Extracting information on CKM phases, electroweak penguin diagrams, and new physics from $B \to VV$ decays

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We derive constraints for $B \to V_1 V_2$ modes ($V_{1,2}$ = vector meson) that allow a model-independent quantitative assessment of the contributions from electroweak penguin diagrams and/or new physics. The interplay of direct CP with oscillation studies then may lead to the extraction of the angle α, using $B \to K^* \omega(\rho)$ and $B \to \rho\omega(\phi)$. Any reservation one may have can be explicitly verified in a model-independent way by assuming only isospin conservation. We also briefly mention how the method can be used to extract γ via B_s decays: $B_s \to K^* \rho$, $\bar{K}^* K^*$. [S0556-2821(98)05323-5]

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I. INTRODUCTION

Recent evidence from CLEO [1] indicates that the long sought after penguin decays occur at the appreciable rate of about 10^{-5}. In both the $b \to s$ and $b \to d$ transition interference between the tree and the strong penguin diagrams is expected to lead to CP violation effects. Two of the most important applications of these rare hadronic decays are (1) determining the phases of the unitarity triangle [2], and (2) testing the presence of nonstandard physics. In this paper we will show that among such B decays those involving two-vector mesons in the final state can be very useful for attaining these goals.

Following Gronau and London [3], a general strategy for extracting Cabibbo-Kobayashi-Maskawa (CKM) angles from modes that result from the interference between tree and penguin diagrams is to exploit the fact that the strong penguin and the tree diagrams have different isospin transformation properties. For example, the strong penguin diagram $b \to d$ has $\Delta I = 1/2$ whereas the interfering tree ($b \to u\bar{u}d$) has both $\Delta I = 1/2$ and 3/2.

In general, the virtual gluon produced by the penguin diagram contributes a net isospin of 0. By a suitable choice of combinations of various exclusive final states that result from the quark level transition, a separation of the pure $\Delta I = 3/2$ piece becomes possible yielding the angle α [4].

If there is a substantial contribution from electroweak penguin (EWP) diagrams, which produce a virtual Z boson or photon, this method fails because the Z boson and the photon, unlike the gluon, can carry the isospin ($I = 1$) component yet it has the same weak phase as the strong penguin [5]. In the $b \to d$ transition then, both the tree and the EWP diagrams will generate $\Delta I = 1/2$ and $\Delta I = 3/2$. Thus in the standard model, from isospin considerations alone, it is not possible to isolate EWP diagrams from the tree amplitude.

In this paper we suggest what might be the next best thing. We consider the decay $B \to V_1 V_2$ (i.e., two-vector particles) and, taking advantage of the information present in the decay distributions of the vector particles [6], we derive a set of three ratios $\{R_1, R_2, R_3\}$ which monitor EWP contamination. In particular if $R_i \neq 1$ then it is implied that an EWP diagram process [or non-standard-model (SM) physics] makes a significant contribution. Decays where EWP diagrams are small may thus be identified as candidates for the extraction of CKM phases (in particular α). In these cases, as we will show, the value of the CKM phase can be determined by combining information from direct CP violation with that from oscillation studies.

We focus on four particular examples where this method may be applied to determine α. In the case of $b \to s$ transitions: $K^* + \omega$ and $K^* + \rho$, while in the case of $b \to d$ transitions: $\rho + \omega$ and $\rho + \phi$. Even if it turns out that every case has significant contamination, important information about the magnitude of these EWP diagrams may still be obtained in these reactions which should be valuable in its own right. For instance, if the contaminating effects are much larger than anticipated, they may represent evidence for physics beyond the SM.

The outline of the rest of this paper is as follows. In Sec. II we describe the crux of our analysis where we determine the phase difference between each of the meson decays and the corresponding tree amplitude. As a byproduct of this analysis we obtain a condition which is sensitive to the effect of EWP amplitudes. In Sec. III we describe the extraction of the necessary information to perform this analysis from the experiment. Then the method for determining the CKM phase α by combining information from oscillation experiments is discussed. In Sec. IV we consider various specific examples: $B \to K^* \omega$, $K^* \rho$, $\rho\omega$, and $\rho\phi$, and in Sec. V we present conclusions and briefly mention how the method may also be used to extract γ from $B_s \to K^* \rho$ and $\bar{K}^* K^*$.

II. GENERAL FORMALISM

We first discuss the general mathematical framework that we will use to find the phase between the meson transitions and the quark-level tree diagram. The cases we will consider here consist of two amplitudes which are related in some way by isospin, in particular a B^- decay which we denote as u_1^h and a $\overline{B}^0$ decay u_2^h. We denote the corresponding final states of the two decays as f_1 and f_2. Here, the superscript h indicates the helicity of the vector particles, that is u_i^h is the amplitude for the decay with the final state $f_i^h = (V_1^h V_2^h)_i$, $h=-1,\ 0,\ +1$. In addition, one has the conjugate amplitudes $\overline{u_1^h}$ and $\overline{u_2^h}$ for B^+ and B^0 decays respectively.

In order to proceed, we need to construct, in the absence of EWP diagrams, a combination of the two amplitudes which receive a contribution only from the tree. Such a component can be isolated since only the tree contributes a term to the effective Hamiltonian with $\Delta I=1$ (for $b\to s$) or $\Delta I=3/2$ (for $b\to d$). In contrast, for these two transitions, the strong penguin diagram contributes to $H_{\rm eff}$ pieces with $\Delta I=0$ and $\Delta I=1/2$, respectively. Therefore, a combination of the amplitudes which has isospin properties as that of the pure tree diagram will contain only the weak phase of the tree diagram. We denote such a combination $c_1 u_1^h + c_2 u_2^h$. This amplitude and its CP conjugate will be related as follows:

$$(c_1 u_1^h + c_2 u_2^h) e^{-i\delta_T} = (c_1 \overline{u_1}^{-h} + c_2 \overline{u_2}^{-h}) e^{+i\delta_T}, \tag{1}$$

where δ_T is the weak phase of the tree diagram. Indeed the value of δ_T will depend on the phase convention for particle versus antiparticle decays. The physical observables will not depend on this convention, though. In practice the same convention will also enter the phase of $B\overline{B}$ oscillation and in combination this dependence will cancel.

