Neutrino magnetic moments in left-right symmetric models

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A closer analysis of the neutrino magnetic moment is provided for various instances of the left-right (LR) symmetric model. We show that it is impossible to generate important Dirac neutrino magnetic moments because of model self-consistency problems. Majorana neutrinos are discussed in the frame of the most natural version of the LR model (with left- and right-handed triplets and a bidoublet in the Higgs sector). We consider two limiting W_L-W_R mixing ξ angles. In the case of a maximal value, the obtained transition magnetic moments could be interesting from an experimental point of view. On the other hand, a vanishing ξ could make the Higgs particle contributions dominate the gauge boson ones. Thus we calculate gauge-Higgs and Higgs-Higgs diagrams, showing that, contrary to naive expectations, they are smaller than those of the standard model. [S0556-2821(98)02923-3]

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I. INTRODUCTION

Interest in the possible magnetic moment of the neutrino has been sustained for four decades [1–3].[1] A nonvanishing magnetic form factor in the electromagnetic current could solve several problems since a neutrino interaction with an external magnetic field would induce a spin flip transforming a left-handed neutrino into a noninteracting (sterile) right-handed one. To explain the supposed deficiency of the solar neutrino flux, its magnitude would have to be of the order of $\mu_\nu \sim 10^{-10}\mu_B$ for the electron neutrino [5]. Supernova and/or neutron star physics would also be heavily influenced by the same kind of effects [6].

Recently, two ideas initiated renewed interest in the field. The first one is about distinguishing Majorana from Dirac neutrinos using the partially polarized solar flux [7], which would necessitate $\mu_{\nu_e} \sim 10^{-12}\mu_B - 10^{-13}\mu_B$ (for strong magnetic fields in the Sun [8]). The second is about testing $\nu_\mu(\nu_e) - \nu_\tau$ oscillations in terrestrial neutrino-electron scattering experiments provided the τ neutrino magnetic moment is large enough and larger than that of $\nu_\mu(\nu_e)$ [9].

The best experimental limits on neutrino magnetic moments are [10]

$$\mu_{\nu_e} \leqslant 1.8\times 10^{-10}\mu_B, \tag{1}$$

$$\mu_{\nu_\mu} \leqslant 7.4\times 10^{-10}\mu_B, \tag{2}$$

$$\mu_{\nu_\tau} \leqslant 5.4\times 10^{-7}\mu_B. \tag{3}$$

The theoretical predictions are highly model dependent. The standard model (with massive Dirac neutrinos) predicts [11]

$$\mu_\nu \leqslant 3.20\times 10^{-19}\left(\frac{m_\nu}{1\ \text{eV}}\right)\mu_B. \tag{4}$$

We know that m_{ν_e} is less than or equal to a few eV. Taking into account also cosmological limits on the total mass of light, stable neutrinos [12], we can see that the above number is much too small to be relevant for any known physics (*vide supra*). It turns out, however, that many of the extensions of the standard model (SM) provide much larger values [13]. We focus here on the case of the left-right symmetric model [14] for two reasons. First, there are papers in which rough estimations have been done with quite large values of neutrino magnetic moments [15]. Usually, however, it is just the opposite that prevails [16]. So we would like to clarify the situation. Second, to our knowledge, the contribution of Higgs particles of the model has not yet been taken into account. Only type I diagrams (see Fig. 1) have been considered in the literature, with the dominant W_1 gauge boson contribution being proportional to the sine of the W_L-W_R mixing angle ξ (required not to vanish to produce a neutrino mass independent magnetic moment). If $\sin\xi$ is not taken to be maximal, then these diagrams are suppressed and, despite their mass ($M_H \gg M_{W_1}$), contributions from Higgs particles, whose couplings are in some cases proportional to $\cos\xi$ and to heavy neutrino masses, seem, by naive order analysis, to be important. Unfortunately, a careful calculation shows that this is not so; both photon-Higgs-gauge boson couplings tend to zero as $\xi \to 0$ and the final total neutrino magnetic moment is proportional to light neutrino masses.

