no phase generated by the anomaly contribution in $B \to \eta^{(\prime)} K^{(*)}$ decays.

Concerning the actual estimates of the $B \to h_1 h_2$ matrix elements in the factorization approximation, we note that they are calculated as in [4] using two different theoretical approaches to calculate the form factors. First, we use the quark model due to Bauer, Stech and Wirbel [32]. The second approach is based on lattice QCD and light-cone QCD sum rules. The specific values of the form factors and decay constants used by us and the references to the literature are given in [4]. The implementation of the $\eta - \eta'$ mixing follows the prescription of [5,6].

Of particular importance for calculating the *CP*-violating asymmetries is the choice of the parameter $k^2$, which appears in the quantities $C_t$, $C_p$ and $C_e$ in the effective coefficients $C_i^{\rm eff}$. Due to the factorization assumption any information on $k^2$ is lost when calculating two-body decays, except for the anomaly contribution as discussed earlier. In a specific model and from simple two-body kinematics the average $k^2$ has been estimated to lie in the range $m_b^2/4 < k^2 < m_b^2/2$ [14]. In [4] it was found that the branching ratios (averaged over $B$ and $\overline{B}$ decays) are not sensitively dependent on $k^2$ if varied in the vicinity of $k^2 = m_b^2/2$. Based on earlier work [15], we do not expect the same result to hold for the asymmetries. Therefore, we calculated the *CP*-violating asymmetries by varying $k^2$ in the range $k^2 = m_b^2/2 \pm 2\,\text{GeV}^2$, which should cover the expected range of $k^2$ in phenomenological models. Quite interestingly, we find that a number of decay modes in the class-I and class-IV decays have asymmetries which are insensitive to the variation of $k^2$. These then provide suitable avenues to test the assumption that strong interaction phases in these decays are dominantly generated perturbatively.

## III. *CP*-VIOLATING ASYMMETRIES IN $B \to h_1 h_2$ DECAYS: FORMALISM

For charged $B^\pm$ decays the *CP*-violating rate-asymmetries in partial decay rates are defined as follows:

$$A_{CP} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}. \quad (6)$$

As these decays are all self-tagging, measurement of these *CP*-violating asymmetries is essentially a counting experiment in well defined final states. Their rate asymmetries require both weak and strong phase differences in interfering amplitudes. The weak phase difference arises from the superposition of amplitudes from various tree (current-current) and penguin diagrams. The strong phase, which are needed to obtain nonzero values for $A_{CP}$ in Eq. (6), are generated by final state interactions. For both $b\to s$ and $b\to d$ transitions, the strong phases are generated in our model perturbatively by taking into account the full NLO corrections, following earlier suggestions along these lines [9].

### A. *CP*-violating asymmetries involving $b\to s$ transitions

For the $b\to s$, and the charge conjugated $\overline{b}\to\overline{s}$, transitions, the respective decay amplitudes $\mathcal{M}$ and $\overline{\mathcal{M}}$, including the weak and strong phases, can be generically written as

$$\mathcal{M}=T\xi_u-P_t\xi_t e^{i\delta_t}-P_c\xi_c e^{i\delta_c}-P_u\xi_u e^{i\delta_u},$$

$$\overline{\mathcal{M}}=T\xi_u^*-P_t\xi_t^* e^{i\delta_t}-P_c\xi_c^* e^{i\delta_c}-P_u\xi_u^* e^{i\delta_u}, \tag{7}$$

where $\xi_i=V_{ib}V_{is}^*$. Here we denote by $T$ the contributions from the current-current operators proportional to the effective coefficients $a_1$ and/or $a_2$; $P_t$, $P_c$ and $P_u$ denote the contributions from penguin operators proportional to the product of the CKM matrix elements $\xi_t$, $\xi_c$ and $\xi_u$, respectively. The corresponding strong phases are denoted by $\delta_t$, $\delta_c$ and $\delta_u$, respectively. Working in the standard model, we can use the unitarity relation $\xi_c=-\xi_u-\xi_t$ to simplify the above equation (7),

$$\mathcal{M}=T\xi_u-P_{tc}\xi_t e^{i\delta_{tc}}-P_{uc}\xi_u e^{i\delta_{uc}},$$

$$\overline{\mathcal{M}}=T\xi_u^*-P_{tc}\xi_t^* e^{i\delta_{tc}}-P_{uc}\xi_u^* e^{i\delta_{uc}}, \tag{8}$$

where we define

$$P_{tc}e^{i\delta_{tc}}=P_t e^{i\delta_t}-P_c e^{i\delta_c},$$

$$P_{uc}e^{i\delta_{uc}}=P_u e^{i\delta_u}-P_c e^{i\delta_c}. \tag{9}$$

Thus, the direct *CP*-violating asymmetry is

$$A_{CP}\equiv a_{\epsilon'}=\frac{A^-}{A^+}, \tag{10}$$

where

$$\begin{aligned}A^-&=\frac{1}{2}\,(|\overline{\mathcal{M}}|^2-|\mathcal{M}|^2)\\&=2TP_{tc}|\xi_u^*\xi_t|\sin\,\gamma\,\sin\,\delta_{tc}\\&\quad+2P_{tc}P_{uc}|\xi_u^*\xi_t|\sin\,\gamma\,\sin(\delta_{uc}-\delta_{tc}),\end{aligned} \tag{11}$$

$$\begin{aligned}A^+&=\frac{1}{2}\,(|\mathcal{M}|^2+|\overline{\mathcal{M}}|^2)\\&=(T^2+P_{uc}^2)|\xi_u|^2+P_{tc}^2|\xi_t|^2\\&\quad-2P_{tc}P_{uc}|\xi_u^*\xi_t|\cos\,\gamma\,\cos(\delta_{uc}-\delta_{tc})\\&\quad-2TP_{uc}|\xi_u|^2\cos\,\delta_{uc}\\&\quad+2TP_{tc}|\xi_u^*\xi_t|\cos\,\gamma\,\cos\,\delta_{tc}.\end{aligned} \tag{12}$$

In the case of $b\to s$ transitions the weak phase entering in $A^-$ is equal to $\gamma$, as we are using the Wolfenstein approximation [30] in which $\xi_t$ has no weak phase and the phase of $\xi_u$ is $\gamma$. Thus, the weak phase dependence factors out in an overall $\sin\,\gamma$ in $A^-$. Despite this, the above equations for the *CP*-violating asymmetry $A_{CP}$ are quite involved due to the fact that several strong phases are present which are in general hard to calculate except in specific models such as the ones being used here. However, there are several small parameters involved in the numerator and denominator given above. Expanding in these small parameters, much simplified forms for $A^-$ and $A^+$ and hence $A_{CP}$ can be obtained in specific decays.

First, we note that $|\xi_u|\ll|\xi_t|\simeq|\xi_c|$, with an upper bound $|\xi_u|/|\xi_t|\leqslant 0.025$. In some channels, such as $B^+\to K^+\pi^0$, $K^{*+}\pi^0$, $K^{*+}\rho^0$, $B^0\to K^+\pi^-$, $K^{*+}\pi^-$, $K^{*+}\rho^-$, typical value of the ratio $|P_{tc}/T|$ is of $O(0.1)$, with both $P_{uc}$ and $P_{tc}$ comparable with typically $|P_{uc}/P_{tc}|=O(0.3)$. The importance of including the contributions proportional to $P_{uc}$ has been stressed earlier in the literature [35] (see, also [36,37]). These estimates are based on perturbation theory but the former inequality $|P_{tc}/T|\ll 1$ should hold generally as the top quark contribution is genuinely short-distance. The other inequality can be influenced by nonperturbative penguin contributions. However, also in this case, for the mentioned transitions, we expect that $|P_{uc}/T|\ll 1$ should hold. Using these approximations, Eqs. (11),(12) become simplified:

$$A^-\simeq 2TP_{tc}|\xi_u^*\xi_t|\sin\,\gamma\,\sin\,\delta_{tc}, \tag{13}$$

$$A^+\simeq P_{tc}^2|\xi_t|^2+T^2|\xi_u|^2+2TP_{tc}|\xi_u^*\xi_t|\cos\,\gamma\,\cos\,\delta_{tc}. \tag{14}$$

The *CP*-violating asymmetry in this case is

$$A_{CP}\simeq\frac{2z_{12}\sin\,\delta_{tc}\sin\,\gamma}{1+2z_{12}\cos\,\delta_{tc}\cos\,\gamma+z_{12}^2}, \tag{15}$$

where $z_{12}=|\xi_u/\xi_t|\times T/P_{tc}$, where we use the notation used in [4]. This relation was suggested in the context of the decay $B\to K\pi$ by Fleischer and Mannel [38]. Because of the circumstance that the suppression due to $|\xi_u/\xi_t|$ is stronger than the enhancement due to $T/P_{tc}$, restricting the value of $z_{12}$, the *CP*-violating asymmetry for these kinds of decays are $O(10\%)$. To check the quality of the approximation made in Eq. (15), we have calculated the *CP*-violating asymmetry using this formula for $B^0\to K^+\pi^-$, which yields $A_{CP}=-7.1\%$ at $N_c=2$, very close to the value $-7.7\%$ in Table V calculated using the full formula, with $\rho=0.12$,

$\eta=0.34$ and $k^2=m_b^2/2$ in both cases. The results for other values of $N_c$ are similar. Thus, we conclude that Eq. (15) holds to a good approximation in the factorization framework for the decays mentioned earlier on. However, the *CP*-violating asymmetries $A_{CP}$ in the mentioned decays are found to depend on $k^2$, making their theoretical predictions considerably uncertain. These can be seen in the various tables for $A_{CP}$. Of course, the relation (15) given above, and others given below, can be modified through SFI—a possibility we are not entertaining here.

There are also some decays with vanishing tree contributions, such as $B^+\to\pi^+K_S^0$, $\pi^+K^{*0}$, $\rho^+K^{*0}$. For these decays, $T=0$, and $|\xi_u|\ll|\xi_t|$, then for these decays

$$A^-=2P_{tc}P_{uc}|\xi_u^*\xi_t|\sin\gamma\sin(\delta_{uc}-\delta_{tc}), \tag{16}$$

$$A^+\simeq P_{tc}^2|\xi_t|^2-2P_{tc}P_{uc}|\xi_u^*\xi_t|\cos\gamma\cos(\delta_{uc}-\delta_{tc}) \tag{17}$$

$$\simeq P_{tc}^2|\xi_t|^2. \tag{18}$$

The *CP*-violating asymmetry is

$$A_{CP}\simeq 2\frac{P_{uc}}{P_{tc}}\left|\frac{\xi_u}{\xi_t}\right|\sin(\delta_{uc}-\delta_{tc})\sin\gamma. \tag{19}$$

Without the $T$ contribution, the suppression due to both $P_{uc}/P_{tc}$ and $|\xi_u/\xi_t|$ is much stronger and the *CP*-violating asymmetries are only around $-(1\text{–}2)\%$. This is borne out by the numerical results obtained with the complete contributions, which can be seen in the tables.

### B. *CP*-violating asymmetries involving $b\to d$ transitions

For $b\to d$ transitions, we have

$$\mathcal{M}=T\zeta_u-P_t\zeta_t e^{i\delta_t}-P_c\zeta_c e^{i\delta_c}-P_u\zeta_u e^{i\delta_u},$$

$$\overline{\mathcal{M}}=T\zeta_u^*-P_t\zeta_t^* e^{i\delta_t}-P_c\zeta_c^* e^{i\delta_c}-P_u\zeta_u^* e^{i\delta_u}, \tag{20}$$

where $\zeta_i=V_{ib}V_{id}^*$, and again using CKM unitarity relation $\zeta_c=-\zeta_t-\zeta_u$, we have

$$\mathcal{M}=T\zeta_u-P_{tc}\zeta_t e^{i\delta_{tc}}-P_{uc}\zeta_u e^{i\delta_{uc}},$$

$$\overline{\mathcal{M}}=T\zeta_u^*-P_{tc}\zeta_t^* e^{i\delta_{tc}}-P_{uc}\zeta_u^* e^{i\delta_{uc}}, \tag{21}$$

$$\begin{aligned}A^-=&-2TP_{tc}|\zeta_u^*\zeta_t|\sin\alpha\sin\delta_{tc}\\&-2P_{tc}P_{uc}|\zeta_u^*\zeta_t|\sin\alpha\sin(\delta_{uc}-\delta_{tc}),\end{aligned} \tag{22}$$

$$\begin{aligned}A^+=&(T^2+P_{uc}^2)|\zeta_u|^2+P_{tc}^2|\zeta_t|^2\\&-2P_{tc}P_{uc}|\zeta_u^*\zeta_t|\cos\alpha\cos(\delta_{uc}-\delta_{tc})\\&-2TP_{uc}|\zeta_u|^2\cos\delta_{uc}\\&+2TP_{tc}|\zeta_u^*\zeta_t|\cos\alpha\cos\delta_{tc}.\end{aligned} \tag{23}$$

For the tree-dominated decays involving $b\to d$ transitions, such as $B^+\to\pi^+\eta^{(\prime)}$, $\rho^+\eta^{(\prime)}$, $\rho^+\omega$; the relation $P_{uc}<P_{tc}\ll T$ holds. This makes the formulas simpler, yielding

$$A^-\simeq-2TP_{tc}|\zeta_u^*\zeta_t|\sin\alpha\sin\delta_{tc}, \tag{24}$$

$$\begin{aligned}A^+\simeq&T^2|\zeta_u|^2-2TP_{uc}|\zeta_u|^2\cos\delta_{uc}\\&+2TP_{tc}|\zeta_u^*\zeta_t|\cos\alpha\cos\delta_{tc}\\\simeq&T'^2|\zeta_u|^2+2TP_{tc}|\zeta_u^*\zeta_t|\cos\alpha\cos\delta_{tc},\end{aligned} \tag{25}$$

with $T'^2\equiv T^2-2TP_{uc}\cos\delta_{uc}$. The *CP*-violating asymmetry is now approximately given by

$$A_{CP}\simeq\frac{-2z_1\sin\delta_{tc}\sin\alpha}{1+2z_1\cos\delta_{tc}\cos\alpha}, \tag{26}$$

with $z_1=|\zeta_t/\zeta_u|\times TP_{tc}/T'^2$. Note, the *CP*-violating asymmetry is approximately proportional to $\sin\alpha$ in this case. Here the suppression due to $P_{tc}T/T'^2$ is accompanied with some enhancement from $|\zeta_t/\zeta_u|$ (the central value of this quantity is about 3 [20]), making the *CP*-violating asymmetry in this kind of decays to have a value $A_{CP}=(10\text{–}20)\%$. We have calculated the *CP*-violating asymmetry of $B^\pm\to\rho^\pm\omega$ using the approximate formula (26). The number we got for $N_c=2$ is $A_{CP}=9.2\%$, which is very close to the value $A_{CP}=8.9\%$ in Table XI calculated using the exact formula, with $\rho=0.12$, $\eta=0.34$ and $k^2=m_b^2/2$.

For the decays with a vanishing tree contribution, such as $B^+\to K^+K_S^0$, $K^+\overline{K}^{*0}$, $K^{*+}\overline{K}^{*0}$, we have $T=0$. Thus,

$$A^-=-2P_{tc}P_{uc}|\zeta_u^*\zeta_t|\sin\alpha\sin(\delta_{uc}-\delta_{tc}), \tag{27}$$

$$\begin{aligned}A^+=&P_{tc}^2|\zeta_t|^2+P_{uc}^2|\zeta_u|^2\\&-2P_{tc}P_{uc}|\zeta_u^*\zeta_t|\cos\alpha\cos(\delta_{uc}-\delta_{tc}).\end{aligned} \tag{28}$$

The *CP*-violating asymmetry is approximately proportional to $\sin\alpha$ again,

$$A_{CP}=\frac{-2z_3\sin(\delta_{uc}-\delta_{tc})\sin\alpha}{1-2z_3\cos(\delta_{uc}-\delta_{tc})\cos\alpha+z_3^2}, \tag{29}$$

with $z_3=|\zeta_u/\zeta_t|\times P_{uc}/P_{tc}$. As the suppressions from $|\zeta_u/\zeta_t|$ and $|P_{uc}/P_{tc}|$ are not very big, the *CP*-violating asymmetry can again be of order (10–20)%. However, being direct *CP*-violating asymmetries, the mentioned asymmetries in the specific $B^\pm\to(h_1h_2)^\pm$ modes depend on $k^2$ and are uncertain.

### C. *CP*-violating asymmetries in neutral $B^0$ decays

For the neutral $B^0(\overline{B^0})$ decays, there is an additional complication due to $B^0-\overline{B^0}$ mixing. These *CP*-asymmetries may require time-dependent measurements, as discussed in the-

TABLE I. $CP$-violating asymmetry parameters $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ (in percent) for the decays $\overset{(-)}{B}{}^0\to h_1h_2$ using $\rho=0.12$, $\eta=0.34$, and $N_c=2$, 3, $\infty$, for $k^2=m_b^2/2\pm2$ GeV$^2$.