Constructing such a relation is straightforward if one of the final particles is an isoscalar. More generally, such a relation can be constructed if and only if (1) at least two amplitudes related by isospin are involved, and (2) the strong penguin effective Hamiltonian can contribute only one isospin amplitude to the final state.

For example, suppose we have a system of decays consisting of n_1 different states from B^- decays and n_2 different states from $\overline{B}^0$ decays where all of the decays in question $B^-\to f_1^i$, $i=1,2\ldots n_1$ and $\overline{B}^0\to f_2^i$, $i=1,2\ldots n_2$ are related by the isospin (for instance, all the various charge combinations of $K^*\rho$). If this is a case which satisfies (1) and (2), then there will be only one strong penguin amplitude, U_P, in the above system. Therefore, one can write the amplitudes for these decays as

$$u(B^-\to f_1^1) = r_1^1 U_P + T_1^1, \tag{2}$$

$$u(B^-\to f_1^2) = r_1^2 U_P + T_1^2,$$

$$\ldots$$

$$u(\overline{B}^0\to f_2^1) = r_2^1 U_P + T_2^1$$

$$u(\overline{B}^0\to f_2^2) = r_2^2 U_P + T_2^2$$

$$\ldots,$$

where U_P is the strong penguin amplitude, T_j^i are the various tree contributions to these amplitudes where the subscript ($j=1,2$) specifies the initial B state, and the superscript ($i=1,2\ldots$) designates the final state. Here r_j^i are coefficients derived from SU(2) of isospin (i.e., Clebsch-Gordon coefficients). If we now take any two amplitudes, for instance, $u(B^-\to f_1^1)$ and $u(\overline{B}^0\to f_2^1)$, we can write a relation of the type:

$$r_2^1 u(B^-\to f_1^1) - r_1^1 u(\overline{B}^0\to f_2^1) = r_2^1 T_1^1 - r_1^1 T_2^1, \tag{3}$$

where now the right-hand side only contains tree amplitudes and so has the weak phase of the tree. This leads to a relation of the form of Eq. (1) where

$$c_1 = r_2^1,$$

$$c_2 = -r_1^1. \tag{4}$$

Let us now survey decays of the type $B\to V_1 V_2$. In the case of $b\to s$ transitions the penguin amplitude is $\Delta I=0$ and therefore in all cases there will be only one penguin amplitude. In particular, this will be true for $K^*\omega$ and $K^*\rho$. In the latter case there are four related final states. Since, in principle, we only need two final states for our analysis, we may, therefore, choose that pair of final states to enter into Eq. (1) which we expect to be the least effected by EWP contributions, as will be discussed in Sec. IV.

In the case of $b\to d$ transitions, the penguin amplitude is $\Delta I=1/2$. In principle, this can transform the B isodoublet into a $I=0$ or $I=1$ final state; thus there are two possible penguin amplitudes. If V_1 is an isovector and V_2 is an isoscalar (e.g., $B\to\rho\omega$) then there is only an $I=1$ final state and a relation of the form Eq. (1) may be constructed. On the other hand, $B\to\omega\omega$ does not work because there is only one amplitude involved while $B\to\rho a_1$ fails since there are two penguin amplitudes leading to $I=0$ and $I=1$ final states.

If Eq. (1) can be established, then the extraction of information about phases in the CKM matrix proceeds in three steps.

(1) First, as discussed in Sec. III, the study of the angular distributions of the decay products of the two-vector particles in each reaction will give us the magnitudes of the helicity amplitudes $|u_i^h|$ and the phases between pairs of helicity amplitudes that lead to a common final state (and thus interfere).

(2) Secondly, as we will describe below, we will use Eq. (1) to obtain the phase difference between u_1^h and u_2^h (and likewise between $\overline{u_1^h}$ and $\overline{u_2^h}$). At this stage three conditions allow us to check the consistency of the assumption that EWP contamination is not significant.

(3) Finally, as we discuss in Sec. III, if u_2 is the decay amplitude to a self-conjugate mode, an oscillation experiment fixes the phase between u_2 and $\overline{u_2}$ so that now the phase between all pairs of amplitudes becomes known and information about δ_T (in combination with the $B\overline{B}$ oscillation phase) may then be recovered.

In order to carry out the third step of this program, the neutral B decay must be to a self-conjugate state. In examples such as $B\to\phi\rho$ this requirement is met since the state $\phi\rho^0$ is self-conjugate. On the other hand, when one of the final-state particles is a K^{0*}, for example $B\to K^*\omega$, this requirement is only met for the K^* decays into a CP eigenstate, that is $K^{0*}\to K_S\pi^0$.

In the case of $B\to K^*\rho$ we can only perform an oscillation experiment on the final state $\overline{B^0}\to\overline{K}^{*0}\rho^0$. We will argue, however, that the amplitude for $\overline{B^0}\to K^{*-}\rho^+$ is more likely to be free of EWP effects. So, in Sec. IV, we will invoke additional isospin relations based on the assumption that the EW Hamiltonian has no $\Delta I=2$ piece in order to interpret the oscillation experiment for $\overline{B^0}\to\overline{K}^{*0}\rho^0$ to obtain the desired weak phase of the quark level transition. This more complicated strategy is necessitated by the realization that the decay $\overline{B^0}\to\overline{K}^{*0}\rho^0$ is more susceptible to EWP contamination than modes containing the charged ρ. Should this not be the case then we may proceed more directly applying our method to this final state (i.e., $\overline{K}^{*0}\rho^0$) together with any one of the other $K^*\rho$ modes.