In the next section we consider the magnetic (diagonal) moment of the Dirac neutrino. Then we proceed to the transition magnetic moments, so Majorana neutrinos are considered (Sec. III). Next we estimate the influence of Higgs particles of the model (diagrams of the second and third type in Fig. 1) in the case when type I diagrams are negligible. Finally, we summarize the discussion in Sec. IV and we describe in the Appendix the relations between masses and couplings of the particles necessary to our discussion.

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[1]A bound on the magnetic moment of the neutrino has been estimated by Pauli in 1930 ($\mu \leqslant 0.02\mu_B$) [4].

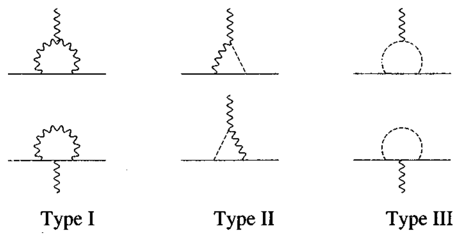

FIG. 1. Topologically different diagrams for the neutrino magnetic moment. Internal wavy lines are for the W_1, W_2 charged bosons, dotted lines are for the H_1, H_2 scalars, and solid lines are for the e, μ, τ charged leptons.

II. MAGNETIC MOMENT OF THE DIRAC NEUTRINO

A magnetic (diagonal) moment does not have to vanish for Dirac neutrinos [11,17]. Taking into account type I diagrams of Fig. 1 and assuming that $m_{W_2} \gg m_{W_1}$, we get (K_R and K_L are transformation matrices from the mass to weak eigenstates for left- and right-handed neutrino states, respectively; $\nu_{\alpha L(R)} = K_{L(R)\alpha a} N_{aL(R)}$) [13][2]

$$\mu_{\nu_a} \simeq \frac{\sqrt{2} G_F}{\pi^2} \sin \xi \cos \xi m_e \times \sum_{\alpha=e,\mu,\tau} m_\alpha \, \mathrm{Re}[(K_R)_{a\alpha}(K_L)^\dagger_{\alpha a}] \mu_B . \tag{5}$$

We are left with the question of the magnitude of ξ and lepton mixing matrices in models with Dirac neutrinos. Let us start with the ξ dependence. It is important to note the difference in the methods used to obtain Majorana and Dirac neutrinos in the framework of left-right symmetric models. From this point of view, there are two kinds of left-right models. If the Higgs sector contains a triplet in addition to the bidoublet, then Majorana neutrinos appear naturally (see [18] for details). Since a Dirac neutrino is a degenerate case of two Majorana particles with opposite CP signatures [19], to get three light neutrinos with at least one Dirac particle among them, we would need at least four light neutrino fields. However, only three of these are connected with the light spectrum (by the seesaw mechanism).[3] So, to get a light Dirac neutrino in a natural way we are left with models without triplets.

Generally, we can differentiate two kinds of such models with [21] and without [22] a bidoublet. In the latter case $W_L - W_R$ mixing is zero. In the first case the $W_L - W_R$ mixing must be very small at the tree level; otherwise the problem of reconciling neutrino masses with other charged fermions would appear. The reason is that without triplet fields the neutrino mass matrix m_D [Eq. (A2)] is proportional to the same vacuum expectation values (VEV's) as the other charged fermion mass matrices [see, e.g., Eq. (A4) for the lepton case] and special symmetries must be imposed to get small neutrino masses (the seesaw type mechanism, which could guarantee small neutrino masses as compared to other charged particles, does not work without triplets). For instance, in [21] a model with a bidoublet, two doublets, and two scalar singlets was considered (all these are needed to reconstruct the existing experimental data) to get a small Dirac neutrino mass. In this case the mixing angle ξ is exactly zero at the tree level and a $\sin \xi$, of the order of 10^{-7}, has been calculated at the loop level. Without triplets K_L and K_R are two independent unitary matrices. Assuming them diagonal, we get, from Eq. (5) ($\sin \xi \simeq 10^{-7}$),