| | $a_{\epsilon'}$ | | | $a_{\epsilon+\epsilon'}$ | | |
|---|---|---|---|---|---|---|
| Channel | $N_c=2$ | $N_c=3$ | $N_c=\infty$ | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
| $\overset{(-)}{B}{}^0\to\pi^+\pi^-$ | $6.9^{+1.6}_{-3.5}$ | $7.0^{+1.6}_{-3.6}$ | $7.0^{+1.7}_{-3.6}$ | $35.3^{-1.6}_{+2.2}$ | $35.0^{-1.6}_{+2.2}$ | $34.5^{-1.7}_{+2.2}$ |
| $\overset{(-)}{B}{}^0\to\pi^0\pi^0$ | $-14.8^{-6.6}_{+14.3}$ | $2.7^{-8.1}_{+14.7}$ | $18.9^{+3.8}_{-7.7}$ | $-90.0^{+3.5}_{-3.8}$ | $-44.6^{-4.0}_{+7.3}$ | $77.8^{-2.7}_{+3.6}$ |
| $\overset{(-)}{B}{}^0\to\eta'\eta'$ | $26.0^{+5.6}_{-11.6}$ | $38.1^{+3.9}_{-6.1}$ | $-17.2^{-7.2}_{+16.0}$ | $62.8^{-4.8}_{+16.0}$ | $78.0^{-2.3}_{+3.6}$ | $-85.7^{+4.5}_{-5.1}$ |
| $\overset{(-)}{B}{}^0\to\eta\eta'$ | $22.9^{+4.3}_{-8.5}$ | $23.3^{+0.5}_{-0.4}$ | $-13.3^{-6.6}_{+14.3}$ | $88.5^{-2.5}_{+3.2}$ | $62.7^{-1.0}_{+2.1}$ | $-96.5^{+1.4}_{-2.3}$ |
| $\overset{(-)}{B}{}^0\to\eta\eta$ | $19.3^{+3.0}_{-5.9}$ | $16.1^{-0.6}_{+1.5}$ | $-10.1^{-5.9}_{+12.5}$ | $97.7^{-0.9}_{+1.2}$ | $50.6^{-0.9}_{+1.8}$ | $-99.5^{+0.9}_{-0.3}$ |
| $\overset{(-)}{B}{}^0\to\pi^0\eta'$ | $31.3^{+0.7}_{-0.8}$ | $22.9^{-3.0}_{+5.1}$ | $9.2^{-7.3}_{+12.6}$ | $59.1^{-2.3}_{+4.0}$ | $29.9^{-3.2}_{+5.6}$ | $-20.1^{-4.3}_{+7.6}$ |
| $\overset{(-)}{B}{}^0\to\pi^0\eta$ | $17.2^{-1.2}_{+2.4}$ | $13.8^{-2.6}_{+5.0}$ | $7.4^{-4.8}_{+9.0}$ | $43.1^{-1.4}_{+2.6}$ | $21.8^{-2.3}_{+3.8}$ | $-15.7^{-3.3}_{+5.4}$ |
| $\overset{(-)}{B}{}^0\to K_S^0\pi^0$ | $0.4^{+0.6}_{-1.3}$ | $-1.2^{+0.0}_{-0.2}$ | $-3.8^{-0.9}_{+1.4}$ | $75.1^{+0.2}_{-0.3}$ | $69.1^{+0.1}_{-0.2}$ | $58.1^{-0.3}_{+0.3}$ |
| $\overset{(-)}{B}{}^0\to K_S^0\eta'$ | $-2.4^{-0.3}_{+0.5}$ | $-1.8^{-0.1}_{+0.2}$ | $-0.9^{+0.2}_{-0.4}$ | $64.7^{-0.0}_{+0.1}$ | $66.9^{-0.0}_{+0.0}$ | $70.2^{+0.1}_{-0.2}$ |
| $\overset{(-)}{B}{}^0\to K_S^0\eta$ | $1.1^{+0.9}_{-1.6}$ | $-1.0^{+0.1}_{-0.3}$ | $-4.3^{-1.1}_{+1.8}$ | $78.0^{+0.2}_{-0.5}$ | $69.7^{+0.1}_{-0.1}$ | $54.1^{-0.4}_{+0.5}$ |
| $\overset{(-)}{B}{}^0\to K^0\overline{K^0}$ | $12.5^{-2.9}_{+5.5}$ | $12.3^{-2.9}_{+5.5}$ | $12.0^{-2.8}_{+5.5}$ | $15.7^{-2.4}_{+4.0}$ | $15.6^{-2.4}_{+3.9}$ | $15.3^{-2.3}_{+3.9}$ |
| $\overset{(-)}{B}{}^0\to\rho^0\pi^0$ | $-6.4^{-4.6}_{+9.8}$ | $-3.1^{-17.0}_{+30.0}$ | $7.8^{+3.4}_{-7.0}$ | $-40.6^{+3.6}_{-4.7}$ | $-99.5^{+1.6}_{+5.1}$ | $36.0^{-3.2}_{+4.5}$ |
| $\overset{(-)}{B}{}^0\to\omega\pi^0$ | $26.2^{+2.6}_{-4.3}$ | $23.4^{-0.6}_{+1.5}$ | $1.0^{+0.2}_{-0.6}$ | $84.7^{-0.5}_{+1.1}$ | $50.1^{-1.7}_{+3.3}$ | $49.8^{-0.2}_{+0.3}$ |
| $\overset{(-)}{B}{}^0\to\rho^0\eta$ | $-19.8^{-3.2}_{+26.7}$ | $12.9^{-8.8}_{+14.5}$ | $30.1^{+5.4}_{-9.9}$ | $-97.9^{+3.6}_{-0.7}$ | $-15.9^{-4.9}_{+9.4}$ | $93.9^{-2.4}_{+3.1}$ |
| $\overset{(-)}{B}{}^0\to\rho^0\eta'$ | $-52.7^{-6.6}_{+26.3}$ | $-55.0^{-1.9}_{+79.7}$ | $38.3^{+8.9}_{-19.1}$ | $37.8^{+10.0}_{-15.7}$ | $-43.5^{+35.2}_{-50.4}$ | $31.8^{-9.4}_{+13.2}$ |
| $\overset{(-)}{B}{}^0\to\omega\eta$ | $16.3^{+3.3}_{-6.8}$ | $25.1^{+3.4}_{-6.1}$ | $1.8^{+0.5}_{-1.0}$ | $74.6^{-2.4}_{+3.3}$ | $94.8^{-0.6}_{+1.0}$ | $9.5^{-0.4}_{+0.7}$ |
| $\overset{(-)}{B}{}^0\to\omega\eta'$ | $17.7^{+4.0}_{-8.5}$ | $43.5^{+9.5}_{-19.2}$ | $1.9^{+0.4}_{-1.0}$ | $46.0^{-3.9}_{+5.2}$ | $55.6^{-9.1}_{+12.3}$ | $37.8^{-0.5}_{+0.6}$ |
| $\overset{(-)}{B}{}^0\to\phi\pi^0$ | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ | $19.0^{-2.9}_{+5.0}$ | $1.6^{-0.3}_{+0.4}$ | $13.8^{-2.1}_{+3.5}$ |
| $\overset{(-)}{B}{}^0\to\phi\eta$ | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ | $19.0^{-2.9}_{+5.0}$ | $1.6^{-0.3}_{+0.4}$ | $13.8^{-2.1}_{+3.5}$ |
| $\overset{(-)}{B}{}^0\to\phi\eta'$ | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ | $19.0^{-2.9}_{+5.0}$ | $1.6^{-0.3}_{+0.4}$ | $13.8^{-2.1}_{+3.5}$ |
| $\overset{(-)}{B}{}^0\to\rho^0K_S^0$ | $2.1^{+0.5}_{-1.4}$ | $0.9^{+0.1}_{-0.4}$ | $-2.0^{-0.8}_{+1.8}$ | $18.7^{+0.6}_{-0.9}$ | $58.0^{+0.2}_{-0.2}$ | $98.6^{-0.2}_{+0.2}$ |
| $\overset{(-)}{B}{}^0\to\phi K_S^0$ | $-1.7^{-0.1}_{+0.1}$ | $-1.8^{-0.1}_{+0.1}$ | $-2.7^{-0.1}_{+0.1}$ | $67.5^{+0.0}_{-0.1}$ | $67.5^{+0.0}_{-0.1}$ | $67.9^{+0.2}_{-0.3}$ |
| $\overset{(-)}{B}{}^0\to\omega K_S^0$ | $-5.3^{-1.5}_{+2.4}$ | $-24.0^{-7.9}_{+13.3}$ | $-3.8^{-0.9}_{+1.6}$ | $50.7^{-0.5}_{+0.7}$ | $19.2^{-0.0}_{+1.1}$ | $54.8^{-0.3}_{+0.5}$ |
| $\overset{(-)}{B}{}^0\to\rho^+\rho^-$ | $4.1^{+1.0}_{-2.2}$ | $4.2^{+1.0}_{-2.3}$ | $4.2^{+1.0}_{-2.3}$ | $17.1^{-1.1}_{+1.5}$ | $16.9^{-1.1}_{+1.5}$ | $16.5^{-1.1}_{+1.5}$ |
| $\overset{(-)}{B}{}^0\to\rho^0\rho^0$ | $-8.0^{-3.9}_{+8.5}$ | $1.9^{-4.0}_{+7.8}$ | $12.1^{+2.2}_{-4.5}$ | $-97.0^{+1.2}_{-1.3}$ | $-41.4^{-2.6}_{+4.1}$ | $88.2^{-1.2}_{+1.6}$ |
| $\overset{(-)}{B}{}^0\to\omega\omega$ | $20.8^{+3.6}_{-6.9}$ | $22.0^{+1.6}_{-2.5}$ | $4.7^{+1.1}_{-2.5}$ | $95.2^{-1.5}_{+1.9}$ | $78.6^{-0.3}_{+0.8}$ | $24.4^{-1.1}_{+1.6}$ |
| $\overset{(-)}{B}{}^0\to K^{*0}\overline{K}{}^{*0}$ | $15.2^{-3.3}_{+6.1}$ | $14.5^{-3.2}_{+5.9}$ | $13.4^{-3.1}_{+5.7}$ | $18.1^{-2.8}_{+4.7}$ | $17.5^{-2.7}_{+4.5}$ | $16.5^{-2.5}_{+4.2}$ |
| $\overset{(-)}{B}{}^0\to\rho^0\omega$ | $8.2^{-6.7}_{+12.0}$ | $4.5^{-0.8}_{+1.6}$ | $8.3^{-3.4}_{+6.5}$ | $-21.4^{-4.1}_{+7.1}$ | $22.8^{-0.6}_{+0.9}$ | $-2.1^{-2.6}_{+4.1}$ |
| $\overset{(-)}{B}{}^0\to\rho^0\phi$ | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ | $19.0^{-2.9}_{+5.0}$ | $1.6^{-0.3}_{+0.4}$ | $13.8^{-2.1}_{+3.5}$ |
| $\overset{(-)}{B}{}^0\to\omega\phi$ | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ | $19.0^{-2.9}_{+5.0}$ | $1.6^{-0.3}_{+0.4}$ | $13.8^{-2.1}_{+3.5}$ |

literature [8,40–42]. Defining the time-dependent asymmetries as

$$A_{CP}(t)=\frac{\Gamma(B^0(t)\to f\,)-\Gamma(\overline{B^0}(t)\to\overline{f}\,)}{\Gamma(B^0(t)\to f\,)+\Gamma(\overline{B^0}(t)\to\overline{f}\,)},\tag{30}$$

there are four cases that one encounters for neutral $B^0(\overline{B^0})$ decays.

***Case (i)***

$B^0\to f$, $\overline{B^0}\to\overline{f}$, where $f$ or $\overline{f}$ is not a common final state of $B^0$ and $\overline{B^0}$, for example $B^0\to K^+\pi^-$.

***Case (ii)***

$B^0 \to (f=\bar{f}\,) \leftarrow \overline{B^0}$ with $f^{CP}=\pm f$, involving final states which are $CP$ eigenstates, i.e., decays such as $\overline{B^0}(B^0) \to \pi^+\pi^-$, $\pi^0\pi^0$, $K_S^0\pi^0$ etc.

***Case (iii)***

$B^0 \to (f=\bar{f}\,) \leftarrow \overline{B^0}$ with $f$, involving final states which are not $CP$ eigenstates. They include decays such as $B^0 \to (VV)^0$, as the $VV$ states are not $CP$-eigenstates.

***Case (iv)***

$B^0 \to (f \& \bar{f}\,) \leftarrow \overline{B^0}$ with $f^{CP} \neq f$, i.e., both $f$ and $\bar{f}$ are common final states of $B^0$ and $\overline{B^0}$, but they are not $CP$ eigenstates. Decays $B^0 \to \rho^+\pi^-$, $\rho^-\pi^+$ and $B^0 \to K^{*0}K_S^0$, $\overline{K^{*0}}K_S^0$ are two examples of interest for us.

Here case (i) is very similar to the charged $B^\pm$ decays. For case (ii), and (iii), $A_{CP}(t)$ would involve $B^0-\overline{B^0}$ mixing. Assuming $|\Delta\Gamma| \ll |\Delta m|$ and $|\Delta\Gamma/\Gamma| \ll 1$, which hold in the standard model for the mass and width differences $\Delta m$ and $\Delta\Gamma$ in the neutral $B$-sector, one can express $A_{CP}(t)$ in a simplified form:

$$A_{CP}(t) \simeq a_{\epsilon'}\cos(\Delta m t) + a_{\epsilon+\epsilon'}\sin(\Delta m t). \tag{31}$$

The quantities $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$, for which we follow the definitions given in [42], depend on the hadronic matrix elements which we have calculated in our model

$$a_{\epsilon'} = \frac{1-|\lambda_{CP}|^2}{1+|\lambda_{CP}|^2}, \tag{32}$$

$$a_{\epsilon+\epsilon'} = \frac{-2\,\mathrm{Im}(\lambda_{CP})}{1+|\lambda_{CP}|^2}, \tag{33}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td}\langle f|H_{\rm eff}|\overline{B^0}\rangle}{V_{tb}V_{td}^*\langle f|H_{\rm eff}|B^0\rangle}. \tag{34}$$

For case (i) decays, the coefficient $a_{\epsilon'}$ determines $A_{CP}(t)$, and since no mixing is involved for these decays, the $CP$-violating asymmetry is independent of time. We shall call these, together with the $CP$-asymmetries in charged $B^\pm$ decays, $CP$-class (i) decays. For cases (ii) and (iii), one has to separate the sin($\Delta mt$) and cos($\Delta mt$) terms to get the $CP$-violating asymmetry $A_{CP}(t)$. The time-integrated asymmetries are

$$A_{CP} = \frac{1}{1+x^2}\,a_{\epsilon'} + \frac{x}{1+x^2}\,a_{\epsilon+\epsilon'}\,, \tag{35}$$

with $x = \Delta m/\Gamma \simeq 0.73$ for the $B^0-\overline{B^0}$ case [39].

Case (iv) also involves mixing but requires additional formulas. Here one studies the four time-dependent decay widths for $B^0(t)\to f$, $\overline{B^0}(t)\to \bar{f}$, $B^0(t)\to\bar{f}$ and $\overline{B^0}(t)\to f$ [40,41,42]. These time-dependent widths can be expressed by four basic matrix elements

$$\begin{aligned} g &= \langle f|H_{\rm eff}|B^0\rangle, \quad & h &= \langle f|H_{\rm eff}|\overline{B^0}\rangle, \\ \bar{g} &= \langle \bar{f}|H_{\rm eff}|\overline{B^0}\rangle, \quad & \bar{h} &= \langle \bar{f}|H_{\rm eff}|B^0\rangle, \end{aligned} \tag{36}$$

which determine the decay matrix elements of $B^0 \to f \& \bar{f}$ and of $\overline{B^0} \to \bar{f} \& f$ at $t=0$. For example, when $f=\rho^-\pi^+$ the ma-

TABLE II. $CP$-violating asymmetry parameters $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ (in percent) for the decays $\overset{(-)}{B^0} \to h_1 h_2$ using $\rho=0.23$, $\eta=0.42$, and $N_c=2$, 3, $\infty$, for $k^2=m_b^2/2$.

| | $a_{\epsilon'}$ | | | $a_{\epsilon+\epsilon'}$ | | |
|---|---|---|---|---|---|---|
| Channel | $N_c=2$ | $N_c=3$ | $N_c=\infty$ | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
| $\overset{(-)}{B^0}\to\pi^+\pi^-$ | 4.9 | 4.9 | 5.0 | 29.3 | 29.1 | 28.7 |
| $\overset{(-)}{B^0}\to\pi^0\pi^0$ | −12.4 | 3.8 | 14.5 | −71.6 | −60.7 | 63.7 |
| $\overset{(-)}{B^0}\to\eta'\eta'$ | 19.3 | 42.3 | −14.0 | 50.8 | 88.3 | −66.3 |
| $\overset{(-)}{B^0}\to\eta\eta'$ | 18.6 | 28.8 | −11.8 | 75.4 | 78.7 | −82.2 |
| $\overset{(-)}{B^0}\to\eta\eta$ | 17.2 | 21.0 | −9.7 | 90.2 | 66.7 | −92.3 |
| $\overset{(-)}{B^0}\to\pi^0\eta'$ | 38.9 | 31.2 | 13.0 | 74.2 | 41.1 | −28.4 |
| $\overset{(-)}{B^0}\to\pi^0\eta$ | 22.8 | 19.2 | 10.5 | 57.9 | 30.5 | −22.3 |
| $\overset{(-)}{B^0}\to K_S^0\pi^0$ | 0.4 | −1.5 | −4.9 | 90.7 | 85.7 | 75.1 |
| $\overset{(-)}{B^0}\to K_S^0\eta'$ | −3.0 | −2.2 | −1.1 | 81.8 | 83.8 | 86.6 |
| $\overset{(-)}{B^0}\to K_S^0\eta$ | 1.3 | −1.2 | −5.6 | 92.7 | 86.3 | 70.9 |
| $\overset{(-)}{B^0}\to K^0\overline{K^0}$ | 17.5 | 17.3 | 16.9 | 22.2 | 22.0 | 21.7 |
| $\overset{(-)}{B^0}\to\rho^0\pi^0$ | −3.6 | 1.9 | 4.8 | −26.1 | −97.0 | 30.7 |
| $\overset{(-)}{B^0}\to\omega\pi^0$ | 28.7 | 30.4 | 0.7 | 94.5 | 65.6 | 40.1 |
| $\overset{(-)}{B^0}\to\rho^0\eta$ | −19.3 | 18.3 | 26.3 | −92.3 | −22.5 | 85.2 |
| $\overset{(-)}{B^0}\to\rho^0\eta'$ | −39.0 | −69.2 | 27.6 | 32.1 | −53.6 | 27.2 |
| $\overset{(-)}{B^0}\to\omega\eta$ | 12.4 | 24.7 | 1.3 | 60.6 | 96.0 | 11.1 |
| $\overset{(-)}{B^0}\to\omega\eta'$ | 12.7 | 32.6 | 1.3 | 37.3 | 45.7 | 31.1 |
| $\overset{(-)}{B^0}\to\phi\pi^0$ | 22.7 | 1.4 | 14.8 | 26.7 | 2.3 | 19.6 |
| $\overset{(-)}{B^0}\to\phi\eta$ | 22.7 | 1.4 | 14.8 | 26.7 | 2.3 | 19.6 |
| $\overset{(-)}{B^0}\to\phi\eta'$ | 22.7 | 1.4 | 14.8 | 26.7 | 2.3 | 19.6 |
| $\overset{(-)}{B^0}\to\rho^0K_S^0$ | 2.9 | 1.1 | −2.0 | 26.3 | 75.1 | 98.7 |
| $\overset{(-)}{B^0}\to\phi K_S^0$ | −2.1 | −2.2 | −3.4 | 84.3 | 84.3 | 84.6 |
| $\overset{(-)}{B^0}\to\omega K_S^0$ | −7.0 | −33.3 | −4.9 | 67.1 | 26.8 | 71.7 |
| $B^0\to\rho^+\rho^-$ | 2.9 | 2.9 | 3.0 | 16.4 | 16.2 | 15.9 |
| $\overset{(-)}{B^0}\to\rho^0\rho^0$ | −7.1 | 2.6 | 9.7 | −82.5 | −56.8 | 74.2 |
| $\overset{(-)}{B^0}\to\omega\omega$ | 17.8 | 25.3 | 3.3 | 84.8 | 91.7 | 21.5 |
| $\overset{(-)}{B^0}\to K^{*0}\overline{K^{*0}}$ | 21.3 | 20.3 | 18.8 | 25.4 | 24.6 | 23.3 |
| $\overset{(-)}{B^0}\to\rho^0\omega$ | 11.6 | 6.3 | 11.8 | −30.4 | 32.1 | −3.0 |
| $\overset{(-)}{B^0}\to\rho^0\phi$ | 22.7 | 1.4 | 14.8 | 26.7 | 2.3 | 19.6 |
| $\overset{(-)}{B^0}\to\omega\phi$ | 22.7 | 1.4 | 14.8 | 26.7 | 2.3 | 19.6 |

trix element $h$ is given in Appendix B of [4] in Eq. (99) and $\overline{g}$ for the decay $\overline{B^0}\to\rho^+\pi^-$ is written down in Eq. (100) in Appendix B of [4]. The matrix elements $\overline{h}$ and $g$ are obtained from $h$ and $\overline{g}$ by changing the signs of the weak phases contained in the products of the CKM matrix elements. We also need to know the *CP*-violating parameter coming from the $B^0-\overline{B^0}$ mixing. Defining

$$B_1=p|B^0\rangle+q|\overline{B^0}\rangle,$$
$$B_2=p|B^0\rangle-q|\overline{B^0}\rangle, \tag{37}$$

with $|p|^2+|q|^2=1$ and $q/p=\sqrt{H_{21}/H_{12}}$, with $H_{ij}=M_{ij}-i/2\Gamma_{ij}$ representing the $|\Delta B|=2$ and $\Delta Q=0$ Hamiltonian [8]. For the decays of $B^0$ and $\overline{B^0}$, we use, as before,

$$\frac{q}{p}=\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}=e^{-2i\beta}. \tag{38}$$

So, $|q/p|=1$, and this ratio has only a phase given by $-2\beta$. Then, the four time-dependent widths are given by the following formulas (we follow the notation of [42]):

$$\Gamma(B^0(t)\to f)=e^{-\Gamma t}\frac{1}{2}(|g|^2+|h|^2)\times\{1+a_{\epsilon'}\cos\Delta mt+a_{\epsilon+\epsilon'}\sin\Delta mt\},$$
$$\Gamma(\overline{B^0}(t)\to\overline{f})=e^{-\Gamma t}\frac{1}{2}(|\overline{g}|^2+|\overline{h}|^2)\times\{1-a_{\overline{\epsilon'}}\cos\Delta mt-a_{\epsilon+\overline{\epsilon'}}\sin\Delta mt\},$$
$$\Gamma(B^0(t)\to\overline{f})=e^{-\Gamma t}\frac{1}{2}(|\overline{g}|^2+|\overline{h}|^2)\times\{1+a_{\overline{\epsilon'}}\cos\Delta mt+a_{\epsilon+\overline{\epsilon'}}\sin\Delta mt\},$$
$$\Gamma(\overline{B^0}(t)\to f)=e^{-\Gamma t}\frac{1}{2}(|g|^2+|h|^2)\times\{1-a_{\epsilon'}\cos\Delta mt-a_{\epsilon+\epsilon'}\sin\Delta mt\}, \tag{39}$$

where

$$a_{\epsilon'}=\frac{|g|^2-|h|^2}{|g|^2+|h|^2},\quad a_{\epsilon+\epsilon'}=\frac{-2\,\mathrm{Im}\left(\frac{q}{p}\frac{h}{g}\right)}{1+|h/g|^2},$$
$$a_{\overline{\epsilon'}}=\frac{|\overline{h}|^2-|\overline{g}|^2}{|\overline{h}|^2+|\overline{g}|^2},\quad a_{\epsilon+\overline{\epsilon'}}=\frac{-2\,\mathrm{Im}\left(\frac{q}{p}\frac{\overline{g}}{\overline{h}}\right)}{1+|\overline{g}/\overline{h}|^2}. \tag{40}$$

By measuring the time-dependent spectrum of the decay rates of $B^0$ and $\overline{B^0}$, one can find the coefficients of the two functions $\cos\Delta mt$ and $\sin\Delta mt$ and extract the quantities $a_{\epsilon'}$, $a_{\epsilon+\epsilon'}$, $|g|^2+|h|^2$, $a_{\overline{\epsilon'}}$, $a_{\epsilon+\overline{\epsilon'}}$ and $|\overline{g}|^2+|\overline{h}|^2$ as well as $\Delta m$ and $\Gamma$, which, however, are already well measured [39]. The signature of CP violation is $\Gamma(\overline{B^0}(t)\to\overline{f})\neq\Gamma(B^0(t)\to\overline{f})$ and $\Gamma(\overline{B^0}(t)\to f)\neq\Gamma(B^0(t)\to f)$ which means, that $a_{\epsilon'}\neq -a_{\overline{\epsilon'}}$ and/or $a_{\epsilon+\epsilon'}\neq -a_{\epsilon+\overline{\epsilon'}}$. In the two examples, $f=\rho^+\pi^-$ and $f=K^{*0}K_S^0$, the amplitudes $g$ and $h$ contain contributions of several terms similar to what we have written down above for the charged $B$ decays. They have weak and strong phases with the consequence that $|g|\neq|\overline{g}|$ and $|h|\neq|\overline{h}|$.