Let us now discuss how to use Eq. (1) in order to extract the phases between the helicity amplitudes u_1^h and u_2^h. To start with, for each specific final state the angular distributions of the vector decays will give the magnitudes $|u_i^h|$ as well as the relative phases between two amplitudes of differing helicities for the same final state (i.e., the phase between $u_i^{h_1}$ and $u_i^{h_2}$). One may obtain this information by fitting the experimental data to the distribution given in the next section [see Eq. (11)]. We will denote the relative phases between the two helicity states (h_1 and h_2) of the same final state (f_i for $i=1,2$) by $\phi_i(h_1,h_2)=\arg(u_i^{h_1}u_i^{h_2^*})$; $\overline{\phi_i}(h_1,h_2)=\arg(\overline{u}_i^{h_1}\overline{u}_i^{h_2^*})$. This information together with Eq. (1) gives us the system of equations we must solve for the relative phases of the amplitudes.

Before proceeding to solve this system, it is useful to factor out the tree weak phase δ_T and rewrite the equations in terms of the quantities v_i^h. We thus define $v_i=e^{-i\delta_T}u_i$; $\overline{v_i}=e^{+i\delta_T}\overline{u_i}$. The system then becomes

$$(c_1v_1^h+c_2v_2^h)=(c_1\overline{v_1}^{-h}+c_2\overline{v_2}^{-h}),\qquad(5)$$

where $|v_i^h|=|u_i^h|$ and $\phi_i(h_1,h_2)=\arg(v_i^{h_1}v_i^{h_2^*})$ (and likewise for the conjugate case) are quantities that may be determined experimentally.

It is convenient to express the above in terms of parity eigenstates, which we denote v^k, where $k=0$, P or S. This basis is defined as $v^S=(v^{+1}+v^{-1})/\sqrt{2}$, $v^P=(v^{+1}-v^{-1})/\sqrt{2}$ and v^0 is common to both bases. The system of equations (5) thus becomes

$$(c_1v_1^{0,S,P}+c_2v_2^{0,S,P})=\pm(c_1\overline{v_1^{0,S,P}}+c_2\overline{v_2^{0,S,P}}),\qquad(6)$$

where $\pm=+$ for the 0 and S cases and $\pm=-$ for the P case.

Experimental data gives us the phase between $v_i^{k_1}$ and $v_i^{k_2}$ for a given final state f_i where $k_1,k_2\in\{0,P,S\}$ and likewise for $\overline{v}$. Thus, all we need to know is the phase of v_1^0, v_2^0, $\overline{v_1^0}$, and $\overline{v_2^0}$ to fix all of the phases of v_i.

Let us denote these phases for the 0-helicity amplitudes by $\psi_i=\arg(v_i^0)$ and $\overline{\psi_i}=\arg(\overline{v_i^0})$ (where $i=1,2$). Clearly, then the system of equations (6) becomes a series of linear conditions on $\{e^{i\psi_i},\ e^{i\overline{\psi_i}}\}$. The solution is given by considering the determinant:

$$\Delta=\begin{vmatrix} x_1 & x_2 & \overline{x_1} & \overline{x_2}\\ c_1|v_1^0| & c_2|v_2^0| & -c_1|\overline{v_1^0}| & -c_2|\overline{v_2^0}|\\ c_1|v_1^S|e^{i\phi_1(S,0)} & c_2|v_2^S|e^{i\phi_2(S,0)} & -c_1|\overline{v_1^S}|e^{i\overline{\phi}_1(S,0)} & -c_2|\overline{v_2^S}|e^{i\overline{\phi}_2(S,0)}\\ c_1|v_1^P|e^{i\phi_1(P,0)} & c_2|v_2^P|e^{i\phi_2(P,0)} & c_1|\overline{v_1^P}|e^{i\overline{\phi}_1(P,0)} & c_2|\overline{v_2^P}|e^{i\overline{\phi}_2(P,0)}\end{vmatrix}\qquad(7)$$

where a solution exists if and only if the three ratios

$$R_1=|\partial\Delta/\partial x_1|/|\partial\Delta/\partial\overline{x_1}|,$$

$$R_2=|\partial\Delta/\partial x_2|/|\partial\Delta/\partial\overline{x_2}|,$$

$$R_3=|\partial\Delta/\partial x_1|/|\partial\Delta/\partial\overline{x_2}|,\qquad(8)$$

satisfy

$$R_1=R_2=R_3=1.\qquad(9)$$

If this condition holds, the required phases are then

$$\psi_i=\xi_0+\arg(\partial\Delta/\partial x_i);\quad \overline{\psi_i}=\xi_0+\arg(\partial\Delta/\partial\overline{x_i})\qquad(10)$$

where ξ_0 is an overall strong phase which cannot be determined (and does not enter into any of the physics discussed here). Clearly Eq. (9) may also be regarded as a test for the presence of EWP diagrams or new physics effects. If there is a contribution from new physics or EWP diagrams, the set of three equations [i.e., Eq. (9)] implies that for each helicity the new contribution satisfies one equation. Unless the new contribution has the same weak phase and also the same isospin transformation properties as the tree, this is rather improbable. Thus, Eq. (9) provides a good test for the presence of EWP diagrams and/or new physics. We must also emphasize that this test of EWP diagrams is completely model independent since it only assumes isospin conservation.