$$\mu_{\nu_e} \leq 0.2 \times 10^{-21} \mu_B , \tag{6}$$

$$\mu_{\nu_\mu} \leq 1.0 \times 10^{-17} \mu_B , \tag{7}$$

$$\mu_{\nu_\tau} \leq 1.0 \times 10^{-16} \mu_B . \tag{8}$$

This means that we are not able to overcome SM mass dependent estimations with experimentally allowed masses of light neutrinos [Eq. (4)]. Consequently, no physically observable signals are to be expected. This is a direct consequence of the phenomenology of left-right models with Dirac neutrinos. We remain with the hope that there is some space left for speculation in the Majorana neutrino case.

III. TRANSITION MAGNETIC MOMENTS

A. Type I diagrams

It is well known that only transition magnetic moments are allowed for Majorana neutrinos. To any diagram with external neutrino legs we have to add its charge conjugate one. For the on-shell electromagnetic current $\Gamma^\mu_{ab}(q)$, this leads to the identity

$$\Gamma^\mu_{ab}(q) = C \Gamma^{\mu T}_{ba}(q) C^{-1}, \tag{9}$$

which makes the diagonal contribution in ab proportional to $\sigma_{\mu\nu} q^\nu$ vanish identically.

Type I diagrams give (again we consider only the dominant contribution from W_1)

$$\mu_{\nu_a \nu_b} \simeq \frac{\sqrt{2} G_F}{\pi^2} \sin \xi \cos \xi m_e \sum_{\alpha=e,\mu,\tau} m_\alpha \times \mathrm{Im}[(K_R)_{a\alpha}(K_L^\dagger)_{\alpha b} + (K_L)_{a\alpha}(K_R^\dagger)_{\alpha b}] \mu_B . \tag{10}$$

In this case one starts to wonder about the magnitude of $\sin \xi$. At the tree level [see Eq. (A11)]

$$\xi \simeq \epsilon \frac{M^2_{W_1}}{M^2_{W_2}} . \tag{11}$$

This is a theoretical prediction. We have also experimental fits; for example, from the muon decay we get [23]

[2]Here and below we give only the neutrino mass independent contributions since the rest is as in the SM [Eq. (4)]. All calculations are made in the unitary gauge.

[3]A Dirac neutrino can appear only in the mass spectrum of heavy neutrinos (see, e.g., [20]).

$$M_{W_2} \geqslant 477 \text{ GeV}, \tag{12}$$

$$\xi \leqslant 0.031 \text{ rad}, \tag{13}$$

where M_{W_2} and ξ were treated as independent parameters. As we can see from Eq. (11), the largest ξ can be obtained for the smallest M_{W_2}. So, taking $M_{W_2}=477$ GeV and maximal $\xi=0.031$ rad, we arrive at $\epsilon \simeq 1$. This means that, taking the largest ξ, a model is realized not only with the smallest M_{W_2} but also with almost equal vacuum expectation values of the Higgs bidoublet ($\kappa_1 \simeq \kappa_2$).

Observed with great precision, lepton number conservation makes the K_L matrix approximately diagonal [18]. Then, with the maximal sin ξ, we get the estimation

$$\mu_{\nu_e \nu_\mu} \simeq 7 \times 10^{-12} (K_R)_{\nu_e \mu} \mu_B, \tag{14}$$

$$\mu_{\nu_e \nu_\tau} \simeq 6.4 \times 10^{-11} (K_R)_{\nu_e \tau} \mu_B, \tag{15}$$

$$\mu_{\nu_\mu \nu_\tau} \simeq 6.4 \times 10^{-11} (K_R)_{\nu_\mu \tau} \mu_B. \tag{16}$$

Our knowledge of the K_R matrix is very unsatisfactory. In the frame of seesaw type models we can estimate $K_R \simeq O(\langle m_D \rangle / m_N)$, where m_N is the mass of a heavy neutrino [18]. Taking $m_N \geqslant 100$ GeV and $\langle m_D \rangle \simeq 1$ GeV, some elements of K_R might be of the order of 0.01. Then the above results are at the edge of physical interest ($\mu_\nu \leqslant 6 \times 10^{-13} \mu_B$).

