High energy photon-neutrino interactions

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A general decomposition of the amplitudes for the $2\to2$ processes $\gamma\nu\to\gamma\nu$ and $\gamma\gamma\to\nu\bar{\nu}$ is obtained using gauge invariance and Bose symmetry. The restrictions implied by this decomposition are investigated for the reaction $\gamma\gamma\to\nu\bar{\nu}$ by computing the one-loop helicity amplitudes in the standard model. In the center of mass, where $\sqrt{s}=2\omega$, the cross section grows roughly as ω^6 up to the threshold for W-boson production, $\sqrt{s}=2m_W$. Astrophysical implications of very high energy photon-neutrino interactions are discussed. [S0556-2821(99)04001-1]

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I. INTRODUCTION

Investigations of the astrophysical importance of the $2\to2$ processes $\gamma\nu\to\gamma\nu$, $\gamma\gamma\to\nu\bar{\nu}$ and $\nu\bar{\nu}\to\gamma\gamma$ have a long history. These reactions have been examined using the four-Fermi interaction [1], charged vector boson theories [2,3], the standard model [4,5] and model-independent parametrizations [6]. Because of the vector–axial-vector nature of the weak coupling, the cross sections are much smaller than a simple counting of powers of the weak coupling, G_F, and the fine structure constant, α, in the diagrams of Fig. 1 would suggest [7,8]. For massless neutrinos, the cross sections vanish to order $G_F^2\alpha^2\omega^2$ [8], and are known to be of order $G_F^4\omega^6$ when $\omega< m_e$ [3,5]. This is due to the fact that the scale of the loop integrals for the diagrams in Fig. 1 is set by the W-boson mass, m_W, rather than the electron mass. As a consequence, for center of mass energies 2ω between 1 keV and 1 MeV, the cross sections for $2\to3$ processes such as $\gamma\nu\to\gamma\gamma\nu$ are larger than the $2\to2$ cross sections [9].

Although of little practical importance in stellar processes where the energy scale is $\lesssim 1$ MeV, the $2\to2$ processes could be important in very high energy reactions such as $\nu\bar{\nu}\to\gamma\gamma$ [10,11], particularly if the ω^6 behavior persists for $\omega> m_e$. The next section contains the development of a general decomposition of the elastic amplitude $\mathcal{A}(s,t,u)$ for $\gamma\nu\to\gamma\nu$. In Sec. III, we apply this decomposition to the crossed channel $\gamma\gamma\to\nu\bar{\nu}$ and obtain numerical results for the complete one-loop helicity amplitudes. This is followed by a discussion of our results.

II. GENERAL DECOMPOSITION OF THE $\gamma\nu$ ELASTIC AMPLITUDE

The amplitude for the process $\gamma\nu\to\gamma\nu$ can be expressed as

$$\mathcal{A}(s,t,u)=\overline{u}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)\mathcal{M}_{\mu\alpha\beta}(\varepsilon_1)_\alpha(\varepsilon_2^*)_\beta, \tag{1}$$

where $\mathcal{M}_{\mu\alpha\beta}$ is a sum of gauge invariant, Bose symmetric tensors constructed from the neutrino momenta p_1,p_2 and the photon momenta k_1,k_2. Construction of these tensors is aided by the fact that, for massless neutrinos, we have

$$(p_1)_\mu\overline{u}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)=0, \tag{2}$$

$$(p_2)_\mu\overline{u}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)=0, \tag{3}$$

$$(k_1-k_2)_\mu\overline{u}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)=0. \tag{4}$$

This suggests the use of the combinations

$$p=p_1+p_2, \tag{5a}$$

$$k_+=k_1+k_2, \tag{5b}$$

$$k_-=k_1-k_2. \tag{5c}$$

From the above, of p_α, p_β and p_μ, only p_α and p_β will result in non-vanishing contributions. Thus, the vector-vector and axial-vector–axial-vector contributions to $\mathcal{M}_{\mu\alpha\beta}$ will consist of combinations of p_α, p_β, $(k_+)_\mu$, $(k_2)_\alpha$ and $(k_1)_\beta$ as well as $\delta_{\mu\alpha}$, $\delta_{\mu\beta}$ and $\delta_{\alpha\beta}$.

