

Decoupling of zero modes and covariance in the light-front formulation of supersymmetric theories

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We show under suitable assumptions that zero modes decouple from the dynamics of non-zero modes in the light-front formulation of some supersymmetric field theories. The implications for Lorentz invariance are discussed. [S0556-2821(98)04922-4]

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I. INTRODUCTION

Although field theories quantized on the light front (LF) have been studied for many years (see [1,2] and also [3,4] for a review), recent developments in non-perturbative string theory have generated additional interest. The first surprise was M(atr)ix theory [5], which was conjectured to be a non-perturbative description of M theory formulated in the infinite momentum frame. Susskind and Motl provided additional insight by suggesting that the finite N version of matrix theory was in fact the discrete light-cone quantization (DLCQ) of M theory [6].

Soon afterwards, the validity of the matrix theory conjecture was seemingly strengthened by the works of Seiberg [7] and Sen [8], but it was pointed out by Hellerman and Polchinski [9] that a correct interpretation of their results required a detailed understanding of the (typically complicated) dynamics of zero longitudinal momentum modes in the light-like compactification limit. In general, it was observed, that “DLCQ is not a free lunch.”

The question we wish to address in this paper is the following: When is the light-like limit a free lunch? Under a reasonable class of assumptions, we argue that the zero-mode degrees of freedom in some supersymmetric field theories decouple, and so omitting them in a DLCQ calculation leads to no inconsistency if the decompactification limit is taken prior to the light-like limit. This observation is intriguing, since it suggests that the complicated zero-mode degrees of freedom studied in [9] might become totally irrelevant in the continuum limit if enough supersymmetry is present. Moreover, the “correctness of matrix theory” argument provided by Seiberg may depend on this special property of supersymmetric theories.

Another issue that we address is Lorentz invariance. We show that in the light-front formulation, Lorentz invariance is maintained after a careful treatment of zero modes. However, for the special case of supersymmetric theories, the boson and fermion zero modes that ensure Lorentz symmetry cancel at least perturbatively. Thus, we are free to exclude them from the outset.

All of these observations suggest that the implementation of DLCQ in the absence of zero modes yields no inconsistency for supersymmetric theories. In general, however, one needs to integrate out the zero-mode degrees of freedom to derive an effective Hamiltonian. We discuss these issues next.

II. TADPOLE-IMPROVED LIGHT-FRONT QUANTIZATION

It has been known for a long time that field theories quantized on a light front $x^+ \equiv (x^0 + x^3)/\sqrt{2} = 0$ lead to a subtle treatment of the zero modes [modes which are independent of $x^- \equiv (x^0 - x^3)/\sqrt{2}$ [10]]. This result holds both in the continuum, when zero modes are discarded but also in DLCQ when the theory is formulated in a finite “box” in the x^- direction with periodic boundary conditions.

Various schemes have been invented to define LF quantization through a limiting procedure in order to investigate these issues. For example, one can study LF perturbation theory by starting from covariant Feynman diagram expressions and then “derive” the LF Hamiltonian perturbation theory by carefully integrating over all energies k^- in loop integrations first (see for example Refs. [12,13] and references therein). An alternative prescription starts by quantizing the fields on a near light-like surface (using so-called ϵ coordinates) and then studying the evolution of the states as one takes the LF limit in an infinite volume [14].

The basic upshot of these investigations is that, at least for theories without massless degrees of freedom, zero modes become high-energy degrees of freedom and “freeze out.” However, this does not mean that zero modes disappear completely, since there is still a strong interaction present among the zero modes, giving rise to non-trivial vacuum structure even in the LF limit. Nevertheless, because of the high energy scale for excitations within the zero-mode sector, one has been able to derive *effective LF Hamiltonians*, where the zero modes have been integrated out, which act only on non-zero-mode degrees of freedom. Thus even though these effective LF Hamiltonians contain only non-

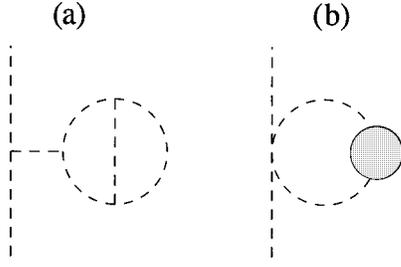


FIG. 1. Typical self-energy diagrams for scalar fields which vanish if zero modes are not included. (a) “Tennis racket” shaped tadpole in ϕ^3 theory, (b) generalized tadpole diagram in ϕ^4 theory. The grey blob represents an arbitrary self-energy insertion.

zero-mode degrees of freedom, they yield the same Green’s functions as a covariant calculation provided one considers only Green’s functions where all external momenta have $k^+ \neq 0$.

