SUSY QM and solitons from two coupled scalar fields in two dimensions

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An analysis and application of the supersymmetry (SUSY) in nonrelativistic quantum mechanics involving two-component wave functions for a stability equation corresponding to two coupled real scalar fields is considered. A general positive potential for two coupled real scalar fields in 1+1 dimensions with a SUSY form is investigated in which the associated two-component normal modes are non-negative, which leads to classically stable soliton solutions, and an example is explicitly considered. [S0556-2821(98)02322-4]

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I. INTRODUCTION

Supersymmetry (SUSY) in nonrelativistic quantum mechanics (QM) was originally formulated in a unidimensional coordinate space [1]. The SUSY algebra has received many applications in order to construct the spectral resolution of solvable potentials and this has been recently reviewed [2]. The formalism of SUSY has also been used to realize superoscillators [3] and to solve the Schrödinger equation of partially solvable potentials so as to yield the eigenfunctions that allow one to compute the eigenvalues using the variational method [4]. Recently, the connection between SUSY QM and the topological and nontopological solitons has been established [5–9]. The shape-invariance conditions in SUSY [10] have been generalized for systems described by two-component wave functions [11].

The soliton solutions have been investigated for field equations defined in a space-time of dimension equal to or higher than 1+1. The kink of a field theory is an example of a soliton in 1+1 dimensions [12–18]. It is a static, nonsingular, *classically stable*, and of finite localized energy solution of the equation of motion, which is sometimes used in quantum corrections to implement the stability of classically unstable solutions [17]. A recent overview [18] shows how a quantum field theory has topological and nontopological soliton solutions in higher spatial dimensions. For solitons of two coupled scalar fields in 1+1 dimensions, there are no general rules for finding analytic solutions since the nonlinearity in the potential increases the difficulties in solving the field equations.

This paper relies on known connections between the theory of Darboux operators in factorizable essentially isospectral partner Hamiltonians (often called as ''SUSY QM'') and the likewise first-order Bogomol'nyi-type classical equations of simple scalar field theories such as the one examined here. It replicates, in a tensor product $2\times 2\otimes 2\times 2$ structure, Witten's evident supersymmetry formulation via the connection between stability of soliton solutions and Hermitian factorization of 2×2 matrix fluctuation operators. This leads to 4×4 supercharges and supersymmetric Hamiltonians whose bosonic sector has one two-component zero-mode ground state associated with the matrix fluctuation operator of two-soliton solutions. Some applications are suggested but only one of them is explicitly considered.

This present work is organized in the following way. In Sec. II we start by summarizing the essential features of the standard supersymmetry in quantum mechanics. We establish in Sec. III the close connection between the SUSY QM for two-component wave functions and stability equations of solitons of bidimensional relativistic systems. When considering only static solutions, we show that the 2×2 matrix Hermitian superpotential can be realized from classical stability equations of nonlinear systems with two coupled real scalar fields in 1+1 dimensions. Section IV contains the concluding remarks.

II. STANDARD SUPERSYMMETRY IN QUANTUM MECHANICS

Let H_- be the Hamiltonian of the unidimensional Schrödinger equation with the zero-mode eigenstate $\psi_-^{(0)}(x)$ for $E_-^{(0)}=0$. Since $\psi_-^{(0)}(x)$ is nodeless and vanishes in the asymptotic region $|x|\rightarrow\infty$, we can realize a factorization of H_-, viz.,

$$H_- = -\frac{d^2}{dx^2} + V_-(x) = A^+A^-, \tag{1}$$

with

$$V_-(x) = W^2(x) + W'(x), \tag{2}$$

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where the prime means a derivative with respect to the argument and the superpotential is given by

$$W(x)=\frac{d}{dx}\ln\psi_-^{(0)}(x). \tag{3}$$

Note that $W^\dagger=W$ because $W(x)$ is a real function. The operator A^- annihilates the ground state of H_-,

$$A^-\psi_-^{(0)}(x)=0, \tag{4}$$

and is formally given by

$$A^-=\psi_-^{(0)}(x)\left(-\frac{d}{dx}\right)\frac{1}{\psi_-^{(0)}(x)}$$
$$=-\frac{d}{dx}+W(x) \tag{5}$$

and A^+ is defined as its Hermitian conjugate.

