Constraining the left-right supersymmetric model from $\mu \to e\gamma$ and $\mu \to 3e$

M. Frank*
Department of Physics, Concordia University, 1455 De Maisonneuve Boulevard West, Montreal, Quebec, Canada H3G 1M8
(Received 5 June 1998; published 20 November 1998)

We present a detailed and complete analysis of the lepton-flavor violating decay $\mu \to e\gamma$ to one loop order (terms proportional to m_μ) in the left-right supersymmetric model. We include the mixing of the scalar partners of the left- and right-handed leptons, and show that it leads to a strong enhancement of the branching ratio. We study the distinctive features of the decay in this model and show that it can be distinguished from the same process in the minimal supersymmetric standard model in most scenarios. [S0556-2821(98)03423-7]

PACS number(s): 13.35.Bv, 12.60.Jv, 14.60.Ef

I. INTRODUCTION

Supersymmetry is the leading candidate among theories beyond the standard model. It has many theoretically attractive features, such as explaining the boson-fermion symmetry and also providing a mechanism for solving two of the fundamental problems in the standard model: stability under radiative corrections and the origin of the electro-weak scale. The most popular realization of supersymmetry, the minimal supersymmetric standard model (MSSM) can be probed experimentally through the production of superpartners.

However, the MSSM, while filling in some of the theoretical gaps of the standard model, fails to explain other phenomena such as the weak mixing angle, the small mass (or masslessness) of the known neutrinos, the origin of *CP* violation, or the absence of rapid proton decay. Extended gauge structures such as grand unified theories, introduced to provide an elegant framework for the unification of forces [1], would connect the standard model with more fundamental structures such as superstrings, and also would resolve the puzzles of the electroweak theory. Such extended structures, while attempting to solve some of the theoretical inconsistencies of the MSSM, would either predict relationships between otherwise independent parameters of the standard model, or predict new interactions (either forbidden or highly suppressed in the MSSM), interactions which would distinguish them from supersymmetry in general. Left-right supersymmetry (LR SUSY) is perhaps the most natural extension of the minimal model [2,3,4,5]. LR SUSY was originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses through the see-saw mechanism [4]. It has received a lot of attention lately because it has been shown that it could offer a solution to both the strong and the weak *CP* problem [6]. LR SUSY can occur as an intermediate scale theory in several SUSY GUT scenarios [2]. The consequences of the left-right supersymmetric model at colliders have been explored extensively [5,7]: it was shown that in most cases it could lead to enhanced production rates for charginos and neutralinos. Another interesting consequence is that the model allows the production of light doubly charged Higgs boson and Higgssinos [8].

Here we propose to investigate a highly suppressed decay in the MSSM, the decay $\mu \to e\gamma$. This decay is a very sensitive probe of physics at the Planck scale, which makes it an essential signal for new physics. In the MSSM flavor violation comes into direct conflict with naturalness [9]. In a previous work [10], we have shown that the LR SUSY model is capable of giving rise to large lepton-flavor decay rates, in much the same way as SO(10) [11,12]. We evaluated $\mu \to e\gamma$ contributions proportional to m_τ coming from potentially large Yukawa couplings for the neutrino, h_ν. We present here a complete one-loop calculation of this process and show that the terms proportional to m_μ can complete with, and be larger than, the dominant m_τ contribution. A complete analysis of this decay is necessary for two purposes. On one hand, we expect the decay to be enhanced compared to the MSSM, because of the rich and interesting structure of the leptonic sector of the LR SUSY model (through the presence of right-handed neutrinos and Higgs bosons that couple to leptons only). On the other hand, the upper limit for both the branching ratios $\mu \to e\gamma$ and $\mu \to 3e$ is severely constrained $BR(\mu \to e\gamma) \leqslant 4.9\times 10^{-11}$ and $BR(\mu \to 3e) \leqslant 1.0\times 10^{-12}$ [13], a test against which all grand unified theories are measured. This justifies the need for more than the rough estimate presented in [10]. This paper is organized as follows: we review the LR SUSY model in Sec. II, we discuss the sources of flavor violation in Sec. III, after which we present our analysis of the decay $\mu \to e\gamma$ and $\mu \to 3e$ in Sec. IV. Our numerical analysis and discussion are included in Sec. V, and we conclude in Sec. VI.

II. THE LEFT-RIGHT SUPERSYMMETRIC MODEL

The LR SUSY model, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right-handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks) [14]. In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^{+,-},W^0)_L$, $(W^{+,-},W^0)_R$ and a singlet gauge boson V corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bi-doublets, $\Phi_u(\frac{1}{2},\frac{1}{2},0)$ and $\Phi_d(\frac{1}{2},\frac{1}{2},0)$, which are required to give masses to both the up and down quarks. The phenomenology of the doublet Higgs is similar

*Email address: mfrank@vax2.concordia.ca

to the non-supersymmetric left-right model [3], except that the second pair of Higgs doublet fields, which provide new contributions to the flavor-changing neutral currents, must be heavy, in the 5–10 TeV range, effectively decoupling from the low-energy spectrum [15]. The spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by the vacuum expectation values of a pair of Higgs triplet fields $\Delta_L(1,0,2)$ and $\Delta_R(0,1,2)$, which transform as the adjoint representation of $SU(2)_R$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated (through the see-saw mechanism) for the right-handed neutrino and a small one for the left-handed neutrino [3]. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets $\delta_L(1,0,-2)$ and $\delta_R(0,1,-2)$, with quantum number $B-L=-2$ to insure cancellation of the anomalies that would otherwise occur in the fermionic sector. Given their strange quantum numbers, the δ_L and δ_R do not couple to any of the particles in the theory, so their contribution is negligible for any phenomenological studies.

The superpotential for the LR SUSY is

$$\begin{aligned} W = {} & \mathbf{h}_q^{(i)} Q_L^T \tau_2 \Phi_i \tau_2 Q_R + \mathbf{h}_l^{(i)} L_L^T \tau_2 \Phi_i \tau_2 L_R \\ & + i(\mathbf{h}_{LR} L_L^T \tau_2 \Delta_L L_L + \mathbf{h}_{LR} L_R^T \tau_2 \Delta_R L_R) \\ & + M_{LR}[\mathrm{Tr}(\Delta_L \overline{\Delta_L}) + \mathrm{Tr}(\Delta_R \overline{\Delta_R})] \\ & + \mu_{ij}\ \mathrm{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j) + \mu_L \mathrm{Tr}(\Delta_L \delta_L) + \mu_R \mathrm{Tr}(\Delta_R \delta_R) \\ & + W_{NR} \end{aligned} \tag{1}$$

where W_{NR} denotes (possible) non-renormalizable terms arising from higher scale physics or Planck scale effects [8]. The presence of these terms insures that, when the SUSY breaking scale is above M_{W_R}, the ground state is *R*-parity conserving [16].