The gauge invariant combinations of p_α and p_β are

$$p_\alpha-\frac{k_+\cdot p}{k_+^2}(k_2)_\alpha \quad\text{and}\quad p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta, \tag{6}$$

and we note that $k_+\cdot p=2k_1\cdot p=2k_2\cdot p$ and $k_+^2=2k_1\cdot k_2$. Gauge invariant second rank tensors involving $\delta_{\mu\alpha}$ and $\delta_{\mu\beta}$ are

$$(k_1\cdot k_2\delta_{\mu\alpha}-(k_1)_\mu(k_2)_\alpha) \quad\text{and}\quad (k_1\cdot k_2\delta_{\mu\beta}-(k_2)_\mu(k_1)_\beta), \tag{7}$$

$$(k_1\cdot p\,\delta_{\mu\alpha}-(k_1)_\mu p_\alpha) \quad\text{and}\quad (k_2\cdot p\,\delta_{\mu\beta}-(k_2)_\mu p_\beta), \tag{8}$$

and $\delta_{\alpha\beta}$ appears in the form

$$(k_1\cdot k_2\delta_{\alpha\beta}-(k_2)_\alpha(k_1)_\beta). \tag{9}$$

Gauge invariant third rank tensors can then be constructed from products of $(k_+)_\mu$, the vectors in Eq. (6), and the second rank tensors in Eqs. (7)–(9). In doing so, it should be noted that combinations such as

$$(k_1\cdot k_2\delta_{\mu\alpha}-(k_1)_\mu(k_2)_\alpha)\left(p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta\right), \tag{10}$$

$$(k_1\cdot p\,\delta_{\mu\alpha}-(k_1)_\mu p_\alpha)\left(p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta\right), \tag{11}$$

$$(k_+)_\mu\left(p_\alpha-\frac{k_+\cdot p}{k_+^2}(k_2)_\alpha\right)\left(p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta\right), \tag{12}$$

are not linearly independent since

$$\frac{k_+\cdot p}{k_1\cdot k_2}[\text{Eq.}(10)]-[\text{Eq. }(12)]=2[\text{Eq. }(11)]. \tag{13}$$

As a consequence, the tensors of Eq. (8) can be omitted when constructing a complete set of gauge invariant third rank tensors.

The requirement of Bose symmetry can easily be included if one constructs gauge invariant tensors that have a definite symmetry under the exchanges $k_1\to -k_2$, $k_2\to -k_1$ and $\alpha\leftrightarrow\beta$. The vector $k_+\to -k_+$ under these exchanges. It is possible to construct four linearly independent, gauge invariant third rank tensors with definite symmetry under the exchange of the photons. They can be chosen to be

$$\begin{aligned}T^{(1)}_{\mu\alpha\beta}=&(k_1\cdot k_2\delta_{\mu\alpha}-(k_1)_\mu(k_2)_\alpha)\left(p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta\right)\\&+(k_1\cdot k_2\delta_{\mu\beta}-(k_2)_\mu(k_1)_\beta)\left(p_\alpha-\frac{k_+\cdot p}{k_+^2}(k_2)_\alpha\right),\end{aligned} \tag{14}$$

$$\begin{aligned}T^{(2)}_{\mu\alpha\beta}=&(k_1\cdot k_2\delta_{\mu\alpha}-(k_1)_\mu(k_2)_\alpha)\left(p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta\right)\\&-(k_1\cdot k_2\delta_{\mu\beta}-(k_2)_\mu(k_1)_\beta)\left(p_\alpha-\frac{k_+\cdot p}{k_+^2}(k_2)_\alpha\right),\end{aligned} \tag{15}$$

$$T^{(3)}_{\mu\alpha\beta}=(k_+)_\mu(k_1\cdot k_2\delta_{\alpha\beta}-(k_2)_\alpha(k_1)_\beta), \tag{16}$$

$$T^{(4)}_{\mu\alpha\beta}=(k_+)_\mu\left(p_\alpha-\frac{k_+\cdot p}{k_+^2}(k_2)_\alpha\right)\left(p_\beta-\frac{k_+\cdot p}{k_+^2}(k_1)_\beta\right). \tag{17}$$

It can be seen that $T^{(1)}_{\mu\alpha\beta}$ is symmetric under the exchange of photons, while $T^{(2)}_{\mu\alpha\beta}$, $T^{(3)}_{\mu\alpha\beta}$ and $T^{(4)}_{\mu\alpha\beta}$ are antisymmetric.