The partner Hamiltonian of H_- is given by

$$H_+=-\frac{d^2}{dx^2}+V_+(x)=A^-A^+, \tag{6}$$

with

$$V_+(x)=W^2(x)-W'(x), \tag{7}$$

which has the same spectrum of H_-, except for the ground state. In fact, in such case we have

$$A^+\psi_+^{(0)}(x)=0,\quad A^+=\psi_+^{(0)}(x)\left(\frac{d}{dx}\right)\frac{1}{\psi_+^{(0)}(x)}, \tag{8}$$

and from Eq. (5) we get

$$A^+=(A^-)^\dagger=\frac{1}{\psi_-^{(0)}(x)}\left(\frac{d}{dx}\right)\psi_-^{(0)}(x).$$

Thus we readily obtain the following relation between the normalizable zero-mode eigenfunction of H_- and the non-normalizable zero-mode solution of H_+:

$$\psi_+^{(0)}(x)\psi_-^{(0)}(x)=C, \tag{9}$$

where C is a real constant.

However, when we consider $\Psi_\pm^{(0)}(x)$ as two-component wave functions, Eqs. (8) and (9) are no longer valid and hence another approach is necessary [9].

III. SOLITONS AND SUSY FROM TWO COUPLED SCALAR FIELDS

We now consider the classical soliton solutions of two coupled real scalar fields in 1+1 dimensions. They are static, nonsingular classically stable solutions of the field equations with finite localized energy. Here we consider an approach to the general case of such two-coupled-field systems, independent of the spatial coordinate, presenting a particular example at the end of this section.

The Lagrangian density for such a nonlinear system in the natural system of units $(c=\hbar=1)$, in $(1+1)$-dimensional space-time with Lorentz invariance is written as

$$\mathcal{L}(\phi,\chi,\partial_\mu\phi,\partial_\mu\chi)=\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}(\partial_\mu\chi)^2-V(\phi,\chi), \tag{10}$$

where $\partial_\mu=\partial/\partial x^\mu$, $x^\mu=(t,x)$ with $\mu=0,1$, $x_\nu=\eta_{\nu\mu}x^\mu$; $\phi=\phi(x,t)$, $\chi=\chi(x,t)$ are real scalar fields, and $\eta^{\mu\nu}$ is the metric tensor given by

$$\eta^{\mu\nu}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{11}$$

Here the potential $V=V(\phi,\chi)$ is any positive semidefinite function of ϕ and χ, which must have at least two different zeros in order to present solitons as solutions. The general classical configurations obey the equations

$$\frac{\partial^2}{\partial t^2}\phi-\frac{\partial^2}{\partial x^2}\phi+\frac{\partial}{\partial\phi}V=0,\quad \frac{\partial^2}{\partial t^2}\chi-\frac{\partial^2}{\partial x^2}\chi+\frac{\partial}{\partial\chi}V=0, \tag{12}$$

which, for static soliton solutions, become the following system of nonlinear differential equations:

$$\phi''=\frac{\partial}{\partial\phi}V,\quad \chi''=\frac{\partial}{\partial\chi}V, \tag{13}$$

where primes represent differentiations with respect to the space variable. There is in the literature a trial orbit method for finding static solutions for certain positive potentials, which constitutes a ''trial and error'' technique. This method yields at best some solutions to Eq. (13) and by no means for all the potentials [13]. Recently the trial orbit method has been applied to systems of two coupled scalar fields containing up to sixth-order powers in the fields [19]. However, the principal result obtained in Ref. [19] is wrong. There the authors have found the minimum of energy as the sum of the asymptotic behavior of the modulus of two real scalar functions, which is not mathematical acceptable.

