Baryons and domain walls in an $\mathcal{N}=1$ superconformal gauge theory

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Coincident D3-branes placed at a conical singularity are related to string theory on $AdS_5 \times X_5$, for a suitable five-dimensional Einstein manifold X_5. For the example of the conifold, which leads to $X_5 = T^{1,1} = [SU(2) \times SU(2)]/U(1)$, the infrared limit of the theory on N D3-branes was constructed recently. This is an $\mathcal{N}=1$ supersymmetric $SU(N)\times SU(N)$ gauge theory coupled to four bifundamental chiral superfields and supplemented by a quartic superpotential which becomes marginal in the infrared. In this paper we consider D3-branes wrapped over the 3-cycles of $T^{1,1}$ and identify them with baryon-like chiral operators built out of products of N chiral superfields. The supergravity calculation of the dimensions of such operators agrees with field theory. We also study the D5-brane wrapped over a 2-cycle of $T^{1,1}$, which acts as a domain wall in AdS_5. We argue that upon crossing it the gauge group changes to $SU(N)\times SU(N+1)$. This suggests a construction of supergravity duals of $\mathcal{N}=1$ supersymmetric $SU(N_1)\times SU(N_2)$ gauge theories. [S0556-2821(98)01624-5]

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I. INTRODUCTION

Over two decades ago 't Hooft showed that gauge theories simplify in the limit where the number of colors, N, is taken to infinity [1]. A number of arguments suggest that, for large N, gauge theories have a dual description in terms of string theory [1,2]. Recently, with some motivation from the D-brane description of black three-branes [3–5], and from studies of the throat geometry [6], Maldacena argued [7] that the $\mathcal{N}=4$ supersymmetric $SU(N)$ gauge theory is related to type-IIB strings on five-dimensional anti–de Sitter space $(AdS_5)\times \mathbf{S}^5$.[1] This correspondence was sharpened in [9,10], where it was shown how to calculate the correlation functions of gauge theory operators from the response of the type IIB theory on $AdS_5 \times \mathbf{S}^5$ to boundary conditions.

According to general arguments presented in [10], type IIB theory on $AdS_5 \times X_5$, where X_5 is a five-dimensional Einstein manifold bearing five-form flux, is expected to be dual to a four-dimensional conformal field theory. Construction of field theories for various manifolds X_5, in addition to the maximally supersymmetric case $X_5 = \mathbf{S}^5$, has become an active area. In one class of examples, X_5 is an $\mathbf{S}^5$ divided by the action of a discrete group. The field theory one thus obtains is the infrared limit of the world volume theory on N D3-branes [11,12] placed at an orbifold [13–16] or 7-brane and orientifold singularity [17–19].

Very recently, a new example of duality was found [20] where X_5 is a smooth Einstein manifold whose local geometry is different from that of $\mathbf{S}^5$. The X_5 for which the dual field theory was constructed in [20] is one of the coset spaces $T^{p,q} = [SU(2)\times SU(2)]/U(1)$ originally considered by Romans in the context of Kaluza-Klein supergravity [21]. The $U(1)$ is a diagonal subgroup of the maximal torus of $SU(2)\times SU(2)$: if $\sigma_i^{L,R}$ are the generators of the left and right $SU(2)$'s, then the $U(1)$ is generated by $p\sigma_3^L + q\sigma_3^R$. Romans found that for $p=q=1$ the compactification preserves 8 supersymmetries, while for other p and q all supersymmetries are broken. Therefore, one expects type IIB theory on $AdS_5 \times T^{1,1}$ to be dual to a certain large N $\mathcal{N}=1$ superconformal field theory in four dimensions. The dual theory constructed in [20] turns out to be a non-trivial infrared fixed point. It is the $SU(N)\times SU(N)$ gauge theory with chiral superfields A_k, $k=1,2$, transforming in the $(N,\overline{N})$ representation and B_l, $l=1,2$, transforming in the $(\overline{N},N)$ representation. These fields acquire infrared anomalous dimensions equal to $-1/4$ determined by the existence of an anomaly-free R-symmetry. A crucial ingredient in the construction of [20] is the quartic superpotential $W = \lambda \mathrm{Tr}(A_1B_1A_2B_2 - A_1B_2A_2B_1)$ which becomes exactly marginal in the infrared.