To obtain tensors which correspond to the vector–axial-vector interference terms, one can use a set of independent momenta together with $\varepsilon_{\alpha\beta\lambda\rho}$ to form gauge invariant tensors with definite symmetry under the exchange of the photons. When this is done, one finds two tensors with the appropriate properties. They have the form

$$T^{(5)}_{\mu\alpha\beta}=(k_+)_\mu\varepsilon_{\alpha\beta\lambda\rho}(k_+)_\lambda(k_-)_\rho, \tag{18}$$

$$\begin{aligned}T^{(6)}_{\mu\alpha\beta}=&(k_+)_\mu\varepsilon_{\alpha\beta\lambda\rho}(k_+)_\lambda p_\rho-k_+^2\varepsilon_{\mu\alpha\beta\lambda}p_\lambda\\&-\delta_{\alpha\beta}\varepsilon_{\mu\nu\lambda\rho}(k_+)_\nu(k_-)_\lambda p_\rho\\&+p_\alpha\varepsilon_{\mu\beta\lambda\rho}(k_+)_\lambda(k_-)_\rho\\&+p_\beta\varepsilon_{\mu\alpha\lambda\rho}(k_+)_\lambda(k_-)_\rho.\end{aligned} \tag{19}$$

The gauge invariance of Eq. (19) can be verified using the identity

$$k_\mu\varepsilon_{\alpha\beta\lambda\rho}+k_\alpha\varepsilon_{\beta\lambda\rho\mu}+k_\beta\varepsilon_{\lambda\rho\mu\alpha}+k_\lambda\varepsilon_{\rho\mu\alpha\beta}+k_\rho\varepsilon_{\mu\alpha\beta\lambda}=0, \tag{20}$$

which holds for any vector k. Using this identity, the factor $\xi_\mu=\overline{u}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)$ in Eq. (1) can always be rewritten so that it is contracted with $\varepsilon_{\mu\nu\alpha\beta}$ instead of with $(k_+)_\mu$. The relation

$$-p_1\cdot p_2\xi_\mu=\varepsilon_{\mu\nu\alpha\beta}\xi_\nu(p_1)_\alpha(p_2)_\beta \tag{21}$$

then enables us to express any tensor containing an $\varepsilon_{\mu\nu\alpha\beta}$ as a sum of scalar products. These are simply combinations of the $T^{(i)}_{\mu\alpha\beta}$ of Eqs. (14)–(17). Explicitly, we find [12]

$$T^{(5)}_{\mu\alpha\beta}\propto T^{(2)}_{\mu\alpha\beta}, \tag{22}$$

$$T^{(6)}_{\mu\alpha\beta}\propto\left(T^{(3)}_{\mu\alpha\beta}+\frac{p\cdot k_+}{k_+^2}T^{(1)}_{\mu\alpha\beta}-T^{(4)}_{\mu\alpha\beta}\right). \tag{23}$$

Consequently, the $T^{(i)}_{\mu\alpha\beta}$, $i=1,\ldots,4$, are sufficient to parametrize the $\gamma\nu$ elastic amplitude.

The tensor $\mathcal{M}_{\mu\alpha\beta}$ can thus be written [13]

$$\begin{aligned}\mathcal{M}_{\mu\alpha\beta}=&\mathcal{M}_1(s,t,u)T^{(1)}_{\mu\alpha\beta}+\mathcal{M}_2(s,t,u)T^{(2)}_{\mu\alpha\beta}\\&+\mathcal{M}_3(s,t,u)T^{(3)}_{\mu\alpha\beta}+\mathcal{M}_4(s,t,u)T^{(4)}_{\mu\alpha\beta},\end{aligned} \tag{24}$$

where Bose symmetry requires

$$\mathcal{M}_1(s,t,u)=\mathcal{M}_1(u,t,s), \tag{25}$$

$$\mathcal{M}_j(s,t,u)=-\mathcal{M}_j(u,t,s),\quad j=2,3,4. \tag{26}$$

Using the center of mass frame, and the explicit forms of all the vectors, we can obtain the following helicity basis

$$\begin{aligned}T^{(1)}_{\mu\alpha\beta}\xi_\mu(\varepsilon_1)_\alpha(\varepsilon_2^*)_\beta=&-s\cos(\theta/2)[t(\lambda_1+\lambda_2+2\lambda_1\lambda_2)\\&+4s\lambda_1\lambda_2],\end{aligned} \tag{27}$$