As in the standard model, in order to preserve $U(1)_{EM}$ gauge invariance, only the neutral Higgs fields acquire non-zero vacuum expectation values (VEV's). These values are:

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

and

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\omega} \end{pmatrix}.$$

$\langle \Phi \rangle$ causes the mixing of W_L and W_R bosons with *CP*-violating phase ω. In order to simplify, we will take the VEV's of the Higgs fields as $\langle \Delta_L \rangle = 0$ and

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$\langle \Phi_d \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}.$$

Choosing $v_L = \kappa' = 0$ satisfies the more loosely required hierarchy $v_R \gg \max(\kappa, \kappa') \gg v_L$ and also the required cancellation of flavor-changing neutral currents. The Higgs fields aquire non-zero VEV's to break both parity and $SU(2)_R$. In the first stage of breaking, the right-handed gauge bosons, W_R and Z_R acquire masses proportional to v_R and become much heavier than the usual (left-handed) neutral gauge bosons W_L and Z_L, which pick up masses proportional to κ_u and κ_d at the second stage of breaking.

The supersymmetric sector of the model, while preserving left-right symmetry, has four singly-charged charginos (corresponding to $\tilde{\lambda}_L$, $\tilde{\lambda}_R$, $\tilde{\phi}_u$, and $\tilde{\phi}_d$), in addition to $\tilde{\Delta}_L^-$, $\tilde{\Delta}_R^-$, $\tilde{\delta}_L^-$ and $\tilde{\delta}_R^-$. The model also has eleven neutralinos, corresponding to $\tilde{\lambda}_Z$, $\tilde{\lambda}_{Z'}$, $\tilde{\lambda}_V$, $\tilde{\phi}_{1u}^0$, $\tilde{\phi}_{2u}^0$, $\tilde{\phi}_{1d}^0$, $\tilde{\phi}_{2d}^0$, $\tilde{\Delta}_L^0$, $\tilde{\Delta}_R^0$, $\tilde{\delta}_L^0$, and $\tilde{\delta}_R^0$. It has been shown that in the scalar sector, the left-triplet Δ_L couplings can be neglected in phenomenological analyses of muon and tau decays [17]. Although Δ_L is not necessary for symmetry breaking [5] and is introduced only for preserving left-right symmetry, both Δ_L^{--} and its right-handed counterpart Δ_R^{--} play very important roles in phenomenological studies of the LR SUSY model. It has been shown that these bosons, and possibly their fermionic counterparts, are light [8]. Also, these doubly charged Higgs and their corresponding Higgsinos lead to an enhancement in lepton-flavor violating decays, as well as enhancing the anomalous magnetic of the muon [4] and the electric dipole moment of the electron [18].

III. SOURCES OF FLAVOR VIOLATION IN LR SUSY

The sources of flavor violation in the LR SUSY model come from either the Yukawa potential or from the trilinear scalar coupling.

The interaction of fermions with scalar (Higgs) fields has the following form:

$$\begin{aligned} \mathcal{L}_Y = {} & \mathbf{h}_u \overline{Q_L} \Phi_u Q_R + \mathbf{h}_d \overline{Q_L} \Phi_d Q_R + \mathbf{h}_\nu \overline{L_L} \Phi_u L_R \\ & + \mathbf{h}_e \overline{L_L} \Phi_d L_R + \text{H.c.}; \\ \mathcal{L}_M = {} & i \mathbf{h}_{LR} (L_L^T C^{-1} \tau_2 \Delta_L L_L + L_R^T C^{-1} \tau_2 \Delta_R L_R) + \text{H.c.}, \end{aligned} \tag{2}$$

where $\mathbf{h}_u$, $\mathbf{h}_d$, $\mathbf{h}_\nu$, and $\mathbf{h}_e$ are the Yukawa couplings for the up and down quarks and neutrino and electron, respectively, and $\mathbf{h}_{LR}$ is the coupling for the triplet Higgs bosons. LR symmetry requires all $\mathbf{h}$-matrices to be Hermitean in the generation space and $\mathbf{h}_{LR}$ matrix to be symmetric. The Yukawa matrices have physical and geometrical significance and cannot be rotated away. Geometrically, they represent misalignment between the particle and sparticle bases in flavor space. Their physical significance is that they cause flavor violation. The trilinear scalar couplings appear in the soft-scalar mass term in the Lagrangian:

$$\mathcal{L}_{soft} = -[\mathbf{A}_q^i \mathbf{h}_q^{(i)} \tilde{Q}_L^T \tau_2 \Phi_i \tau_2 \tilde{Q}_R + \mathbf{A}_l^i \mathbf{h}_l^{(i)} \tilde{L}_L^T \tau_2 \Phi_i \tau_2 \tilde{L}_R + i \mathbf{A}_{LR}^i \mathbf{h}_{LR} (\tilde{L}_L^T \tau_2 \Delta_L L_L + L_R^T \tau_2 \Delta^c \tilde{L}_R)] - [M_L \tilde{W}_L \tilde{W}_L + M_R \tilde{W}_R \tilde{W}_R + M_V \tilde{V} \tilde{V}] - M_\Delta^2 [\mathrm{Tr}(\Delta_L \overline{\Delta_L}) + \mathrm{Tr}(\Delta_R \overline{\Delta_R})] - B \mu_{ij} \Phi_i \Phi_j - \mu_{ij}^2 \Phi_i \Phi_j \quad (3)$$

where the **A**-matrices (A_u, A_d, A_ν, and A_e) are of a similar form to the Yukawa couplings and provide additional sources of flavor violation; and B is a mass term. The inter-generational slepton mixing ($\tilde{e}$, $\tilde{\mu}$, and $\tilde{\tau}$) and also left-right slepton mixing ($\tilde{e}_L$, $\tilde{e}_R$) cause the off-diagonal nature of the matrices, and therefore are responsible for flavor violation. We shall analyze both of these in turn.

In the scalar matter sector, the LR SUSY contains two left-handed and two right-handed scalar fermions as partners of the ordinary leptons and quarks, which themselves come in left- and right-handed doublets. In general the left- and right-handed scalar leptons will mix together. Some of the effects of this mixings, such as the enhancement of the anomalous magnetic moment of the muon, have been discussed elsewhere [4]. Only global lepton-family-number conservation would prevent $\tilde{e}$, $\tilde{\mu}$, and $\tilde{\tau}$ to mix arbitrarily. Permitting this mixing to occur, we could expect small effects to occur in the non-supersymmetric sector, such as radiative muon or tau decays, in addition to other nonstandard effects such as massive neutrino oscillations and violation of lepton number itself. Allowing general mixings leads to six charged-scalar lepton states (involving 15 real angles and 10 complex phases) and six scalar neutrinos (also involving 15 real angles and 10 complex phases). In order to reduce the (large) number of parameters we shall assume in what follows that two types of mixings dominate.