The construction of the field theory in [20] was guided by the observation that it is the infrared limit of the world volume theory on coincident Dirichlet three-branes placed at a conical singularity of a non-compact Calabi-Yau (CY) threefold (this is a special case of the connection between compactification on Einstein manifolds and the metric of three-branes placed at conical singularities [22,20,23,24]). The CY manifold relevant here is the simplest non-compact threefold with a conical singularity. This is the conifold [25,26], which for our purposes is the complex manifold C:

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0, \qquad (1)$$

with a "double point" singularity at $z_i = 0$. The metric on the conifold may be written as

$$ds_6^2 = dr^2 + r^2 g_{ij}(x) dx^i dx^j \quad (i,j=1,\ldots,5). \qquad (2)$$

Here g_{ij} is the metric on the base of the cone, which is precisely the Romans manifold $T^{1,1}$ [26,20]. The isometries of $T^{1,1}$, which form the group $SU(2)\times SU(2)\times U(1)$, are realized very simply in terms of the z-coordinates. The z_k transform in the four-dimensional representation of $SO(4) = SU(2)\times SU(2)$, while, under the $U(1)$, $z_k \to e^{i\alpha} z_k$. The metric on $T^{1,1}$ may be written down explicitly by utilizing the fact that it is a $U(1)$ bundle over $\mathbf{S}^2 \times \mathbf{S}^2$. Choosing the

[1] This type IIB background was originally considered in [8].

coordinates (θ_1,ϕ_1) and (θ_2,ϕ_2) to parametrize the two spheres in a conventional way, and the angle $\psi\in[0,4\pi)$ to parametrize the $U(1)$ fiber, the metric may be written as [26]

$$g_{ij}(x)dx^i dx^j=\frac{1}{9}(d\psi+\cos\theta_1\, d\phi_1+\cos\theta_2\, d\phi_2)^2+\frac{1}{6}\sum_{i=1}^{2}[d\theta_i^2+\sin^2\theta_i\, d\phi_i^2]. \tag{3}$$

Explicit calculation shows that this metric is indeed Einsteinian, $R_{ij}=4g_{ij}$ [26,30].

It turns out that the coset space $T^{1,1}=[SU(2)\times SU(2)]/U(1)$ may be obtained by blowing up the fixed circle of $\mathbf{S}^5/\mathbf{Z}_2$ (an operation that breaks $\mathcal{N}=2$ to $\mathcal{N}=1$) [20]. On the field theory side, the $\mathcal{N}=2$ superconformal theory corresponding to $\mathbf{S}^5/\mathbf{Z}_2$ flows to the $\mathcal{N}=1$ IR fixed point corresponding to $T^{1,1}$. The necessary relevant perturbation of the superpotential is odd under the $\mathbf{Z}_2$ and therefore corresponds to a blowup mode of the orbifold [27,28]. It is interesting to examine how various quantities change under the renormalization group (RG) flow from the $\mathbf{S}^5/\mathbf{Z}_2$ theory to the $T^{1,1}$ theory. The behavior of the conformal anomaly [which is equal to the $U(1)_R^3$ anomaly] was studied in [29]. Using the values of the R-charges deduced in [20], on the field theory side it was found that

$$\frac{c_{IR}}{c_{UV}}=\frac{27}{32}. \tag{4}$$

On the other hand, all 3-point functions calculated from supergravity on $\mathrm{AdS}_5\times X_5$ carry a normalization factor inversely proportional to Vol (X_5). Thus, on the supergravity side,

$$\frac{c_{IR}}{c_{UV}}=\frac{\mathrm{Vol}(\mathbf{S}^5/\mathbf{Z}_2)}{\mathrm{Vol}(T^{1,1})}. \tag{5}$$

Since [29]

$$\mathrm{Vol}(T^{1,1})=\frac{16\pi^3}{27},\quad \mathrm{Vol}\ (\mathbf{S}^5/\mathbf{Z}_2)=\frac{\pi^3}{2}, \tag{6}$$

the supergravity calculation is in exact agreement with the field theory result (4). This is a striking and highly sensitive test of the $\mathcal{N}=1$ dual pair constructed in [20].