B. The $\epsilon \simeq 0$ case: Diagrams with Higgs bosons

We have shown that taking the angle ξ maximally allowed by experimental data the model with $\kappa_1 \simeq \kappa_2$ is realized. However, at least two arguments exist against such models. In order to generate a large mass ratio for m_t/m_b it is most natural that κ_1 be significantly different from κ_2 [24]. Also the problem of flavor changing neutral currents supports this statement [25]. If $\kappa_2 \ll \kappa_1$ or $\kappa_1 \ll \kappa_2$ then $\epsilon \to 0$ and $\xi \to 0$ [Eq. (11)]. In this case contributions to the transition magnetic moments from diagrams of type I proportional to sin ξ become negligible.

The $H_1 W_1 \gamma$ and $H_1 W_2 \gamma$ couplings of the type II diagrams are proportional to v_L, which we assumed to be negligible,[4] and the $H_2 W_1 \gamma$ coupling vanishes in the $\epsilon=0$ limit [see Eqs. (A17)–(A19)]. Also the $\Gamma(H_2 W_2 \gamma)$ coupling vanishes in the $\epsilon \to 0$ limit. Let us see how large μ_{ab} can be in the case of nonzero, but small ϵ. Using the relation (9) and taking nonzero couplings of W_2 and H_2 particles with leptons [see Eqs. (A14), (17), and (A27) for notation] we get[5] ($\Omega \equiv K_L^\dagger K_L$)

$$\mu_{ab} \simeq \frac{1}{\sqrt{2} \times 4\pi^2} \Gamma(H_2 W_2 \gamma) \cos \xi m_e \frac{1}{M_{H_2}^2 - M_{W_2}^2} \times \left[\frac{2 M_{H_2}^2}{M_{H_2}^2 - M_{W_2}^2} \ln\left(\frac{M_{H_2}}{M_{W_2}} \right) - 1 \right] (m_a^N + m_b^N) \text{Im } \Omega_{ab} \mu_B, \tag{17}$$

with $\Gamma(H_2 W_2 \gamma) < 1$ and $\Gamma(H_2 W_2 \gamma) \to 0$ for $\epsilon \to 0$.

Let us note that although couplings of H_2 to leptons are proportional to the masses of all neutrinos [see Eq. (A27)], a cancellation occurs, leaving only a light mass contribution. Numerically, taking $M_{W_2}=1$ TeV and $M_{H_2} \simeq 1660$ GeV [see Eq. (A13)], we get

$$\mu_{ab} \leqslant 10^{-22} \left(\frac{m_a^N + m_b^N}{eV} \right) \mu_B, \tag{18}$$

which is smaller than the SM contribution [Eq. (4)].

Type III diagrams vanish since the right- (left-) handed coupling of H_1 (H_2) with leptons is zero in the limit $\epsilon=0$ [Eqs. (A24)–(A27)]. For nonzero ϵ left-handed coupling of H_2 also contributes, but again the total contribution using Eq. (9) turns out to be of the form of Eq. (18) multiplied by an extra factor ϵ.

IV. CONCLUSION

In conclusion, we have estimated the magnetic moments of Dirac and Majorana neutrinos in the left-right symmetric models. Since both left and right charged currents occur in the model, large magnetic moments, independent of the (small) neutrino masses, are possible. However, we have argued that in the Dirac case they must be small because light Dirac neutrinos are possible only in models without triplets. Then internal consistency of the model implies a small $W_L - W_R$ mixing angle and as a consequence a small magnetic moment. In the Majorana case we can get physically interesting results in models with $\epsilon \simeq 1$. In addition, some elements of the K_R matrix would have to be large to generate a sizable effect.