$$T^{(2)}_{\mu\alpha\beta}\xi_\mu(\varepsilon_1)_\alpha(\varepsilon_2^*)_\beta=st\cos(\theta/2)(\lambda_1-\lambda_2), \tag{28}$$

$$T^{(3)}_{\mu\alpha\beta}\xi_\mu(\varepsilon_1)_\alpha(\varepsilon_2^*)_\beta=st\cos(\theta/2)(1-\lambda_1\lambda_2), \tag{29}$$

$$T^{(4)}_{\mu\alpha\beta}\xi_\mu(\varepsilon_1)_\alpha(\varepsilon_2^*)_\beta=-8\frac{s^2u}{t}\cos(\theta/2)\lambda_1\lambda_2, \tag{30}$$

where θ is the angle between the incoming and outgoing neutrinos, λ_1 is the helicity of the initial photon, λ_2 is the helicity of the final photon and it is unnecessary to specify the neutrino helicities since they have a definite handedness. Notice that the first, third and fourth of these amplitudes are symmetric under the exchange of λ_1 and λ_2, while the second is antisymmetric under this exchange. Since time reversal invariance implies that the amplitude should be symmetric under the exchange of initial and final helicities, the second amplitude is T-violating. There are thus three helicity amplitudes if we require time reversal symmetry. This is consistent with simple helicity counting arguments.

III. THE CHANNEL $\gamma\gamma\to\nu\overline{\nu}$

The results of Sec. II can be expressed in the crossed channel $\gamma\gamma\to\nu\overline{\nu}$ using the relations $p_1\to -p_1$ and $k_2\to -k_2$, which imply the interchange $s\leftrightarrow t$. The combinations k_+ and p become

$$k_+=k_1+k_2\to k_1-k_2\equiv k_- \tag{31a}$$

$$p=p_1+p_2\to -p_1+p_2\equiv -p. \tag{31b}$$

These transformations allow us to write the cross channel counterparts of Eqs. (14)–(17) as

$$\tilde{T}^{(1)}_{\mu\alpha\beta}=(k_1\cdot k_2\delta_{\mu\alpha}-(k_1)_\mu(k_2)_\alpha)\left(p_\beta-\frac{k_-\cdot p}{k_-^2}(k_1)_\beta\right)+(k_1\cdot k_2\delta_{\mu\beta}-(k_2)_\mu(k_1)_\beta)\left(p_\alpha+\frac{k_-\cdot p}{k_-^2}(k_2)_\alpha\right), \tag{32}$$

$$\tilde{T}^{(2)}_{\mu\alpha\beta}=(k_1\cdot k_2\delta_{\mu\alpha}-(k_1)_\mu(k_2)_\alpha)\left(p_\beta-\frac{k_-\cdot p}{k_-^2}(k_1)_\beta\right)-(k_1\cdot k_2\delta_{\mu\beta}-(k_2)_\mu(k_1)_\beta)\left(p_\alpha+\frac{k_-\cdot p}{k_-^2}(k_2)_\alpha\right), \tag{33}$$

$$\tilde{T}^{(3)}_{\mu\alpha\beta}=-(k_-)_\mu(k_1\cdot k_2\delta_{\alpha\beta}-(k_2)_\alpha(k_1)_\beta), \tag{34}$$

$$\tilde{T}^{(4)}_{\mu\alpha\beta}=(k_-)_\mu\left(p_\alpha+\frac{k_-\cdot p}{k_-^2}(k_2)_\alpha\right)\left(p_\beta-\frac{k_-\cdot p}{k_-^2}(k_1)_\beta\right). \tag{35}$$

The amplitude can be expanded as

$$\tilde{\mathcal{M}}_{\mu\alpha\beta}=\mathcal{M}_1(t,s,u)\tilde{T}^{(1)}_{\mu\alpha\beta}+\mathcal{M}_2(t,s,u)\tilde{T}^{(2)}_{\mu\alpha\beta}+\mathcal{M}_3(t,s,u)\tilde{T}^{(3)}_{\mu\alpha\beta}+\mathcal{M}_4(t,s,u)\tilde{T}^{(4)}_{\mu\alpha\beta}, \tag{36}$$