In Ref. [19] a positive potential $V(\phi,\chi)$ is considered in the following SUSY form, analogous to the case with one single field only [8]:

$$V(\phi,\chi)=\frac{1}{2}\left(F+\phi\frac{\partial F}{\partial\phi}+\chi\frac{\partial G}{\partial\phi}\right)^2$$
$$+\frac{1}{2}\left(G+\phi\frac{\partial F}{\partial\chi}+\chi\frac{\partial G}{\partial\chi}\right)^2. \tag{14}$$

Let us define

$$\Gamma=\Gamma(\phi,\chi)=\mathcal{F}+\mathcal{G}, \tag{15}$$

where $\mathcal{F}=\mathcal{F}(\phi,\chi)=\phi F$ and $\mathcal{G}=\mathcal{G}(\phi,\chi)=\chi G$. In this case Bogomol'nyi form of the energy consisting of a sum of squares and the surface term becomes

$$E_B=\int dx\,\frac{\partial}{\partial x}\Gamma[\phi(x),\chi(x)] \tag{16}$$

if we require that ϕ and χ satisfy the Bogomol'nyi conditions

$$\phi'=-M(\phi,\chi),\quad \chi'=-N(\phi,\chi) \tag{17}$$

and the function $\Gamma[\phi(x),\chi(x)]$ satisfy

$$\frac{\partial\Gamma}{\partial\phi}=M(\phi,\chi),\quad \frac{\partial\Gamma}{\partial\chi}=N(\phi,\chi), \tag{18}$$

where the function $\Gamma(\phi,\chi)$ leads to the correct value for a Bogomol'nyi minimum energy.

Then we see that only if $\Gamma[\phi(x),\chi(x)]_{x=-\infty}>\Gamma[\phi(x),\chi(x)]_{x=+\infty}$, one may put

$$E_B^{min}=|\Gamma[\phi(x),\chi(x)]_{x=+\infty}-\Gamma[\phi(x),\chi(x)]_{x=-\infty}|. \tag{19}$$

Therefore, only in the particular case that $\mathcal{F}[\phi(x),\chi(x)]_{x=+\infty}<\mathcal{F}[\phi(x),\chi(x)]_{x=-\infty}$ and $\mathcal{G}[\phi(x),\chi(x)]_{x=+\infty}<\mathcal{G}[\phi(x),\chi(x)]_{x=-\infty}$ may we put[1]

$$E_B^{min}=|\mathcal{F}[\phi(x),\chi(x)]_{x=+\infty}-\mathcal{F}[\phi(x),\chi(x)]_{x=-\infty}| +|\mathcal{G}[\phi(x),\chi(x)]_{x=+\infty}-\mathcal{G}[\phi(x),\chi(x)]_{x=-\infty}|,$$

where $\Gamma[\phi(x),\chi(x)]=\mathcal{F}[\phi(x),\chi(x)]+\mathcal{G}[\phi(x),\chi(x)]$.

Since the conserved topological current $(\partial_\mu j^\mu=0)$ can be written in terms of the continuously twice differentiable function $\Gamma(\phi,\chi)$, viz.,

$$j^\mu=\epsilon^{\mu\nu}\partial_\nu\Gamma(\phi,\chi),\quad \epsilon^{00}=\epsilon^{11}=0,\quad \epsilon^{10}=-\epsilon^{01}=-1, \tag{20}$$

the topological charge of such a system is equivalent to the aforementioned minimum value of the energy.

Let $V(\phi,\chi)$ be written in the following SUSY form, analogous to the case with one single field only [8]:

$$V(\phi,\chi)=\frac{1}{2}M^2(\phi,\chi)+\frac{1}{2}N^2(\phi,\chi). \tag{21}$$

Now let us analyze the classical stability of the soliton solutions in this nonlinear system [8,15], which is ensured by considering small perturbations around $\phi(x)$ and $\chi(x)$:

$$\phi(x,t)=\phi(x)+\eta(x,t) \tag{22}$$

and

$$\chi(x,t)=\chi(x)+\xi(x,t). \tag{23}$$

[1]This issue has been treated incorrectly in Eq. (8) in Ref. [19] as the minimum value of the Bogomol'nyi energy. A full analysis of the work considered in Ref. [19] has been submitted for publication in another relevant journal [20].