## IV. NUMERICAL RESULTS FOR *CP*-VIOLATING COEFFICIENTS AND $A_{CP}$

Given the amplitudes $\mathcal{M}$ and $\overline{\mathcal{M}}$, one can calculate the *CP*-violating asymmetry $A_{CP}$ for all the $B\to PP, B\to PV$ and $B\to VV$ decay modes and their charged conjugates whose branching ratios were calculated by us recently in the factorization approach [4]. The asymmetries depend on several variables, such as the CKM parameters, $N_c$, the virtuality $k^2$ discussed earlier, and the scale $\mu$. The scale dependence of $A_{CP}$ is important in only a few decays and we shall estimate it by varying $\mu$ between $\mu=m_b/2$ and $\mu=m_b$ at the end of this section for these decays and fix the scale at $\mu=m_b/2$. The dependence on the rest of the parameters is worked out explicitly. We show the results for $N_c=2,3,\infty$, for two representative choices of the CKM parameters in the tables: central values emerging from the CKM unitarity fits

TABLE III. *CP*-violating asymmetry parameters $a_{\epsilon'}$, $a_{\overline{\epsilon'}}$, $a_{\epsilon+\epsilon'}$, $a_{\epsilon+\overline{\epsilon'}}$ defined in Eq. (40) for the decays $\overset{(-)}{B^0}\to\rho^-\pi^+$, $\overset{(-)}{B^0}\to\rho^+\pi^-$, and $\overset{(-)}{B^0}\to\overline{K}^{*0}K_S^0$, $\overset{(-)}{B^0}\to K^{*0}K_S^0$ (in percent), using $\rho=0.12$, $\eta=0.34$, and $k^2=m_b^2/2\pm 2\ \mathrm{GeV}^2$.

| Channel | $N_c$ | $a_{\epsilon'}$ | $a_{\overline{\epsilon'}}$ | $a_{\epsilon+ge'}$ | $a_{\epsilon+\overline{\epsilon'}}$ |
|---|---|---|---|---|---|
| | $N_c=2$ | $-54.9^{+0.6}_{-1.3}$ | $58.6^{+0.4}_{-0.8}$ | $6.0^{-0.4}_{+0.4}$ | $6.2^{-0.7}_{+1.2}$ |
| $\overset{(-)}{B^0}\to\rho^-\pi^+,\ \rho^+\pi^-$ | $N_c=3$ | $-54.9^{+0.6}_{-1.3}$ | $58.7^{+0.3}_{-0.9}$ | $5.8^{-0.4}_{+0.5}$ | $6.0^{-0.7}_{+1.3}$ |
| | $N_c=\infty$ | $-54.9^{+0.6}_{-1.3}$ | $58.7^{+0.3}_{-0.9}$ | $5.6^{-0.5}_{+0.4}$ | $5.8^{-0.8}_{+1.2}$ |
| | $N_c=2$ | $99.3^{-0.2}_{+0.2}$ | $-99.1^{+0.4}_{-0.5}$ | $-5.3^{-2.9}_{+4.8}$ | $10.0^{+2.2}_{-3.9}$ |
| $\overset{(-)}{B^0}\to\overline{K}^{*0}K_S^0,\ K^{*0}K_S^0$ | $N_c=3$ | $99.9^{-0.1}_{+0.0}$ | $-99.6^{+0.1}_{-0.2}$ | $-3.2^{-2.3}_{+3.9}$ | $8.8^{+1.5}_{-2.6}$ |
| | $N_c=\infty$ | $99.8^{-0.1}_{+0.0}$ | $-99.1^{-0.1}_{+0.2}$ | $-0.4^{-1.5}_{+2.7}$ | $7.2^{+0.5}_{-0.9}$ |

TABLE IV. *CP*-violating asymmetry parameters $a_{\epsilon'}$, $a_{\overline{\epsilon'}}$, $a_{\epsilon+\epsilon'}$, $a_{\epsilon+\overline{\epsilon'}}$ defined in Eq. (40) for the decays $\overset{(-)}{B^0}\to\rho^-\pi^+$, $\overset{(-)}{B^0}\to\rho^+\pi^-$, and $\overset{(-)}{B^0}\to\overline{K}^{*0}K_S^0$, $\overset{(-)}{B^0}\to K^{*0}K_S^0$ (in percent), using $\rho=0.23$, $\eta=0.42$, and $k^2=m_b^2/2$.

| Channel | $N_c$ | $a_{\epsilon'}$ | $a_{\overline{\epsilon'}}$ | $a_{\epsilon+ge'}$ | $a_{\epsilon+\overline{\epsilon'}}$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0}\to\rho^-\pi^+,\ \rho^+\pi^-$ | $N_c=2$ | $-55.5$ | 58.1 | 7.8 | 8.1 |
| | $N_c=3$ | $-55.5$ | 58.1 | 7.6 | 8.0 |
| | $N_c=\infty$ | $-55.5$ | 58.1 | 7.5 | 7.8 |
| $\overset{(-)}{B^0}\to\overline{K}^{*0}K_S^0,\ K^{*0}K_S^0$ | $N_c=2$ | 99.4 | $-99.0$ | $-4.4$ | 11.1 |
| | $N_c=3$ | 99.9 | $-99.5$ | $-2.2$ | 10.3 |
| | $N_c=\infty$ | 99.8 | $-98.8$ | 0.8 | 9.0 |

of the existing data, yielding $\rho=0.12$, $\eta=0.34$ [20]; for values of $\rho$ and $\eta$ which correspond to their central values $+1\sigma$, yielding $\rho=0.23$ and $\eta=0.42$ [20].

For each decay mode and given a value of $N_c$, the errors shown on the numbers in the tables reflect the uncertainties due to the variation of $k^2$ in the range $k^2=(m_b^2/2\pm 2)$ GeV$^2$. For some selected *CP*-asymmetries, we show in figures, however, the dependence on the CKM parameters for a wider range of $\rho$ and $\eta$ which are allowed by the present 95% C.L. unitarity fits [20].

TABLE V. *CP*-violating asymmetries $A_{CP}$ in $\overset{(-)}{B}\to PP$ decays (in percent) using $\rho=0.12$, $\eta=0.34$, and $N_c=2$, 3, $\infty$, for $k^2=m_b^2/2\pm 2$ GeV$^2$.

| Channel | Class | *CP* class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0}\to\pi^+\pi^-$ | I | (ii) | $21.3^{+0.3}_{-1.2}$ | $21.2^{+0.3}_{-1.2}$ | $21.0^{+0.3}_{-1.3}$ |
| $\overset{(-)}{B^0}\to\pi^0\pi^0$ | II | (ii) | $-52.5^{-2.6}_{+7.5}$ | $-19.4^{-7.2}_{+13.0}$ | $49.3^{+1.2}_{-3.3}$ |
| $\overset{(-)}{B^0}\to\eta'\eta'$ | II | (ii) | $46.9^{+1.4}_{-4.5}$ | $62.0^{+1.4}_{-2.3}$ | $-52.0^{-2.6}_{+8.0}$ |
| $\overset{(-)}{B^0}\to\eta\eta'$ | II | (ii) | $57.1^{+1.6}_{-4.1}$ | $45.1^{-0.2}_{+0.7}$ | $-54.6^{-3.1}_{+8.3}$ |
| $\overset{(-)}{B^0}\to\eta\eta$ | II | (ii) | $59.0^{+1.6}_{-3.2}$ | $34.6^{-0.8}_{+1.8}$ | $-53.9^{-3.5}_{+8.0}$ |
| $B^\pm\to\pi^\pm\pi^0$ | III | (i) | $0.1^{+0.0}_{-0.1}$ | $0.0^{+0.1}_{-0.0}$ | $0.0^{+0.0}_{-0.0}$ |
| $B^\pm\to\pi^\pm\eta'$ | III | (i) | $12.0^{+2.6}_{-5.9}$ | $14.5^{+3.2}_{-6.7}$ | $21.3^{+4.2}_{-8.4}$ |
| $B^\pm\to\pi^\pm\eta$ | III | (i) | $11.8^{+2.4}_{-5.3}$ | $14.0^{+2.8}_{-5.9}$ | $19.1^{+3.3}_{-6.4}$ |
| $\overset{(-)}{B^0}\to\pi^0\eta'$ | V | (ii) | $48.6^{-0.7}_{+1.4}$ | $29.2^{-3.5}_{+6.0}$ | $-3.6^{-6.7}_{+11.9}$ |
| $\overset{(-)}{B^0}\to\pi^0\eta$ | V | (ii) | $31.7^{-1.4}_{+2.9}$ | $19.4^{-2.8}_{+5.0}$ | $-2.6^{-4.8}_{+8.4}$ |
| $B^\pm\to K^\pm\pi^0$ | IV | (i) | $-7.1^{-2.1}_{+3.7}$ | $-6.3^{-1.8}_{+3.2}$ | $-4.9^{-1.3}_{+2.3}$ |
| $\overset{(-)}{B^0}\to K^\pm\pi^\mp$ | IV | (i) | $-7.7^{-2.3}_{+4.0}$ | $-7.9^{-2.3}_{+4.2}$ | $-8.2^{-2.4}_{+4.4}$ |
| $\overset{(-)}{B^0}\to K_S^0\pi^0$ | IV | (ii) | $36.0^{+0.5}_{-1.0}$ | $32.0^{+0.1}_{-0.2}$ | $25.1^{-0.7}_{+1.1}$ |
| $B^\pm\to K^\pm\eta'$ | IV | (i) | $-4.9^{-1.2}_{+2.1}$ | $-4.1^{-1.0}_{+1.6}$ | $-3.0^{-0.5}_{+1.0}$ |
| $\overset{(-)}{B^0}\to K_S^0\eta'$ | IV | (ii) | $29.2^{-0.2}_{+0.4}$ | $30.7^{-0.1}_{+0.0}$ | $32.8^{+0.1}_{-0.3}$ |
| $B^\pm\to K^\pm\eta$ | IV | (i) | $8.5^{+3.4}_{-6.3}$ | $6.2^{+2.6}_{-4.8}$ | $2.8^{+1.4}_{-2.6}$ |
| $\overset{(-)}{B^0}\to K_S^0\eta$ | IV | (ii) | $37.8^{+0.7}_{-1.3}$ | $32.5^{+0.1}_{-0.3}$ | $22.9^{-0.9}_{+1.5}$ |
| $B^\pm\to\pi^\pm K_S^0$ | IV | (i) | $-1.4^{-0.1}_{+0.0}$ | $-1.4^{-0.1}_{+0.0}$ | $-1.4^{-0.0}_{+0.1}$ |
| $B^\pm\to K^\pm K_S^0$ | IV | (i) | $12.5^{-2.9}_{+5.5}$ | $12.3^{-2.9}_{+5.5}$ | $12.0^{-2.8}_{+5.5}$ |
| $\overset{(-)}{B^0}\to K^0\overline{K^0}$ | IV | (ii) | $15.6^{-3.0}_{+5.6}$ | $15.5^{-3.1}_{+5.4}$ | $15.1^{-2.9}_{+5.5}$ |

The results are presented taking into account the following considerations. For decays belonging to the *CP* class-(i), the *CP*-asymmetry is time-independent. Hence for this class, $A_{CP}=a_{\epsilon'}$. For the *CP* class-(ii) and *CP* class-(iii) decays, the measurement of $A_{CP}$ will be done in terms of the coefficients $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$, which are the measures of direct and indirect (or mixing-induced) *CP*-violation, respectively. In view of this, we give in Tables I and II these coefficients for the thirty decay modes of the $B^0$ and $\overline{B^0}$ mesons, which belong to these classes, for the two sets of CKM parameters given above. For the four decays belonging to the *CP* class-(iv) decays, one would measure by time-dependent decay rates the quantities $a_{\epsilon'}, a_{\epsilon+\epsilon'}, a_{\overline{\epsilon'}}$ and $a_{\epsilon+\overline{\epsilon'}}$. They are given in Tables III and IV for the two sets of CKM parameters, respectively. Having worked out these quantities, we then give the numerical values of the *CP*-violating asymmetries for all the seventy six decays $B\to PP$, $B\to PV\,(b\to d$ transition), $B\to PV\,(b\to s$ transition) and $B\to VV$ in Tables V–XII.

### A. Parametric dependence of *CP*-violating parameters and $A_{CP}$

We now discuss the *CP* asymmetries given in Tables I–XII and their parametric dependence.

#### *Form factor dependence of* $A_{CP}$

The *CP*-violating asymmetries depend very weakly on the form factors. We have calculated the *CP*-violating asymme-

TABLE VI. *CP*-violating asymmetries $A_{CP}$ in $\overset{(-)}{B^0}\to PP$ decays (in percent) using $\rho=0.23$, $\eta=0.42$, and $N_c=2$, 3, $\infty$ for $k^2=m_b^2/2$.

| Channel | Class | *CP* class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0}\to\pi^+\pi^-$ | I | (ii) | 17.2 | 17.1 | 16.9 |
| $\overset{(-)}{B^0}\to\pi^0\pi^0$ | II | (ii) | $-42.2$ | $-26.4$ | 39.8 |
| $\overset{(-)}{B^0}\to\eta'\eta'$ | II | (ii) | 36.8 | 69.7 | $-40.7$ |
| $\overset{(-)}{B^0}\to\eta\eta'$ | II | (ii) | 48.0 | 56.3 | $-46.8$ |
| $\overset{(-)}{B^0}\to\eta\eta$ | II | (ii) | 54.2 | 45.4 | $-50.2$ |
| $B^\pm\to\pi^\pm\pi^0$ | III | (i) | 0.0 | 0.0 | 0.0 |
| $B^\pm\to\pi^\pm\eta'$ | III | (i) | 8.5 | 10.5 | 16.7 |
| $B^\pm\to\pi^\pm\eta$ | III | (i) | 8.7 | 10.6 | 16.2 |
| $\overset{(-)}{B^0}\to\pi^0\eta'$ | V | (ii) | 60.7 | 39.9 | $-5.0$ |
| $\overset{(-)}{B^0}\to\pi^0\eta$ | V | (ii) | 42.5 | 27.1 | $-3.7$ |
| $B^\pm\to K^\pm\pi^0$ | IV | (i) | $-9.8$ | $-8.6$ | $-6.5$ |
| $\overset{(-)}{B^0}\to K^\pm\pi^\mp$ | IV | (i) | $-10.5$ | $-10.8$ | $-11.2$ |
| $\overset{(-)}{B^0}\to K_S^0\pi^0$ | IV | (ii) | 43.4 | 39.8 | 32.5 |
| $B^\pm\to K^\pm\eta'$ | IV | (i) | $-6.3$ | $-5.3$ | $-3.8$ |
| $\overset{(-)}{B^0}\to K_S^0\eta'$ | IV | (ii) | 36.9 | 38.4 | 40.5 |
| $B^\pm\to K^\pm\eta$ | IV | (i) | 8.4 | 6.4 | 3.1 |
| $\overset{(-)}{B^0}\to K_S^0\eta$ | IV | (ii) | 45.0 | 40.2 | 30.0 |
| $B^\pm\to\pi^\pm K_S^0$ | IV | (i) | $-1.8$ | $-1.7$ | $-1.7$ |
| $B^\pm\to K^\pm K_S^0$ | IV | (i) | 17.5 | 17.3 | 16.9 |
| $\overset{(-)}{B^0}\to K^0\overline{K^0}$ | IV | (ii) | 22.1 | 21.8 | 21.4 |

TABLE VII. $CP$-violating asymmetries $A_{CP}$ in $\overset{(-)}{B}\to PV$ decays ($b\to d$ transition) (in percent) using $\rho$=0.12, $\eta$=0.34, and $N_c$=2, 3, $\infty$ for $k^2=m_b^2/2\pm 2\ \mathrm{GeV}^2$.

| Channel | Class | $CP$ class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $B^0/\overline{B^0}\to\rho^-\pi^+/\rho^+\pi^-$ | I | (iv) | $3.6^{-0.7}_{+1.2}$ | $3.5^{-0.7}_{+1.2}$ | $3.3^{-0.7}_{+1.2}$ |
| $B^0/\overline{B^0}\to\rho^+\pi^-/\rho^-\pi^+$ | I | (iv) | $6.0^{+0.7}_{-1.9}$ | $5.9^{+0.8}_{-1.8}$ | $5.9^{+0.7}_{-1.9}$ |
| $\overset{(-)}{B^0}\to\rho^0\pi^0$ | II | (ii) | $-23.5^{-1.3}_{+4.2}$ | $-49.4^{-10.3}_{+22.1}$ | $22.2^{+0.7}_{-2.4}$ |
| $\overset{(-)}{B^0}\to\omega\pi^0$ | II | (ii) | $57.5^{+1.4}_{-2.3}$ | $39.2^{-1.3}_{+2.5}$ | $24.3^{+0.1}_{-0.2}$ |
| $\overset{(-)}{B^0}\to\rho^0\eta$ | II | (ii) | $-59.5^{-7.0}_{+17.1}$ | $0.9^{-8.1}_{+14.0}$ | $64.4^{+2.4}_{-5.0}$ |
| $\overset{(-)}{B^0}\to\rho^0\eta'$ | II | (ii) | $-16.5^{+0.4}_{+9.7}$ | $-56.7^{+15.5}_{+28.2}$ | $40.2^{+1.4}_{-6.2}$ |
| $\overset{(-)}{B^0}\to\omega\eta$ | II | (ii) | $46.2^{+0.9}_{-2.9}$ | $61.5^{+1.9}_{-3.5}$ | $5.7^{+0.1}_{-0.3}$ |
| $\overset{(-)}{B^0}\to\omega\eta'$ | II | (ii) | $33.5^{+0.8}_{-3.1}$ | $54.9^{+1.9}_{-6.7}$ | $19.2^{+0.1}_{-0.4}$ |
| $B^\pm\to\rho^0\pi^\pm$ | III | (i) | $-3.9^{-1.1}_{+2.6}$ | $-5.2^{-1.5}_{+3.5}$ | $-11.0^{-3.8}_{+8.8}$ |
| $B^\pm\to\rho^\pm\pi^0$ | III | (i) | $2.7^{+0.6}_{-1.5}$ | $3.0^{+0.7}_{-1.6}$ | $3.6^{+0.9}_{-1.9}$ |
| $B^\pm\to\omega\pi^\pm$ | III | (i) | $9.8^{+2.2}_{-4.8}$ | $7.9^{+1.9}_{-4.0}$ | $-1.8^{-0.6}_{+1.2}$ |
| $B^\pm\to\rho^\pm\eta$ | III | (i) | $3.9^{+0.9}_{-2.2}$ | $4.4^{+1.1}_{-2.4}$ | $5.7^{+1.4}_{-3.0}$ |
| $B^\pm\to\rho^\pm\eta'$ | III | (i) | $3.8^{+1.0}_{-2.2}$ | $4.3^{+1.2}_{-2.5}$ | $5.6^{+1.4}_{-3.2}$ |
| $\overset{(-)}{B^0}\to\overline{K^{*0}}K^0_S/K^{*0}K^0_S$ | IV | (iv) | $15.9^{-3.4}_{+6.2}$ | $15.3^{-3.3}_{+6.0}$ | $14.3^{-3.2}_{+5.9}$ |
| $\overset{(-)}{B^0}\to K^{*0}K^0_S/\overline{K^{*0}}K^0_S$ | V | (iv) | $-12.2^{+3.0}_{-5.7}$ | $-10.6^{+2.6}_{-5.2}$ | $-8.2^{+2.2}_{-4.3}$ |
| $B^\pm\to K^\pm\overset{(-)}{K^{*0}}$ | IV | (i) | $15.2^{-3.3}_{+6.1}$ | $14.5^{-3.2}_{+5.9}$ | $13.4^{-3.1}_{+5.7}$ |
| $B^\pm\to K^{*\pm}K^0_S$ | V | (i) | $-1.2^{+5.6}_{-32.8}$ | $46.8^{-13.2}_{-3.3}$ | $48.1^{-5.6}_{+4.8}$ |
| $B^\pm\to\phi\pi^\pm$ | V | (i) | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ |
| $\overset{(-)}{B^0}\to\phi\pi^0$ | V | (ii) | $19.6^{-3.6}_{+6.5}$ | $1.4^{-0.3}_{+0.7}$ | $13.4^{-2.7}_{+5.0}$ |
| $\overset{(-)}{B^0}\to\phi\eta$ | V | (ii) | $19.6^{-3.6}_{+6.5}$ | $1.4^{-0.3}_{+0.7}$ | $13.4^{-2.7}_{+5.0}$ |
| $\overset{(-)}{B^0}\to\phi\eta'$ | V | (ii) | $19.6^{-3.6}_{+6.5}$ | $1.4^{-0.3}_{+0.7}$ | $13.4^{-2.7}_{+5.0}$ |

tries for the form factors based on both the BSW [32] and the hybrid lattice-QCD/QCD-SR models. The form factor values used are given in [4]. However, the dependence of $A_{CP}$ on the form factors is weak. Hence, we show results only for the former case.