In this paper we carry out further studies of this dual pair. In particular, we consider various branes wrapped over the cycles of $T^{1,1}$ and attempt to identify these states in the field theory. Wrapped D3-branes turn out to correspond to baryon-like operators A^N and B^N where the indices of both $SU(N)$ groups are fully antisymmetrized. For large N the dimensions of such operators calculated from the supergravity are found to be $3N/4$. This is in complete agreement with the fact that the dimension of the chiral superfields at the fixed point is 3/4 and may be regarded as a direct supergravity calculation of an anomalous dimension in the dual gauge theory.

We further argue that a domain wall made out of a D5-brane wrapped over a 2-cycle of $T^{1,1}$ separates a $SU(N)\times SU(N)$ gauge theory from a $SU(N)\times SU(N+1)$ gauge theory. Indeed, passing a wrapped D3-brane through such a domain wall produces a fundamental string stretched between the D3-brane and the domain wall. On the $SU(N)\times SU(N+1)$ side we find baryon-like operators which transform in the fundamental representation of one of the gauge groups, and identify them with the wrapped D3-brane attached to a string.

II. THREE-CYCLES AND BARYON-LIKE OPERATORS

After placing a large number N of coincident D3-branes at the singularity of the conifold and taking the near-horizon limit, the metric becomes that of $\mathrm{AdS}_5\times X_5$:

$$ds_{10}^2=\frac{r^2}{L^2}\eta_{\mu\nu}dy^\mu dy^\nu+L^2\left(\frac{dr^2}{r^2}+g_{ij}(x)dx^i dx^j\right). \tag{7}$$

The scale L is related to N and the gravitational constant κ through[2] [29]

$$L^4=\frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(X_5)}. \tag{8}$$

Now consider wrapping a D3-brane over a 3-cycle of $T^{1,1}$. Topologically, $T^{1,1}$ is $\mathbf{S}^2\times\mathbf{S}^3$ which establishes the existence of a 3-cycle [26]. In fact, this is a supersymmetric cycle; this fact was used in [31] where a three-brane wrapped over this cycle was argued to give rise to a massless black hole. An immediate guess for a 3-cycle of minimum volume is to consider the subspace at a constant value of (θ_2,ϕ_2) in the metric (3). We have checked that the three-brane wrapped over (ψ,θ_1,ϕ_1) coordinates indeed satisfies the equations of motion and is thus a minimum volume configuration.[3] To calculate the 3-volume, we need to find the determinant of the following metric:

$$\frac{L^2}{9}(d\psi+\cos\theta\, d\phi)^2+\frac{L^2}{6}[d\theta^2+\sin^2\theta\, d\phi^2]. \tag{9}$$

Integrating the square root of the determinant over the three coordinates, we find $V_3=8\pi^2L^3/9$. The mass of the three-brane wrapped over the 3-cycle is, therefore,

$$m=V_3\frac{\sqrt{\pi}}{\kappa}=\frac{8\pi^{5/2}L^3}{9\kappa}. \tag{10}$$

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[2]An easy way to derive this relation is by equating, as in [3], the Arnowitt-Deser-Misner (ADM) tension of the three-brane solution, $2\,\mathrm{Vol}(X_5)L^4/\kappa^2$, to the tension of N D3-branes, $N\sqrt{\pi}/\kappa$.

[3]Equivalently, we could consider the subspaces at constant (θ_1,ϕ_1) and the significance of this in the dual field theory will become clear shortly.

To relate this to the dimension Δ of the corresponding operator in the dual field theory, we use the results of [9,10] which for large mL imply $\Delta = mL$. Using Eqs. (8) and (6) for the case of $T^{1,1}$, we find

$$\Delta = mL = \frac{8\pi^{5/2}L^4}{9\kappa} = \frac{3}{4}N. \tag{11}$$

What are the operators in the dual field theory whose dimensions grow as N? The answer is clear: since the fields $A^{\alpha}_{k\beta}$ carry an index α in the N of $SU(N)_1$ and an index β in the $\overline{\mathrm{N}}$ of $SU(N)_2$, we can construct a baryon-like color-singlet operator by antisymmetrizing completely with respect to both groups. The resulting operator has the form

$$\mathcal{B}_{1l} = \epsilon_{\alpha_1 \ldots \alpha_N}\epsilon^{\beta_1 \ldots \beta_N} D_l^{k_1 \ldots k_N} \prod_{i=1}^{N} A^{\alpha_i}_{k_i \beta_i}, \tag{12}$$