First, the scalar lepton (selectron, smuon and stau) mix, but we shall assume for simplicity that only two generations of scalar leptons (the lightest) mix significantly. The third family will be suppressed by a small, extra mixing angle, important only if mass splittings involving the third family are much larger than the mass splittings between the first two families. But, from universality, we expect the mass splittings to be the same for all three families, i.e.,

$$m_{\tilde{e}}^2 - m_{\tilde{\mu}}^2 \approx m_{\tilde{\mu}}^2 - m_{\tilde{\tau}}^2 \approx m_{\tilde{\tau}}^2 - m_{\tilde{e}}^2 \quad (4)$$

(although of course there will be additional effects coming from the large Yukawa coupling h_τ below the flavor scale). The scalar lepton mixing is described as follows: $\tilde{\mu}_{L,R}$ and $\tilde{e}_{L,R}$ with angle $\theta_{L,R}$; $\tilde{\nu}_{\mu_{L,R}}$ and $\tilde{\nu}_{e_{L,R}}$ with angle $\alpha_{L,R}$; so that, for example:

$$\tilde{l}_{L_1} = \tilde{\mu}_L \cos\,\theta_L + \tilde{e}_L \sin\,\theta_L, \quad (5)$$

$$\tilde{l}_{L_2} = -\tilde{\mu}_L \sin\,\theta_L + \tilde{e}_L \cos\,\theta_L, \quad (6)$$

and similarly for $\tilde{l}_{R_{1,2}}$ and $\tilde{\nu}_{L_{1,2}}$ and $\tilde{\nu}_{R_{1,2}}$.

Second, we have the mixing of the scalar partners of the left- and right-handed leptons; which is independent of the above mixing. We parametrize this second mixing as follows:

$$\tilde{e}_L = \cos\,\theta_{\tilde{e}} \tilde{e}_1 - \sin\,\theta_{\tilde{e}} \tilde{e}_2, \quad (7)$$

$$\tilde{e}_R = \sin\,\theta_{\tilde{e}} \tilde{e}_1 + \cos\,\theta_{\tilde{e}} \tilde{e}_2, \quad (8)$$

and similarly for the other generations.

In order to protect the decay $\mu \to e\gamma$ from a large contribution from the A term, we must assume all the A terms are *approximately* proportional and that the scalar masses are approximately universal. Since with approximate proportionality the splittings among the A-terms are comparable to the splittings among the scalar mass term, then individual contributions from $\mu \to e\gamma$ diagrams involving the A terms will be of the same order of magnitude as the leading graphs. We encounter the same situations as in the MSSM [9], in which when all the leading diagrams are added, the A terms make a negligible contribution, so as another simplifying approximation we set $A = 0$.

Next we consider the implications of these flavor changing mechanisms in LR SUSY in lepton-flavor violating decays $\mu \to e\gamma$ and $\mu \to 3e$.

IV. THE AMPLITUDE FOR THE PROCESS $\mu \to e\gamma$ AND $\mu \to 3e$

A. $\mu \to e + \gamma$

The amplitude of the $\mu \to e\gamma$ transition can be written in the form of the usual dipole-type interaction:

$$\mathcal{M}_{\mu \to e\gamma} = \frac{1}{2} \overline{\psi}_e (d_L P_L + d_R P_R) \sigma^{\mu\nu} F_{\mu\nu} \psi_\mu. \quad (9)$$

It leads to the branching ratio:

$$\Gamma_{\mu \to e\gamma} = \frac{1}{16\pi} \tau_\mu (|d|_L^2 + |d|_R^2) m_\mu^3. \quad (10)$$

Comparing it with the standard decay width, $\Gamma_{\mu \to e\nu\overline{\nu}} = (1/192\pi^3) G_F^2 m_\mu^5$ and using the experimental constraint on the branching ratio, we get the following limit on the dipole amplitude:

$$|d| = \sqrt{(|d_L|^2 + |d_R|^2)/2} < 3.5 \times 10^{-26}\ e\ \mathrm{cm}. \quad (11)$$

The most complete calculation of $\mu \to e + \gamma$ branching ratio in the MSSM is that of reference [9], which calculates all leading one loop contributions, including several previously omitted contributions.

The contributions to the decay $\mu \to e + \gamma$ in the left-right supersymmetric model are presented in the diagrams of Fig. 1. The evaluation of these graphs includes the elements of 2×2 mass matrices: the Yukawa matrix, the left-handed and

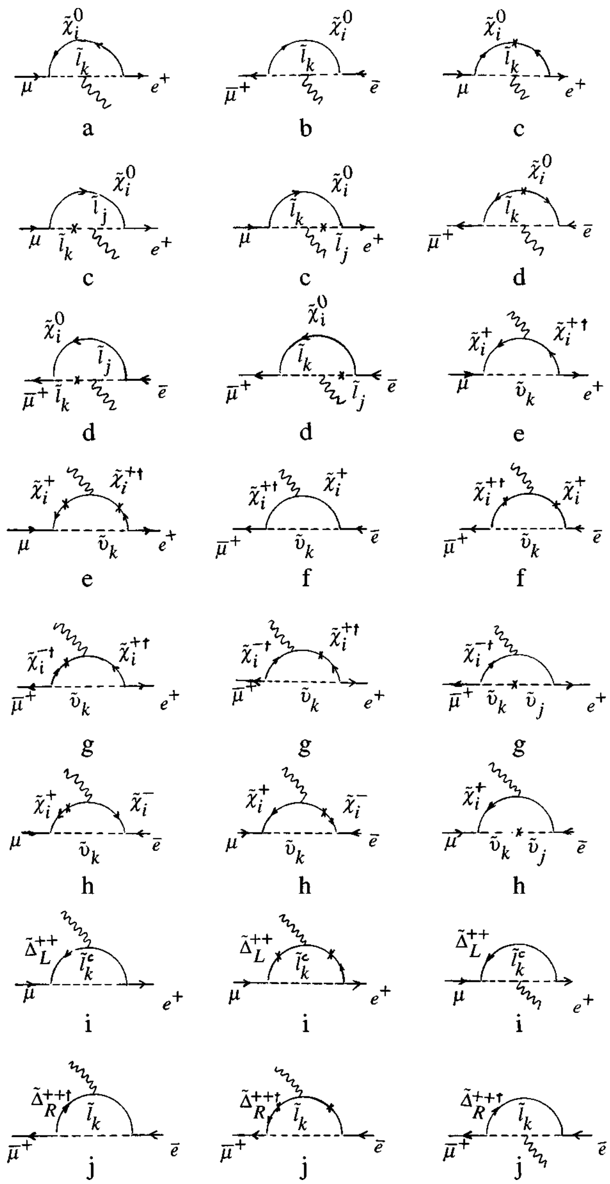

FIG. 1. One loop contributions to the decay $\mu \to e\gamma$ in the left-right supersymmetric model: (a) neutralinos, left-handed fermions, external chirality flip; (b) neutralinos, right-handed fermions, external chirality flip; (c) neutralinos, left-handed fermions, internal chirality flip; (d) neutralinos, right-handed fermions, internal chirality flip; (e) charginos, left-handed fermions, external chirality flip; (f) charginos, right-handed fermions, external chirality flip; (g) charginos, left-handed fermions, internal chirality flip; (h) charginos, right-handed fermions, internal chirality flip; (i) doubly charged higgsinos, left-handed fermions, external chirality flip; (j) doubly charged Higgsinos, right-handed fermions, external chirality flip; (k) doubly charged Higgsinos, left-handed fermions, internal chirality flip; (l) doubly charged Higgsinos, right-handed fermions, internal chirality flip.

the right-handed lepton doublet scalar mass matrix. The associated physical parameters are the Yukawa eigenvalues; the mass eigenvalues for the left- and right-handed lepton doublets; and mixing angles for both left lepton doublets, θ_L, and right handed lepton doublets, θ_R, as well as angles $\theta_{\tilde{e}}$ and $\theta_{\tilde{\mu}}$ between the left and right scalar leptons of the same family. These angles describe the rotation between the sparticle and particle mass eigenbases.