### $N_c$ dependence of $A_{CP}$

The classification of the $B\to h_1h_2$ decays using $N_c$-dependence of the rates that we introduced in [4] is also very useful in discussing $A_{CP}$. We see that the $CP$-asymmetries in the class-I and class-IV decays have mild dependence on $N_c$, reflecting very much the characteristics of the decay rates. As already remarked, this can be traced back to the $N_c$-dependence of the effective coefficients. However, in some decays classified as class-IV decays in [4], we have found that $A_{CP}$ shows a marked $N_c$ dependence. All these cases are on the borderline as far as the $N_c$-sensitivity of the decay rates is concerned due to the presence of several competing amplitudes. The decays, which were classified in [4] as class-IV decays but are now classified as class-V decays, are as follows:

$B\to PP$ decays: $B^0\to\pi^0\eta^{(\prime)}$.

$B\to PV$ decays involving $b\to s$ transitions: $B^0\to K^{*0}\eta$. (The decay mode $B^0\to K^{*0}\eta'$ was already classified in [4] as a class-V decay.)

$B\to VV$ decays: $B^0\to K^{*0}\rho^0$.

With this, we note that the $N_c$-dependence of $A_{CP}$ in the class-I and class-IV decays is at most $\pm 20\%$, as one varies $N_c$ in the range $N_c=2$ to $N_c=\infty$.

Concerning class-III decays, in most cases $A_{CP}$ is found to vary typically by a factor 2 as $N_c$ is varied, with one exception: $B^+\to\omega\pi^+$, in which case the estimate for $A_{CP}$ is uncertain by an order of magnitude. This is in line with the observation made on the decay rate for this process in [4]. Both $CP$-violating asymmetries and decay rates for the class-II and class-V decays are generally strongly $N_c$-dependent. We had shown this for the decay rates in [4] and for the $CP$-violating asymmetries this feature can be seen in the tables presented here.

### $k^2$ dependence of $A_{CP}$

The $CP$-violating asymmetries depend on the value of $k^2$, as discussed in the literature [15]. The value of $k^2$ relative to the charm threshold, i.e., $k^2\leqslant(\geqslant)4m_c^2$, plays a central role here. For the choice $k^2\leqslant 4m_c^2$, the $c\bar{c}$ loop will not generate a strong phase. We treat $k^2$ as a variable parameter and have studied the sensitivity of the $CP$-asymmetries by varying it in the range $k^2=m_b^2/2\pm 2\ \mathrm{GeV}^2$. This range may appear somewhat arbitrary, however, it is large enough to test which of

TABLE VIII. $CP$-violating asymmetries $A_{CP}$ in $\overset{(-)}{B}\to PV$ decays ($b\to d$ transition) (in percent) using $\rho=0.23$, $\eta=0.42$, and $N_c=2$, 3, $\infty$ for $k^2=m_b^2/2$.

| Channel | Class | $CP$ class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $B^0/\overline{B^0}\to\rho^-\pi^+/\rho^+\pi^-$ | I | (iv) | 5.3 | 5.2 | 5.1 |
| $B^0/\overline{B^0}\to\rho^+\pi^-/\rho^-\pi^+$ | I | (iv) | 5.4 | 5.4 | 5.4 |
| $\overset{(-)}{B^0}\to\rho^0\pi^0$ | II | (ii) | $-14.8$ | $-44.9$ | 17.8 |
| $\overset{(-)}{B^0}\to\omega\pi^0$ | II | (ii) | 63.7 | 51.1 | 19.5 |
| $\overset{(-)}{B^0}\to gr^0\eta$ | II | (ii) | $-56.5$ | 1.3 | 57.7 |
| $\overset{(-)}{B^0}\to\rho^0\eta'$ | II | (ii) | $-10.2$ | $-70.8$ | 31.0 |
| $\overset{(-)}{B^0}\to\omega\eta$ | II | (ii) | 36.9 | 61.8 | 6.1 |
| $\overset{(-)}{B^0}\to\omega\eta'$ | II | (ii) | 26.1 | 43.0 | 15.7 |
| $B^\pm\to\rho^0\pi^\pm$ | III | (i) | $-2.7$ | $-3.7$ | $-8.3$ |
| $B^\pm\to\rho^\pm\pi^0$ | III | (i) | 1.9 | 2.1 | 2.6 |
| $B^\pm\to\omega\pi^\pm$ | III | (i) | 7.0 | 5.6 | $-1.3$ |
| $B^\pm\to\rho^\pm\eta$ | III | (i) | 2.7 | 3.1 | 4.0 |
| $B^\pm\to\rho^\pm\eta'$ | III | (i) | 2.7 | 3.0 | 3.9 |
| $B^0/\overline{B^0}\to\overline{K^{*0}}K_S^0/K^{*0}K_S^0$ | IV | (iv) | 22.3 | 21.4 | 20.1 |
| $B^0/\overline{B^0}\to K^{*0}K_S^0/\overline{K^{*0}}K_S^0$ | V | (iv) | $-17.0$ | $-14.9$ | $-11.5$ |
| $B^\pm\to K^\pm\overset{(-)}{K^{*0}}$ | IV | (i) | 21.3 | 20.3 | 18.8 |
| $B^\pm\to K^{*\pm}K_S^0$ | V | (i) | $-1.6$ | 54.6 | 62.5 |
| $B^\pm\to\phi\pi^\pm$ | V | (i) | 22.7 | 1.4 | 14.8 |
| $\overset{(-)}{B^0}\to\phi\pi^0$ | V | (ii) | 27.5 | 2.0 | 19.0 |
| $\overset{(-)}{B^0}\to\phi\eta$ | V | (ii) | 27.5 | 2.0 | 19.0 |
| $\overset{(-)}{B^0}\to\phi\eta'$ | V | (ii) | 27.5 | 2.0 | 19.0 |

TABLE X. $CP$-violating asymmetries $A_{CP}$ in $\overset{(-)}{B}\to PV$ decays ($b\to s$ transition) (in percent) using $\rho=0.23$, $\eta=0.42$, and $N_c=2$, 3, $\infty$ for $k^2=m_b^2/2$.

| Channel | Class | $CP$ class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0}\to\rho^\mp K^\pm$ | I | (i) | $-11.5$ | $-11.5$ | $-11.4$ |
| $B^\pm\to K^{*\pm}\eta'$ | III | (i) | $-55.2$ | $-71.7$ | $-75.2$ |
| $\overset{(-)}{B^0}\to K^{*\pm}\pi^\mp$ | IV | (i) | $-22.1$ | $-22.6$ | $-23.6$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\pi^0$ | V | (i) | 1.6 | $-1.6$ | $-6.3$ |
| $B^\pm\to K^{*\pm}\pi^0$ | IV | (i) | $-18.2$ | $-17.1$ | $-14.8$ |
| $B^\pm\to\rho^0K^\pm$ | IV | (i) | $-14.5$ | $-15.9$ | $-10.7$ |
| $B^\pm\to K^{*\pm}\eta$ | IV | (i) | $-13.0$ | $-13.3$ | $-13.5$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\eta$ | V | (i) | $-3.1$ | $-1.7$ | 0.7 |
| $B^\pm\to\overset{(-)}{K^{*0}}\pi^\pm$ | IV | (i) | $-2.1$ | $-2.0$ | $-1.9$ |
| $\overset{(-)}{B^0}\to\rho^0K_S^0$ | V | (ii) | 14.4 | 36.4 | 45.6 |
| $B^\pm\to\rho^\pm K_S^0$ | V | (i) | 3.7 | 5.6 | $-5.3$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\eta'$ | V | (i) | $-47.7$ | $-16.3$ | 9.0 |
| $B^\pm\to\phi K^\pm$ | V | (i) | $-2.1$ | $-2.2$ | $-3.4$ |
| $\overset{(-)}{B^0}\to\phi K_S^0$ | V | (ii) | 38.7 | 38.7 | 38.0 |
| $\overset{(-)}{B^0}\to\omega K_S^0$ | V | (ii) | 27.3 | $-9.1$ | 30.9 |
| $B^\pm\to\omega K^\pm$ | V | (i) | $-18.6$ | $-15.1$ | 1.1 |

TABLE IX. $CP$-violating asymmetries $A_{CP}$ in $\overset{(-)}{B}\to PV$ decays ($b\to s$ transition) (in percent) using $\rho=0.12$, $\eta=0.34$, and $N_c=2$, 3, $\infty$ for $k^2=m_b^2/2\pm2$ GeV$^2$.

| Channel | Class | $CP$ class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0}\to\rho^\mp K^\pm$ | I | (i) | $-16.4^{-3.9}_{+9.7}$ | $-16.4^{-3.9}_{+9.8}$ | $-16.3^{-3.9}_{+9.7}$ |
| $B^\pm\to K^{*\pm}\eta'$ | III | (i) | $-72.6^{-5.1}_{+35.9}$ | $-84.3^{-8.6}_{+44.0}$ | $-61.5^{-16.1}_{+36.5}$ |
| $\overset{(-)}{B^0}\to K^{*\pm}\pi^\mp$ | IV | (i) | $-15.5^{-5.0}_{+8.9}$ | $-15.9^{-5.2}_{+9.2}$ | $-16.6^{-5.4}_{+9.6}$ |
| $\overset{(-)}{B^0}\to K^{*0}\pi^0$ | V | (i) | $1.4^{+1.2}_{-2.2}$ | $-1.3^{+0.1}_{-0.4}$ | $-4.9^{-1.3}_{+2.0}$ |
| $B^\pm\to K^{*\pm}\pi^0$ | IV | (i) | $-12.8^{-3.9}_{+7.3}$ | $-12.0^{-3.7}_{+6.8}$ | $-10.5^{-3.2}_{+5.8}$ |
| $B^\pm\to\rho^0K^\pm$ | IV | (i) | $-13.2^{-3.2}_{+6.8}$ | $-12.8^{-3.2}_{+6.7}$ | $-7.5^{-2.0}_{+3.8}$ |
| $B^\pm\to K^{*\pm}\eta$ | IV | (i) | $-9.1^{-2.7}_{+5.1}$ | $-9.3^{-2.8}_{+5.2}$ | $-9.6^{-2.9}_{+5.3}$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\eta$ | V | (i) | $-2.4^{-0.5}_{+0.9}$ | $-1.4^{-0.1}_{+0.2}$ | $0.6^{+0.5}_{-1.1}$ |
| $B^\pm\to\overset{(-)}{K^{*0}}\pi^\pm$ | IV | (i) | $-1.7\mp0.1$ | $-1.6\mp0.1$ | $-1.5^{-0.1}_{+0.0}$ |
| $\overset{(-)}{B^0}\to\rho^0K_S^0$ | V | (ii) | $10.2^{+0.7}_{-1.3}$ | $28.2^{+0.1}_{-0.3}$ | $45.5^{-0.5}_{+1.4}$ |
| $B^\pm\to\rho^\pm K_S^0$ | V | (i) | $3.0^{-0.4}_{+0.8}$ | $4.6^{-1.1}_{+4.8}$ | $-4.4^{+0.6}_{-0.5}$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\eta'$ | V | (i) | $-44.0^{+11.9}_{-48.0}$ | $-13.3^{+2.1}_{+0.5}$ | $8.0^{+4.6}_{-7.5}$ |
| $B^\pm\to\phi K^\pm$ | V | (i) | $-1.7\mp0.1$ | $-1.8\mp0.1$ | $-2.7^{-0.1}_{+0.1}$ |
| $\overset{(-)}{B^0}\to\phi K_S^0$ | V | (ii) | $31.0^{-0.1}_{+0.0}$ | $30.9^{-0.0}_{+0.0}$ | $30.5^{+0.0}_{-0.1}$ |
| $\overset{(-)}{B^0}\to\omega K_S^0$ | V | (ii) | $20.6^{-1.2}_{+1.9}$ | $-6.6^{-5.2}_{+9.2}$ | $23.6^{-0.8}_{+1.3}$ |
| $B^\pm\to\omega K^\pm$ | V | (i) | $-13.1^{-4.1}_{+7.4}$ | $-19.6^{-4.7}_{+11.1}$ | $0.9^{+0.7}_{-1.3}$ |

TABLE XI. $CP$-violating asymmetries $A_{CP}$ in $\overset{(-)}{B}\to VV$ decays (in percent) using $\rho=0.12$, $\eta=0.34$, and $N_c=2$, 3, $\infty$ for $k^2=m_b^2/2\pm 2$ GeV$^2$.

| Channel | Class | $CP$ class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0}\to\rho^+\rho^-$ | I | (iii) | $10.8^{+0.2}_{-0.7}$ | $10.8^{+0.1}_{-0.8}$ | $10.6^{+0.1}_{-0.8}$ |
| $\overset{(-)}{B^0}\to\rho^0\rho^0$ | II | (iii) | $-51.4^{-1.9}_{+5.0}$ | $-18.5^{-3.8}_{+7.1}$ | $49.9^{+0.8}_{-2.2}$ |
| $\overset{(-)}{B^0}\to\omega\omega$ | II | (iii) | $58.9^{+1.6}_{-3.6}$ | $51.8^{+0.9}_{-1.2}$ | $14.7^{+0.2}_{-0.9}$ |
| $B^\pm\to\rho^\pm\rho^0$ | III | (i) | $0.2^{+0.1}_{-0.1}$ | $0.2^{+0.1}_{-0.0}$ | $0.3^{+0.0}_{-0.1}$ |
| $B^\pm\to\rho^\pm\omega$ | III | (i) | $8.9^{+1.9}_{-4.4}$ | $7.7^{+1.7}_{-3.9}$ | $4.0^{+1.0}_{-2.2}$ |
| $\overset{(-)}{B^0}\to K^{*\pm}\rho^\mp$ | IV | (i) | $-15.5^{-5.0}_{+8.9}$ | $-15.9^{-5.2}_{+9.2}$ | $-16.6^{-5.4}_{+9.6}$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\rho^0$ | V | (i) | $5.1^{+2.8}_{-4.8}$ | $-0.8^{+0.5}_{-0.9}$ | $-9.2^{-2.8}_{+4.8}$ |
| $B^\pm\to K^{*\pm}\rho^0$ | IV | (i) | $-11.8^{-3.6}_{+6.6}$ | $-10.3^{-3.1}_{+5.7}$ | $-7.3^{-2.1}_{+3.8}$ |
| $B^\pm\to\rho^\pm\overset{(-)}{K^{*0}}$ | IV | (i) | $-1.7\mp 0.1$ | $-1.6\mp 0.1$ | $-1.5^{-0.1}_{+0.0}$ |
| $B^\pm\to K^{*\pm}\overset{(-)}{K^{*0}}$ | IV | (i) | $15.2^{-3.3}_{+6.1}$ | $14.5^{-3.2}_{+5.9}$ | $13.4^{-3.1}_{+5.7}$ |
| $\overset{(-)}{B^0}\to K^{*0}\overline{K}^{*0}$ | IV | (iii) | $18.6^{-3.5}_{+6.2}$ | $17.8^{-3.4}_{+6.0}$ | $16.6^{-3.2}_{+5.7}$ |
| $\overset{(-)}{B^0}\to\rho^0\omega$ | V | (iii) | $-4.8^{-6.4}_{+11.2}$ | $13.8^{-0.8}_{+1.5}$ | $4.4^{-3.5}_{+6.2}$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\omega$ | V | (i) | $-3.1^{-0.7}_{+1.1}$ | $-2.1^{-0.3}_{+0.4}$ | $-11.7^{-3.7}_{+6.8}$ |
| $B^\pm\to K^{*\pm}\omega$ | V | (i) | $-9.6^{-2.9}_{+5.2}$ | $-14.3^{-4.6}_{+8.2}$ | $7.2^{+2.6}_{-5.1}$ |
| $B^\pm\to K^{*\pm}\phi$ | V | (i) | $-1.7\mp 0.1$ | $-1.8\mp 0.1$ | $-2.7^{-0.1}_{+0.1}$ |
| $\overset{(-)}{B^0}\to\overset{(-)}{K^{*0}}\phi$ | V | (i) | $-1.7\mp 0.1$ | $-1.8\mp 0.1$ | $-2.7^{-0.1}_{+0.1}$ |
| $B^\pm\to\rho^\pm\phi$ | V | (i) | $16.2^{-3.4}_{+6.2}$ | $1.0^{-0.4}_{+0.7}$ | $10.5^{-2.6}_{+5.1}$ |
| $\overset{(-)}{B^0}\to\rho^0\phi$ | V | (iii) | $19.6^{-3.6}_{+6.5}$ | $1.4^{-0.3}_{+0.7}$ | $13.4^{-2.7}_{+5.0}$ |
| $\overset{(-)}{B^0}\to\omega\phi$ | V | (iii) | $19.6^{-3.6}_{+6.5}$ | $1.4^{-0.3}_{+0.7}$ | $13.4^{-2.7}_{+5.0}$ |

the asymmetries are sensitive to the choice of $k^2$. One sees from the tables, that $A_{CP}$ as well as the $CP$-violating parameters are sensitive to $k^2$ in most cases. Fortunately, there are some decays which have large $A_{CP}$ with only moderate theoretical errors from the $k^2$-dependence.

### *$\mu$ dependence of $A_{CP}$*

It should be remarked that the $CP$-asymmetries depend on the renormalization scale $\mu$. Part of this dependence is due to the fact that the strong phases are generated only by the explicit $\mathcal{O}(\alpha_s)$ corrections. This can be seen in the numerator $A^-$ given in Eq. (11). In other words, NLO corrections to $A_{CP}$, which are of of $O(\alpha_s^2)$ remain to be calculated. Despite this, the scale-dependence of $A_{CP}$ in $B\to h_1h_2$ decays is not very marked, except for a few decays for which the relevant Wilson coefficients do show some scale dependence. We give a list of these decays in Table XIII. This feature is quantitatively different from the scale-dependence of $A_{CP}$ in the inclusive radiative decays $B\to X_s\gamma$ and $B\to X_d\gamma$ [43], for which the $\mu$-dependence of the Wilson coefficient in the electromagnetic penguin operator introduces quite significant scale dependence in the $CP$-asymmetries. In contrast, the Wilson coefficients contributing to $A_{CP}$ in the decays $B\to h_1h_2$ are less $\mu$-dependent. Of course, there is still some residual scale dependence in the quark masses. For all numbers and figures shown here, we use $\mu=m_b/2$, a scale suggested by NLO corrections in the decay rates for $B\to X_s\gamma$ and $B\to X_d\gamma$ [43], for which NLO corrections are small.