where

$$\mathcal{M}_1(t,s,u)=\mathcal{M}_1(u,s,t), \tag{37}$$

$$\mathcal{M}_j(t,s,u)=-\mathcal{M}_j(u,s,t),\quad j=2,3,4. \tag{38}$$

In this case, the center of mass helicity basis is

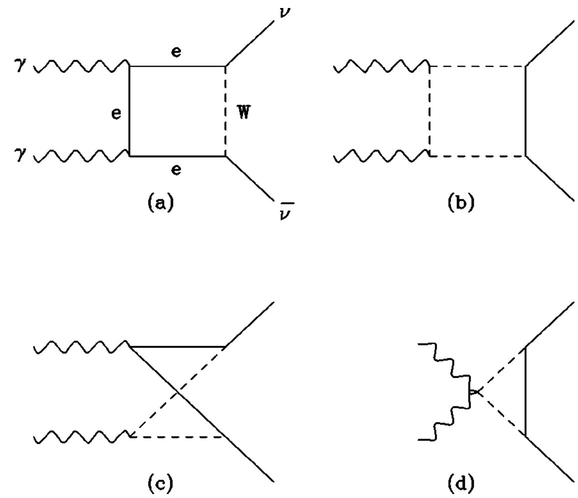

FIG. 1. Diagrams for the process $\gamma\gamma\to\nu_e\overline{\nu_e}$. For each of (a), (b), (c) there is also a diagram with the photons interchanged.

$$\tilde{T}^{(1)}_{\mu\alpha\beta}\tilde{\xi}_\mu(\varepsilon_1)_\alpha(\varepsilon_2)_\beta=\frac{1}{2}s\ \sin\ \theta[s(\lambda_1-\lambda_2+2\lambda_1\lambda_2)+4t\lambda_1\lambda_2], \tag{39}$$

$$\tilde{T}^{(2)}_{\mu\alpha\beta}\tilde{\xi}_\mu(\varepsilon_1)_\alpha(\varepsilon_2)_\beta=-\frac{1}{2}s^2\ \sin\ \theta(\lambda_1+\lambda_2), \tag{40}$$

$$\tilde{T}^{(3)}_{\mu\alpha\beta}\tilde{\xi}_\mu(\varepsilon_1)_\alpha(\varepsilon_2)_\beta=\frac{1}{2}s^2\ \sin\ \theta(1+\lambda_1\lambda_2), \tag{41}$$

$$\tilde{T}^{(4)}_{\mu\alpha\beta}\tilde{\xi}_\mu(\varepsilon_1)_\alpha(\varepsilon_2)_\beta=4tu\ \sin\ \theta\lambda_1\lambda_2. \tag{42}$$

Here, $\tilde{\xi}_\mu=\overline{u}(p_1)\gamma_\mu(1+\gamma_5)v(p_2)$ and θ is the angle between photon 1, which is in the z direction, and the outgoing neutrino. Using the helicity basis, we can express the helicity amplitudes $\tilde{\mathcal{A}}_{\lambda_1\lambda_2}(s,z)$, where $z=\cos\,\theta$, as

$$\tilde{\mathcal{A}}_{++}(s,z)=\sin\ \theta[s(t-u)\mathcal{M}_1(t,s,u)+s^2\mathcal{M}_3(t,s,u)+4tu\mathcal{M}_4(t,s,u)], \tag{43}$$

$$\tilde{\mathcal{A}}_{+-}(s,z)=\sin\ \theta[-2st\mathcal{M}_1(t,s,u)-4tu\mathcal{M}_4(t,s,u)], \tag{44}$$

$$\tilde{\mathcal{A}}_{-+}(s,z)=\sin\ \theta[2su\mathcal{M}_1(t,s,u)-4tu\mathcal{M}_4(t,s,u)], \tag{45}$$

$$\tilde{\mathcal{A}}_{--}(s,z)=\sin\ \theta[s(t-u)\mathcal{M}_1(t,s,u)+s^2\mathcal{M}_3(t,s,u)+4tu\mathcal{M}_4(t,s,u)], \tag{46}$$

where we have assumed time reversal symmetry and omitted $\mathcal{M}_2(t,s,u)$. From Eqs. (43) and (46), it is clear that $\tilde{\mathcal{A}}_{++}(s,z)=\tilde{\mathcal{A}}_{--}(s,z)$. Moreover, since $t=-\frac{1}{2}s(1-z)$ and $u=-\frac{1}{2}s(1+z)$, the symmetries of Eqs. (37) and (38) imply