The phases of v_i^k are physically meaningful modulo the overall strong phase ξ_0 above. For instance, in the case of $B \to K^* \omega$, we can interpret $\arg(v_1^k)$ and $\arg(v_2^k)$ as the phases between the quark level $b \to s\overline{u}u$ transition and the meson decays $B^- \to K^{*-}\omega$ and $\overline{B}^0 \to \overline{K}^{*0}\omega$, respectively, in the helicity combination indicated by (k). Likewise $\arg(\overline{v_1^k})$ and $\arg(\overline{v_2^k})$ are the phases between $\overline{b} \to \overline{s}u\overline{u}$ and $B^+ \to K^{*+}\omega$ and $B^0 \to K^{*0}\omega$.

Although at this point we know the phases of all the v_i^k and $\overline{v_i^k}$ $(i=1,2,\ k \in \{0,P,S\})$ we still do not know the phase differences between u_i^k and $\overline{u_i^k}$ since we do not know δ_T. Indeed in the context of the standard model it is δ_T which we wish to know since it is derived from the CKM matrix. Since δ_T cannot be obtained from the experimental information which we have included so far, one needs some additional data which depends, in particular, on the phase difference between a particle and antiparticle decay.

In the examples we consider, the decay from the neutral B meson provides an opportunity to do this, since in that instance, oscillation effects allow the interference of B^0 and $\overline{B}^0$ decay amplitudes. Thus, we are able to interfere with the amplitudes u_2^k and $\overline{u_2^k}$ if the final state f_2 is a CP eigenstate, for instance, B^0, $\overline{B}^0 \to \phi\rho^0$. In the following section we show that from observing the decay of B^0 and $\overline{B}^0$ as a function of time, it is possible to extract the quantity $\sin(\zeta - 2(\hat{\beta}+\delta_T))$ where the angle ζ is a function only of v_i^k which can be determined as described above and $\hat{\beta}$ is the phase from the CKM matrix inherent in neutral B oscillations (using the same convention in which δ_T is defined). If ζ is determined then the quark-level quantity $\hat{\beta}+\delta_T$ ($\hat{\beta}+\delta_T=\beta+\gamma=\pi-\alpha$ in the examples we consider in the standard convention of [7]), which depends only on the CKM matrix [4], may thus be extracted up to the ambiguity of the sine function. Specifically, if we adopt the CKM phase convention of [7], then in the standard model $\hat{\beta}=\beta$ and $\delta_T=\gamma$.

In summary, the extraction of $\hat{\beta}+\delta_T$ proceeds as follows. First we must determine from the angular distributions of the decays the magnitudes of $|v_i^{(0,P,S)}|$ and $|\overline{v_i^{(0,P,S)}}|$ as well as the phases $\phi_i(V,0)$, $\phi_i(A,0)$, $\overline{\phi}_i(V,0)$, and $\overline{\phi}_i(A,0)$. We then check that there is no EWP contamination via Eq. (9). If this is satisfied, then for any value of i, we can obtain the phases of v_i and $\overline{v_i}$ from Eq. (10) from which we can calculate ζ as we will describe below. Observing oscillation effects in the neutral B decay will then allow us to obtain $\hat{\beta}+\delta_T$ which yields the phase α of the unitarity triangle.

III. EXPERIMENTAL PROCEDURE

Let us now discuss the experimental observables which are needed to perform the above analysis. The basic ingredient will be the study of correlations between the decay distributions of the two-vector mesons or, equivalently, the correlation of their polarizations.

First, let us consider the case where the vector meson V decays to two pseudoscalars $V \to P_1 P_2$; for instance, $\rho \to \pi\pi$, $\phi \to K\overline{K}$ and $K^* \to K\pi$. Then the polarization vector $\mathcal{E}_V$ in the rest frame of V can be taken to be parallel to the momentum of one of the pseudoscalars, $\vec{\mathcal{E}}_V \propto \vec{P}_{P_1}$. We are not concerned about the sign of $\vec{\mathcal{E}}_V$ since it will not enter into the analysis below. The case of ω decaying to 3π is similarly self-analyzing since if $\omega \to \pi^+\pi^-\pi^0$, the polarization is related to the momenta of the pions by $\vec{\mathcal{E}}_\omega \propto (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$, in the rest frame of the ω.

In the V_i rest frame, denote the angle between $\vec{\mathcal{E}}_i$ and $-\vec{P}_B$ (the three momentum of the B meson) by θ_i. Let us define Φ to be the azimuthal angle from $\vec{\mathcal{E}}_1$ to $\vec{\mathcal{E}}_2$ in the rest frame of the B about $\vec{P}_{V_1}$ such that $\sin\Phi \propto (\vec{\mathcal{E}}_1 \times \vec{P}_{V_1}) \cdot \vec{\mathcal{E}}_2$. If we define $y_i = \sin\theta_i$ and $z_i = \cos\theta_i$ then the angular distribution of the decays in terms of $\{\theta_1,\ \theta_2,\ \Phi\}$ is

$$d^3\Gamma/(dz_1 dz_2 d\Phi) = |u^0|^2 z_1^2 z_2^2 + y_1^2 y_2^2 (|u^S|^2 \cos^2\Phi + |u^P|^2 \sin^2\Phi) + 2\mathrm{Re}(u^0 (u^S)^*) y_1 y_2 z_1 z_2 \cos\Phi - 2\mathrm{Im}((u^0)^* u^P) y_1 y_2 z_1 z_2 \sin\Phi + 2\mathrm{Im}(u^S (u^P)^*) y_1^2 y_2^2 \sin\Phi\cos\Phi. \tag{11}$$