### B. Decay modes with stable $A_{CP}$ in the factorization approach

We use the parametric dependence of $A_{CP}$ just discussed to pick out the decay modes which are stable against the variation of $N_c$, $k^2$ and the scale $\mu$. As only class-I and class-IV (and some class III) decays are stable against $N_c$, we need concentrate only on decays in these classes. With the help of the entries in Tables V–XIII, showing the $k^2$ and $\mu$ dependence, we find that the following decay modes have measurably large asymmetries, i.e., $|A_{CP}|\geqslant 20\%$ [except for $A_{CP}(\rho^+\rho^-)$ which is estimated to be more like $O(10\%)$] with large branching ratios, typically $O(10^{-5})$ [except for $B^0\to K_S^0\eta$, which is estimated to be of $O(10^{-6})$ [4]].

#### $\overset{(-)}{B^0}\to\pi^+\pi^-$, $\overset{(-)}{B^0}\to K_S^0\pi^0$, $\overset{(-)}{B^0}\to K_S^0\eta'$, $\overset{(-)}{B^0}\to K_S^0\eta$ *and* $\overset{(-)}{B^0}\to\rho^+\rho^-$

We discuss these cases in detail showing the CKM-parametric dependence of $A_{CP}$ in each case. Since these decays measure, ideally, one of the phases in the unitarity triangle, we shall also plot $A_{CP}$ as a function of the relevant phase, which is $\sin 2\alpha$ for $A_{CP}(\pi^+\pi^-)$, and $\sin 2\beta$ for $A_{CP}(K_S^0\pi^0)$, $A_{CP}(K_S^0\eta)$ and $A_{CP}(K_S^0\eta')$.

TABLE XII. *CP*-violating asymmetries $A_{CP}$ in $\overset{(-)}{B} \to VV$ decays (in percent) using $\rho=0.23$, $\eta=0.42$, and $N_c=2$, 3, $\infty$ for $k^2 = m_b^2/2$.

| Channel | Class | *CP* class | $N_c=2$ | $N_c=3$ | $N_c=\infty$ |
|---|---|---|---|---|---|
| $\overset{(-)}{B^0} \to \rho^+\rho^-$ | I | (iii) | 9.7 | 9.6 | 9.5 |
| $\overset{(-)}{B^0} \to \rho^0\rho^0$ | II | (iii) | $-43.9$ | $-25.3$ | 41.6 |
| $\overset{(-)}{B^0} \to \omega\omega$ | II | (iii) | 52.0 | 60.2 | 12.4 |
| $B^\pm \to \rho^\pm\rho^0$ | III | (i) | 0.2 | 0.2 | 0.2 |
| $B^\pm \to \rho^\pm\omega$ | III | (i) | 6.3 | 5.4 | 2.8 |
| $\overset{(-)}{B^0} \to K^{*\pm}\rho^\mp$ | IV | (i) | $-22.1$ | $-22.6$ | $-23.6$ |
| $\overset{(-)}{B^0} \to \overset{(-)}{K^{*0}}\rho^0$ | V | (i) | 5.7 | $-1.0$ | $-12.3$ |
| $B^\pm \to K^{*\pm}\rho^0$ | IV | (i) | $-16.8$ | $-14.6$ | $-10.1$ |
| $B^\pm \to \rho^\pm \overset{(-)}{K^{*0}}$ | IV | (i) | $-2.1$ | $-2.0$ | $-1.9$ |
| $B^\pm \to K^{*\pm} \overset{(-)}{K^{*0}}$ | IV | (i) | 21.3 | 20.3 | 18.8 |
| $\overset{(-)}{B^0} \to K^{*0}\overline{K}^{*0}$ | IV | (iii) | 26.1 | 25.0 | 23.4 |
| $\overset{(-)}{B^0} \to \rho^0\omega$ | V | (iii) | $-6.8$ | 19.4 | 6.3 |
| $\overset{(-)}{B^0} \to \overset{(-)}{K^{*0}}\omega$ | V | (i) | $-4.0$ | $-2.6$ | $-16.7$ |
| $B^\pm \to K^{*\pm}\omega$ | V | (i) | $-13.3$ | $-20.4$ | 6.6 |
| $B^\pm \to K^{*\pm}\phi$ | V | (i) | $-2.1$ | $-2.2$ | $-3.4$ |
| $\overset{(-)}{B^0} \to \overset{(-)}{K^{*0}}\phi$ | V | (i) | $-2.1$ | $-2.2$ | $-3.4$ |
| $B^\pm \to \rho^\pm\phi$ | V | (i) | 22.7 | 1.4 | 14.8 |
| $\overset{(-)}{B^0} \to \rho^0\phi$ | V | (iii) | 27.5 | 2.0 | 19.0 |
| $\overset{(-)}{B^0} \to \omega\phi$ | V | (iii) | 27.5 | 2.0 | 19.0 |

### *CP-violating asymmetry in* $\overset{(-)}{B^0} \to \pi^+\pi^-$

We show in Figs. 1(a) and 1(b) the *CP*-asymmetry parameters $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$, defined in Eqs. (32) and (33), respectively, plotted as a function of the CKM-Wolfenstein parameter $\rho$ with the indicated values of $\eta$. The shadowed area in this and all subsequent figures showing the $\rho$-dependence corresponds to the range $0<\rho<0.23$, which is the $\pm 1\sigma$ allowed values of this parameter from the unitarity fits [20]. The three curves in Figs. 1(a) and 1(b) represent three different values of the CKM-Wolfenstein parameter: $\eta=0.26$ (solid curve), $\eta=0.34$ (dashed curve), and $\eta=0.42$ (dotted curve). The time-integrated asymmetry $A_{CP}$ calculated with the help of Eq. (35) is shown for three values of $\eta$ ($\eta=0.42$, 0.34, 0.26) with $k^2=m_b^2/2$ in Fig. 2(a). One notes that the CKM-dependence of $A_{CP}$ is very significant. The $k^2$-dependence of $A_{CP}(\pi^+\pi^-)$ is found to be very weak as shown is Fig. 2(b), where we plot this quantity as a function of $\rho$ for $\eta=0.34$ by varying $k^2$ in the range $k^2=m_b^2/2 \pm 2\,\mathrm{GeV}^2$. Hence, there is a good case for $A_{CP}(\pi^+\pi^-)$ yielding information on the CKM parameters.

To have a closer look at this, we plot in Figs. 3(a) and 3(b), the asymmetry $A_{CP}(\pi^+\pi^-)$ as a function of $\sin 2\alpha$ to study the effect of the penguin contribution (called in the jargon ''penguin pollution'') and the dependence on $|V_{ub}|$, respectively. The lower (upper) curve in Fig. 3(a) corresponds to keeping only the tree contribution in the decays $\overset{(-)}{B^0} \to \pi^+\pi^-$ (tree+penguin). We see that in the entire $\pm 1\sigma$ expected range of $\sin 2\alpha$, depicted as a shadowed region, the ''penguin pollution'' is quite significant, chang-

TABLE XIII. *CP*-violating asymmetries $A_{CP}$ in $\overset{(-)}{B} \to h_1 h_2$ decays (in percent) using $\rho=0.12$, $\eta=0.34$, and $N_c=2$, 3, $\infty$, $k^2=m_b^2/2$ for $\mu=m_b/2$ and $\mu=m_b$.

| | $N_c=2$ | | $N_c=3$ | | $N_c=\infty$ | |
|---|---|---|---|---|---|---|
| Channel | $\mu=m_b/2$ | $\mu=m_b$ | $\mu=m_b/2$ | $\mu=m_b$ | $\mu=m_b/2$ | $\mu=m_b$ |
| $\overset{(-)}{B^0} \to \pi^0\pi^0$ | $-42.0$ | $-37.8$ | $-15.1$ | $-32.2$ | 43.9 | 45.5 |
| $B^\pm \to K^\pm\eta$ | 8.5 | 12.2 | 6.2 | 9.0 | 2.8 | 4.4 |
| $\overset{(-)}{B^0} \to \rho^0\pi^0$ | $-23.5$ | $-18.3$ | $-49.4$ | $-49.9$ | 22.2 | 20.1 |
| $\overset{(-)}{B^0} \to \omega\pi^0$ | 57.5 | 61.5 | 39.2 | 48.4 | 24.3 | 23.4 |
| $\overset{(-)}{B^0} \to \rho^0\eta$ | $-59.5$ | $-61.0$ | 0.9 | $-21.4$ | 64.4 | 64.7 |
| $\overset{(-)}{B^0} \to \rho^0\eta'$ | $-16.5$ | $-10.0$ | $-56.7$ | $-59.3$ | 40.2 | 35.8 |
| $\overset{(-)}{B^0} \to \omega\eta'$ | 33.5 | 29.1 | 54.9 | 41.3 | 19.2 | 17.9 |
| $B^\pm \to K^{*\pm}K_S^0$ | $-1.2$ | $-0.9$ | 46.8 | 35.0 | 48.1 | 46.8 |
| $\overset{(-)}{B^0} \to \overset{(-)}{K^{*0}}\pi^0$ | 1.4 | 3.5 | $-1.3$ | $-0.3$ | $-4.9$ | $-5.3$ |
| $\overset{(-)}{B^0} \to \omega K_S^0$ | 20.6 | 18.3 | $-6.6$ | $-14.6$ | 23.6 | 23.3 |
| $\overset{(-)}{B^0} \to \rho^0\rho^0$ | $-51.4$ | $-49.5$ | $-18.5$ | $-33.5$ | 49.9 | 50.5 |
| $\overset{(-)}{B^0} \to \overset{(-)}{K^{*0}}\rho^0$ | 5.1 | 10.0 | $-0.8$ | 1.3 | $-9.2$ | $-11.2$ |
| $\overset{(-)}{B^0} \to \rho^0\omega$ | $-4.8$ | $-14.8$ | 13.8 | 18.5 | 4.4 | 2.8 |

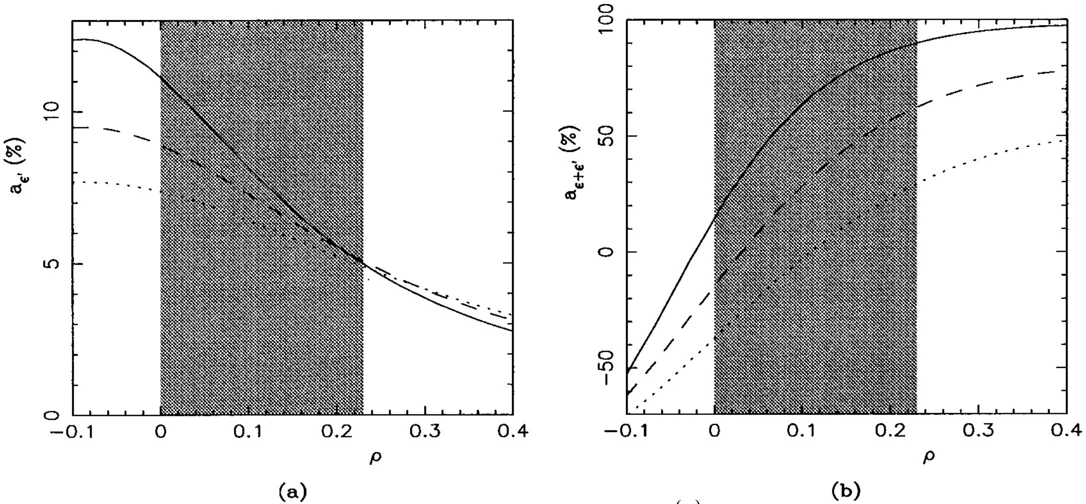



FIG. 1. *CP*-violating asymmetry parameters $a_{\epsilon'}$ (a) and $a_{\epsilon+\epsilon'}$ (b) for the decay $\overset{(-)}{B^0} \to \pi^+\pi^-$ as a function of the CKM parameter $\rho$ with $k^2 = m_b^2/2$. The dotted, dashed, and solid curves correspond to the CKM parameter values $\eta=0.42$, $\eta=0.34$, and $\eta=0.26$, respectively.

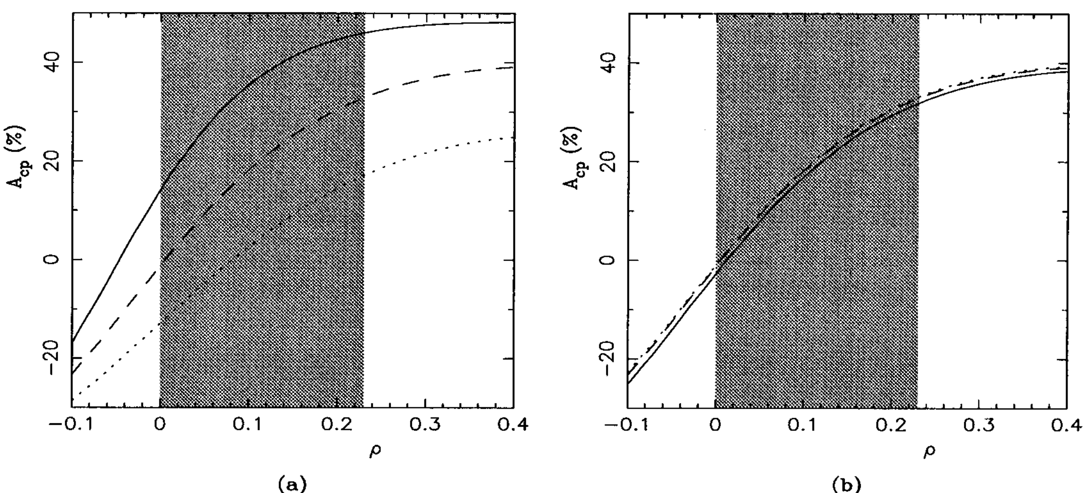



FIG. 2. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to \pi^+\pi^-$ as a function of the CKM parameter $\rho$. (a) $k^2 = m_b^2/2$. The dotted, dashed, and solid curves correspond to the CKM parameter values $\eta=0.42$, $\eta=0.34$, and $\eta=0.26$, respectively; (b) $\eta=0.34$. The dotted, dashed, and solid lines correspond to $k^2 = m_b^2/2 + 2\ \mathrm{GeV}^2$, $k^2 = m_b^2/2$, and $k^2 = m_b^2/2 - 2\ \mathrm{GeV}^2$, respectively.

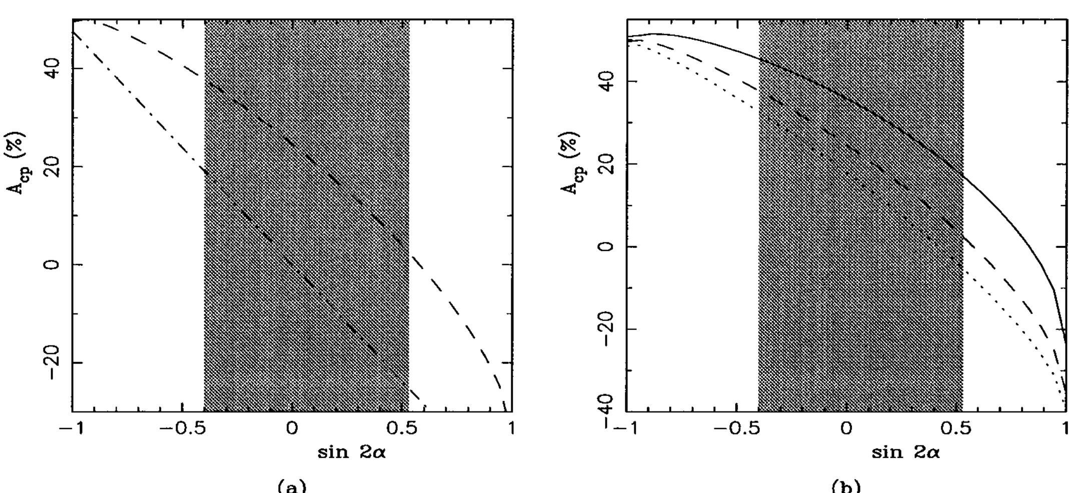
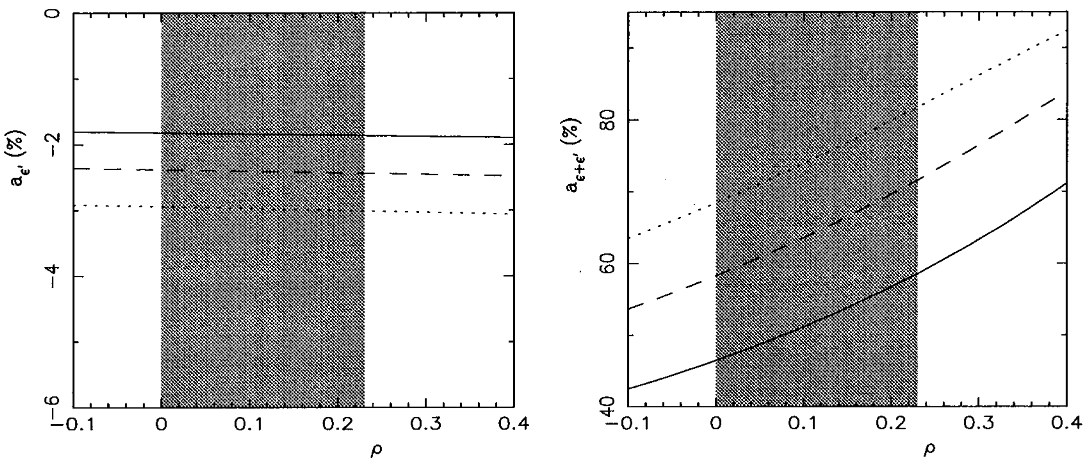
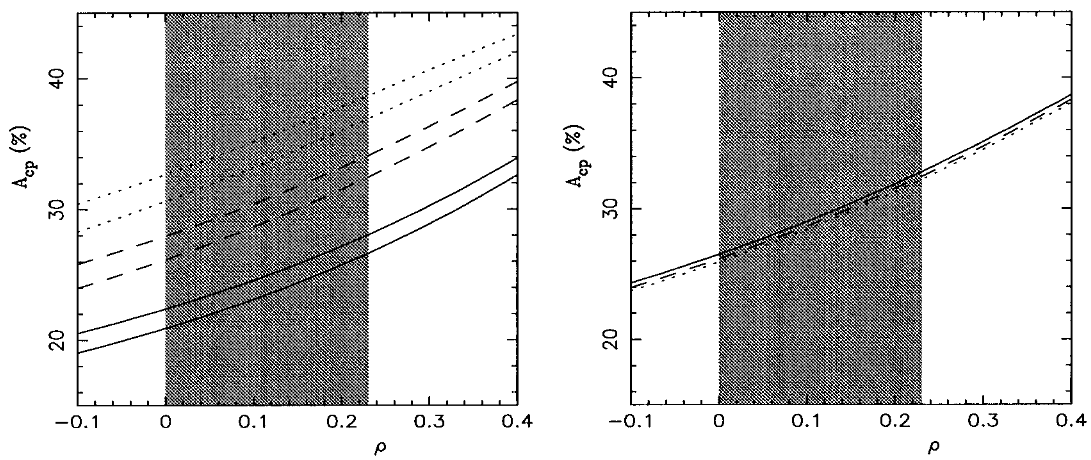



FIG. 3. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to \pi^+\pi^-$ as a function of $\sin 2\alpha$ for $k^2 = m_b^2/2$. (a) Effect of the ''penguin pollution'': the lower (upper) curve corresponds to keeping only the tree contribution (the complete amplitude, tree+penguin). Note that $|V_{ub}| = 0.003$. (b) Dependence on $|V_{ub}|$: $|V_{ub}| = 0.002$ (solid curve), $|V_{ub}| = 0.003$ (dashed curve), $|V_{ub}| = 0.004$ (dotted curve).