A. Self-interacting scalar fields

As an example, let us consider a scalar field theory with cubic (plus higher order) self-interactions. The presence of cubic self-interactions gives rise to “tennis racket” Feynman diagrams [Fig. 1(a)].

If zero modes are excluded, then obviously all tennis racket diagrams (which do contribute to Feynman perturbation theory) have no analogue in LF Hamiltonian perturbation theory. However, the crucial observation is that tennis racket diagrams are momentum independent and only lead to a mass renormalization proportional to $\langle 0|\phi|0\rangle$. Similarly, all tennis racket insertions into n -point interactions only lead to a renormalization of the $(n-1)$ -point interaction term; i.e., all these diagrams can be easily integrated out.

More generally, one can show that for self-interacting scalar fields,¹ zero modes contribute only to diagrams with generalized tadpole topology. As a result, zero modes can be integrated out easily. For a polynomial self-interaction

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \quad (2.1)$$

where

$$V(\phi) = \sum_{k \leq n} c_k \frac{\phi^k}{k!} \quad (2.2)$$

one thus finds, for the effective LF Hamiltonian [13],

$$P^- = V_{eff}(\phi), \quad (2.3)$$

where the effective potential is also a polynomial of the same degree,

¹Here and in the following we will implicitly restrict ourselves to Green’s functions where all external momenta as well as arbitrary sums and differences of external momenta have a non-vanishing plus component so that zero modes can only appear within loops.

$$V_{eff}(\phi) = \sum_{k \leq n} c_k^{eff} \frac{\phi^k}{k!}, \quad (2.4)$$

but with coefficients that are renormalized due to integrating out zero-mode degrees of freedom:

$$c_k^{eff} = \sum_{l=k}^n c_l \langle 0 | \frac{\phi^{l-k}}{(l-k)!} | 0 \rangle. \quad (2.5)$$

As an illustration of how Eq. (2.5) arises, let us consider a theory with quartic self-interactions,² i.e. $V(\phi) = \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda^2 \phi^4$. In this case, the only Feynman diagrams which are improperly handled (they are set to zero) when the zero-mode region ($k^+ = 0$) is cut out are the generalized tadpole diagrams [Fig. 1(b)]. In order to see why these diagrams give only a zero-mode contribution, let us consider the sum of all generalized tadpole diagrams, which can be easily done by using the full propagator for the scalar fields for which we write down a spectral representation [15]

$$\Delta_F(p) = \int_0^\infty dM^2 \frac{i\rho(M^2)}{p^2 - M^2 + i\varepsilon} \quad (2.6)$$

with spectral density $\rho(M^2)$.

As a side remark, for later use, we would like to point out that the spectral density has a very simple representation in terms of the LF Fock states. Upon inserting a complete set of eigenstates of the LF Hamiltonian into the scalar two-point function [15], one finds (see the Appendix)

$$\begin{aligned} \rho(M^2) &= 2\pi \sum_n \delta\left(\frac{M^2}{2P^+} - P_n^-\right) |\langle 0|\phi(0)|n, P^+\rangle|^2 \\ &= 2\pi \sum_n \delta(M^2 - M_n^2) 2P^+ |\langle 0|\phi(0)|n, P^+\rangle|^2 \\ &= \sum_n \delta(M^2 - M_n^2) b_n \end{aligned} \quad (2.7)$$

where $|n, P^+\rangle$ is a complete set of eigenstates of P^- (with eigenvalues $P_n^- = M_n^2/2P^+$) which we take to be normalized to 1 and where b_n is the probability that the state n is in its one boson Fock component (one boson which carries the whole momentum P^+). The sum can be evaluated at arbitrary but fixed total momentum P^+ (assuming we work in the continuum limit).