From an experimental study of the distribution of the decays, one can extract the quantities $|u^0|$, $|u^S|$, $|u^P|$ as well as $\cos\phi(0,S)$, $\sin\phi(P,0)$, and $\sin\phi(S,P)$; the latter three correspond to interference terms of the type $u^{(0)}u^{(S)*}$. Note that there is a twofold ambiguity in the determination of the actual phase differences since either $\{\phi(0,S),\ \phi(P,0),\ \phi(S,P)\}$ or $\{-\phi(0,S),\ \pi-\phi(P,0),\ \pi-\phi(S,P)\}$ will explain a given set of data. When the data for all the helicities and for the decays of the neutral and charged B to specific modes is considered together, however, Eq. (9) should only work for one of the two cases. Note also that these phase angles satisfy the condition, $\phi(0,V)+\phi(A,0)+\phi(V,A) \equiv 0 \mod 2\pi$, which is a useful constraint on interpreting the experimental data.

In the above distribution, CP violation will be manifest by the difference between $d^3\Gamma(z_1,z_2,\Phi)$ for a $\overline{B}^0$ or B^- meson and $d^3\Gamma(z_1,z_2,-\Phi)$ for the conjugate meson decay. The two manifestly P-odd interference terms $\propto u^P$ represent CP-violating effects which are P odd C even. Further, these terms are odd under "naive time reversal (T_N)," defined as the inversion of momenta and spins without the interchange of initial and final states required under T. Such effects are present even if there are no absorptive phases. In contrast, the other four CP-violating terms are even under T_N and so only present if there are absorptive phases. Two possible sources for such phases are (1) the result of rescattering at short distances [8] or (2) at long distances [9,10].

Let us now discuss the problem of extracting the CKM phase $(\hat{\beta}+\delta_T)$ through the observation of oscillations effects in the decay of neutral B mesons assuming that, through the use of Eq. (9), it has been demonstrated that EWP contributions are negligible. As indicated above, we assume that u_2 represents the decay from the neutral B meson, i.e., $\overline{B^0}\to V_1V_2$ while $\overline{u_2}$ the decay $B^0\to\overline{V_1}\,\overline{V_2}$. In such an oscillation experiment, we will assume that at a point in time, which we define to be $t=0$, the flavor of the neutral B meson is known to be either $\overline{B^0}$ or B^0 due to some tagging event. At an e^+e^- machine, sitting at the $\Upsilon(4S)$, this tagging event would be the decay of the associated meson to a final state of unambiguous flavor. For instance, if the partner decayed to $e^+\nu_e D^-$ at $t=0$ then the meson in question must be a $\overline{B^0}$ at $t=0$. At a hadron collider, the tagging event would generally occur at the moment of creation. For example, if $p+p\to B^+\overline{B^0}+X$ at $t=0$ then the flavor of the neutral B meson is unambiguously fixed at that point in time. In the following, therefore, negative values of t are allowed in e^+e^- experiments while only positive values will apply to hadronic collisions.

Below we will consider only the total decay rate as a function of time t. The generalization to decay distributions as a function of time is straightforward but it is probably much more difficult experimentally to use such information. In any case the extraction of phases of the CKM angle can be made from the inclusive time-dependent rate.

Let us denote Γ_B to be the total width of the neutral B meson and $g(t)=d\Gamma(\overline{B^0}(t)\to V_1V_2)/dt$ to mean the differential rate that a meson, identified as a $\overline{B^0}$ at $t=0$, decays to V_1V_2 at time t. Likewise we denote $\overline{g}(t)=d\Gamma(B^0(t)\to\overline{V_1}\,\overline{V_2})/dt$. At $t=0$ let us define $\hat{g}\Gamma_B=rg(0)$ and $\hat{\overline{g}}\Gamma_B=r\overline{g}(0)$, where, $r=1$ in cases when only $t\geqslant 0$ (i.e., hadronic colliders) is allowed and $r=2$ when both signs of t are present (i.e., e^+e^- colliders). Here $\hat{g}$ and $\hat{\overline{g}}$ are the decay rates that would be present in the absence of oscillations. These may also be obtained in self-tagging situations which apply to some cases as discussed below.

Clearly, interference is only possible if the states V_1V_2 and $\overline{V_1}\,\overline{V_2}$ eventually cascade down to the same final state. The simplest situation where this applies, and the case we shall consider here is when V_1V_2 is an eigenstate of C (charge conjugation) with eigenvalue $\lambda=\pm1$.

From the analysis of Sec. II we know the phase of each of the meson decay amplitudes with respect to the tree graph. Using this information we can now write the following expression for the total decay rate to the final state under consideration as a function of time:

$$[g(t)+\overline{g}(t)]/2=(\hat{g}+\hat{\overline{g}})\Gamma_B e^{-\Gamma_B|t|}/2,$$

$$[g(t)-\overline{g}(t)]/2=(\hat{g}+\hat{\overline{g}})\Gamma_B e^{-\Gamma_B|t|}[T_c\cos(\Delta m t)+T_s\sin(\Delta m t)]/2, \tag{12}$$

where Δm is the $B_0\overline{B}_0$ mass difference and

$$T_c=\sum_k\,(|v_2^k|^2-|\overline{v_2^k}|^2)/\mathcal{V};$$

$$T_s=-\lambda Im\left[e^{-2(\hat{\beta}+\delta_T)}\sum_k\,\overline{v_2^k}v_2^{k*}/\mathcal{V}\right], \tag{13}$$