Because the scalar mass splittings are required to be small, we will parametrize the scalar mass eigenvalues by the average masses, $m_{\tilde{L}}^2$ and $m_{\tilde{R}}^2$, and the mass splittings, $\delta\tilde{m}_L^2$ and $\delta\tilde{m}_R^2$. We will also keep only the leading contribution in both the mass splittings and the mixing angles. Equation (12) gives the branching ratio for the process $\mu \to e + \gamma$. The functions X_L and X_R are given below:

$$BR(\mu \to e+\gamma) = \frac{3e^2}{2\pi^2}\left\{ \theta_L^2 \left(\frac{M_W}{m_{\tilde{L}}}\right)^4 (X_L)^2 \left(\frac{\delta\tilde{m}_L^2}{m_{\tilde{L}}^2}\right)^2 + \theta_R^2 \left(\frac{M_W}{m_{\tilde{R}}}\right)^4 (X_R)^2 \left(\frac{\delta\tilde{m}_R^2}{m_{\tilde{R}}^2}\right)^2 \right\}. \quad (12)$$

We now consider the implications of these mixings in the the lepton-flavor violating decay $\mu \to e\gamma$. We use the modified loop functions which are defined below. The argument of these loop functions is $r_{pk} = M_k^2/m_p^2$ where k represents the chargino or neutralino, and p represents the slepton. The chargino and neutralino masses enter the theory via their mass eigenvalues and mixing matrices. The explicit form of these matrices is found in [20].

Following [20], we employ the following notation: the U, N matrices rotate the gaugino/Higgsino interaction basis into the neutralino/chargino mass basis. N^0 is the matrix for the neutralinos; U^+ is the matrix for the charginos $\tilde{W}_{L,R}^+$ and $\tilde{H}_u^+$; and U^- is for the charginos $\tilde{W}^-$ and $\tilde{H}_d^-$. $U_{\Delta_{L,R}}^{--}$, $U_{\Delta_{L,R}}^{++}$ are mixing matrices for the doubly charged $\tilde{\Delta}_{L,R}$ and $\tilde{\delta}_{L,R}$ Higgsino mixing:

$$X_L = X_{Lf} + X_{Lh} + X_{Lg} + X_{Lj} + X_{L\Delta_f} + X_{L\Delta_h} \quad (13)$$

$$X_R = X_{Rf} + X_{Rh} + X_{Rg} + X_{Rj} + X_{R\Delta_f} + X_{R\Delta_h}. \quad (14)$$

Here X_L represents the left-handed contribution, and X_R the contribution from the right-handed sector. The individual contributions are as follows:
For neutralinos, left-handed fermions, with an external chirality flip (Fig. 1a):

$$X_{Lg} = \frac{1}{2}\left(N_{W_L k}^0 + \frac{g_1}{g_2} N_{Bk}^0\right)^2 f_g(r_{e_L k}). \quad (15)$$

For the neutralinos, left-handed fermions, with an internal chirality flip (Fig. 1c):

$$X_{Lh}=\frac{(A+\mu\ \tan\beta)M}{m^2_{\tilde{e}_L}}\left(N^0_{W_Rk}+\frac{g_1}{g_2}N^0_{Bk}\right)\times\left(N^0_{W_Lk}+\frac{g_1}{g_2}N^0_{Bk}\right)h_k(r_{e_Lk},r_{e_Rk})-\frac{M}{\sqrt{2}g_2v_1}(N^0_{Hk})\times\left(N^0_{W_Lk}+\frac{g_1}{g_2}N^0_{Bk}\right)h_g(r_{e_Lk})+\left(\frac{m^4_{\tilde{e}_L}}{\delta m^2_{\tilde{\nu}_L}}\right)\frac{\delta A_{\overline{\mu e}}M}{m^2_{\tilde{e}_L}-m^2_{\tilde{e}_R}}\times\left(N^0_{W_Rk}+\frac{g_1}{g_2}N^0_{Bk}\right)\left(N^0_{W_Lk}+\frac{g_1}{g_2}N^0_{Bk}\right)\times\left[\frac{h(r_{e_Lk})}{m^2_{\tilde{e}_L}}-\frac{h(r_{e_Rk})}{m^2_{\tilde{e}_R}}\right]. \tag{16}$$

For charginos, left-handed fermions, with an external chirality flip (Fig. 1e):

$$X_{Lf}=-\left(\frac{m^4_{\tilde{e}_L}}{m^4_{\tilde{\nu}_L}}\right)(U^+_{W_Lk})^2f_g(r_{\nu_Rk}). \tag{17}$$

For charginos, left-handed fermions, with an internal chirality flip (Fig. 1g):

$$X_{Lj}=\left(\frac{m^4_{\tilde{e}}}{m^4_{\tilde{\nu}}}\right)(U^-_{Hk})(U^+_{W_Lk})j_g(r_{\nu_Rk})+\frac{(A+\mu\ \tan\beta)M}{m^2_{\tilde{e}}}(U^-_{W_Lk})\times(U^+_{W_Rk})j_k(r_{e_Lk},r_{e_Rk})+\left(\frac{m^4_{\tilde{e}_L}}{\delta m^2_{\tilde{\nu}_L}}\right)\frac{\delta A_{\overline{\nu}_\mu\nu_e}M}{m^2_{\tilde{e}_L}-m^2_{\tilde{e}_R}}(U^-_{W_Lk})\times(U^+_{W_Rk})\left[\frac{j(r_{e_Lk})}{m^2_{\tilde{e}_L}}-\frac{j(r_{e_Rk})}{m^2_{\tilde{e}_R}}\right]. \tag{18}$$

For doubly-charged Higgsinos, left-handed fermions, with an external chirality flip (Fig. 1i):

$$X_{L\Delta_f}=-\frac{M_{LR}}{2g_2v_R}(U^{--}_{\Delta_R})^2\frac{(A+\mu\ \tan\beta)M}{m^2_{\tilde{e}_L}}\times(2\,f_g(r_{e_L\Delta},r_{e_R\Delta})+g_g(r_{e_L\Delta},r_{e_R\Delta})). \tag{19}$$

There are no contributions for doubly-charged Higgsinos with internal chirality flip because $\tilde{\Delta}_L$ only couples to left-handed fermions and $\tilde{\Delta}_R$ only couples to right-handed fermions.