Next let us expand the fluctuations $\eta(x,t)$ and $\xi(x,t)$ in terms of the normal modes:

$$\eta(x,t)=\sum_n \epsilon_n\eta_n(x)e^{i\omega_n t} \tag{24}$$

and

$$\xi(x,t)=\sum_n c_n\xi_n(x)e^{i\omega'_n t}, \tag{25}$$

where ϵ_n and c_n are chosen so that $\eta_n(x)$ and $\chi_n(x)$ are real. By choosing $\omega'_n=\omega_n$ the equation for the field becomes a Schrödinger-like equation for two-component wave functions $\tilde{\Psi}_n$:

$$\mathcal{H}\tilde{\Psi}_n=\omega_n^2\tilde{\Psi}_n,\ n=0,1,2,\ldots, \tag{26}$$

where

$$\mathcal{H}=\left.\begin{pmatrix} -\frac{d^2}{dx^2}+\frac{\partial^2}{\partial\phi^2}V & \frac{\partial^2}{\partial\chi\partial\phi}V \\ \frac{\partial^2}{\partial\phi\partial\chi}V & -\frac{d^2}{dx^2}+\frac{\partial^2}{\partial\chi^2}V\end{pmatrix}\right|_{\phi=\phi(x),\chi=\chi(x)} \tag{27}$$

and

$$\tilde{\Psi}_n=\begin{pmatrix}\eta_n(x)\\ \xi_n(x)\end{pmatrix}. \tag{28}$$

The two-component normal modes in Eq. (28) satisfy $\omega_n^2\geqslant 0$ so that the stability of the Schrödinger-like equation is ensured. Note that, if

$$\frac{\partial^2}{\partial\chi\partial\phi}V=\left(\frac{\partial}{\partial\chi}M\right)\left(\frac{\partial}{\partial\phi}M\right)+M\frac{\partial^2}{\partial\chi\partial\phi}M +\left(\frac{\partial}{\partial\chi}N\right)\left(\frac{\partial}{\partial\phi}N\right)+N\frac{\partial^2}{\partial\chi\partial\phi}N =\frac{\partial^2}{\partial\phi\partial\chi}V, \tag{29}$$

then $\mathcal{H}$ is Hermitian. Hence the eigenvalues ω_n^2 of $\mathcal{H}$ are all real ones. We will now show that ${\omega_n}^2$ are non-negative, the proof of which takes us to a realization of the SUSY QM algebra. Making an extension for the case of only one single real scalar field we can realize, *a priori*, the 2×2 matrix superpotential in the following manner:

$$\mathbf{W}=-\left.\begin{pmatrix}\frac{\partial}{\partial\phi}M & \frac{\partial}{\partial\chi}M\\ \frac{\partial}{\partial\phi}N & \frac{\partial}{\partial\chi}N\end{pmatrix}\right|_{\phi=\phi(x),\chi=\chi(x)}. \tag{30}$$

But according to Ref. [9] we must impose the Hermiticity condition ($\mathbf{W}^\dagger=\mathbf{W}$) on it so that it is satisfied if and only if

$$\frac{\partial}{\partial\chi}M=\frac{\partial}{\partial\phi}N, \tag{31}$$

which is in accordance with Eq. (18). In this case we have a bilinear form for $\mathcal{H}$ given by

$$\mathcal{H}=\mathcal{A}^+\mathcal{A}^-, \tag{32}$$

where

$$\begin{aligned}\mathcal{A}^-&=-\mathbf{1}\frac{d}{dx}+\mathbf{W}\\ &=\left.\begin{pmatrix} a^- & -\frac{\partial}{\partial\phi}N\\ -\frac{\partial}{\partial\phi}N & b^-\end{pmatrix}\right|_{\phi=\phi(x),\chi=\chi(x)}\\ &=(\mathcal{A}^+)^\dagger,\end{aligned} \tag{33}$$

with the following first-order differential operators that appear in the analysis of classical stability associated with only one single field [8]:

$$a^-=-\frac{d}{dx}-\frac{\partial}{\partial\phi}M,\quad b^-=-\frac{d}{dx}-\frac{\partial}{\partial\chi}N. \tag{34}$$