where $D_l^{k_1 \ldots k_N}$ is the completely symmetric $SU(2)$ Clebsch-Gordon coefficient corresponding to forming the $N+1$ of $SU(2)$ out of N 2's. Thus the $SU(2)\times SU(2)$ quantum numbers of B_{1l} are $(N+1,1)$. Similarly, we can construct baryon-like operators which transform as $(1,\ N+1)$:

$$\mathcal{B}_{2l} = \epsilon^{\alpha_1 \ldots \alpha_N}\epsilon_{\beta_1 \ldots \beta_N} D_l^{k_1 \ldots k_N} \prod_{i=1}^{N} B^{\beta_i}_{k_i \alpha_i}. \tag{13}$$

Under the duality these operators map to D3-branes classically localized at a constant (θ_1, ϕ_1). Thus, the existence of two types of baryon operators is related on the supergravity side to the fact that the base of the $U(1)$ bundle is $\mathbf{S}^2 \times \mathbf{S}^2$.

We can further explain why one of the $SU(2)$ quantum numbers is precisely $N+1$. As shown in [32] in the context of an analogous construction of Pfaffian operators, it is necessary to carry out collective coordinate quantization of the wrapped D3-brane. While classically the wrapped D3-brane is localized at a point in the remaining two dimensions, quantum mechanically we have to find its collective coordinate wave function. In the present case the wrapped D3-brane acts as a charged particle, while the 5-form field flux through $T^{1,1}$ effectively gives rise to ordinary magnetic flux through the $\mathbf{S}^2$. We need to ask how many different ground states there are for a charged particle on a sphere with N units of magnetic flux. The answer to this problem is well-known: $N+1$ (in fact, one is dealing here with a supersymmetric quantum mechanics on $\mathbf{S}^2$). This degeneracy is due to the fact that the lowest possible angular momentum of a non-relativistic charged particle in the field of a monopole carrying N elementary units of magnetic charge is $N/2$ [33]. Thus, the ground state collective coordinate wave functions form an $(N+1)$-dimensional representation of the $SU(2)$ that rotates the $\mathbf{S}^2$ which is not wrapped by the D3-brane [the D3-brane is obviously a singlet under the other $SU(2)$]. The infinity of classical ground states is turned into $N+1$ quantum mechanical ground states. The $SU(2)\times SU(2)$ quantum numbers of the collective-coordinate quantized wrapped D3-branes are exactly the same as those of the baryon-like operators (12), (13). This can be regarded as a new test of the AdS conformal field theory (CFT) duality at finite N.

Finally, let us compare the actual dimensions of the operators. Since the A's and the B's have infrared dimension 3/4 in the construction of [20], we see that the dimension of the baryon-like operator is indeed $3N/4$, in perfect agreement with supergravity.[4] We regard this as a highly non-trivial check of both the AdS (CFT) correspondence and of the construction of the dual $\mathcal{N}=1$ superconformal field theory in [20].

As a slight digression, and also to check the consistency of our approach, we show following [32] that an analogous calculation with a wrapped D3-brane produces agreement with the field theoretic dimension of the Pfaffian operator in $SO(2N)$ gauge theory:

$$\epsilon_{a_1 \ldots a_{2N}} \Phi^{a_1 a_2} \ldots \Phi^{a_{2N-1} a_{2N}}. \tag{14}$$

Since the dimension of Φ is not renormalized in this case, we see that the dimension of the Pfaffian operator is equal to N.

The $SO(2N)$ theory is dual to supergravity on $\mathrm{AdS}_5 \times \mathbf{RP}^5$, and now

$$L^4 = \frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(\mathbf{RP}^5)} = \frac{\kappa N}{\pi^{5/2}}. \tag{15}$$

The object dual to the Pfaffian operator is the D3-brane wrapped over $\mathbf{RP}^3 = \mathbf{S}^3/\mathbf{Z}_2$, whose volume is $V_3 = \pi^2 L^3$. Thus,

$$\Delta = L V_3 \frac{\sqrt{\pi}}{\kappa} = N,$$

once again in perfect agreement with the field theory.