We have also estimated the effect of Higgs particles in the left-right model with Majorana neutrinos. When $\epsilon \simeq 1$, their contribution is smaller than that of type I diagrams for kinematic reasons ($m_W \ll m_{H_{1,2}}$). For $\epsilon \simeq 0$ (vanishing contribution of type I diagrams) type II and type III diagrams turn out to be unimportant, either.

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[4]For nonvanishing v_L, diagrams with H_1 in the loop give maximal results that differ by the factor $v_L \kappa_1 / v_R^2$ from those given for the H_2 particle [see Eq. (18) in the H_2 case].

[5]Neglecting whatever is not important, e.g., terms proportional to sin ξ.

APPENDIX

1. Masses of particles

We consider the classical left-right symmetric model with two left- and right-handed triplets and a bidoublet in the Higgs sector [26]. For the reader's convenience we present an excerpt of the complete set of formulas with our notation, the full version of which can be found in [18,27]. The neutrino 6×6 mass matrix is of the form

$$M_\nu=\begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix}, \tag{A1}$$

where

$$m_D=\frac{1}{\sqrt{2}}(h\kappa_1+\tilde{h}\kappa_2), \quad M_{R,L}=\sqrt{2}h_{R,L}v_{R,L} \tag{A2}$$

are Hermitian and symmetric matrices of dimension 3, respectively.[6] $\kappa_{1,2},v_{R,L}$ are VEV's of the neutral components of the bidoublet and triplets, respectively. $h,\tilde{h}$ are Yukawa couplings. This matrix is diagonalized with the help of the unitary matrix U,

$$U=\begin{pmatrix} K_L^T \\ K_R^\dagger \end{pmatrix}. \tag{A3}$$

Masses of charged leptons are

$$m_l=\frac{1}{\sqrt{2}}(h\kappa_2+\tilde{h}\kappa_1). \tag{A4}$$

In this model we have two single charged Higgs bosons with masses [27]

$$M^2_{H_1^\pm}=\frac{1}{2}\left[v_R^2+\frac{1}{2}\kappa_+^2\sqrt{1-\epsilon^2}\right], \tag{A5}$$

$$M^2_{H_2^\pm}=\frac{1}{2}\left[v_R^2\frac{1}{\sqrt{1-\epsilon^2}}+\frac{1}{2}\kappa_+^2\sqrt{1-\epsilon^2}\right], \tag{A6}$$

where

$$0\leq\epsilon=\frac{2\kappa_1\kappa_2}{\kappa_1^2+\kappa_2^2}\leq1, \quad \kappa_\pm=\sqrt{\kappa_1^2\pm\kappa_2^2}. \tag{A7}$$

Masses of charged gauge bosons are

$$M^2_{W_1}\simeq\frac{g^2}{4}\kappa_+^2, \tag{A8}$$

$$M^2_{W_2}\simeq\frac{g^2}{2}v_R^2, \tag{A9}$$

which come from diagonalization of the appropriate charged gauge boson mass matrix with the mixing angle ξ given by

$$\tan 2\xi=\frac{2\kappa_1\kappa_2}{v_R^2}. \tag{A10}$$

Taking into account Eqs. (A7)–(A9) we get

$$\xi\simeq\epsilon\frac{M^2_{W_1}}{M^2_{W_2}}. \tag{A11}$$

This relation holds to first order in the expansion parameter $M^2_{W_1}/M^2_{W_2}$ (or, equivalently, κ_+^2/v_R^2). We can also obtain an interesting relation between ϵ and physical masses M_{W_2},M_{H_2},

$$\epsilon=\sqrt{1-\left(\frac{M_{W_2}}{gM_{H_2}}\right)^4}, \tag{A12}$$

from which we deduce the following very restrictive relation between gauge and Higgs boson masses:

$$M_{W_2}\simeq gM_{H_2} \tag{A13}$$

if only $\epsilon\simeq0$, as is discussed in the text.