$$\tilde{\mathcal{A}}_{++}(s,z)=-\tilde{\mathcal{A}}_{++}(s,-z). \tag{47}$$

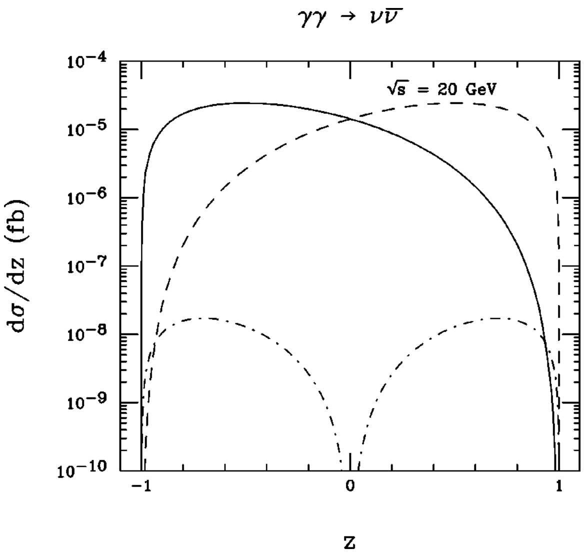

FIG. 2. The helicity dependent differential cross sections for $\gamma\gamma \to \nu\bar{\nu}$ are shown for $\sqrt{s}=20$ GeV. The solid line is $d\sigma_{+-}/dz$, the dashed line is $d\sigma_{-+}/dz$ and the dot-dashed line is $d\sigma_{++}/dz$.

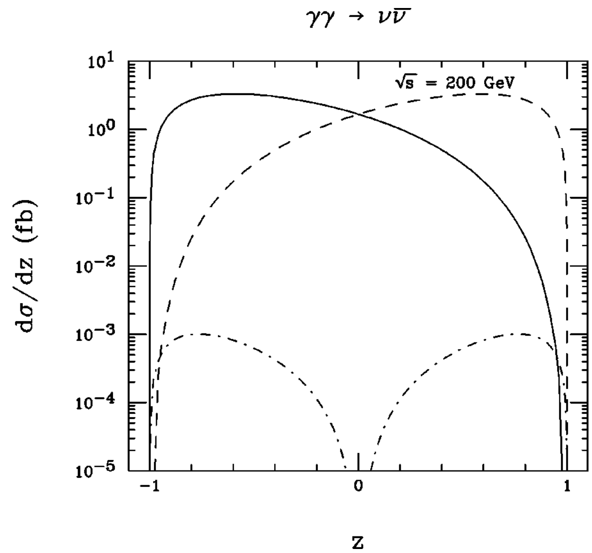

FIG. 3. Same as Fig. 2 with $\sqrt{s}=200$ GeV.

Similarly, we find that the helicity flip amplitudes satisfy

$$\tilde{\mathcal{A}}_{+-}(s,z) = -\tilde{\mathcal{A}}_{-+}(s,-z). \tag{48}$$

To explore these general results, we calculated the standard model diagrams of Fig. 1 using a nonlinear R_ξ gauge such that the coupling between the photon, the W-boson and the Goldstone boson vanishes [5,14,15]. These diagrams can be decomposed in terms of scalar two-point, three-point and four-point functions, which are expressible in terms of dilogarithms [16]. The expressions for the helicity amplitudes in terms of dilogarithms were evaluated numerically and the results checked against FORTRAN codes developed for one-loop integrals [17,18]. In doing so, we assumed that $s \gg m_\ell^2$, where m_ℓ is the lepton mass, and showed that the dependence on $\ln(m_\ell^2)$ or $\ln^2(m_\ell^2)$ from individual diagrams actually cancels.