In many orbifold theories [14] there are analogues closer than the Pfaffian of the $SO(2N)$ theory to the baryons considered in Eqs. (12) and (13). Namely, from a bifundamental matter field A charged under two gauge groups of the same size, one can make a singlet operator by completely antisymmetrizing both upper and lower color indices.[5] We will mention the two simplest examples. An $\mathcal{N}=1$ theory results from the transitive $\mathbf{Z}_3$ orbifold action on $\mathbf{S}^5$ defined by coordinatizing $\mathbf{R}^6$ by three complex numbers z_1, z_2, z_3 and considering the map $z_k \rightarrow e^{2\pi i/3} z_k$ for all k. The theory has gauge group $SU(N)^3$ with three $(N,\overline{N})$ representations between each pair of gauge groups. Baryons formed as in Eqs. (12) and (13) from the bifundamental matter have dimension N. Minimal area 3-cycles on $\mathbf{S}^5/\mathbf{Z}_3$ can be constructed by inter-

[4] It is possible that there are $1/N$ corrections to the field theory result which would be difficult to see in supergravity.

[5] There are more exotic possibilities involving k_2 lower index ϵ-tensors and k_1 upper index ϵ-tensors with $k_1 k_2 N$ powers of a field A in a $(k_1 N, \overline{k_2 N})$ of $SU(k_1 N)\times SU(k_2 N)$ where k_1 and k_2 are relatively prime.

secting the 4-plane $z_k=0$ for any particular k with the sphere $|z_1|^2+|z_2|^2+|z_3|^2=1$. Now we have

$$L^4=\frac{3}{2}\frac{\kappa N}{\pi^{5/2}}, \quad V_3=\frac{2}{3}\pi^2 L^3, \tag{16}$$

and we find that $\Delta=LV_3\sqrt{\pi}/\kappa=N$ as expected.

As a second example we may consider the $\mathcal{N}=2$ $\mathbf{S}^5/\mathbf{Z}_2$ theory. In this case the orbifold group does not act freely, but has a circle of fixed points on $\mathbf{S}^5$. The blowup of the orbifold can be depicted as an $\mathbf{S}^3$ bundle over $\mathbf{S}^2$ [20]. The $\mathbf{S}^3$ fibers in this bundle would be three-dimensional analogues of great circles on $\mathbf{S}^5$, except that the $\mathbf{Z}_2$ acts on them by identifying a point with its image under a 180° rotation. Their volume is thus cut in half, and for a D3-brane wrapping a fiber we have

$$L^4=\frac{\kappa N}{\pi^{5/2}}, \quad V_3=\pi^2 L^3. \tag{17}$$

Once again, $\Delta=LV_3\sqrt{\pi}/\kappa=N$ in agreement with the field theory.

III. DOMAIN WALLS IN AdS$_5$

Domain walls in a holographic theory come from three-branes in AdS$_5$ [32]. The simplest example is a D3-brane which is not wrapped over the compact manifold. Through an analysis of the five-form flux carried over directly from [32] one can conclude that when one crosses the domain wall, the effect in field theory is to change the gauge group from $SU(N)\times SU(N)$ to $SU(N+1)\times SU(N+1)$.

The field theory interpretation of a D5-brane wrapped around $\mathbf{S}^2$ is less obvious. Recall that $T^{1,1}$ has the topology of $\mathbf{S}^3\times\mathbf{S}^2$; so there is topologically only one way to wrap the D5-brane. If on one side of the domain wall we have the original $SU(N)\times SU(N)$ theory, then we claim that on the other side the theory is $SU(N)\times SU(N+1)$.[6] The matter fields A_k and B_k are still bifundamentals, filling out $2(N,\overline{N+1})\oplus 2(\overline{N},N+1)$. An anti-D5-brane wrapped around $\mathbf{S}^2$ will act as a domain wall which decrements the rank of one gauge group, so that traversing a D5 and then an anti-D5 leads one back to the original $SU(N)\times SU(N)$ theory.