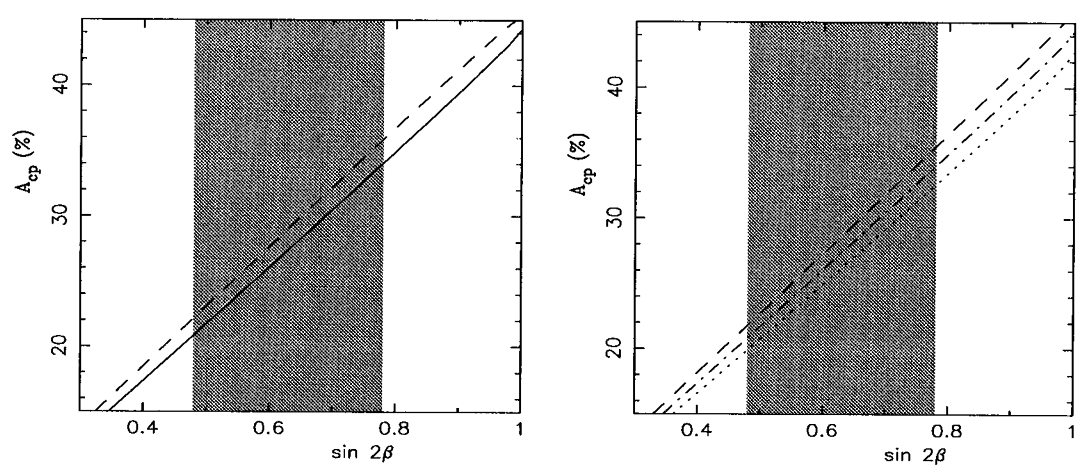

(a) (b)

FIG. 4. *CP*-asymmetry parameters $a_{\epsilon'}$ (a) and $a_{\epsilon+\epsilon'}$ (b) for $\overset{(-)}{B^0} \to K_S^0 \eta'$ as a function of the CKM parameter $\rho$. The dotted, dashed, and solid curves correspond to the CKM parameter values $\eta = 0.42$, $\eta = 0.34$, and $\eta = 0.26$, respectively.




(a) (b)

FIG. 5. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K_S^0 \eta'$ decays as a function of the CKM parameter $\rho$. (a) $k^2 = m_b^2/2$. The dotted, dashed, and solid curves correspond to the CKM parameter values $\eta = 0.42$, $\eta = 0.34$, and $\eta = 0.26$, respectively. In all three groups, the upper (lower) curve corresponds to neglecting the tree contributions (with the complete amplitude). (b) $\eta = 0.34$. The dotted, dashed, and solid curves correspond to $k^2 = m_b^2/2 + 2\ \mathrm{GeV}^2$, $k^2 = m_b^2/2$, and $k^2 = m_b^2/2 - 2\ \mathrm{GeV}^2$, respectively.




(a) (b)

FIG. 6. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K_S^0 \eta'$ as a function of $\sin 2\beta$ for $k^2 = m_b^2/2$. (a) "Tree shadow": the solid (dashed) curve corresponds to the full amplitude (neglecting the tree contribution). (b) $|V_{td}|$ dependence: dashed curve ($|V_{td}| = 0.004$), dashed-dotted curve ($|V_{td}| = 0.008$), dotted curve ($|V_{td}| = 0.012$).

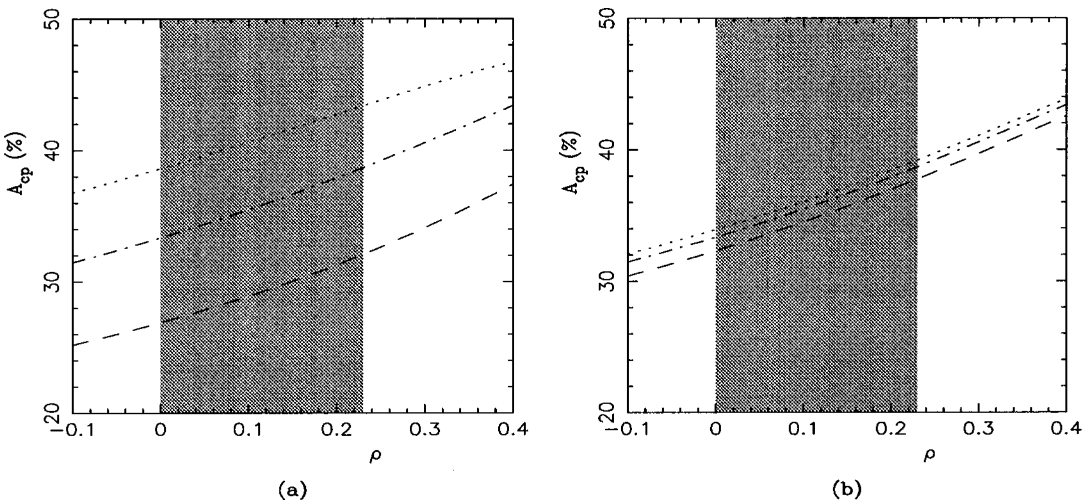



FIG. 7. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K_S^0 \pi^0$ decays as a function of the CKM parameter $\rho$. (a) $k^2 = m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta$=0.42, $\eta$=0.34, and $\eta$=0.26, respectively. (b) $\eta$=0.34. The dotted, dashed-dotted, and dashed curves correspond to $k^2 = m_b^2/2 + 2$ GeV$^2$, $k^2 = m_b^2/2$, and $k^2 = m_b^2/2 - 2$ GeV$^2$, respectively.

ing both $A_{CP}(\pi^+\pi^-)$ and its functional dependence on $\sin 2\alpha$. Based on Fig. 3(b), we estimate $-10\% \leq A_{CP}(\pi^+\pi^-) \leq +45\%$, with $A_{CP}(\pi^+\pi^-)=0$ as an allowed solution, varying $\sin 2\alpha$ in the $\pm 1\sigma$ range: $-0.40 \leq \sin 2\alpha \leq 0.53$ [20].

### *CP-violating asymmetry in* $\overset{(-)}{B^0} \to K_S^0 \eta'$.

The parameters $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ for the decays $\overset{(-)}{B^0} \to K_S^0 \eta'$ are shown in Figs. 4(a) and 4(b), respectively, for $\eta = 0.42$, 0.34, 0.26 with fixed $k^2 = m_b^2/2$. As can be seen from these figures, the time-integrated *CP*-violating asymmetry $A_{CP}(B^0 \to K_S^0 \eta')$ is completely dominated by the $a_{\epsilon+\epsilon'}$ term. The *CP*-violating asymmetry $A_{CP}(K_S^0 \eta')$ is shown in Fig. 5(a) for three values of $\eta$ ($\eta$=0.42, 0.34, 0.26). The upper curve for each value of $\eta$ is obtained by neglecting the tree contribution in $\overset{(-)}{B^0} \to K_S^0 \eta'$ and the lower curves represent the corresponding full (tree+penguin) contribution. Figure 5(b) shows the $k^2$-dependence of $A_{CP}(K_S^0 \eta')$ with the three (almost) overlapping curves corresponding to $k^2 = m_b^2/2$ and $k^2 = m_b^2/2 \pm 2$ GeV$^2$ for fixed value, $\eta$=0.34. As we see from this set of figures, the CKM-parametric dependence of $A_{CP}(K_S^0 \eta')$ is marked and the effect of the ''tree shadow'' is relatively small. To illustrate this further, we plot in Figs. 6(a) and 6(b) this asymmetry as a function of $\sin 2\beta$, showing the effect of the ''tree-shadowing'' and dependence of $A_{CP}(K_S^0 \eta')$ on$|V_{td}|$, respectively. Restricting to the range $0.48 \leq \sin 2\beta \leq 0.78$, which is the $\pm 1\sigma$ range for this quantity from the unitarity fits [20], we find that $A_{CP}(K_S^0 \eta')$ has a value in the range $20\% < A_{CP} < 36\%$. This decay has been measured by the CLEO Collaboration with a branching ratio $\mathcal{B}(B^0 \to K_S^0 \eta') = (4.7^{+2.7}_{-2.0} \pm 0.9) \times 10^{-5}$ and is well accounted for in the factorization-based approach [4–6]. As the ''tree shadow'' is small in the decay $B^0 \to K_S^0 \eta'$ and the electroweak penguin contribution is also small [4], $A_{CP}(K_S^0 \eta')$ is a good measure of $\sin 2\beta$. This was anticipated by London and Soni [44], who also advocated $A_{CP}(K_S^0 \phi)$ as a measure of the angle $\beta$, following the earlier suggestion of the same in Ref. [45]. The *CP*-asymmetry for this decay, like $A_{CP}(K_S^0 \eta')$, is dominated by the $a_{\epsilon+\epsilon'}$ term. The quantity $A_{CP}(K_S^0 \phi)$ is found to be stable against variation in $N_c$ and $k^2$ (see Tables IX and X). However, being a class-V decay, the branching ratio for $B^0 \to K_S^0 \phi$ (and its charged conjugate) is very sensitively dependent on $N_c$, with $\mathcal{B}(B^0 \to K_S^0 \phi) = (0.2-9) \times 10^{-6}$, with the lower (higher) range corresponding to $N_c = \infty$ ($N_c = 2$) [4]. Moreover, the electroweak penguin effect in this decay is estimated to be rather substantial. The present upper bound on this decay is $\mathcal{B}(B^0 \to K_S^0 \phi) < 6.2 \times 10^{-5}$ [2]. Depending on $N_c$, the above experimental bound is between one and two orders of magnitude away from the expected rate. Despite the large and stable value of $A_{CP}(K_S^0 \phi)$, it may turn out not to be measurable in the first generation of $B$ factory experiments.

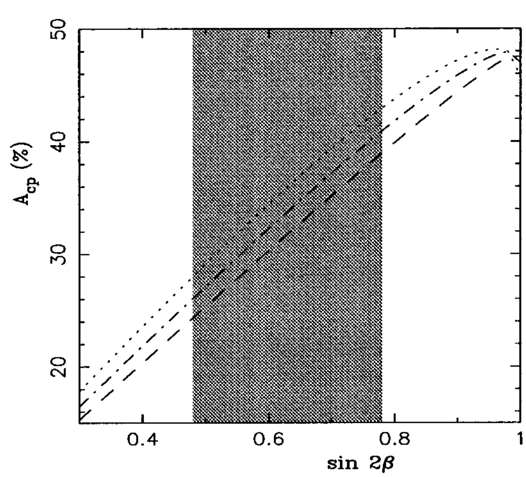



FIG. 8. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K_S^0 \pi^0$ as a function of $\sin 2\beta$ for $k^2 = m_b^2/2$. The three curves correspond to the following values of the CKM matrix element $|V_{td}|$: dashed curve ($|V_{td}| = 0.004$), dashed-dotted curve ($|V_{td}| = 0.008$), dotted curve ($|V_{td}| = 0.012$).

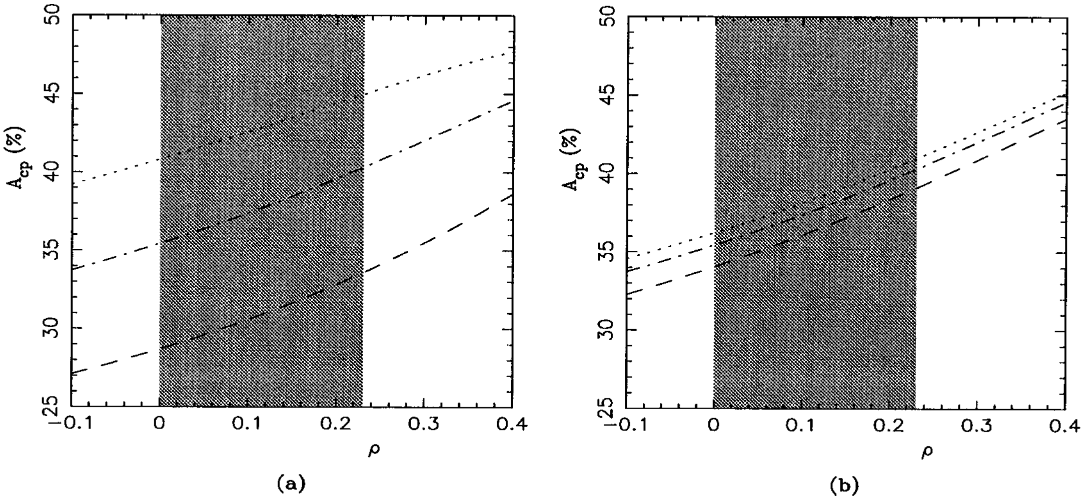



FIG. 9. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B}{}^0 \to K_S^0 \eta$ decays as a function of the CKM parameter $\rho$. (a) $k^2 = m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta = 0.42$, $\eta = 0.34$, and $\eta = 0.26$, respectively. (b) $\eta = 0.34$. The dotted, dashed-dotted, and dashed lines correspond to $k^2 = m_b^2/2 + 2\ \mathrm{GeV}^2$, $k^2 = m_b^2/2$, and $k^2 = m_b^2/2 - 2\ \mathrm{GeV}^2$, respectively.

### *CP-violating asymmetry in* $\overset{(-)}{B}{}^0 \to K_S^0 \pi^0$

The decay $B^0 \to K_S^0 \pi^0$ is dominated by the penguin diagrams, with significant electroweak penguin contribution [4]. The estimated decay rate in the factorization approach is $\mathcal{B}(B^0 \to K_S^0 \pi^0) = (2.5 - 5) \times 10^{-6}$, with the present experimental bound being $\mathcal{B}(B^0 \to K_S^0 \pi^0) < 4.1 \times 10^{-5}$ [1], with these numbers to be understood as averages over the charge conjugated decays. We expect that with $10^8$ $B\overline{B}$ events, several hundred $K_S^0 \pi^0$ decays will be measured. The *CP*-asymmetry $A_{CP}(K_S^0 \pi^0)$ is dominated by the $a_{\epsilon + \epsilon'}$ term (see Tables I and II), which is large, stable against variation in $k^2$ and shows only a mild dependence on $N_c$. The quantities $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ for this decay [together with the others in the $B \to \pi\pi, KK$ and $B^\pm \to (K\pi)^\pm$ decays] were worked out by Kramer and Palmer in [15]. As remarked already, there are detailed differences in the underlying theoretical framework used here and in [15] and also in the values of the CKM and other input parameters, but using identical values of the various input parameters for the sake of comparison, the agreement between the two is fair. We show in Fig. 7(a), $A_{CP}(K_S^0 \pi^0)$ as a function of $\rho$ for three values of $\eta$: $\eta = 0.42$, 0.34, 0.26 and note that this dependence is quite marked. The $k^2$-dependence of $A_{CP}(K_S^0 \pi^0)$ is found to be small, as shown in Fig. 7(b). Thus, we expect that $A_{CP}(K_S^0 \pi^0)$ is also a good measure of $\sin 2\beta$. This is shown in Fig. 8, with the three curves showing the additional dependence of $A_{CP}(K_S^0 \pi^0)$ on $|V_{td}|$. Restricting the value of $\sin 2\beta$ in the $\pm 1\sigma$ range shown by the shadowed region, we find $24\% \leqslant A_{CP}(K_S^0 \pi^0) \leqslant 44\%$.

### *CP-violating asymmetry in* $\overset{(-)}{B}{}^0 \to K_S^0 \eta$

The decay $B^0 \to K_S^0 \eta$, like the preceding decay, is dominated by the penguins diagrams with significant electroweak penguin contribution [4]. The branching ratio for this mode is estimated to be about a factor 3 too small compared to $B^0 \to K_S^0 \pi^0$, with $\mathcal{B}$ $(B^0 \to K_S^0 \eta) \simeq (1-2) \times 10^{-6}$. The *CP*-violating asymmetry $A_{CP}(K_S^0 \eta)$ is, however, found to be very similar to $A_{CP}(K_S^0 \pi^0)$. This is shown in Fig. 9(a) where we plot $A_{CP}(K_S^0 \eta)$ as a function of $\rho$ for the three indicated values of $\eta$, keeping $k^2 = m_b^2/2$ fixed. The $k^2$-dependence of $A_{CP}(K_S^0 \eta)$ is shown in Fig. 9(b) and is found to be moderately small in the range $k^2 = m_b^2/2 \pm 2\ \mathrm{GeV}^2$. We show in Fig. 10 $A_{CP}(K_S^0 \eta)$ as a function of $\sin 2\beta$, with the three curves showing three different values of $|V_{td}|$. Restricting again to the $\pm 1\sigma$ range of $\sin 2\beta$, we estimate: $24\% \leqslant A_{CP}(K_S^0 \eta) \leqslant 46\%$.

### *CP-violating asymmetry in* $\overset{(-)}{B}{}^0 \to K_S^0 h^0$*, with* $h^0 = \pi^0, K_S^0, \eta, \eta'$.

As the CKM-parametric dependence of the *CP*-violating asymmetries $A_{CP}(K_S^0 \pi^0)$, $A_{CP}(K_S^0 \eta)$, $A_{CP}(K_S^0 \eta')$ are very similar, one could combine these asymmetries. We estimate

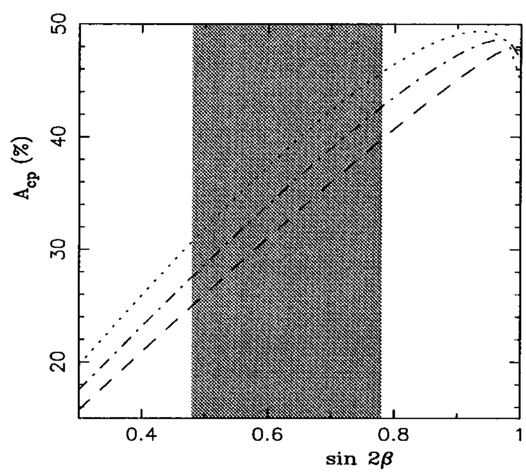



FIG. 10. *CP*-violating asymmetry $A_{CP}$ in $\overset{(-)}{B}{}^0 \to K_S^0 \eta$ as a function of $\sin 2\beta$ for $k^2 = m_b^2/2$. The three curves correspond to the following values of the CKM matrix element $|V_{td}|$: $|V_{td}| = 0.004$ (dashed curve), $|V_{td}| = 0.008$ (dashed-dotted curve), $|V_{td}| = 0.012$ (dotted curve).

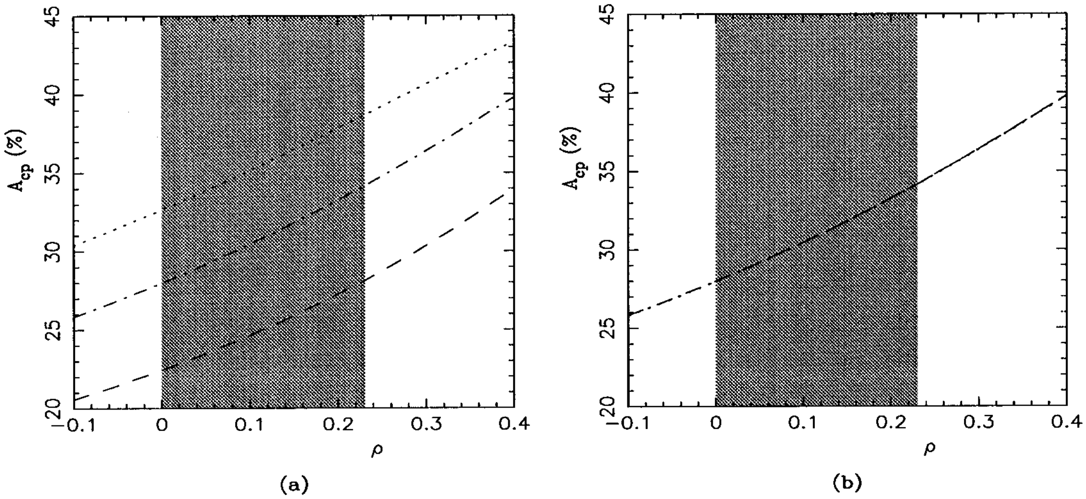



FIG. 11. $CP$-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K_S^0 h^0$ decays with $h^0 = \pi^0, K_S^0, \eta, \eta'$ as a function of the CKM parameter $\rho$. (a) $k^2 = m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta$=0.42, $\eta$=0.34, and $\eta$=0.26, respectively. (b) $\eta$=0.34. The overlapping curves correspond to $k^2 = m_b^2/2 + 2$ GeV$^2$, $k^2 = m_b^2/2$, and $k^2 = m_b^2/2 - 2$ GeV$^2$.