where, again, k can be taken to be 0, S, P with $\mathcal{V}=\Sigma_k(|v_2^k|^2+|\overline{v_2^k}|^2)$. If we denote $R\exp(i\zeta)=\Sigma_k\overline{v_2^k}v_2^{k*}$ then the above expression for T_s may be rewritten

$$T_s=-\lambda(R/\mathcal{V})\sin(\zeta-2(\hat{\beta}+\delta_T)), \tag{14}$$

where the values of R and ζ may be obtained once the phases of v_i^k are determined from Eq. (10). Thus, from the experimental determination of T_s, one obtains, up to a fourfold ambiguity, the value of $\hat{\beta}+\delta_T$. The additional solutions which produce identical results to a given value of $\hat{\beta}+\delta_T$ are $\{\pi+\hat{\beta}+\delta_T,\ \pm\pi/2+\zeta-\hat{\beta}-\delta_T\}$. The latter two possible solutions which involve ζ could be eliminated if a different mode with a different value of ζ were considered. The other spurious solution requires that the quadrant of $\hat{\beta}+\delta_T$ be separately known and cannot be eliminated via this kind of oscillation experiment since the angle enters as $2(\hat{\beta}+\delta_T)$.

IV. SPECIFIC EXAMPLES

Consider now the application of this method to a few cases relevant to $b\to s$ and $b\to d$ penguin transitions. The first example is $B\to\omega K^*$. Here the underlying process is a $b\to su\overline{u}$ or $b\to sd\overline{d}$ transition. The strong penguin diagram is $\Delta I=0$ and the tree diagram $b\to su\overline{u}$ has both $\Delta I=0$, 1. Define $u_1=\mathcal{M}(B^-\to K^{*-}\omega)$ and $u_2=\mathcal{M}(\overline{B}\to\overline{K}^{*0}\omega)$. From isospin considerations we obtain $c_1=-c_2=1$ since (u_1-u_2) is proportional to the $\Delta I=1$ amplitude only and so must have only the weak phase of the tree diagram. In this case $\delta_T=\gamma$, so that $\hat{\beta}+\delta_T=\beta+\gamma=\pi-\alpha$, in the above. Thus, if the contamination of EWP diagram is small, the angle α may be extracted following the procedure outlined above. The degree to which such contamination is present may be gauged by checking the condition in Eq. (9).

One feature of the neutral B meson in this case is that one may control whether oscillation effects are present or not by selecting the decay mode of the K^{0*}; thus if $\overline{B^0}\to\omega\overline{K}^{*0}[\to K_s\pi^0]$ the final state is an eigenstate of C so this mode may be used to extract T_s. If, on the other hand, $\overline{B^0}\to\omega\overline{K}^{*0}[\to K^-\pi^+]$ then clearly the flavor of the initial state is determined from the final state and oscillation effects are absent allowing the direct determination of $|v_2^k|$. Unfortunately, it is not quite clear that the EWP diagrams are small; some estimates [5] of color allowed EWP diagrams to such final states indicate that the contamination may be $O(10\%)$.

It may, however be possible to select final meson states where EWP effects are likely to be small based on the assumption that color suppression tends to render them unimportant. With that in mind, observe that the contribution to $K^*\omega$ by the EWP diagram is color allowed when both the quarks that result from the virtual Z or γ form the ω. However, note that for this unsuppressed contribution, the EWP diagram has $\Delta I=0$ since the Z or γ are then converting to an $I=0$ object (i.e., ω). The failure of the condition in Eq. (9) and the problem of extracting α which these diagrams cause comes only from their $\Delta I=1$ component. Therefore, this manifestation of the color-allowed EWP diagram does not effect the determination of α given that (as is the case in the SM) the electroweak and strong penguins diagrams have the same weak phase. The effects which result from the $\Delta I=1$ component arise from hadronization where one of the quarks from the Z, γ goes with the ω and the other with the K^*. Such diagrams are color suppressed and so their contamination on the ability to determine α are expected to be only $O(1\%)$. However, since our understanding of color suppression is not reliable it would be very useful to quantitatively ascertain the EWP diagram through the use of Eq. (9). Thus, based on all that we know so far it seems very likely that $K^*\omega$ would be a very good mode for the extraction of α. The mild reservation regarding the presence of EWP diagrams can and should be verified through Eq. (9).

Another example where color suppression may reduce the effect of EWP diagrams is in the class of decays $B\to K^*\rho$. First, consider the case when there were no EWP diagrams. Then each helicity combination, 0, S, and P behaves like the analogous $K\pi$ system which is discussed in [10,12]. Furthermore, the cases where EWP diagrams would be color suppressed are those which contain $\rho^{\pm}$. Thus, if we denote, $u_1=\mathcal{M}(B^-\to K^{*-}\rho^0)$, $u_2=\mathcal{M}(B^-\to \overline{K}^{*0}\rho^-)$, $u_3=\mathcal{M}(\overline{B}^0\to K^{*-}\rho^+)$, and $u_4=\mathcal{M}(\overline{B}^0\to\overline{K}^{*0}\rho^0)$, the assumption that electroweak penguin diagrams are color suppressed and are negligible is equivalent to saying that u_2 and u_3 are free of EWP diagrams.

In this case the application of the isospin is somewhat more complicated than in the previous case where one of the final-state mesons was an isosinglet. We can, however construct a relationship of the desired form between the two amplitudes, u_2 and u_3, by noting that if only $\Delta I=0$ contributions were present, $u_2+u_3=0$. This means that more generally u_2+u_3 is proportional to the $\Delta I=1$ transition amplitude and will have the weak phase of the tree graph although it will be a combination of the amplitude going to a $I=1/2$ and $I=3/2$ final state. It thus follows that

$$v_2^h+v_3^h=\overline{v}_2^{\,h}+\overline{v}_3^{\,h}, \tag{15}$$

which is a relation of the form of Eq. (5), and so one obtains all the results that follow from it; in particular, the test for EWP contamination Eq. (9), and the determination of the phases of the amplitudes Eq. (10). The phases of $v_{2,3}$ and $\overline{v}_{2,3}$ may thus be determined.