The immediate evidence for this claim is the way the baryons considered in Sec. II behave when crossing the D5-brane domain wall. In homology there is only one $\mathbf{S}^3$, but for definiteness let us wrap the D3-brane around a particular three-sphere $\mathbf{S}^3_{(1)}$ which is invariant under the group $SU(2)_B$ under which the fields B_k transform. The corresponding state in the $SU(N)\times SU(N)$ field theory is $\mathcal{B}_1$ of Eq. (13). In the $SU(N)\times SU(N+1)$ theory, one has instead

$$\epsilon_{\alpha_1\ldots\alpha_N}\epsilon^{\beta_1\ldots\beta_{N+1}}A^{\alpha_1}_{\beta_1}\ldots A^{\alpha_N}_{\beta_N} \tag{18a}$$

or

$$\epsilon_{\alpha_1\ldots\alpha_N}\epsilon^{\beta_1\ldots\beta_{N+1}}A^{\alpha_1}_{\beta_1}\ldots A^{\alpha_N}_{\beta_N}A^{\alpha_{N+1}}_{\beta_{N+1}} \tag{18b}$$

where we have omitted $SU(2)$ indices. Either the upper index β_{N+1}, indicating a fundamental of $SU(N+1)$, or the upper index α_{N+1}, indicating a fundamental of $SU(N)$, is free.

How can this be in supergravity? The answer is simple: the wrapped D3-brane must have a string attached to it. In the $\mathbf{S}^5/\mathbf{Z}_2$ theory from which our original $SU(N)\times SU(N)$ theory descends via RG flow, it is clear that a string ending on the holographic world-volume transforms in the $(N,1)\oplus(1,\ N)$ of the gauge group. The same then should be true of the $T^{1,1}$ theory. The new feature of the domain wall is that a string must stretch from it to the wrapped D3-brane. There are two roughly equivalent ways to see that this string must be present. Most directly, one can recall that a D3-brane crossing a D5-brane completely orthogonal to it leads to the production of a string stretched between the two. In flat space this effect was discussed in detail in [35,36] and is U-dual to the brane creation process discovered in [37].[7] Equivalently, one can proceed along the lines of [32], noting first that there is a discontinuity of $\int_{\mathbf{S}^3}H_{RR}$ across a D5-brane. Since we have assumed that on one side the theory is the original $SU(N)\times SU(N)$ theory, all the H_{RR}-flux should be through three-spheres on the other side. More precisely, on the $SU(N)\times SU(N+1)$ side, H_{RR} is an element of the third cohomology group $H^3(T^{1,1})$, which is one-dimensional. Using the basis one-forms generated by the vielbeins of $T^{1,1}$,

$$\begin{gathered}
e^{\psi}=\tfrac{1}{3}(d\psi+\cos\theta_1\,d\phi_1+\cos\theta_2\,d\phi_2)\\
e^{\theta_1}=\frac{1}{\sqrt{6}}d\theta_1, \quad e^{\phi_1}=\frac{1}{\sqrt{6}}\sin\theta_1\,d\phi_1\\
e^{\theta_2}=\frac{1}{\sqrt{6}}d\theta_2, \quad e^{\phi_2}=\frac{1}{\sqrt{6}}\sin\theta_2\,d\phi_2,
\end{gathered} \tag{19}$$

we can express the harmonic representatives of the second and third cohomology groups as

$$\begin{gathered}
e^{\theta_1}\wedge e^{\phi_1}-e^{\theta_2}\wedge e^{\phi_2}\in H^2(T^{1,1})\\
e^{\psi}\wedge e^{\theta_1}\wedge e^{\phi_1}-e^{\psi}\wedge e^{\theta_2}\wedge e^{\phi_2}\in H^3(T^{1,1}).
\end{gathered} \tag{20}$$

The D3-brane wrapping $\mathbf{S}^3_{(1)}$ needs a fundamental string attached to it to compensate for the flux of H_{RR} from the D5-brane: $B_{RR}\to B_{RR}-\tilde{f}$ on the D3-brane where $d\tilde{f}=2\pi\delta_P$ and $\tilde{f}$ is the dual of the $U(1)$ world-volume field strength on the D3-brane.

[6]We are grateful to O. Aharony for useful discussions on this possibility.

[7]Note that while the D5-brane and the D3-brane may not be strictly orthogonal in our setup, they do traverse complementary directions in $T^{1,1}$; so together they fill out eight spatial dimensions. This is sufficient for the arguments of [37,35,36] to apply.