For neutralinos, right-handed fermions, with an external chirality flip (Fig. 1b):

$$X_{Rg}=\frac{1}{2}\left(N^0_{W_Rk}+\frac{g_1}{g_2}N^0_{Bk}\right)^2f_g(r_{e_Rk}). \tag{20}$$

For neutralinos, right-handed fermions, with an internal chirality flip (Fig. 1d):

$$X_{Rh}=\frac{(A+\mu\ \tan\beta)M}{m^2_{\tilde{e}_R}}\left(N^0_{W_Lk}+\frac{g_1}{g_2}N^0_{Bk}\right)\times\left(N^0_{W_Rk}+\frac{g_1}{g_2}N^0_{Bk}\right)h_k(r_{e_Rk},r_{e_Lk})-\frac{M}{g_2v_1}(N^0_{Hk})\left(N^0_{Wk}+\frac{g_1}{g_2}N^0_{Bk}\right)h_g(r_{e_Rk})+\left(\frac{m^4_{\tilde{e}_R}}{\delta m^2_{\tilde{\nu}_R}}\right)\frac{\delta A_{\mu\overline{e}}M}{m^2_{\tilde{e}_L}-m^2_{\tilde{e}_R}}\times\left(N^0_{W_Rk}+\frac{g_1}{g_2}N^0_{Bk}\right)\left(N^0_{W_Lk}+\frac{g_1}{g_2}N^0_{Bk}\right)\times\left[\frac{h(r_{e_Lk})}{m^2_{\tilde{e}_L}}-\frac{h(r_{e_Rk})}{m^2_{\tilde{e}_R}}\right]. \tag{21}$$

For charginos, right-handed fermions, with an external chirality flip (Fig. 1f):

$$X_{Rf}=-\left(\frac{m^4_{\tilde{e}_R}}{m^4_{\tilde{\nu}_R}}\right)(U^+_{W_Rk})^2f_g(r_{\nu_Rk}). \tag{22}$$

For charginos, right-handed fermions, with an internal chirality flip (Fig. 1h):

$$X_{Rj}=\left(\frac{m^4_{\tilde{e}_R}}{m^4_{\tilde{\nu}_R}}\right)(U^-_{Hk})(U^+_{W_Rk})j_g(r_{\nu_Rk})+\frac{(A+\mu\ \tan\beta)M}{m^2_{\tilde{\nu}_R}}\times(U^-_{W_Lk})(U^+_{W_Rk})j_k(r_{\nu_R},r_{\nu_L})+\left(\frac{m^4_{\tilde{e}_R}}{\delta m^2_{\tilde{\nu}_R}}\right)\frac{\delta A_{\overline{\nu}_e\nu_\mu}M}{m^2_{\tilde{\nu}_L}-m^2_{\tilde{\nu}_R}}\times(U^-_{W_Lk})(U^+_{W_Rk})\left[\frac{j(r_{\nu_Lk})}{m^2_{\tilde{\nu}_L}}-\frac{j(r_{\nu_Rk})}{m^2_{\tilde{\nu}_R}}\right]. \tag{23}$$

For doubly-charged Higgssinos, right-handed fermions, with an external chirality flip (Fig. 1j):

$$X_{R\Delta_f}=-\frac{M_{LR}}{2g_2v_R}(U^{--}_{\Delta_R})^2(2\,f_g(r_{e_R\Delta},r_{e_L\Delta})+g_g(r_{e_R\Delta},r_{e_L\Delta})). \tag{24}$$

Although the equations depend very weakly on A, justifying setting $A=0$, we include the complete A dependence here. Equations (25)–(28) give definitions for the non-universality of the A terms used above.

$$\delta A_{\bar{\mu}e}=A_{\bar{\mu}e}-A_{\bar{\mu}\mu} \tag{25}$$

$$\delta A_{\bar{e}\mu}=A_{\bar{e}\mu}-A_{\bar{\mu}\mu} \tag{26}$$

$$\delta A_{\bar{\nu}_\mu \nu_e}=A_{\bar{\nu}_\mu \nu_e}-A_{\bar{\nu}_\mu \nu_\mu} \tag{27}$$

$$\delta A_{\bar{\nu}_e \nu_\mu}=A_{\bar{\nu}_e \nu_e}-A_{\bar{\nu}_e \nu_\mu}. \tag{28}$$

The $\mu\to e+\gamma$ loop functions are

$$f(r)=\frac{1}{12(1-r)^4}\,(2r^3+3r^2-6r+1-6r^2\ \log\ r) \tag{29}$$

$$g(r)=\frac{1}{12(1-r)^4}\,(r^3-6r^2+3r+2+6r\ \log\ r) \tag{30}$$

$$h(r)=\frac{1}{2(1-r)^3}\,(-r^2+1+2r\ \log\ r) \tag{31}$$

$$j(r)=\frac{1}{2(1-r)^3}\,(r^2-4r+3+2\log\ r) \tag{32}$$

and the functions which appear in the amplitudes are not *f*, *g*, *h*, and *j*, but modifications that result from expansion in the inter-family mass difference. Equation (33) defines the *g* subscript, and Eq. (34) defines the *k* subscript. *F* represents any of the four functions *f*, *g*, *h*, or *j*:

$$F_g\left(\frac{M^2}{m^2}\right)\equiv m^4\ \frac{d}{dm^2}\left\{\frac{1}{m^2}\ F\left(\frac{M^2}{m^2}\right)\right\} \tag{33}$$

$$F_k\left(\frac{M^2}{m_a^2},\ \frac{M^2}{m_b^2}\right)\equiv m_a^6\ \frac{d}{dm_a^2}\left\{\frac{1}{m_a^2-m_b^2}\left[\frac{1}{m_a^2}\ F\left(\frac{M^2}{m_a^2}\right)-\frac{1}{m_b^2}\ F\left(\frac{M^2}{m_b^2}\right)\right]\right\}. \tag{34}$$

B. $\mu\to e+\bar{e}+e$

For $\Gamma(\mu\to 3e)$ we use [17]

$$\frac{\Gamma(\mu\to 3e)}{\Gamma(\mu\to e\gamma)}=\frac{\alpha}{3\pi}\left[\ln\left(\frac{m_\mu^2}{m_e^2}\right)-\frac{11}{4}\right]. \tag{35}$$

Using $\alpha=1/127.9$ we obtain $BR(\mu\to 3e)/BR(\mu\to e\gamma)=6.6\times 10^{-3}$, so the process $\mu\to e+\bar{e}+e$, although significant in restricting the parameters of LR SUSY, is not as stringent a constraint as $\mu\to e\gamma$.