Using Eq. (2.6), one finds, for the sum of all generalized tadpole diagrams,³

$$-i\Sigma^{tadpole} = \frac{\lambda^2}{2} \int_0^\infty dM^2 \rho(M^2) \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - M^2 + i\varepsilon}. \quad (2.8)$$

²For the general case, see Ref. [13].

³Note that this result holds regardless of whether or not fermions pairs contribute to the spectral density of the bosons.

The crucial point is that for $k^+ \neq 0$, all poles lie only on one side of the real k^- axis and the result is thus zero (up to a contribution from the semi-circle at infinity, which disappears if one subtracts the one loop result).

In order to compensate for the omission of all generalized tadpole diagrams in naive LF quantization, we thus add a counter-term equal to the sum of all these omitted diagrams; i.e., a calculation that omits all explicit zero-mode degrees of freedom, but adds a mass counter-term $\delta\Sigma = \Sigma^{tadpole}$, will give the same results as a calculation that includes all zero modes explicitly. The connection with Eq. (2.5) can now be seen by noting that the vacuum expectation value of ϕ^2 is (up to a combinatoric factor) identical to the right hand side (RHS) of Eq. (2.8).

In summary, one finds the following (for self-interacting scalar fields) [13]:

- (i) Zero modes contribute to n -point functions involving only $k^+ \neq 0$ modes only through generalized tadpole (sub-)diagrams. By generalized tadpole diagrams we mean diagrams where a sub-diagram is connected to the rest of the diagram only at one single point and hence there is no momentum transfer through that point.
- (ii) n -point functions calculated with the ‘‘tadpole-improved’’ effective LF Hamiltonian (2.3),(2.4), (2.5) and without explicit zero-mode degrees of freedom are equivalent to covariant perturbation theory generated by \mathcal{L} , Eq. (2.1), to all orders in perturbation theory.

B. Yukawa interactions

As a generic example for a theory with fermions, let us now consider a Yukawa theory with scalar couplings:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_F - g\phi)\psi - \frac{1}{2}\phi(\square + m_B^2)\phi. \quad (2.9)$$

If zero modes are excluded, then two classes of Feynman diagrams (to be discussed below) are treated improperly in the LF Hamiltonian perturbation series.

Obviously, LF theory without zero modes cannot generate any tadpole (i.e. tennis racket) self-energies for the fermions. Since the above Lagrangian contains a scalar Yukawa coupling, such diagrams are in general non-zero. Their omission in naive LF quantization can be easily compensated by replacing

$$m_F \rightarrow m_F^{eff} \equiv m_F + g\langle 0|\phi|0\rangle. \quad (2.10)$$

The second class of diagrams which cannot be generated by a zero-mode free LF field theory is more subtle. As an example, let us consider the one loop fermion self-energy⁴

⁴For simplicity, we will write down the expressions only in 1 + 1 dimensions, but it should be emphasized that the conclusions are also valid in 3 + 1 dimensions [12].

$$\begin{aligned} -i\Sigma(p) &= g^2 \int \frac{d^2k}{(2\pi)^2} \frac{\gamma^\mu k_\mu + m_F}{k^2 - m_F^2 + i\epsilon} \frac{1}{(p-k)^2 - m_B^2 + i\epsilon} \\ &= -i\Sigma_{LF} + g^2 \int \frac{d^2k}{(2\pi)^2} \frac{\gamma^+}{2k^+} \frac{1}{(p-k)^2 - m_B^2 + i\epsilon} \end{aligned} \quad (2.11)$$

where

$$-i\Sigma_{LF} = g^2 \int \frac{d^2k}{(2\pi)^2} \frac{\gamma^\mu \tilde{k}_\mu + m_F}{k^2 - m_F^2 + i\epsilon} \frac{1}{(p-k)^2 - m_B^2 + i\epsilon} \quad (2.12)$$

and $\tilde{k}^+ = k^+$ while $\tilde{k}^- = m_F^2/2k^+$ is the on mass shell energy for the fermion. Obviously, Eq. (2.11) is a mere algebraic rewriting of the original Feynman self-energy. The important point is that the second term on the RHS of Eq. (2.11) has the same pole structure as a tadpole diagram and thus cannot be generated by a LF Hamiltonian. Indeed, as one can easily verify, second order perturbation theory with the canonical LF Hamiltonian yields only (the matrix elements of) Σ_{LF} and a disagreement between self-energies calculated in covariant perturbation theory and those calculated in LF Hamiltonian perturbation theory (without zero modes) emerges. Before we proceed to analyze more general diagrams which suffer from a similar problem, let us understand intuitively how this second term arises:

In the LF formulation, not all components of the fermion field are independent degrees of freedom. Multiplying the Dirac equation

$$(i\gamma^\mu \partial_\mu - m_F - g\phi)\psi = 0 \quad (2.13)$$

by γ^+ one finds that

$$2i\partial_- \psi_{(-)} = (m_F + g\phi)\gamma^+ \psi_{(+)}, \quad (2.14)$$

where $\psi_{(\pm)} \equiv \frac{1}{2}\gamma^\mp \gamma^\pm \psi$. Equation (2.14) is a constraint equation and it is often used to eliminate the dependent component $\psi_{(-)}$ prior to quantization. This gives rise to ‘‘induced’’ four point interactions

$$\mathcal{L}^{(4)} = -g^2 \psi_{(+)}^\dagger \phi \frac{1}{i\sqrt{2}\partial_-} \phi \psi_{(+)} \quad (2.15)$$

in the Lagrangian after eliminating the constrained field $\psi_{(-)}$ and hence it is possible to generate ‘‘induced tadpoles’’ diagrams by contracting for example the two scalar fields in Eq. (2.15).

Before discussing the general case, it is very instructive to investigate the one loop fermion self-energy in more detail. First one notes that the 2nd order perturbation theory result (2.12) is divergent at $k^+ \rightarrow 0$:

$$\Sigma_{LF} = \frac{g^2}{8\pi} \int_0^{p^+} \frac{dk^+}{k^+(p^+ - k^+)} \frac{k^+ \gamma^- + \frac{m_F^2}{2k^+} \gamma^+ + m_F}{p^- - \frac{m_F^2}{2k^+} - \frac{m_B^2}{2(p^+ - k^+)}}. \quad (2.16)$$

This divergence is cancelled by the self-induced inertia term, which arises from normal ordering, Eq. (2.15),

$$\Sigma_{n.o.} = \frac{g^2}{8\pi} \int_0^{p^+} \frac{dk^+}{k^+}, \quad (2.17)$$

yielding

$$\begin{aligned} \Sigma_{LF} + \Sigma_{n.o.} &= \frac{g^2}{4\pi} \int_0^1 dx \frac{xp^\mu \gamma_\mu + m_F}{x(1-x)p^2 - m_F^2(1-x) - m_B^2 x} \\ &+ \frac{g^2}{4\pi} \frac{\gamma^+}{p^+} \ln \frac{m_B^2}{m_F^2}. \end{aligned} \quad (2.18)$$

Several important observations can be made from Eq. (2.18). First of all, even though including the normal ordering term renders the self-energy finite, the final result disagrees in general with the covariantly calculated result [the first term on the RHS in Eq. (2.18)]. Furthermore, the additional term breaks covariance (parity invariance).⁵ However, most importantly, the unwanted term vanishes for $m_F = m_B$, which indicates already a crucial cancellation between bosonic zero modes and fermionic zero modes. In the rest of this paper, we will demonstrate, for the case of certain supersymmetric theories, that this cancellation goes beyond the one loop result.

After this more intuitive discussion of zero-mode effects for fermions, let us now formally derive the counter-terms that arise for a theory with Yukawa interactions. For this purpose, it is useful to identify those Feynman diagrams (external momenta non-zero) where zero modes in internal lines give a non-zero contribution to the total amplitude. Diagrams which suffer from the same problem as the one-loop fermion self-energy are all diagrams where the internal lines in the fermion self-energy are dressed by arbitrary self-interactions (Fig. 2).