The baryon $\mathcal{B}_2$ corresponding to a D3-brane wrapped around the three-sphere which is the orbit of the other $SU(2)$, $\mathbf{S}^3_{(2)}$, also becomes a non-singlet in the $SU(N)\times SU(N+1)$ theory: it transforms in the $(\overline{N},1)\oplus(1,\overline{N+1})$. This is appropriate because $\mathbf{S}^3_{(2)}$ is opposite $\mathbf{S}^3_{(1)}$ in homology, as one can see from the minus sign in Eqs. (20). Thus the three-form flux through the D3-brane changes sign, and the fundamental string that runs from it to the domain wall must be of opposite orientation to the previous case. In effect, a D3-brane around $\mathbf{S}^3_{(2)}$ is topologically equivalent to an anti-D3-brane around $\mathbf{S}^3_{(1)}$.

It may be objected at this point that nothing selects which gauge group gets changed as one crosses a D5 domain wall. There is no problem here because in fact nothing in the original $T^{1,1}$ solution distinguishes the two gauge groups. Our only claim is that crossing a domain wall increments (or, for anti-D5's, decrements) the rank of one gauge group: we do not attempt to distinguish between $SU(N)\times SU(N+1)$ and $SU(N+1)\times SU(N)$, if indeed there is any difference other than pure convention.

The domain wall in AdS_5 made out of M wrapped D5-branes has the following structure: on one side of it the 3-form field H_{RR} vanishes, while on the other side there are M units of flux of H_{RR} through the $\mathbf{S}^3$. Thus, the supergravity dual of the $SU(N)\times SU(N+M)$ theory involves adding M units of RR 3-form flux to the $AdS_5\times T^{1,1}$ background. If M is held fixed while $N\to\infty$, then the additional 3-form field will not alter the gravity and the 5-form background. In particular, the presence of the AdS_5 factor signals that the theory remains conformal to leading order in N. This agrees with the fact that assigning R-charge 1/2 to the bifundamental fields A_k and B_l guarantees that the beta functions for both $SU(N)$ and $SU(N+M)$ factors vanish to leading order in N (for $M\neq 0$ there are, however, $1/N$ corrections to the beta functions).

A different situation occurs if the large N limit is taken with fixed M/N. Then it is obvious that addition of M flux quanta of H_{RR} will have a back-reaction on the geometry even at leading order in N. Some solutions with both 5-form and 3-form field strengths were discussed in [21], but they were found to break all supersymmetry. For comparison with $\mathcal{N}=1$ supersymmetric field theory, one presumably needs to find a static supergravity background with the same degree of supersymmetry. We leave the search for such backgrounds as a problem for the future. Some interesting physics motivates this search: for $N_1\neq N_2$, it is impossible to choose the R-charges so that the beta functions for both $SU(N_1)$ and $SU(N_2)$ vanish. Correspondingly, in supergravity, the presence of three-form flux generically necessitates a dilaton profile (and even the converse is true for static, supersymmetric, bosonic type IIB backgrounds). Furthermore, the quartic superpotential $W=\lambda\,\mathrm{Tr}(A_1B_1A_2B_2-A_1B_2A_2B_1)$ is no longer marginal.[8] Therefore, the corresponding supergavity background is not expected to have the $AdS_5\times X_5$ structure.

[8]We thank M. Strassler for pointing this out to us.

The $N_1\neq N_2$ field theories are somewhat analogous to the standard model, where $SU(2)$ and $SU(3)$ have positive and negative beta functions, respectively, and left-handed quarks form bifundamentals under these gauge groups. Two salient differences between the standard model and our theories are the chiral coupling of the weak interactions and the presence of matter fields (leptons) which are neutral under the larger gauge group. An analysis along the lines of [38] may help elucidate the possible $\mathcal{N}=1$ field theories. Supergravity compactifications which have both 5-form and 3-form fields turned on and which preserve $\mathcal{N}=1$ supersymmetry appear to be good candidates for their dual description.

IV. OTHER WRAPPED BRANES

In this section we list other admissible ways of wrapping branes over cycles of $T^{1,1}$ and discuss their field theory interpretation. Our discussion is quite analogous to that given by Witten for $AdS_5\times\mathbf{RP}^5$ [32].