$\mathcal{B}(B^0 \to K_S^0 h^0) \simeq (2.7\text{–}4.6) \times 10^{-5}$, with $A_{CP}(K_S^0 h^0) \simeq (22\text{–}36)\%$, for $h^0 = \pi^0$, $\eta$ and $\eta'$. As the branching ratio for the decay $B^0 \to K_S^0 \overline{K^0}$ is estimated to be small, typically $\mathcal{B}(B^0 \to K_S^0 \overline{K^0}) \simeq 5 \times 10^{-7}$, the above estimates of $\mathcal{B}(B^0 \to K_S^0 h^0)$ and $A_{CP}(K_S^0 h^0)$ hold also to a very good approximation if we now also include $K_S^0$ in $h^0$. The dependence of $A_{CP}(K_S^0 h^0)$ on the CKM parameters $\rho$ and $\eta$ is shown in Fig. 11(a) and the $k^2$-dependence in Fig. 11(b). Interestingly, the $k^2$-dependence in various components which is already small gets almost canceled in the sum, yielding $A_{CP}(K_S^0 h^0)$ which is practically independent of $k^2$. We show the dependence of $A_{CP}(K_S^0 h^0)$ on $\sin 2\beta$ in Fig. 12, with the three curves representing each a different value of $|V_{td}|$. Thus, we predict $A_{CP}(K_S^0 h^0) \simeq (22\text{–}36)\%$, for $h^0 = \pi^0, K_S^0$, $\eta$ and $\eta'$ for the $\pm 1\sigma$ range of $\sin 2\beta$.

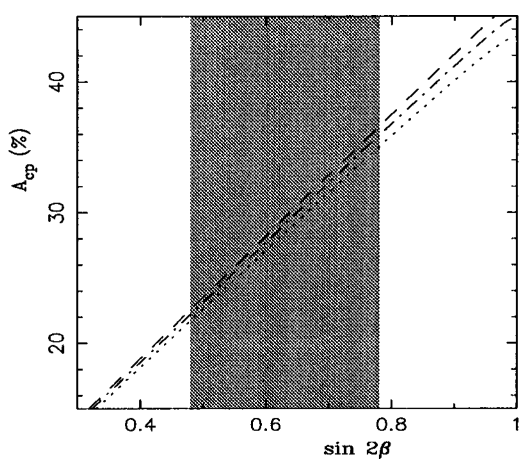



FIG. 12. $CP$-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K_S^0 h^0$ decays with $h^0 = \pi^0, K_S^0, \eta, \eta'$ as a function of $\sin 2\beta$ for $k^2 = m_b^2/2$. The three curves correspond to the following values of the CKM matrix element $|V_{td}|$: dashed curve ($|V_{td}| = 0.004$), dashed-dotted curve ($|V_{td}| = 0.008$), dotted curve ($|V_{td}| = 0.012$).

***CP*-violating asymmetry in $\overset{(-)}{B^0} \to \rho^+ \rho^-$.**

As another example of the decay whose $A_{CP}$ is stable against variation in $N_c$ and $k^2$, we remark that the decay mode $B^0 \to \rho^+ \rho^-$ is estimated to have an asymmetry $A_{CP} \simeq 10\%$, as can be seen in Tables XI and XII. This decay mode is dominated by the tree amplitudes (like $B^0 \to \pi^+ \pi^-$) and belongs to the $CP$ class (iii) decays. Estimated branching ratio for this mode is $\mathcal{B}(B^0 \to \rho^+ \rho^-) \simeq (2-3) \times 10^{-5}$.

### C. The decays $B^0 \to \rho^+ \pi^-$, $B^0 \to \rho^- \pi^+$ and $CP$-violating asymmetries

Next, we discuss decay modes which belong to the $CP$ class (iv) decays. There are four of them $B^0 \to \overline{K^{*0}} K_S^0$, $B^0 \to K^{*0} K_S^0$, $B^0 \to \rho^+ \pi^-$ and $B^0 \to \rho^- \pi^+$. Of these the decay $B^0 \to K^{*0} K_S^0$ belongs to the class-V decay and is estimated to have a very small branching ratio in the factorization approach $\mathcal{B}(B^0 \to K^{*0} K_S^0) \simeq O(10^{-9})$ [4]. The other $B^0 \to \overline{K^{*0}} K_S^0$ is a class-IV decay but is expected to have also a small branching ratio, with $\mathcal{B}(B^0 \to \overline{K^{*0}} K_S^0) \simeq (2-3) \times 10^{-7}$. In view of this, we concentrate on the decays $B^0 \to \rho^+ \pi^-$ and $\overline{B^0} \to \rho^- \pi^+$.

With $f = \rho^+ \pi^-$ and $\bar{f} = \rho^- \pi^+$, the time evolution of the four branching ratios is given in Eq. (40). They have each three components with characteristic time-dependences proportional to $e^{-\Gamma t}$, $e^{-\Gamma t} \cos \Delta m t$ and $e^{-\Gamma t} \sin \Delta m t$, with the relative and overall normalization explicitly stated there. The time dependence of the branching ratio $\mathcal{B}(B^0(t) \to \rho^- \pi^+)$ and of the branching ratio for the charge conjugate decay $\mathcal{B}(\overline{B^0}(t) \to \rho^+ \pi^-)$ is shown in Figs. 13(a) and 13(b), respectively. The time dependence of the branching ratio $\mathcal{B}(B^0(t) \to \rho^+ \pi^-)$ and of $\mathcal{B}(\overline{B^0}(t) \to \rho^- \pi^+)$ is shown in Figs. 14(a) and 14(b), respectively. The three components and the sum are depicted by the four curves.

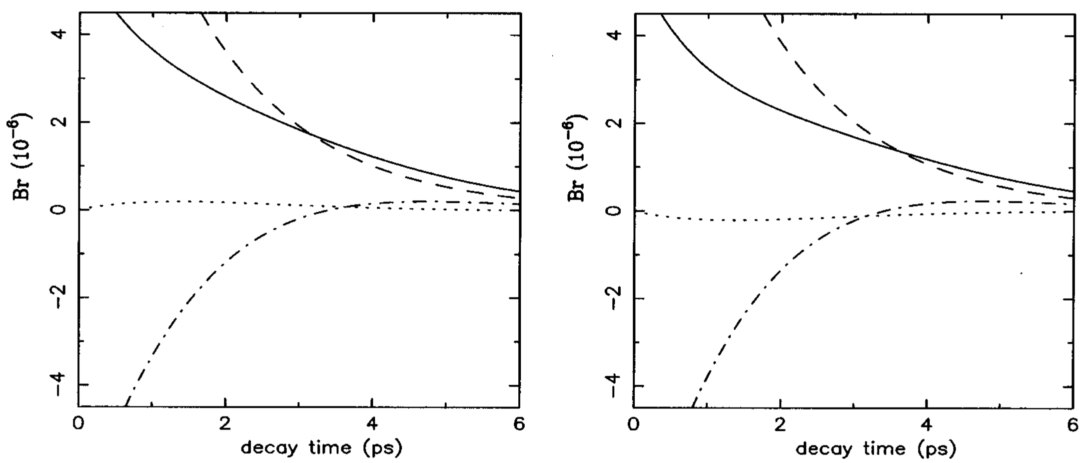



FIG. 13. Time-dependent branching ratio for the decays $B^0 \to \rho^- \pi^+$ (left) and $\overline{B^0} \to \rho^+ \pi^-$ (right) as a function of the decay time. The dashed, dashed-dotted, and dotted curves correspond to the contributions from the exponential decay term $e^{-\Gamma t}$, $e^{-\Gamma t}\cos \Delta m t$, and $e^{-\Gamma t}\sin \Delta m t$ in Eq. (39), respectively. The solid curve is the sum of the three contributions.

The resulting time-dependent $CP$-violating asymmetry $A_{CP}(t)$ for $B^0 \to \rho^- \pi^+$ defined as

$$A_{CP}(t;\rho^- \pi^+) \equiv \frac{\Gamma(B^0(t) \to \rho^- \pi^+) - \Gamma(\overline{B^0}(t) \to \rho^+ \pi^-)}{\Gamma(B^0(t) \to \rho^- \pi^+) + \Gamma(\overline{B^0}(t) \to \rho^+ \pi^-)}, \tag{41}$$

is shown in Fig. 15 through the solid curve. The corresponding asymmetry $A_{CP}(t;\rho^+ \pi^-)$ defined in an analogous way as for $A_{CP}(t;\rho^- \pi^+)$ is given by the dashed curve in this figure.

We recall that the decay rate for $B^0 \to \rho^+ \pi^-$ averaged over its charge conjugated decay $\overline{B^0} \to \rho^- \pi^+$ is estimated to have a value in the range $\mathcal{B}(B^0 \to \rho^+ \pi^-) \simeq (2-4) \times 10^{-5}$ [4]; the time-integrated $CP$-asymmetry is estimated to be $A_{CP}(\rho^+ \pi^-) \simeq (4-7)\%$. Being a class-I decay, both the branching ratio $\mathcal{B}(B^0 \to \rho^+ \pi^-)$ and $A_{CP}(\rho^+ \pi^-)$ are $N_c$-stable. In addition, $A_{CP}(\rho^+ \pi^-)$ is also $k^2$-stable, as shown in Table VII.

The branching ratio for the decay $B^0 \to \rho^- \pi^+$, averaged over its charge conjugate decay $\overline{B^0} \to \rho^+ \pi^-$, is expected to be $\mathcal{B}(B^0 \to \rho^- \pi^+) \simeq (6-9) \times 10^{-6}$ [4], i.e., typically a factor 4 smaller than $\mathcal{B}(B^0 \to \rho^+ \pi^-)$. Also, $A_{CP}(\rho^- \pi^+)$ is estimated somewhat smaller for the central value of the CKM-parameter $\rho = 0.12$, $\eta = 0.34$. For these CKM parameters, we estimate $A_{CP}(\rho^- \pi^+) \simeq (3-4)\%$. For $\rho = 0.23$, $\eta = 0.42$, $A_{CP}(\rho^- \pi^+) \simeq A_{CP}(\rho^+ \pi^-) \simeq O(5\%)$ (see Table VIII).

We note that our estimate of the ratio $\mathcal{B}(B^0 \to \rho^+ \pi^-)/\mathcal{B}(B^0 \to \pi^+ \pi^-) \simeq 2.3$ derived in [4] is in reasonable agreement with the corresponding ratio estimated in [41] but we also find $\mathcal{B}(B^0 \to \rho^- \pi^+)/\mathcal{B}(B^0 \to \rho^+ \pi^-) \simeq 0.27$, which is drastically different from the estimates presented in [41].

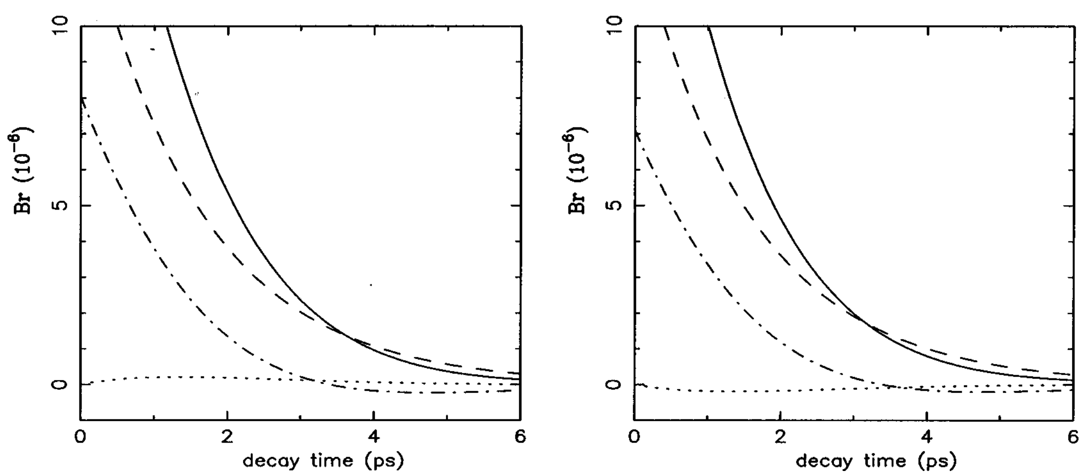



FIG. 14. Time-dependent branching ratio for the decays $B^0 \to \rho^+ \pi^-$ (left) and $\overline{B^0} \to \rho^- \pi^+$ (right) as a function of the decay time. The dashed, dashed-dotted, and dotted curves correspond to the contributions from the exponential decay term $e^{-\Gamma t}$, $e^{-\Gamma t}\cos \Delta m t$, and $e^{-\Gamma t}\sin \Delta m t$ in Eq. (39), respectively. The solid curve is the sum of the three contributions.

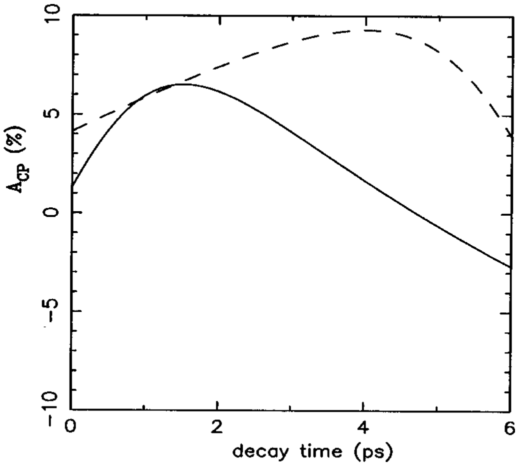



FIG. 15. Time-dependent *CP*-violating asymmetry $A_{CP}(t;\rho^-\pi^+)$ (solid curve) and $A_{CP}(t;\rho^+\pi^-)$ (dashed curve) as a function of the decay time, with $\rho$=0.12, $\eta$=0.34, and $k^2=m_b^2/2$.

### D. Decay modes with measurable but $k^2$-dependent $A_{CP}$

In addition to the decay modes discussed above, the following decay modes have $A_{CP}$ which are $N_c$-and $\mu$-stable but show significant or strong $k^2$-dependence. However, we think that further theoretical work and/or measurements of $A_{CP}$ in one or more of the following decay modes will greatly help in determining $k^2$ and hence in reducing the present theoretical dispersion on $A_{CP}$.

$$B^\pm\to\pi^\pm\eta',\ \overset{(-)}{B^0}\to K^{*\pm}\pi^\mp,\ B^\pm\to K^{*\pm}\pi^0,\ B^\pm\to K^{*\pm}\eta,\ B^\pm\to K^{*\pm}\eta',\ \overset{(-)}{B^0}\to K^{*\pm}\rho^\mp,\ B^\pm\to K^{*\pm}\rho^0$$

These decays have branching ratios which are estimated to be several multiples of $10^{-5}$ to several multiples of $10^{-6}$ and may have $|A_{CP}|$ at least of $O(5\%)$, but being uncertain due to the $k^2$-dependence may reach rather large values. The *CP*-violating asymmetries in these cases belong to the class (i), i.e., they are direct *CP*-violating asymmetries.

In Figs. 16(a) and 16(b), we show the *CP*-violating asymmetry $A_{CP}(K^{*\pm}\pi^0)$ as a function of $\rho$. The three curves in Fig. 16(a) correspond to the three choices of $\eta$, with $k^2=m_b^2/2$, whereas the three curves in Fig. 16(b) correspond to using $k^2=m_b^2/2+2\ \mathrm{GeV}^2$ (dotted curve), $k^2=m_b^2/2$ (dashed-dotted curve), $k^2=m_b^2/2-2\ \mathrm{GeV}^2$ (dashed curve) with $\eta=0.34$. Depending on the value of $k^2$, $A_{CP}(K^{*\pm}\pi^0)$ could reach a value $-25\%$. The branching ratio is estimated to lie in the range $\mathcal{B}(B^+\to K^{*+}\pi^0)\simeq(4-7)\times10^{-6}$. The decay mode $B^+\to K^{*+}\rho^0$ has very similar CKM and $k^2$-dependence, which is shown in Figs. 17(a) and 17(b), respectively, where we plot the *CP*-asymmetry $A_{CP}(K^{*\pm}\rho^0)$. Also, the branching ratio $\mathcal{B}(B^+\to K^{*+}\rho^0)\simeq(5-8)\times10^{-6}$ estimated in [4] is very similar to $B^+\to K^{*+}\pi^0$.

In Figs. 18(a) and 18(b), we show the *CP*-violating asymmetry $A_{CP}(K^{*\pm}\eta')$ in the decays $B^\pm\to K^{*\pm}\eta'$. This is a class-III decay dominated by the tree amplitude and is expected to have a branching ratio $\mathcal{B}(B^+\to K^{*+}\eta')\simeq3\times10^{-7}$, where an average over the charge conjugated decays is implied. However, depending on the value of $k^2$ this decay mode may show a large *CP*-violating asymmetry, reaching $A_{CP}(K^{*+}\eta')\simeq-90\%$ for $\rho=0.12$, $\eta=0.34$ and $k^2=m_b^2/2+2\ \mathrm{GeV}^2$. For $k^2=m_b^2/2-2\ \mathrm{GeV}^2$, the *CP*-asymmetry comes down to a value $A_{CP}(K^{*\pm}\eta')\simeq-20\%$. All of these values are significantly higher than the ones reported in [19]. Large but $k^2$-sensitive values of this quantity have also been reported earlier in [15].

We also mention here the decay modes $B^\pm\to K^{*\pm}\eta$, whose branching ratio is estimated as $\mathcal{B}(B^+\to K^{*+}\eta)\simeq(2-3)\times10^{-6}$ [5,6,4] and which may have *CP*-violating asymmetry in the range $A_{CP}(K^{*\pm}\eta)\simeq-(4-15)\%$ depending on the CKM parameters and $k^2$ (see Tables IX and X).

Finally, we mention two more decay modes $B^0\to K^{*+}\pi^-$ and $B^0\to K^{*+}\rho^-$ which are both class-IV decays, with branching ratios estimated as $\mathcal{B}(B^0\to K^{*+}\pi^-)\simeq(6-9)\times10^{-6}$ and $\mathcal{B}(B^0\to K^{*+}\rho^-)\simeq(5-8)\times10^{-6}$ [4].

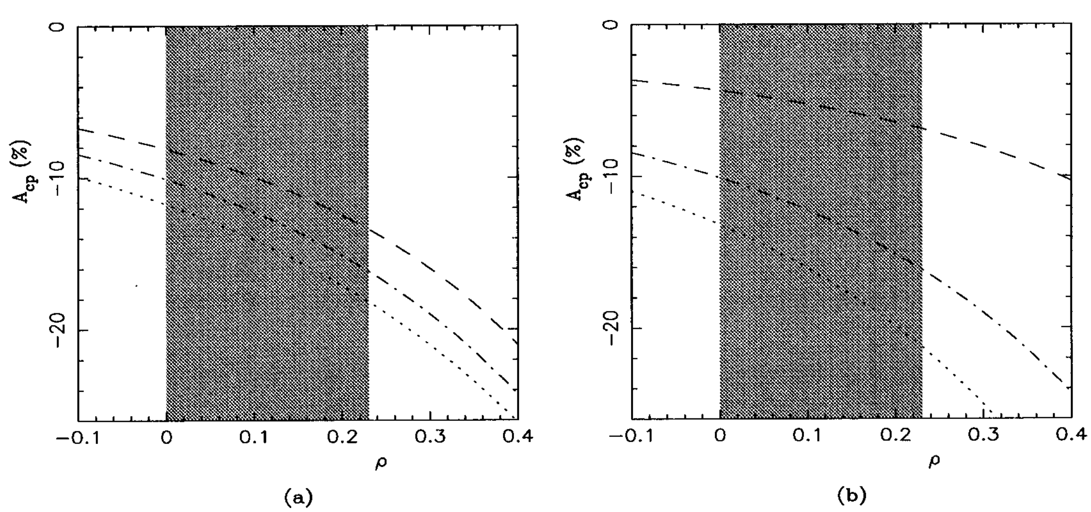



FIG. 16. *CP*-violating asymmetry $A_{CP}$ in the decays $B^\pm\to K^{*\pm}\pi^0$ as a function of the CKM parameter $\rho$. (a) $k^2=m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta$=0.42, $\eta$=0.34, and $\eta$=0.26, respectively. (b) $\eta$=0.34. The dotted, dashed-dotted, and dashed curves correspond to $k^2=m_b^2/2+2\ \mathrm{GeV}^2$, $k^2=m_b^2/2$, and $k^2=m_b^2/2-2\ \mathrm{GeV}^2$, respectively.