2. Couplings

The charged current has (with appropriate weights) left- and right-handed parts both for light and heavy charged gauge bosons

$$L_{CC}=\frac{g}{\sqrt{2}}\sum_{i=1}^{2}\bar{N}\gamma^\mu[A_L^{(i)}P_L+A_R^{(i)}P_R]\hat{l}W^+_{i\mu}+\text{H.c.}, \tag{A14}$$

where [a stands for leptons ($a=e,\mu,\tau$) and β stands for neutrinos ($\beta=1,\dots,6$)]

$$(A_L^{(1)})_{a\beta}=\cos\xi(K_L)_{a\beta}, \quad (A_R^{(1)})_{a\beta}=-\sin\xi(K_R)_{a\beta}, \tag{A15}$$

$$(A_L^{(2)})_{a\beta}=\sin\xi(K_L)_{a\beta}, \quad (A_R^{(2)})_{a\beta}=\cos\xi(K_R)_{a\beta}. \tag{A16}$$

The coupling of the Higgs particles $H_{1,2}$ to the photon γ equals $-ie$. Higgs–charged gauge boson–photon couplings are ($c\equiv\cos\xi$, $s\equiv\sin\xi$, and $\sin\Theta_W\equiv s_W$)

[6] For simplicity we assume $v_L=0$.

$$i\Gamma(H_2W_2\gamma)=g^2 s_W\left(\frac{v_R}{\sqrt{2}}ca_1+\frac{1}{2}[(s\kappa_2-c\kappa_1)a_2 -(s\kappa_1-c\kappa_2)a_3]\right), \tag{A17}$$

$$i\Gamma(H_2W_1\gamma)=g^2 s_W\left(\frac{v_R}{\sqrt{2}}sa_1+\frac{1}{2}[(c\kappa_2-s\kappa_1)a_2 -(c\kappa_1-s\kappa_2)a_3]\right), \tag{A18}$$

$$i\Gamma(H_1W_1\gamma)=\Gamma(H_1W_2\gamma)=0, \tag{A19}$$

where

$$a_1=\frac{1}{\sqrt{1+\left(\frac{\sqrt{2}\kappa_+v_R}{\kappa_-^2}\right)^2}}, \tag{A20}$$

$$a_2=\frac{\kappa_1}{\kappa_+\sqrt{1+\left(\frac{\kappa_-^2}{\sqrt{2}\kappa_+v_R}\right)^2}}, \tag{A21}$$

$$a_3=\frac{\kappa_2}{\kappa_+\sqrt{1+\left(\frac{\kappa_+^2}{\sqrt{2}\kappa_+v_R}\right)^2}}. \tag{A22}$$

Couplings of Higgs particles to leptons are ($\alpha_2=\sqrt{2}/\kappa_+\sqrt{1-\epsilon^2}$) (see [27] for details)

$$L_H\equiv\sum_{i=1}^{2}\bar{N}[B_L^{(i)}P_L+B_R^{(i)}P_R]\hat{l}H_i^+ +\text{H.c.}, \tag{A23}$$

$$(B_L^{(1)})_{ae}=\frac{1}{v_R}\sum_{c=1,\dots,6}(K_L^\dagger K_R^T M_{diag}^\nu K_R K_L^*)_{ac}(K_L)_{ce}P_L, \tag{A24}$$

$$(B_R^{(1)})_{ae}=0, \tag{A25}$$

$$(B_L^{(2)})_{ae}=-m_a^N(K_L)_{ae}\epsilon\alpha_2, \tag{A26}$$

$$(B_R^{(2)})_{be}\simeq\left[-\sum_{c=1,\dots,6}(\Omega)_{bc}m_c^N(K_R)_{ce}\alpha_2\right]. \tag{A27}$$