If EWP contamination were absent from all of $B\to K^*\rho$ amplitudes then one would also have a similar expression involving v_1 and v_4 and could obtain these phases in the same way. In particular, we need to know the phase of v_4 to obtain α through an oscillation experiment since $\overline{K}^{*0}\rho^0$ is the only case where the final state may be a CP eigenstate (if the neutral K^* decays to K_s, for instance). Fortunately, we may obtain an isospin relation which gives us the required phase in terms of v_2 and v_3 independent of the possibility of EWP contamination since the magnitudes $|v_{1,4}^h|$ and $|\overline{v}_{1,4}^{\,h}|$ are known and even in the presence of EWP contamination, v_1 and v_4 are related to v_2 and v_3 through

$$v_4^h-v_1^h=(v_2^h-v_3^h)/\sqrt{2};\quad \overline{v}_4^{\,h}-\overline{v}_1^{\,h}=(\overline{v}_2^{\,h}-\overline{v}_3^{\,h})/\sqrt{2}. \tag{16}$$

These relations follow since the left and the right side of each is proportional to the $\Delta I=0$ component of the transition, assuming that there is no $\Delta I=2$ transition. This latter assumption would be valid in the SM (to the extent that isospin is conserved and we are working up to the lowest order in weak interactions) and in most of its extensions. It follows that in the SM these apply even with an arbitrary amount of electroweak penguin contamination to v_1 and v_4. Note also that unlike [10,12], the measured phases between the helicity amplitudes are essential to fix the phases of the amplitudes v_2^h and v_3^h because with the EWP contamination to $v_{1,4}$ only the relation (15) between v_2 and v_3 can be formed. Relation (15) between v_2 and v_3 forms the basis for finding their relative phase and Eq. (16) allows us then to find the relative phases of the $v_{1,4}$ independent of EWP contamination to these amplitudes.

Since the magnitude and phase of the right-hand side of each of these Eqs. (16) is known, one can solve for the phases of $v_{1,4}^h$ and $\overline{v}_{1,4}^{\,h}$ up to a twofold ambiguity. The observed relative phases between the various helicities in the v_1 and v_4 channels eliminates this ambiguity. We can then use the phase differences between v_4^h and $\overline{v}_4^h$ and find α from the oscillation data for $\overline{B}^0\to\overline{K}^{0}{}^*\rho^0$ as previously described. Once again we stress that for this analysis for α, through $B\to K^*\rho$ modes, to work, we must assume only that EWP diagrams are small in the color-suppressed instances, which are states containing $\rho^{\pm}$ (i.e., u_2, u_3). This is clearly highly plausible, but in any case is verifiable through Eq. (9). No corresponding assumption regarding ρ^0 modes (u_1 and u_4) is required.

In the case where the EWP diagrams are negligible, it is interesting to compare the information that may be learned from the $B\to VV$ decays where there are three helicity amplitudes with that from $B\to PP$ and $B\to VP$ decays where there is only one. Cases of the latter type would include $B\to K\eta(\eta')$ or $B\to K\omega$, for instance.

For $B\to K\omega$, we define $u_1=\mathcal{M}(B^-\to K^-\omega)$ and $u_2=\mathcal{M}(\overline{B}\to\overline{K}^0\omega)$ but here there is no helicity dependence and all angular distributions are isotropic so one only knows the magnitudes of the amplitudes but not their phases. In the absence of EWP diagrams, $v_1 - v_2 = \overline{v}_1 - \overline{v}_2$, which still leaves free one degree (aside from an overall strong phase) of freedom, the magnitude $f = |v_1 - v_2|$.

We can, however, infer some inequalities which will apply in these cases. The equation among the complex amplitudes: $v_1 - v_2 = \overline{v}_1 - \overline{v}_2$ implies that

$$|v_1| \leq |v_2| + |\overline{v}_1| + |\overline{v}_2|, \tag{17}$$

where the four amplitudes may be permuted. If these inequalities are not satisfied, then it would mean that there is significant contamination from EWP diagrams or from some source of new physics.

In decays to scalars with more complicated structure, for instance $B \to K\pi$, it is also possible to detect the presence of EWP diagrams using the equation [10]:

$$2|m_1|^2 - |m_2|^2 - |m_3|^2 + 2|m_4|^2 = 2|\overline{m}_1|^2 - |\overline{m}_2|^2 - |\overline{m}_3|^2 + 2|\overline{m}_4|^2. \tag{18}$$

where m_1, etc., are the amplitudes of the four $K\pi$ modes [10]. If with this relation the EWP diagrams are confirmed to be negligible, it will then be possible to extract α as described in [12,10].

Returning to the case of $B \to K^*\rho$, we can construct similar identities for each helicity:

$$2|u_1^h|^2 - |u_2^h|^2 - |u_3^h|^2 + 2|u_4^h|^2 = 2|\overline{u}_1^{-h}|^2 - |\overline{u}_2^{-h}|^2 - |\overline{u}_3^{-h}|^2 + 2|\overline{u}_4^{-h}|^2. \tag{19}$$

Again, it is worth noting that these relations [Eqs. (17)–(19)] are completely model independent, they assume only isospin conservation. It may, for instance, be of particular interest to consider the case involving u^{+1} and $\overline{u}^{-1}$ since these would require the final-state s quark to be right handed and so may be suppressed in the SM. On the other hand, effects from new physics which couple to right-handed fermions may be enhanced in this channel and so in that case Eq. (19) may be sensitive to such contributions.