V. NUMERICAL RESULTS AND DISCUSSION

In the previous section we presented analytical expressions for the branching ratio of $\mu\to e\gamma$ in the LR SUSY model. Unfortunately, the large number of parameters makes an exact solution impossible. Before making some approximations, we will discuss all the parameters involved. The electroweak sector of LR SUSY contains three gauge couplings and three gaugino masses. (We are not concerned here with complex parameters since $\mu\to e\gamma$ is always proportional to the real part of the parameters.) The Higgs sector of the Lagrangian contains the scalar masses m_{Φ_u}, m_{Φ_d}, m_{Δ_L} and m_{Δ_R} as well as the parameters μ_{ij} and *B*. The remaining part of the Lagrangian contains, in the flavor sector, fermion Yukawa matrices for both left and right-handed fermions, four tri-linear scalar coupling matrices and scalar mass matrices. Of particular interest for flavor violation in the leptonic sector is the slepton mass matrix, which arises as a result of the renormalization group evolution from the Λ_{GUT} scale and is caused by the admixture of the neutrino Yukawa couplings. The form of this matrix, which could be calculated only numerically, is, incorporating some elements of the left-right symmetry [10]:

$$\mathcal{L}_{Y_{\tilde{e}}}=(\tilde{e}_L^\dagger\tilde{e}_R^\dagger)\begin{pmatrix} m_{\tilde{L}}^2+c_e h_e^2+c_\nu h_\nu^2 & \mathcal{A}_e \\ \mathcal{A}_e^\dagger & m_{\tilde{R}}^2+c'_e h_e^2+c'_\nu h_\nu^2, \end{pmatrix}\begin{pmatrix}\tilde{e}_L\\ \tilde{e}_R\end{pmatrix}, \tag{36}$$

where $\mathcal{A}_e=A(m_e+a_e h_e^2 m_e+a_\nu h_\nu^2 m_e+a'_\nu m_e h_\nu^2)-m_e\mu\ \tan\beta$.

The coefficients c_ν, c'_ν, c_e, c'_e, a_e, a_ν, a'_ν appear either at tree level or in the one-loop renormalization from Λ_{GUT}. The requirement of the L-R symmetry is

$$m_{\tilde{L}}=m_{\tilde{R}},\quad c_e=c'_e,\quad c_\nu=c'_\nu,\quad a_\nu=a'_\nu. \tag{37}$$

As a result, the mass matrix (36) differs from that of the MSSM where $c'_\nu=0$ and $a'_\nu=0$. The values of all these coefficients depend on many additional parameters and typically one assumes, as an estimate:

$$c_\nu\sim c'_\nu\sim m_{susy}^2(16\pi^2)^{-1}\ \ln(\Lambda_{GUT}^2/M_{W_R}^2)\sim\mathcal{O}(m_{susy}^2). \tag{38}$$

When the left-right symmetry is broken, the relations (37) become approximate one obtains the estimate $(m_{\tilde{L}}^2-m_{\tilde{R}}^2)/\bar{m}^2\sim 10^{-2}-10^{-1}$ [6,15].

The equations (5)–(8) (which we will use) represent simplified versions of the complete mixings. As an estimate for the mixing angle we shall take the usual assumption [9] that each is equal to the square root of the masses of the two particles it relates: for the leptons, $\theta_L = \theta_R = \sqrt{e/\mu}$. This result is inspired by GUT theories, quark-lepton universality, and the successful quark counterpart form: $\theta_{Cabbibo} = \sqrt{d/s}$. We will also assume the same for $\theta_{\tilde{e}}$ and $\theta_{\tilde{\mu}}$, a very conservative left-right mixing, consistent with the estimate $(m_{\tilde{L}}^2 - m_{\tilde{R}}^2)/\tilde{m}^2 \sim 10^{-2} - 10^{-1}$, and which simplifies our calculation. This differs from previous analyses [19], which assumed maximal mixing ($\pi/4$), in which case the final branching ratio would be 3.6 times larger. We shall also assume $(\delta\tilde{m}_L^2/m_{\tilde{L}}^2)^2 \approx (\delta\tilde{m}_R^2/m_{\tilde{R}}^2)^2 = (\delta\tilde{m}^2/\tilde{m}^2)^2$, which gives, for the branching ratio:

$$BR(\mu \to e + \gamma) = 2.15 \times 10^{-4} \left(\frac{\tilde{m}}{100 \text{ GeV}} \right)^4 \times \{(X_L)^2 + (X_R)^2\}. \qquad (39)$$

The matrix elements X_L and X_R are defined by the equations (15)–(26). These matrices depend on the following parameters: M_L, M_R, μ (only one μ parameter is sufficient since as we explained one Φ Higgs is heavy and decouples) and $\tan\beta$, as well as M_{LR} and h_{LR} in the doubly charged Higgs-Higgssino sector. We shall use the analytical expressions obtained in [20]. We assume the GUT-inspired relations: $M_R = \frac{5}{3}(g_1^2/g_L^2) M_L$, where g_1, g_L are the couplings of $U(1)_{B-L}$ and $SU(2)_L$, respectively and we assume left-right symmetry in gauge couplings: $g_L = g_R$. We take $g_L^2 \geqslant 2 g_1^2$ [6].

We are still left with parameters h_{LR} and M_{LR} from the doubly-charged Higgs-Higgsino sector, as well as $M_L = M$, μ and $\tan\beta = (\kappa_u/\kappa_d)(\equiv v_1/v_2)$. The first two determine the contribution from the doubly charged sector, which was previously investigated [19]. We shall use the constraint $h_{LR}^2 < (10^{-7} - 10^{-6}) M_{LR}^2$ (GeV), constraint found when maximal electron-muon ($\theta_{\tilde{e}} = \pi/4$) mixing was assumed. Note also that M_{LR} is unconstrained by phenomenology and is likely to be of the order of electroweak scales [8].

Fixing the parameters in this fashion we are able to investigate the constraints on the left-right supersymmetric model coming from $\mu \to e\gamma$. As in MSSM the following terms are found to be dominant:

One Yukawa coupling at the vertex;

Chirality flip inside the loop;

The dominant functions are h_g and j_g, which result from expansions in the small mass difference between the sleptons.

In particular, we are interested in constraints put on the scalar lepton mass splitting. In Figs. 2 and 3 we present several scenarios for the splittings, for various values of the parameters M and μ in the range (0, 400 GeV) and (-200 GeV, 200 GeV) for $\tan\beta = 2$. We have set the scalar mass $\tilde{m} = 100$ GeV and $M_{LR} = 100$ GeV. It is interesting to note that in all scenarios the scalar mass splitting is constrained to be $\leqslant 0.07$, a more relaxed constraint than in the MSSM, and this value is independent of the values chosen for M and μ in that range. By comparison, one could obtain a larger splitting in the MSSM for larger $M \approx 300$ GeV or 400 GeV and smaller than ours for the rest of the mass range. This value is also insensitive to values of M_{LR} as long as the constraint $h_{LR}^2 < (10^{-7} - 10^{-6}) M_{LR}^2$ is maintained.