Since $\pi_1(T^{1,1})$ is trivial, there are no states associated with wrapping the 1-branes. For D3-branes there are two types of wrapping. One of them, discussed in the previous section, involves 3-cycles and produces particles on AdS_5 related to the baryon-like operators (12) and (13). The other involves wrapping a D3-brane over an $\mathbf{S}^2$ and produces a string in AdS_5. The tension of such a ''fat'' string scales as $L^2/\kappa\sim N(g_sN)^{-1/2}/\alpha'$. The non-trivial dependence of the tension on the 't Hooft coupling g_sN indicates that such a string is not a Bogomol'nyi-Prasad-Sommerfield (BPS) saturated object. This should be contrasted with the tension of a BPS string obtained by wrapping a D5-brane over $\mathbf{RP}^4$ in [32], which is $\sim N/\alpha'$.

In discussing wrapped 5-branes, we will limit explicit statements to D5-branes: since a (p,q) 5-brane is an $SL(2,\mathbf{Z})$ transform of a D5-brane, our discussion may be immediately generalized to wrapped (p,q) 5-branes using the $SL(2,\mathbf{Z})$ symmetry of type IIB string theory. If a D5-brane is wrapped over the entire $T^{1,1}$, then according to the arguments in [32,34], it serves as a vertex connecting N fundamental strings. Since each string ends on a charge in the fundamental representation of one of the $SU(N)$'s, the resulting field theory state is a baryon built out of external quarks. A D5-brane wrapped over an $\mathbf{S}^2$ produces a domain wall discussed in the previous section.

If a D5-brane is wrapped over an $\mathbf{S}^3$, then we find a membrane in AdS_5. Although we have not succeeded in finding its field theoretic interpretation, let us point out the following interesting effect. Consider positioning a ''fat'' string made of a wrapped D3-brane orthogonally to the membrane. As the string is brought through the membrane, a fundamental string stretched between the ''fat'' string and the membrane is created. The origin of this effect is, once again, the creation of fundamental strings by crossing D5- and D3-branes, as discussed in [35,36].

V. CONCLUSIONS

The $AdS_5\times T^{1,1}$ model of [20] is the first example of a supersymmetric holographic theory based on a compact manifold which is not locally $\mathbf{S}^5$. Correspondingly, the quantum field theory description in terms of an $\mathcal{N}=1$ $SU(N)\times SU(N)$ gauge theory is in no way a projection of the $\mathcal{N}=4$ theory.

In the context of this model, we have provided a string theory description of baryon-like operators formed from a symmetric product of N bifundamental matter fields, fully antisymmetrized on upper and lower color indices separately. The dual representation of such an operator is a D3-brane wrapped around an $\mathbf{S}^3$ embedded in $T^{1,1}$. Two natural ways of embedding $\mathbf{S}^3$ are as orbits of either of the two $SU(2)$ global symmetry groups of the theory. A D3-brane wrapping an orbit of one $SU(2)$ can be regarded classically as a charged particle allowed to move on the $\mathbf{S}^2$ which parametrizes the inequivalent orbits. The five-form flux supporting the $AdS_5\times T^{1,1}$ geometry acts as a magnetic field through this $\mathbf{S}^2$, and the quantum mechanical ground states fill out an $(N+1)$-dimensional representation of the other $SU(2)$. All this meshes beautifully with the field theory because the N matter fields are doublets of the $SU(2)$'s. Moreover, the 3-volume of the $SU(2)$ orbits gives a dimension for the operators, $3N/4$, which is precisely matched by the field theory.

We have used this baryon construction to argue that D5-branes wrapped around the 2-cycle of $T^{1,1}$ act as domain walls separating the original $SU(N)\times SU(N)$ theory from a $SU(N)\times SU(N+1)$ theory. The essential point is that crossing a wrapped D3-brane through a D5-brane creates a string stretched between the two, so that the baryon is no longer a singlet, but rather a fundamental of one of the gauge groups. This tallies with the field theory, because when one attempts to antisymmetrize the color indices on a product of N or $N+1$ bifundamentals of $SU(N)\times SU(N+1)$, one is always left with one free index. Our treatment of the domain walls has been restricted to the test brane approximation. Further evidence for the fact that the D5-brane domain walls lead to $SU(N_1)\times SU(N_2)$ gauge theories, as well as perhaps some new phenomena, may arise when one understands the full supergravity solution.

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