From Eqs. (21) and (31) we have

$$\begin{aligned}\frac{\partial^2}{\partial\phi^2}V&=\left(\frac{\partial}{\partial\phi}M\right)^2+M\frac{\partial^2}{\partial\phi^2}M+\left(\frac{\partial}{\partial\phi}N\right)^2+N\frac{\partial^2}{\partial\phi^2}N,\\ \frac{\partial^2}{\partial\chi^2}V&=\left(\frac{\partial}{\partial\chi}M\right)^2+M\frac{\partial^2}{\partial\chi^2}M+\left(\frac{\partial}{\partial\chi}M\right)^2+N\frac{\partial^2}{\partial\chi^2}N.\end{aligned} \tag{35}$$

Since $a^+=(a^-)^\dagger, b^+=(b^-)^\dagger$, and using the aforementioned conditions of Hermiticity, we obtain

$$\begin{aligned}a^+a^-+\left(\frac{\partial}{\partial\phi}N\right)^2&=-\frac{d^2}{dx^2}+\frac{\partial^2}{\partial\phi^2}V,\\ b^+b^-+\left(\frac{\partial}{\partial\phi}N\right)^2&=-\frac{d^2}{dx^2}+\frac{\partial^2}{\partial\chi^2}V,\end{aligned} \tag{36}$$

which are exactly the diagonal elements of $\mathcal{H}$. Therefore, it is easy to show that the linear stability is satisfied, i.e., $\omega_n^2=\langle\mathcal{H}\rangle=\langle\mathcal{A}^+\mathcal{A}^-\rangle=(\mathcal{A}^-\tilde{\Psi}_n)^\dagger(\mathcal{A}^-\tilde{\Psi}_n)=|\mathcal{A}^-\tilde{\Psi}_n|^2\geqslant 0$, as has been affirmed.

Indeed the bosonic sector Hamiltonian of H_{SUSY} is given exactly by $\mathcal{H}$, which as obtained in stability equation (26) has the following ground state:

$$\mathcal{A}^-\tilde{\Psi}_-^{(0)}(x)=0\Rightarrow\tilde{\Psi}_-^{(0)}(x)=\begin{pmatrix}\eta_0(x)\\ \xi_0(x)\end{pmatrix}=-\begin{pmatrix}M(\phi(x),\chi(x))\\ N(\phi(x),\chi(x))\end{pmatrix}, \tag{37}$$

which represents the two-component zero mode.

Now, we consider an example. Let $N(\phi,\chi)$ and $M(\phi,\chi)$ be given by

$$\begin{aligned}N(\phi,\chi)&=\mu\phi^2\chi+\gamma\chi,\\ M(\phi,\chi)&=\lambda\phi\left(\phi^2-\frac{m}{\lambda}\right)+\mu\phi\chi^2,\end{aligned} \tag{38}$$

where $\gamma\geqslant 0,\mu\geqslant 0$ and $\lambda>0$. In this case, a simple typical potential for two coupled real scalar fields becomes

$$\begin{aligned}V(\phi,\chi)=&\frac{1}{2}\left\{\lambda^2\phi^2\left(\phi^2-\frac{m}{\lambda}\right)^2+2\mu\lambda\chi^2\phi^2\left(\phi^2-\frac{m}{\lambda}\right)\right\}\\ &+\frac{1}{2}\{\chi^2(\gamma^2+\mu^2\phi^2\chi^2)+\mu^2\phi^4\chi^2+2\mu\gamma\phi^2\chi^2\}.\end{aligned} \tag{39}$$

Note that this potential has a discrete symmetry as $\phi\to-\phi$ and $\chi\to-\chi$ so that we have a necessary condition (but not sufficient) that it must have at least two zeros in order that solitons can exist.

In this case we have the zero-mode ground state given by

$$\tilde{\Psi}_-^{(0)}(x)=-\begin{pmatrix}\lambda\phi(x)\left(\phi^2(x)-\frac{m}{\lambda}\right)+\mu\phi(x)\chi^2(x)\\ \mu\phi^2(x)\chi(x)+\gamma\chi(x)\end{pmatrix}, \tag{40}$$

associated with the following 2×2 matrix superpotential:

$$\mathbf{W}=-\begin{pmatrix}3\lambda\phi^2(x)+\mu\chi^2(x)-m & 2\mu\phi(x)\chi(x)\\ 2\mu\phi(x)\chi(x) & \mu\phi^2(x)+\gamma\end{pmatrix}, \tag{41}$$

where the soliton solutions satisfy the following system of two first-order differential equations, analogous to the Bogomol'nyi conditions for only one single soliton [8,15,21], viz.,

$$\begin{aligned}\frac{d}{dx}\phi+\lambda\phi\left(\phi^2-\frac{m}{\lambda}\right)+\mu\phi\chi^2&=0,\\ \frac{d}{dx}\chi+\mu\phi^2\chi+\gamma\chi&=0.\end{aligned} \tag{42}$$