Let us assume that all counter-terms that are necessary to achieve agreement between LF perturbation theory (no zero modes) and covariant perturbation theory have been added to all sub-loops in Fig. 2; i.e., we assume that there exists a

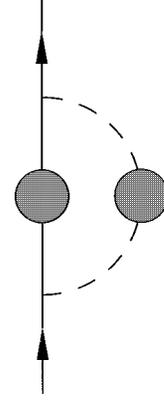


FIG. 2. Fermion self-energy diagram, which is treated improperly when zero modes are excluded. The shaded blobs represent arbitrary self-energy insertions.

covariant spectral representation for fermion propagators within the loop

$$S_F(p) = i \int_0^\infty dM^2 \frac{\gamma^\mu p_\mu \rho_1(M^2) + M \rho_2(M^2)}{p^2 - M^2 + i\varepsilon}. \quad (2.19)$$

Similar to the scalar case, the fermion spectral density has a very simple representation in terms of the eigenstates of the LF Hamiltonian as well (see the Appendix):

$$\begin{aligned} \rho_1(M^2) &= \frac{2\pi}{2P^+} \sum_n \delta\left(\frac{M^2}{2P^+} - P_n^-\right) |\langle 0 | \Psi_-(0) | n, P^+ \rangle|^2 \\ &= \sum_n \delta(M^2 - M_n^2) f_n. \end{aligned} \quad (2.20)$$

The spectral representation for bosons, Eq. (2.6), from the previous section is also still valid (of course with a different spectral function since we now deal with a different theory). For later use, we also note that completeness of the LF eigenstates implies the normalization condition

$$\int_0^\infty dM^2 \rho_1(M^2) = \int_0^\infty d\mu^2 \rho(\mu^2) = 1 \quad (2.21)$$

for the spectral densities.

Using the above spectral representations [Eqs. (2.6) and (2.20)] for the internal propagators, we now calculate the necessary counter-term self-consistently. The covariant self-energy for the diagram in Fig. 2 thus reads

$$-i\Sigma_F = g^2 \int_0^\infty dM^2 \int_0^\infty d\mu^2 \int \frac{d^2k}{(2\pi)^2} \frac{\gamma^\nu k_\nu \rho_1(M^2) + M \rho_2(M^2)}{k^2 - M^2 + i\varepsilon} \frac{\rho(\mu^2)}{(p-k)^2 - \mu^2 + i\varepsilon}. \quad (2.22)$$

We will now calculate the piece which is missed when the vicinity of both $k^+ = 0$ and $p^+ - k^+ = 0$ is omitted in the integration

⁵This fact has been used in Ref. [16] to determine the necessary counterterm non-perturbatively by demanding covariance for physical amplitudes.

in Eq. (2.22) (naive LF quantization with omission of fermion and boson zero modes respectively). Using the one-loop analysis as a guide, it is clear that the only problems arise in the γ^+ component of the self-energy. In order to further isolate the troublemaker, we use the algebraic identity

$$\frac{k^-}{k^2 - M^2 + i\epsilon} \frac{1}{(p-k)^2 - \mu^2 + i\epsilon} = \frac{1}{2p^+} \frac{2(p^+ - k^+)p^- + M^2 - \mu^2}{(k^2 - M^2 + i\epsilon)[(p-k)^2 - \mu^2 + i\epsilon]} + \frac{1}{2p^+} \left[\frac{1}{k^2 - M^2 + i\epsilon} - \frac{1}{(p-k)^2 - \mu^2 + i\epsilon} \right]. \quad (2.23)$$

Obviously, the first term on the RHS of Eq. (2.23) can be straightforwardly integrated over k^- and, for this term, the ‘‘zero-mode regions’’ ($k^+ = 0$ and $p^+ - k^+ = 0$) can be omitted without altering the result of the integration. However, the two last terms on the RHS of Eq. (2.23) have the pole structure of simple tadpoles and hence their only contribution to the k integration is from zero modes of the fermions $k^+ = 0$ as well as the bosons $p^+ - k^+ = 0$. This simple observation implies that the zero-mode counter-term from the class of diagrams in Fig. 2 reads [11]

$$-i\delta\Sigma_F = \frac{g^2\gamma^+}{2p^+} \int \frac{d^2k}{(2\pi)^2} \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{k^2 - \mu^2 + i\epsilon} - \frac{g^2\gamma^+}{2p^+} \int \frac{d^2k}{(2\pi)^2} \int_0^\infty dM^2 \frac{\rho_1(M^2)}{k^2 - M^2 + i\epsilon} \quad (2.24)$$

where we made use of the normalization of the spectral functions (2.21).