[1] F. Reines and C. L. Cowan, Jr., Nature (London) **178**, 446 (1956).
[2] A. Cisneros, Astrophys. Space Sci. **10**, 87 (1971).
[3] M. B. Voloshin, M. I. Vysotskii, and L. B. Okun, Sov. J. Nucl. Phys. **44**, 440 (1986); M. B. Voloshin and M. I. Vysotskii, *ibid.* **44**, 544 (1986).
[4] W. Pauli, in *Neutrino Physics*, edited by K. Winter (Cambridge University Press, Cambridge, 1991), p. 4.
[5] P. Davis (unpublished); P. Raychaudhuri, Mod. Phys. Lett. A **3**, 1319 (1988).
[6] K. Fujikawa and R. Shrock, Phys. Rev. Lett. **45**, 963 (1980); P. B. Pal, astro-ph/9803312, and references therein.
[7] V. A. Semikoz, Nucl. Phys. **B498**, 39 (1997).
[8] E. Kh. Akhmedov, S. T. Petcov, and A. Yu. Smirnov, Phys. Rev. D **48**, 2167 (1993); G. Raffelt and A. Weiss, Astron. Astrophys. **264**, 536 (1992).
[9] S. N. Gninenko, Phys. Lett. B **427**, 206 (1998).
[10] D. A. Krakauer *et al.*, Phys. Lett. B **252**, 177 (1990); A. M. Cooper-Sarkar *et al.*, *ibid.* **280**, 153 (1992).
[11] W. Marciano and A. Sanda, Phys. Lett. **67B**, 303 (1977).
[12] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996), p. 116.
[13] For a review see, e.g., *Physics and Astrophysics of Neutrinos*, edited by M. Fukugita and A. Suzuki (Springer-Verlag, Tokyo, 1994), p. 1.
[14] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975); G. Senjanovic and R. N. Mohapatra, *ibid.* **12**, 1502 (1975); G. Senjanovic, Nucl. Phys. **B153**, 334 (1979).
[15] M. A. B. Beg, W. J. Marciano, and M. Ruderman, Phys. Rev. D **17**, 1395 (1978); see also [3].
[16] P. Pal and R. N. Mohapatra, *Massive Neutrinos in Physics and Astrophysics* (World Scientific, Singapore, 1991); M. Fukugita and T. Yanagida, Phys. Rev. Lett. **58**, 1807 (1987).
[17] B. W. Lee and R. E. Shrock, Phys. Rev. D **16**, 1444 (1977).
[18] J. Gluza and M. Zrałek, Phys. Rev. D **48**, 5093 (1993).
[19] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
[20] D. Wyler and L. Wolfenstein, Nucl. Phys. **B218**, 205 (1983).
[21] D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987).
[22] S. Rajpoot, Phys. Lett. B **191**, 122 (1987); A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); B. S. Balakrishna, *ibid.* **60**, 1602 (1988); K. S. Babu and R. N. Mohapatra, *ibid.* **62**, 1079 (1989); R. N. Mohapatra, Phys. Lett. B **201**, 517 (1988); B. S. Balakrishna, A. Kagan, and R. N. Mohapatra, *ibid.* **205**, 345 (1988); B. S. Balakrishna, *ibid.* **214**, 267 (1988); E. Ma, Phys. Rev. Lett. **63**, 1042 (1989).

[23] J. Polak and M. Zrałek, Nucl. Phys. **B363**, 385 (1991).
[24] G. Ecker, W. Grimus, and W. Konetschny, Phys. Lett. **94B**, 381 (1980); Nucl. Phys. **B250**, 517 (1985).
[25] N. G. Deshpande, J. F. Gunion, B. Kayser, and F. Olness, Phys. Rev. D **44**, 837 (1991).
[26] J. F. Gunion *et al.*, Phys. Rev. D **40**, 1546 (1989), and references therein.
[27] J. Gluza and M. Zrałek, Phys. Rev. D **51**, 4695 (1995).