The results for the helicity amplitudes are shown in Figs. 2 and 3, where the cross sections for various helicities are plotted for $\sqrt{s}=20$ GeV and $\sqrt{s}=200$ GeV using

$$\frac{d\sigma_{\lambda_1\lambda_2}}{dz} = \frac{1}{32\pi s}|\tilde{\mathcal{A}}_{\lambda_1\lambda_2}|^2. \tag{49}$$

These figures clearly illustrate the symmetries of Eqs. (47) and (48), particularly the vanishing of the non-flip amplitudes at $z=0$. The total cross section for unpolarized photons is given by

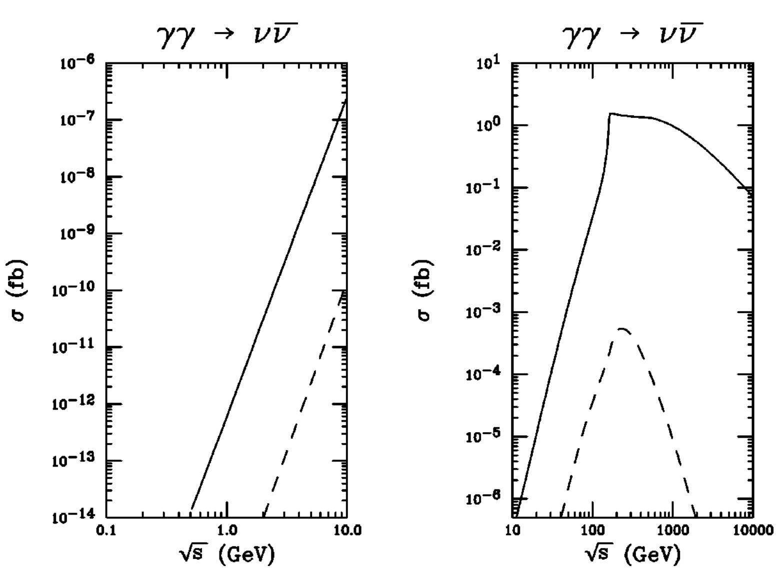

FIG. 4. The solid line is the total cross section $\sigma_{\gamma\gamma\to\nu\bar{\nu}}$ and the dashed line is the contribution from the helicity non-flip amplitudes.

$$\sigma_{\gamma\gamma\to\nu\bar{\nu}} = \frac{1}{64\pi s}\int_{-1}^{1} dz[|\tilde{\mathcal{A}}_{++}(s,z)|^2 + |\tilde{\mathcal{A}}_{+-}(s,z)|^2], \tag{50}$$

and is plotted in Fig. 4. This figure illustrates the roughly s^3 behavior of the total cross section up to the threshold for W pair production. Also shown in dashes is the helicity non-flip contribution to the cross section, which can be seen to be much smaller than the helicity flip contribution. This feature does not appear to be related to any symmetry, but it is reminiscent of the low energy case, where the non-flip amplitudes vanish [5].

IV. DISCUSSION AND CONCLUSIONS

As mentioned in the Introduction, the low energy $2\to2$ photon-neutrino cross sections are much smaller than the corresponding $2\to3$ cross sections for center of mass energies between 1 keV and 1 MeV. Recently, the $2\to3$ cross sections have been computed for a range of center of mass energies from well below $2m_e$, where the cross sections vary as ω^{10}, to well above $2m_e$, but less than m_W [19]. For $m_e \ll \omega \ll m_W$, the only scale in the $2\to3$ processes is m_W or, equivalently, G_F. From dimensional considerations, the center of mass cross section must behave as $G_F^2\omega^2$ in this range of energy. Explicitly, it is found that $\sigma_{\gamma\gamma\to\nu\bar{\nu}\gamma}$ can be expressed as

$$\sigma_{\gamma\gamma\to\nu\bar{\nu}\gamma} = 5.68\times10^{-14}\left(\frac{\omega}{m_e}\right)^2 \text{ fb}. \tag{51}$$

For the $2\to2$ process $\gamma\gamma\to\nu\bar{\nu}$, we expect an ω^6 behavior, and a fit to the points in Fig. 4 yields

$$\sigma_{\gamma\gamma\to\nu\bar{\nu}} = 4.0\times10^{-31}\left(\frac{\omega}{m_e}\right)^6 \text{ fb}. \tag{52}$$

These expressions are equal for $\omega = 1.94\times10^4 m_e$ or $\sqrt{s} \sim 20$ GeV. Thus, for sufficiently high energies, the $2\to2$ process dominates the $2\to3$ process.