Note that Eq. (19) also applies if h represents one of the parity eigenstates 0, V or A. If one of these cases has a small EWP contribution, α may be extracted from that case also using the same method as in the $B \to K\pi$ system (with a sign change for barred amplitudes in the A case).

We can also consider the analogous case where there is a $b \to d$ transition, for example, $B \to \rho\omega$. Now, the strong penguin diagram is $\Delta I = 1/2$ and the tree process ($b \to du\overline{u}$) contains both $\Delta I = 1/2$ and $\Delta I = 3/2$ components. Likewise, possible electroweak penguin processes are $\Delta I = 1/2$, $I = 3/2$.

Here again $\delta_T = \gamma$ so that the CKM phase we may hope to recover through our method is still α. If we define $u_1 = \mathcal{M}(B^- \to \rho^- \omega)$ and $u_2 = \mathcal{M}(\overline{B} \to \rho^0 \omega)$, isospin gives $c_1 = -c_2 = 1$. Now, the color-allowed EWP contribution (i.e. $Z \to \omega$) will not cause any problem for them $\Delta I = 1/2$ as in the case of the strong penguin diagram. However, the other color-allowed EWP diagram (i.e., $Z \to \rho^0$) will be problematic. Thus $B \to \rho\omega$ can only become a viable method for extracting α, if it can be shown, through Eq. (9), that EWP contamination is small.

It has been suggested [9,10] that rescattering effects in exclusive states of the type that we are considering may be large due to the presence of many intermediate states which rescatter to such a final state. If this is true, the quark content in the final state may differ from that initially present in the weak decay. For instance, the tree-level transition $b \to du\overline{u}$ could lead to decays like $B \to \rho\phi$ on the meson level. Here, the EWP contamination from $b \to ds\overline{s}$ will, again, not be a problem since it has the same isospin properties as the strong penguin diagram. The contamination that might cause a problem will come from rescattering of the EWP modes $b \to du\overline{u}$ and $b \to dd\overline{d}$ to $b \to ds\overline{s}$. This is expected to be extremely tiny as it originates from Zweig suppressed conversion of the EWP amplitude.

In this example the strong penguin diagrams and the $b \to ds\overline{s}$ EWP diagrams are $\Delta I = 1/2$, while the tree process has $\Delta I = 1/2$ and $\Delta I = 3/2$. If we define $u_1 = \mathcal{M}(B^- \to \rho^- \phi)$ and $u_2 = \mathcal{M}(\overline{B} \to \rho^0 \phi)$, the isospin structure is clearly the same as $\rho\omega$; so $c_1 = -c_2 = 1$. Again $\delta_T = \gamma$ so that the analysis will give α. Thus final-state rescattering of tree amplitudes in exclusive channels has a nice application here as it leads to a clean method for obtaining α.

V. CONCLUSIONS

To summarize, in this work we have provided a systematic, model-independent technique for quantitatively assessing the importance of electroweak penguin diagrams and/or new physics by studying B decays to two-vector particles resulting from penguin and tree interferences. Our tests only assume isospin conservation; note also that these tests make no assumption about rescattering contributions. The modes that do not exhibit such effects can then be used for extracting the angles of the unitarity triangle. $B \to K^*\omega(\rho)$, $\rho\omega(\phi)$ can all be used for extracting α.

We close with the following two brief remarks.

(1) Our first comment concerns the expected branching ratio. In this regard, we note a weak indication for two related modes [13]:

$$\mathcal{B}(B^+ \to \omega K^+) = (1.5^{+0.7}_{-0.6} \pm 0.2) \times 10^{-5}, \tag{20}$$

$$\mathcal{B}(B \to \phi K^*) = (1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}, \tag{21}$$

in which the penguin contribution is expected to be dominant. Based on these results we should also expect

$$\mathcal{B}(B \to \omega K^*) \simeq \mathcal{B}(B \to \rho K^*) \simeq 1 \times 10^{-5}, \tag{22}$$

which is also quite close to the $\mathcal{B}(B \to K\pi)$ found by CLEO [1].

For $B^+ \to \rho^+ \omega$ we expect the tree graph to dominate. CLEO [14] reports weak ($\approx 2\sigma$) signal in the $\pi\pi$ modes with the upper bound [15]:

$$\mathcal{B}(B^+ \to \pi^+ \pi^0) < 1.6 \times 10^{-5}. \tag{23}$$

Therefore, it is quite likely that $\mathcal{B}(B\to\rho\omega)$ is also in the same ballpark (i.e., $\sim 10^{-5}$). On the other hand, $B\to\rho\phi$ results from final-state rescattering effects. Most likely the branching ratio of this mode will, therefore, be smaller than 10^{-5}, by factors of order (3–10).

(2) Our second comment deals with the application of our method for extracting the CKM phase γ. Since in the standard phase convention [7] $\delta_T=\gamma$ and the B_s-$\overline{B_s}$ oscillation phase $\hat{\beta}_{B_s\text{-}\overline{B_s}}=0$, it is therefore clear that to determine γ through the use of our VV method will require the study of B_s decays. As in the case of B decays reported in this work, an interplay of direct and mixing-induced CP (through $B_s - \overline{B_s}$ oscillations) will have to be involved. By inspection of the tree process $b\to u\overline{u}s$, which donates δ_T, one immediately arrives at two examples:

$$(1) B_s\to K^*\rho,$$

$$(2) B_s\to \overline{K^*}K^*, \tag{24}$$

which can be used. We hope to return to these in a future publication.

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