This generalized system can be solved by the trial orbit development considered in [13]. However, a possible soliton solution occurs when we choose $\chi=0$, so that it implies a soliton solution in 1+1 dimensions, which is the soliton of the ϕ^6 model, viz., $\phi(x)=\sqrt{\frac{1}{2}(m/\lambda)\{1+\tanh[m(x+x_0)]\}}$, where x_0 is obtained from the integration constant that represents the soliton center. Therefore the superpotential and the zero-mode ground state for our example become

$$\mathbf{W}(x)=\begin{pmatrix} \frac{m}{2}\{1-\tanh[m(x+x_0)]\} & 0 \\ 0 & -\frac{\mu m}{2\lambda}\{1+\tanh[m(x+x_0)]\}-\gamma \end{pmatrix} \tag{43}$$

and

$$\tilde{\Psi}_{-}^{(0)}(x)=N\begin{pmatrix} \frac{m\lambda}{2}\sqrt{\frac{m}{2}\{1+\tanh[m(x+x_0)]\}}\{\tanh[m(x+x_0)]-1\} \\ 0 \end{pmatrix}, \tag{44}$$

where N is the normalization constant. A detailed analysis of this application will be published elsewhere.

The graded Lie algebra of the supersymmetry in quantum mechanics can be readily realized as

$$H_{SUSY}=[Q_-,Q_+]_+=\begin{pmatrix} \mathcal{A}^+\mathcal{A}^- & 0 \\ 0 & \mathcal{A}^-\mathcal{A}^+ \end{pmatrix}_{4\times 4} =\begin{pmatrix} \mathcal{H}_- & 0 \\ 0 & \mathcal{H}_+ \end{pmatrix}, \tag{45}$$

$$[H_{SUSY},Q_\pm]_-=0=(Q_-)^2=(Q_+)^2. \tag{46}$$

The supercharges $Q_\pm$ written in terms of the operators $a^\pm$ and $b^\pm$ become 4×4 matrix differential operators, i.e.,

$$Q_-=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a^- & -\frac{\partial}{\partial\phi}N & 0 & 0 \\ -\frac{\partial}{\partial\phi}N & b^- & 0 & 0 \end{pmatrix}, \quad Q_+=Q_-^\dagger, \tag{47}$$

which establishes the connection of SUSY QM with two soliton solutions.

IV. CONCLUSION

The connection between supersymmetric quantum mechanics with two-component wave functions and the stability equations associated with soliton solutions of simple models of two coupled real scalar fields in $1+1$ dimensions has been presented and an application has been given.

In [9] we have seen that if $\Psi_-^{(0)}$ is a normalizable two-component eigenstate, one cannot write $\Psi_+^{(0)}$ in terms of $\Psi_-^{(0)}$ in a similar manner to ordinary supersymmetric quantum mechanics [as in Eq. (9)]. This is can be seen in the example treated here of the classical stability analysis for two coupled real scalar fields. The Hermiticity condition satisfied by the superpotential associated with the positive potentials with a SUSY form considered here leads to the two-component normal modes to be non-negative ($\omega_n^2 \geqslant 0$, analogous to the case with only one single field [8]) so that the linear stability of the Schrödinger-like equations is ensured. The Bogomol'nyi condition leads us to a set of first-order differential equations (17) which have solutions which are also solutions of the second-order differential equations (13).

Our approach can easily be applied to the soliton solutions to specific systems considered in terms of two coupled scalar fields in [13,15,19]. However, we have considered only an application for a particular case associated with a generalization of the ϕ^6 model in $1+1$ dimensions which, in the general case, can be solved by the trial orbit method treated in [13].

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