A similar zero-mode counter-term arises from ‘‘vacuum-polarization’’ type self-energies for the bosons where the fermion and anti-fermion lines may be dressed but where there is no interaction among the fermion and anti-fermion (Fig. 3). Using again the above spectral representation (2.19) one finds, for this class of Feynman diagrams,

$$-i\Sigma = -2g^2 \int_0^\infty dM_A^2 \int_0^\infty dM_B^2 \int \frac{d^2k}{(2\pi)^2} \frac{k \cdot (k-p) \rho_1(M_A^2) \rho_1(M_B^2) + M_A M_B \rho_2(M_A^2) \rho_2(M_B^2)}{(k^2 - M_A^2 + i\epsilon)[(p-k)^2 - M_B^2 + i\epsilon]}. \quad (2.25)$$

The part of Eq. (2.25) where zero-mode contributions are crucial arises from the $k \cdot (p-k)$ term in the numerator. In order to see this, let us write

$$2k \cdot (k-p) = (M_A^2 + M_B^2 - p^2) + (k^2 - M_A^2) + [(p-k)^2 - M_B^2] \quad (2.26)$$

and we note that the first term on the RHS yields no zero-mode contribution when inserted in Eq. (2.25). However, the other two terms on the RHS of Eq. (2.26) cancel one of the energy denominators and thus again yield a tadpole-like pole structure. One thus finds, for the contribution from the zero modes,

$$-i\delta\Sigma = -g^2 \int_0^\infty dM^2 \int \frac{d^2k}{(2\pi)^2} \frac{\rho_1(M^2)}{k^2 - M^2 + i\epsilon}. \quad (2.27)$$

Empirical studies of Feynman diagrams up to three loops [12] in Yukawa theories have shown that zero modes play a role for $k^+ \neq 0$ modes only in 2-point functions (except of course through sub-diagrams). Furthermore, of all the diagrams contributing to the two point functions, only the very simple sub-class of diagrams discussed above seems to be affected when zero modes are cut out. Diagrams with a more complicated topology, such as crossed diagrams (except of course through sub-diagrams), require no zero-mode counter-terms when the region $k^+ = 0$ is cut out. Although no rigorous analytical proof exists at this stage, it is reasonable to assume that these are the only diagrams yielding contri-

butions from zero modes. In the following, we discuss the consequences of this assumption for supersymmetric field theories.

III. ZERO MODES IN SUPERSYMMETRIC THEORIES

In order to study the implications of supersymmetry on zero-mode renormalization, let us consider a concrete example, namely a matrix model in 1+1 dimensions with the action [17]

$$S = \int d^2x \text{Tr} \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \bar{\Psi} i \not{\partial} \Psi - \frac{1}{2} V^2(\phi) - \frac{1}{2} V'(\phi) \bar{\Psi} \Psi \right] \quad (3.1)$$

where $V(\phi) = \mu\phi - (\lambda/\sqrt{N})\phi^2$. The canonical LF Hamiltonian for this model has been discussed in Ref. [17] and we refer the reader to this paper for details.

Obviously this model contains both three-point and four-point interactions for the scalar field as well as a Yukawa coupling between the scalar field and the (Majorana) fermion field; i.e., we can now directly apply the above zero-mode analysis to this model.

First we note that ‘‘tennis racket’’ tadpole diagrams [Fig. 1(a)] must all vanish in a covariant calculation, since $\langle \phi \rangle \neq 0$ would break the global matrix symmetry of the model. On the LF, without zero modes, these diagrams are automatically zero for simple kinematic reasons and, therefore, there

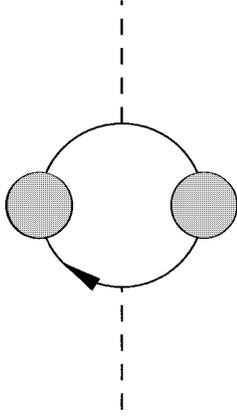


FIG. 3. Boson self-energy diagram, which is treated improperly when zero modes are excluded. The solid and dashed lines are fermion and boson propagators respectively. The shaded blobs represent arbitrary self-energy insertions.

is no need to add any zero-mode counter-terms for tennis racket diagrams to the LF Hamiltonian.

Since we have already seen that all tennis racket diagrams vanish in this model, the only diagrams that could still give rise to zero-mode counter-terms are the classes of self-energy diagrams depicted in Figs. 1(b), 2 and 3.