To assess the importance of very high energy photon-neutrino interactions in processes of interest in cosmology, consider the scattering of high energy neutrinos from the cosmic neutrino background. This process has been studied by assuming that the neutrino collision produces a Z at resonance whose decay chain contains photons and protons which could account for the flux of $>10^{20}$ eV cosmic rays [10,11]. For the processes considered here, the $\nu\bar{\nu}$ collision would directly produce high energy photons. The cross section for $\nu\bar{\nu}\to\gamma\gamma$ can be obtained from Eq. (52) by supplying a factor of 2. Assuming the relic neutrinos have a small mass [10,11,20], the cross section in the rest frame of the target is

$$\sigma_{\nu\bar{\nu}\to\gamma\gamma} = 5.6\times10^{-42}(\tilde{m}_\nu \tilde{E}_\nu)^3 \text{ cm}^2, \tag{53}$$

where $\tilde{m}_\nu$ is $m_\nu/1$ eV and $\tilde{E}_\nu$ is $E_\nu/10^{21}$ eV. The interaction rate on the cosmic neutrino background is $\sigma_{\nu\bar{\nu}\to\gamma\gamma} n_\nu c$, where n_ν is the relic neutrino density. If this is multiplied by t_0, the age of the universe, the condition that at least one interaction occur is

$$\sigma_{\nu\bar{\nu}\to\gamma\gamma} n_\nu c t_0 = 1, \tag{54}$$

or

$$\tilde{m}_\nu \tilde{E}_\nu = \frac{5.6\times10^{13} \text{ cm}^{-2/3}}{(n_\nu c t_0)^{1/3}}. \tag{55}$$

Taking $n_\nu = 56$ cm^{-3} and $t_0 = 15\times10^9$ years, $\tilde{m}_\nu \tilde{E}_\nu = 6.07\times10^3$, which translates into $\sqrt{s} = 3.5$ TeV. Even if there were sources of such high energy neutrinos (which seems unlikely), this is well above the region for which the s^3 behavior is valid. In fact, the relation $(n_\nu c t_0)^{-1} = 1.26\times10^{-30}$ cm^2 together with Fig. 4 show that $\sigma_{\nu\bar{\nu}\to\gamma\gamma}$ is not large enough for any value of s to attenuate a high energy neutrino flux given the present values of n_ν and the age of the universe.

The temperature at which the reaction $\nu\bar{\nu}\to\gamma\gamma$ ceases to occur can be determined from the reaction rate per unit volume

$$\rho = \frac{1}{(2\pi)^6}\int \frac{d^3p_1}{e^{E_1/T}+1}\int \frac{d^3p_2}{e^{E_2/T}+1}\sigma|\vec{v}|, \tag{56}$$

where $\vec{p}_1$ and $\vec{p}_2$ are the neutrino and antineutrino momenta, E_1 and E_2 their energies, $|\vec{v}|$ is the flux and T the temperature. Using the invariance of $\sigma E_1 E_2 |\vec{v}|$, the relationship between $\sigma|\vec{v}|$ in the center of mass frame and any other frame is

$$\sigma|\vec{v}| = \sigma_{CM}\frac{2E_{\mathrm{CM}}^2}{E_1 E_2}. \tag{57}$$

If the angle between $\vec{p}_1$ and $\vec{p}_2$ is θ_{12}, Eq. (57) gives

$$\sigma_{\nu\bar{\nu}\to\gamma\gamma}|\vec{v}| = 1.6\times10^{-30}\frac{E_1^3 E_2^3}{m_e^6}\sin^8(\theta_{12}/2) \text{ fb}. \tag{58}$$

The integration is straightforward and Eq. (56) gives

$$\rho_{\nu\bar{\nu}\to\gamma\gamma} = \frac{1.6\times10^{-30} \text{ fb}}{5\pi^4}\left(\frac{31}{32}\Gamma(6)\zeta(6)\right)^2\frac{T^{12}}{m_e^6}, \tag{59}$$

where $\zeta(x)$ is the Riemann zeta function. The interaction rate $R_{\nu\bar{\nu}\to\gamma\gamma}$ can be obtained by dividing $\rho_{\nu\bar{\nu}\to\gamma\gamma}$ by the neutrino density $n_\nu = 3\zeta(3)T^3/4\pi^2$, and we find

$$R_{\nu\bar{\nu}\to\gamma\gamma} = 7.3\times10^{-24} T_{10}^9 \text{ s}^{-1}, \tag{60}$$

with T_{10} denoting $T/10^{10}$ K. Expressing the age of the universe as $t = 2T_{10}^{-2}$ s, the condition for at least one interaction, $R_{\nu\bar{\nu}\to\gamma\gamma} t \sim 1$ gives $T \sim 1.6$ GeV, which is within the region of validity of the s^3 behavior.

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