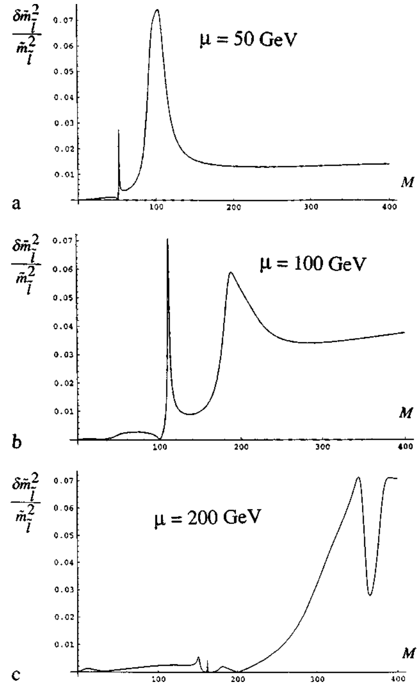

FIG. 2. Fractional scalar splittings as a function of M, the gaugino mass parameter for the following values of μ, the Higgsino mass parameter: (a) $\mu = 50$ GeV; (b) $\mu = 100$ GeV; and (c) $\mu = 200$ GeV.

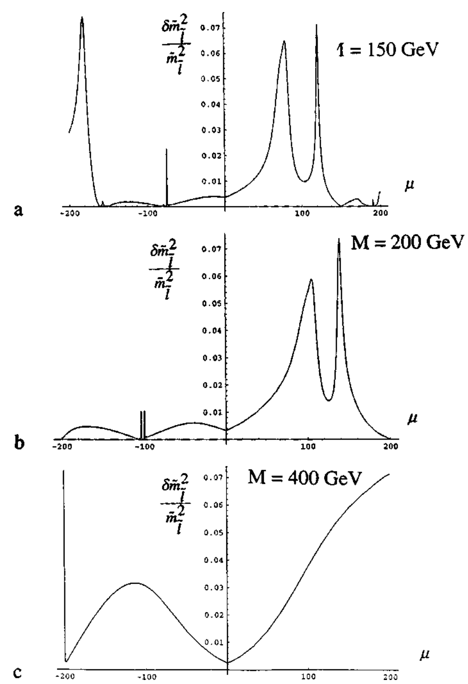

FIG. 3. Fractional scalar splittings as a function of μ, the Higgsino mass parameter for the following values of M, the gaugino mass parameter: (a) $M = 150$ GeV; (b) $M = 200$ GeV and (c) $M = 400$ GeV.

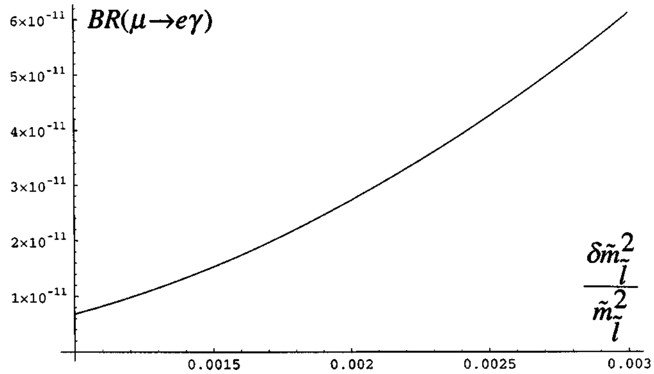

FIG. 4. Branching ratio $BR(\mu\to e\gamma)$ as a function of fractional scalar lepton mass splittings for $\mu=200$ GeV, and $M=120$ GeV.

If, turning the question around, one asks what are the restrictions on the branching ratio of $\mu\to e\gamma$ on the parameters of the left-right supersymmetric model, from the dependence of the branching ratio on the mass splittings, then the branching ratio is seen to exceed the present experimental bound for small mass splittings. In particular, if we take the upper limit of $M=120$ GeV $\mu=200$ GeV from the 10% fine tunning criterion, the branching ratio can reach its present bound for fractional slepton mass splittings of only 0.025 (Fig. 4). The branching ratio obtained from all 1-loop contributions (proportional to m_μ) to the decay $\mu\to e\gamma$ can be, and in general are, larger than the contributions proportional to m_τ coming from diagrams proportional to a large Yukawa coupling of ν_τ [10].

As in the MSSM, the slepton mass splitting depends very weakly on the value of the parameter A, justifying the approximation $A=0$. Although A appears in the graphs where the flip between left- and right-handed particles occurs in the slepton mass matrix through the term proportional to $A+\mu\tan\beta$, the contributions from these terms are smaller because of the loop functions involved are smaller.

However, as in the MSSM [9], as well as in SUSY GUT models [21] the branching ratio is strongly enhanced by large values of $\tan\beta$ (Fig. 5). Both the slepton mass matrix and the gaugino-Higgsino mass matrix are affected strongly by $\tan\beta$. In particular, $\tan\beta=50$ imposes very strict constraints on the fractional slepton splittings.

Lastly, we note that the analysis assumed a particular value of the slepton mass, that is we assumed a low-lying slepton spectrum $\tilde{m}\approx 100$ GeV. Our constraints for the splittings hold for scalar masses $\tilde{m}\approx 200$ GeV, 400 GeV, assuming correspondingly higher chargino-neutralino mass states.

VI. CONCLUSION

We presented a complete analysis of the branching ratio for the process $\mu\to e\gamma$ in the left-right supersymmetric model. The branching ratio is found to be enhanced over the (already large) result found from terms proportional to m_τ [10], and much larger than the contribution obtained from the doubly charged Higgs bosons and Higgsinos [19]. The branching ratio is always larger than the one obtained in MSSM studies. This enhancement is due to the contribution of right-handed particles in both the gauge and matter sec-