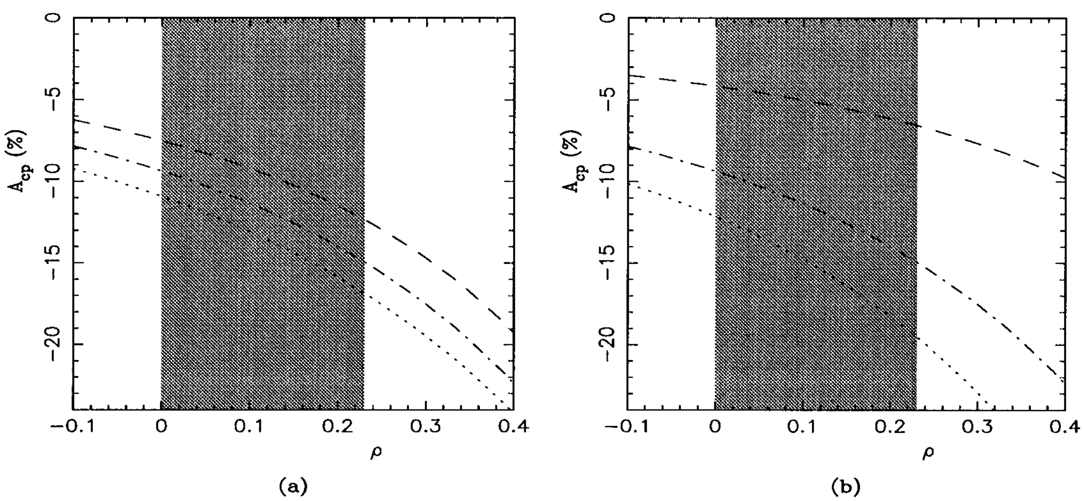



FIG. 17. *CP*-violating asymmetry $A_{CP}$ in $B^{\pm}\rightarrow K^{*\pm}\rho^{0}$ decays as a function of the CKM parameter $\rho$. (a) $k^2=m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta$=0.42, $\eta$=0.34, and $\eta$=0.26, respectively. (b) $\eta$=0.34. The dotted, dashed-dotted, and dashed curves correspond to $k^2=m_b^2/2+2$ GeV$^2$, $k^2=m_b^2/2$, and $k^2=m_b^2/2-2$ GeV$^2$, respectively.

The *CP*-violating asymmetries in these decays are estimated to lie in the range $A_{CP}(K^{*\pm}\pi^{\mp})=A_{CP}(K^{*\pm}\rho^{\mp})\simeq -(6-30)\%$. In Figs. 19(a) and 19(b), we show $A_{CP}(K^{*\pm}\pi^{\mp})$ as a function of $\rho$ by varying $\eta$ and $k^2$, respectively.

## V. SUMMARY AND CONCLUSIONS

Using the NLO perturbative framework and a generalized factorization approach discussed in detail in [4], we have calculated the *CP*-violating asymmetries in partial decay rates of all the two-body nonleptonic decays $B\rightarrow h_1 h_2$, where $h_1$ and $h_2$ are the light pseudoscalar and vector mesons. Our results can be summarized as follows.

We find that the decay classification scheme presented in [4] for the branching ratios is also very useful in discussing the *CP*-violating asymmetries. In line with this, class-I and class-IV decays yield asymmetries which are stable against the variation of $N_c$. There are two exceptions, $A_{CP}(K^{\pm}\eta)$ and $A_{CP}(K_S^0\eta)$, which vary by a factor 3 and 1.65, respectively, for $2\leqslant N_c\leqslant\infty$.

Estimates of *CP*-violating asymmetries in class-II and class-V decays depend rather sensitively on $N_c$ and hence are very unreliable. There is one notable exception $A_{CP}(\phi K_S^0)$, which is parametrically stable and large. However being a class-V decay, the branching ratio $\mathcal{B}(B^0\rightarrow\phi K_S^0)$ is uncertain in the factorization approach by at least an order of magnitude [4].

The *CP*-asymmetries in class-III decays vary by approximately a factor 2, as one varies $N_c$ in the range $2\leqslant N_c\leqslant\infty$, with the exception of $A_{CP}(\omega\pi^{\pm})$ and $A_{CP}(\rho^0\pi^{\pm})$ which are much more uncertain. The $N_c$-sensitivity of $\mathcal{B}(B^{\pm}\rightarrow\omega\pi^{\pm})$ was already pointed out in [4].

The *CP*-violating asymmetries worked out here are in

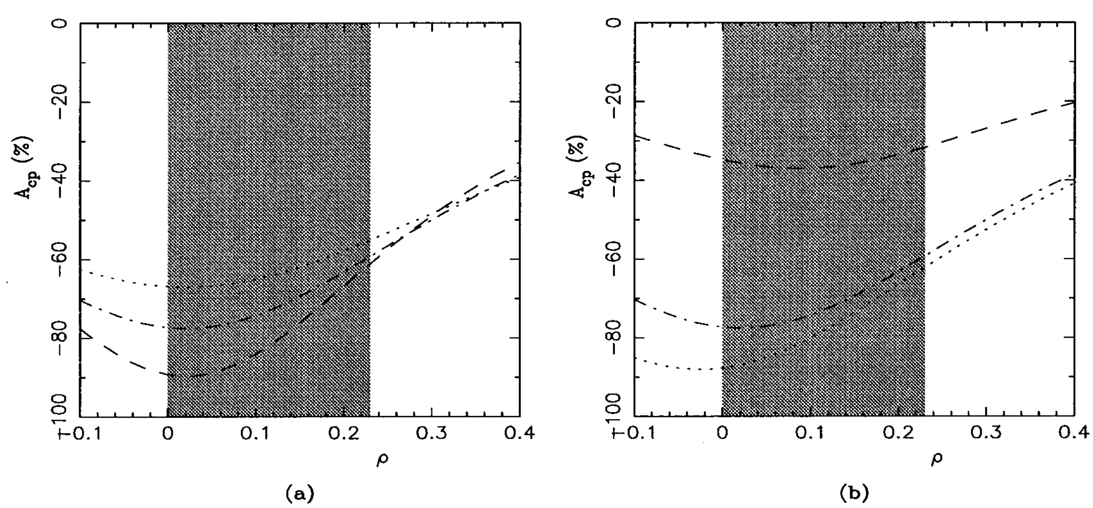



FIG. 18. *CP*-violating asymmetry $A_{CP}$ in $B^{\pm}\rightarrow K^{*\pm}\eta'$ decays as a function of the CKM parameter $\rho$. (a) $k^2=m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta$=0.42, $\eta$=0.34, and $\eta$=0.26, respectively. (b) $\eta$=0.34. The dotted, dashed-dotted, and dashed curves correspond to $k^2=m_b^2/2+2$ GeV$^2$, $k^2=m_b^2/2$, and $k^2=m_b^2/2-2$ GeV$^2$, respectively.

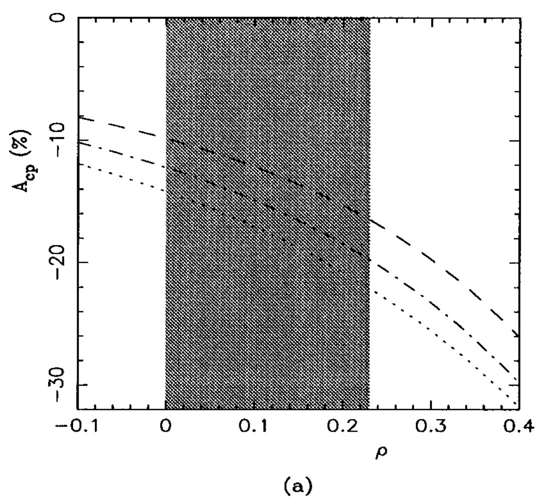



FIG. 19. $CP$-violating asymmetry $A_{CP}$ in $\overset{(-)}{B^0} \to K^{*\pm}\pi^{\mp}$ decays as a function of the CKM parameter $\rho$. (a) $k^2 = m_b^2/2$. The dotted, dashed-dotted, and dashed curves correspond to the CKM parameter values $\eta = 0.42$, $\eta = 0.34$, and $\eta = 0.26$, respectively. (b) $\eta = 0.34$. The dotted, dashed-dotted, and dashed curves correspond to $k^2 = m_b^2/2 + 2\ \mathrm{GeV}^2$, $k^2 = m_b^2/2$, and $k^2 = m_b^2/2 - 2\ \mathrm{GeV}^2$, respectively.

most cases relatively insensitive to the scale $\mu$, i.e., this dependence is below $\pm 20\%$, for $m_b/2 \leqslant \mu \leqslant m_b$, except in some decays which we have listed in Table XIII.

As opposed to the branching ratios, asymmetries do not depend in the first approximation on the form factors and decay constants. However, in most cases, they depend on the parameter $k^2$, the virtuality of the $g$, $\gamma$ and $Z^0$ decaying into $q\bar{q}$ from the penguin contributions. This has been already studied in great detail in [15], a behavior which we have also confirmed.

Interestingly, we find that a number of $B \to h_1 h_2$ decays have $CP$-violating asymmetries which can be predicted within a reasonable range in the factorization approach. They include: $A_{CP}(\pi^+\pi^-)$, $A_{CP}(K_S^0\eta')$, $A_{CP}(K_S^0\pi^0)$, $A_{CP}(K_S^0\eta)$ and $A_{CP}(\rho^+\rho^-)$. The decay modes involved have reasonably large branching ratios and the $CP$-violating asymmetries are also measurably large in all these cases. Hence, their measurements can be used to put constraints on the CKM parameters $\rho$ and $\eta$. Likewise, these decay modes are well suited to test the hypothesis that strong phases in these decays are generated dominantly by perturbative QCD. This, in our opinion, is difficult to test in class-II and class-V decays. Of particular interest is $A_{CP}(K_S^0\eta')$, which is expected to have a value $A_{CP}(K_S^0\eta') \simeq (20\text{–}36)\%$. This decay mode has already been measured by the CLEO Collaboration [1] and estimates of its branching ratio in the factorization approach are in agreement with data [4–6].

The $CP$-asymmetry $A_{CP}(K_S^0 h^0)$, where $h^0 = \pi^0, K_S^0, \eta, \eta'$ is found to be remarkably stable in $k^2$, due to the compensation in the various channels. The resulting $CP$-asymmetry is found to be large, with $A_{CP}(K_S^0 h^0) \simeq (20\text{–}36)\%$, with the range reflecting the CKM-parametric dependence.

We have studied the dependence of $A_{CP}(\pi^+\pi^-)$ on $\sin 2\alpha$, studying the effect of the "penguin pollution," which we find to be significant. The effect of the "tree-shadowing" in $A_{CP}(K_S^0\eta')$ is, however, found to be small. Thus, $A_{CP}(K_S^0\eta')$, likewise $A_{CP}(K_S^0\pi^0)$, $A_{CP}(K_S^0\eta)$ and $A_{CP}(K_S^0 h^0)$ are good measures of $\sin 2\beta$.

We have studied time-dependent $CP$-violating asymmetries $A_{CP}(t;\rho^+\pi^-)$ and $A_{CP}(t;\rho^-\pi^+)$, working out the various characteristic components in the time evolution of the individual branching ratios. With the branching ratio averaged over the charge-conjugated modes $\mathcal{B}(B^0 \to \rho^+\pi^-) = (2-4)\times 10^{-5}$ and time-integrated $CP$-violating asymmetry $A_{CP}(\rho^+\pi^-) = (4-7)\%$, for the central values $\rho = 0.12$ and $\eta = 0.34$, it is an interesting process to measure, as stressed in [41]. The branching ratio $\mathcal{B}(B^0 \to \rho^-\pi^+)$ is estimated by us as typically a factor 4 below $\mathcal{B}(B^0 \to \rho^+\pi^-)$ and hence $A_{CP}(\rho^-\pi^+)$ is a relatively more difficult measurement.

There are several class-IV decays whose $CP$-asymmetries are small but stable against variation in $N_c$, $k^2$ and $\mu$. They include: $A_{CP}(K^\pm\eta')$, $A_{CP}(\pi^\pm K_S^0)$ and $A_{CP}(\rho^\pm \overset{(-)}{K^{*0}})$. $CP$-asymmetries well over 5% in these decay modes can arise through SFI and/or new physics. We argue that the role of SFI can be disentangled already in decay rates and through the measurements of a number of $CP$-violating asymmetries which are predicted to be large. As at this stage it is hard to quantify the effects of SFI, one can not stress too strongly that a measurement of $CP$-violating asymmetry in any of these partial rates significantly above the estimates presented here will be a sign of new physics.

There are quite a few other decay modes which have measurably large $CP$-violating asymmetries, though without constraining the parameter $k^2$ experimentally, or removing this dependence in an improved theoretical framework, they are at present rather uncertain. A good measurement of the $CP$-asymmetry in any one of these could be used to determine $k^2$. We list these potentially interesting asymmetries below:

$$A_{CP}(K^{*\pm}\pi^\mp), \quad A_{CP}(K^{*\pm}\pi^0), \quad A_{CP}(K^{*\pm}\eta),$$

$$A_{CP}(K^{*\pm}\eta'), \quad A_{CP}(K^{*\pm}\rho^\mp) \text{ and } A_{CP}(K^{*\pm}\rho^0).$$

In conclusion, by systematically studying the $B\to h_1 h_2$ decays in the factorization approach, we hope that we have found classes of decays where the factorization approach can be tested as it makes predictions within a reasonable range. If the predictions in the rates in these decays are borne out by data, then it will strengthen the notion based on color transparency that nonfactorization effects in decay rates are small and QCD dynamics in $B\to h_1 h_2$ decays can be largely described in terms of perturbative QCD and factorized amplitudes. This will bring in a number of *CP*-violating asymmetries under quantitative control of the factorization-based theory. If these expectations did not stand the experimental tests, attempts to quantitatively study two-body nonleptonic decays would have to wait for a fundamental step in the QCD technology enabling a direct computation of the four-quark matrix elements in the decays $B\to h_1 h_2$. However, present data on $B\to h_1 h_2$ decays are rather encouraging and perhaps factorization approach is well poised to becoming a useful theoretical tool in studying nonleptonic *B* decays—at least in class-I and class-IV decays. We look forward to new experimental results where many of the predictions presented here and in [4] will be tested in terms of branching ratios and *CP*-violating asymmetries in partial decay rates.

## ACKNOWLEDGMENTS

We thank Christoph Greub and Jim Smith for helpful discussions. We would also like to thank Alexey Petrov for clarifying some points in his paper [19]. One of us (A.A.) would like to thank Matthias Neubert for discussions on the factorization approach, and the CERN-TH Division for hospitality where this work was completed.

G.K. was supported by Bundesministerium für Bildung und Forschung, Bonn, under Contract 057HH92P(0) and EEC Program ''Human Capital and Mobility'' through Network ''Physics at High Energy Colliders'' under Contract CHRX-CT93-0357 (DG12COMA). C.-D.L. thanks the Alexander-von-Humboldt Foundation for financial support.

[1] CLEO Collaboration, R. Godang *et al.*, Phys. Rev. Lett. **80**, 3456 (1998); CLEO Collaboration, B. H. Behrens *et al.*, *ibid.* **80**, 3710 (1998); J. G. Smith, in Proceedings of the Seventh International Symposium on Heavy Flavor Physics, Santa Barbara, California, 1997, COLO-HEP-395, 1998.
[2] CLEO Collaboration, T. Bergfeld *et al.*, Phys. Rev. Lett. **81**, 272 (1998).
[3] For a recent experimental review on penguin diagrams in *B* meson decays and references to experimental and theoretical work, see K. Lingel, T. Skwarnicki, and J. G. Smith, Annu. Rev. Nucl. Part. Sci. (to be published), Report No. COLO-HEP-395, SLAC-PUB-7796, HEPSY 98-1, hep-ex/9804015.
[4] A. Ali, G. Kramer, and C. D. Lü, Phys. Rev. D **58**, 094009 (1998).
[5] A. Ali and C. Greub, Phys. Rev. D **57**, 2996 (1998).
[6] A. Ali, J. Chay, C. Greub, and P. Ko, Phys. Lett. B **424**, 161 (1998).
[7] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[8] For reviews, see I. I. Bigi *et al.*, in *CP Violation*, edited by C. Jarlskog, Advanced Series on Directions in High Energy Physics Vol. 3 (World Scientific, Singapore, New Jersey, London, Hong Kong, 1989); Y. Nir and H. R. Quinn, in *B Decays*, edited by S. Stone (World Scientific, Singapore, New Jersey, London, Hong Kong, 1989); R. Fleischer, Int. J. Mod. Phys. A **12**, 2459 (1997).
[9] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979); J. M. Gerard and W. S. Hou, Phys. Rev. D **43**, 2909 (1991); H. Simma, G. Eilam, and D. Wyler, Nucl. Phys. **B352**, 367 (1991).
[10] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
[11] M. Gronau, O. Hernandez, D. London, and J. Rosner, Phys. Rev. D **50**, 4529 (1994).
[12] M. Gronau, O. Hernandez, D. London, and J. Rosner, Phys. Rev. D **52**, 6374 (1995).
[13] N. G. Deshpande and X.-G. He, Phys. Lett. B **336**, 471 (1994); Phys. Rev. Lett. **74**, 26 (1995); N. G. Deshpande, X.-G. He, and J. Trampetic, Phys. Lett. B **345**, 547 (1995).
[14] H. Simma and D. Wyler, Phys. Lett. B **272**, 395 (1991).
[15] G. Kramer, W. F. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994); Z. Phys. C **66**, 429 (1995); G. Kramer and W. F. Palmer, Phys. Rev. D **52**, 6411 (1995); G. Kramer, W. F. Palmer, and Y. L. Wu, Commun. Theor. Phys. **27**, 457 (1997).
[16] D. Du and L. Guo, Z. Phys. C **75**, 9 (1997).
[17] N. G. Deshpande, B. Dutta, and S. Oh, Phys. Rev. D **57**, 5723 (1998).
[18] N. G. Deshpande, B. Dutta, and S. Oh, Report No. OITS-644, COLO-HEP-394, hep-ph/9712445.
[19] A. Petrov, Phys. Rev. D **58**, 054004 (1998).
[20] A. Ali, to be published in Proceedings of the First APCTP Workshop, Pacific Particle Physics Phenomenology, Seoul, South Korea, DESY Report 97-256, hep-ph/9801270.
[21] F. Parodi, P. Roudeau, and A. Stocchi, hep-ph/9802289.
[22] J. F. Donoghue *et al.*, Phys. Rev. Lett. **77**, 2178 (1996).
[23] D. Atwood and A. Soni, Phys. Rev. D **58**, 036005 (1998).
[24] A. F. Falk *et al.*, Phys. Rev. D **57**, 4290 (1998).
[25] M. Neubert, Phys. Lett. B **424**, 152 (1998).
[26] D. Delepine *et al.*, Phys. Lett. B **429**, 106 (1998).
[27] D.-S. Du *et al.*, hep-ph/9805260.
[28] J. D. Bjorken, in *New Developments in High Energy Physics*, edited by E. G. Floratos and A. Verganelakis [Nucl. Phys. B (Proc. Suppl.) **11**, 321 (1989)].
[29] M. Ciuchini, E. Franco, G. Martinelli, and L. Silvestrini, Nucl. Phys. **B501**, 271 (1997); M. Ciuchini *et al.*, *ibid.* **B512**, 3 (1998).
[30] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[31] M. Neubert and B. Stech, in Heavy Flavors, edited by A. J. Buras and M. Lindner, 2nd ed. (World Scientific, Singapore, 1998).
[32] M. Bauer and B. Stech, Phys. Lett. **152B**, 380 (1985); M.

Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).
[33] H. Simma and D. Wyler, Nucl. Phys. **B344**, 283 (1990).
[34] M. B. Voloshin, Phys. Lett. B **397**, 275 (1997).
[35] A. J. Buras and R. Fleischer, Phys. Lett. B **341**, 379 (1995).
[36] R. Fleischer, Phys. Lett. B **321**, 259 (1994).
[37] R. Fleischer and T. Mannel, Phys. Lett. B **397**, 269 (1997).
[38] R. Fleischer and T. Mannel, Phys. Rev. D **57**, 2752 (1998); Report No. TTP-97-22, hep-ph/9706261.
[39] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[40] M. Gronau, Phys. Lett. B **233**, 479 (1989).
[41] R. Aleksan, I. Dunietz, B. Kayser, and F. Le Diberder, Nucl. Phys. **B361**, 141 (1991).
[42] W. F. Palmer and Y. L. Wu, Phys. Lett. B **350**, 245 (1995).
[43] A. Ali, H. Asatrian, and C. Greub, Phys. Lett. B **429**, 87 (1998).
[44] D. London and A. Soni, Phys. Lett. B **407**, 61 (1997).
[45] D. London and R. D. Peccei, Phys. Lett. B **223**, 257 (1989). See also, Nir and Quinn [8].