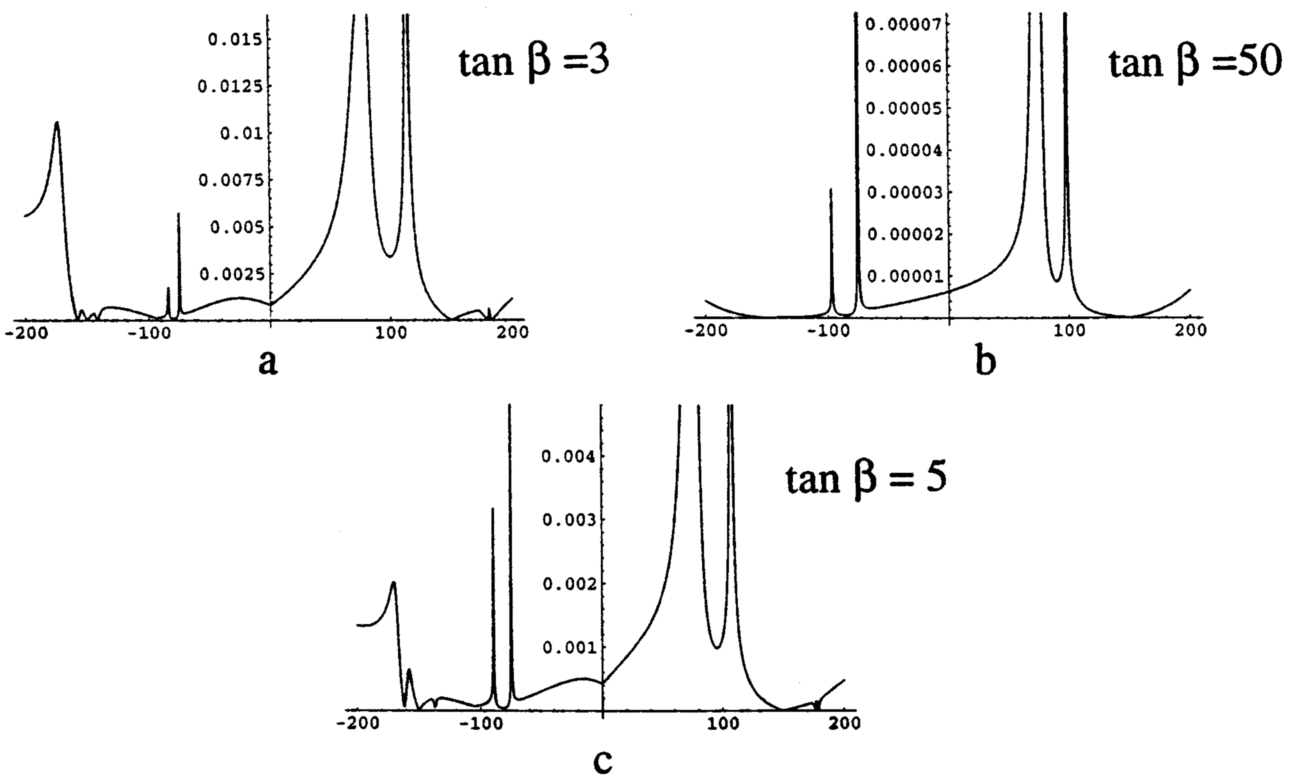

FIG. 5. Fractional scalar splittings as a of function μ for different values of $\tan\beta=v_1/v_2$, (a) for $\tan\beta=3$ and (b) for $\tan\beta=5$ and (c) $\tan\beta=50$ for $M=150$ GeV.

tors, and to a richer Higgs-Higgsino sector. It has some distinctive features from the same signal in the MSSM: it allows fractional scalar lepton splittings larger by about a factor of 7, and, most importantly, these splittings are independent of the values of the parameters M (of the gaugino sector) and μ (of the Higgsino sector). It shows the same sensitivity to large values of $\tan\beta$ and is qualitatively approximately independent of the value of A, the tri-linear scalar coupling. The left-right supersymmetric model, while allowing for larger splittings, does not resolve the flavor violation versus naturalness of the MSSM. The lepton flavor violation is allowed to differ from the flavor violation in the quark sector through some diagrams that contribute only in the lepton sector (the diagrams with doubly charged Higgsinos and Higgs) and the possibility of un-related sources of flavor violation in squark and slepton mass matrices. [The latter occurs in the case where LR SUSY is not imbedded into a SUSY GUT structure, such as SO(10).]

The contribution of $\mu \rightarrow e+\gamma$ provides a sensitive test for LR SUSY, especially given that the present experimental limit may be improved by the forthcoming Mega Experiment to an upper limit of 5×10^{-13} [22].

ACKNOWLEDGMENTS

I would like to thank G. Couture, H. Hamidian, H. König, M. Pospelov, and H. Saif for past collaborations and discussions. This work was funded in part by NSERC of Canada.

[1] P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991); G. Ross and R. G. Roberts, Nucl. Phys. **B377**, 571 (1992); J. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B **260**, 131 (1995).
[2] R. N. Mohapatra and A. Rašin, Phys. Rev. D **54**, 5835 (1996); R. Kuchimanchi, Phys. Rev. Lett. **79**, 3486 (1996); R. N. Mohapatra, A. Rašin, and G. Senjanović, *ibid.* **79**, 4744 (1997); C. S. Aulakh, K. Benakli, and G. Senjanović, *ibid.* **79**, 2188 (1997); C. Aulakh, A. Melfo, and G. Senjanović, Phys. Rev. D **57**, 4174 (1998).
[3] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975); G. Senjanovič and R. N. Mohapatra, *ibid.* **12**, 1502 (1975); R. N. Mohapatra and R. E. Marshak, Phys. Lett. **91B**, 222 (1980).
[4] R. Francis, M. Frank, and C. S. Kalman, Phys. Rev. D **43**, 2369 (1991).
[5] K. Huitu and J. Maalampi, Phys. Lett. B **344**, 217 (1995); K. Huitu, J. Maalampi, and M. Raidal, *ibid.* **328**, 60 (1994); Nucl. Phys. **B420**, 449 (1994).
[6] Mohapatra and Rašin [2].
[7] M. Frank and H. Saif, Z. Phys. C **67**, 32 (1995); **69**, 673 (1996); Mod. Phys. Lett. A **11**, 2443 (1996); J. Phys. G **22**, 1653 (1996).
[8] Z. Chacko and R. N. Mohapatra, Phys. Rev. D **58**, 015001 (1998); B. Dutta and R. N. Mohapatra, *ibid.* (to be published), hep-ph/9804277.
[9] S. Dimopoulos and D. Sutter, Nucl. Phys. **B452**, 496 (1995).
[10] M. Frank and H. Hamidian, Phys. Rev. D **54**, 6790 (1996); G. Couture, M. Frank, H. König, and M. Pospelov, hep-ph/9701299.
[11] R. Barbieri, L. Hall, and A. Strumia, Nucl. Phys. **B445**, 219 (1995); **B449**, 437 (1995).
[12] R. Barbieri and L. Hall, Phys. Lett. B **338**, 212 (1994).
[13] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[14] M. Frank and H. N. Saif, Z. Phys. C **65**, 337 (1995).
[15] M. E. Pospelov, Phys. Lett. B **391**, 324 (1996).
[16] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. D **48**, 4352 (1993).
[17] A. Pilaftsis, Phys. Rev. D **52**, 459 (1995); A. Pliaftsis and J. Bernabéu, Phys. Lett. B **351**, 235 (1995).
[18] M. Frank, Mod. Phys. Lett. A **12**, 3131 (1997).
[19] G. Couture, M. Frank, and H. König, Phys. Rev. D **56**, 4219 (1997).
[20] M. Frank, C. S. Kalman, and H. N. Saif, Z. Phys. C **59**, 655 (1993).
[21] J. Hisano, D. Nomura, Y. Okada, Y. Shimizu, and M. Tanaka, Phys. Rev. D (to be published), hep-ph/9805367.
[22] J. F. Amann *et al.*, Mega Experiment Report No. VPI-IHEP-93/7 (1993).