For the zero-mode contributions to the fermion self-energy, matrix symmetry is not sufficient to prove that the zero-mode counter-term vanishes and we have to invoke supersymmetry. Using the explicit expression for the supercharge Q_- in terms of the LF fields,

$$Q_- \equiv \int dx^- : \text{Tr}[\sqrt{2}(\partial_- \phi)\Psi_-] :, \quad (3.2)$$

we obtain the supersymmetry transformation

$$[Q_-, \phi] = -\frac{i}{\sqrt{2}}\Psi_-, \quad (3.3)$$

$$[Q_-, \Psi_-] = \sqrt{2}\partial_- \phi, \quad (3.4)$$

which gives rise to $Q_-^2 = P^+$. Let us show that the spectral densities ρ and ρ_1 defined in the previous section are equal owing to the supersymmetry. First note that the states of non-zero energy are paired by the action of supercharge. Namely,

$$Q_-|n, P^+\rangle_B = \sqrt{P^+}|n, P^+\rangle_F, \quad (3.5)$$

$$Q_-|n, P^+\rangle_F = \sqrt{P^+}|n, P^+\rangle_B, \quad (3.6)$$

where the B and F denote bosonic and fermionic state, respectively. The fermionic state is normalized if the bosonic state is, i.e. ${}_B\langle n, P^+ | n, P^+ \rangle_B = 1$, since $Q_-^2 = P^+$. Now we can easily find

$$\begin{aligned} \rho_1(M^2) &= 2\pi \sum_n \delta(M^2 - M_n^2) |\langle 0 | \Psi_-(0) | n, P^+ \rangle|^2 \\ &= 2\pi \sum_n \delta(M^2 - M_n^2) |\langle 0 | \sqrt{2}[Q_-, \phi(0)] | n, P^+ \rangle|^2 \\ &= 2\pi \sum_n \delta(M^2 - M_n^2) 2P^+ |\langle 0 | \phi(0) | n, P^+ \rangle|^2, \\ &= \rho(M^2). \end{aligned} \quad (3.7)$$

Therefore the spectral densities ρ and ρ_1 must be equal.

Since fermions and bosons contribute with opposite signs (but equal strength) to the zero-mode part of the fermion self-energy [Eq. (2.24)], the zero-mode contributions from bosons and fermions to the fermion self-energy cancel exactly.⁶

The boson self-energy is more complicated, since we have to consider two different classes of diagrams where zero modes contribute: tadpoles from ϕ^4 interactions [Fig. 1(b)] as well as the vacuum polarization type graphs (Fig. 3). Using the results from the previous two sections, we find that the zero-mode contribution from tadpoles to the mass reads, Eqs. (2.8),

$$\delta\mu_{boson}^2{}_{ZM} = 4\lambda^2 \int_0^\infty dM^2 \rho(M^2) \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - M^2 + i\epsilon}. \quad (3.8)$$

For the contribution from zero modes in fermion loops to the boson self-energy one finds instead

$$\begin{aligned} \delta\mu_{fermion}^2{}_{ZM} &= -4\lambda^2 \int_0^\infty dM^2 \rho_1(M^2) \\ &\quad \times \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - M^2 + i\epsilon}, \end{aligned} \quad (3.9)$$

and invoking again supersymmetry, we find that the contributions from boson and fermion zero modes again cancel. Note that supersymmetry has played a dual role in obtaining this fundamental result. First of all, it relates the Yukawa coupling and the scalar four-point coupling and thus the coefficients of Eqs. (3.8) and (3.9) are the same. But the cancellation between Eqs. (3.8) and (3.9) happens only because the spectral densities are the same.

IV. SUMMARY

Even for theories with massive particles, where zero modes are high energy degrees of freedom, they cannot be

⁶Note that there is a flaw in the discussion of the two loop fermion self-energy for the supersymmetry (SUSY) Wess-Zumino model in Ref. [12] which arises because the subtraction procedure employed in Ref. [12] breaks the supersymmetry. The unsubtracted result in Ref. [12] is consistent with the above findings of a cancellation between bosonic and fermionic zero-mode contributions.

completely discarded. However, they can be integrated out, which gives rise to an effective (tadpole improved) LF Hamiltonian. In supersymmetric theories, there is scope for a complete cancellation between effective interactions induced by bosonic zero modes and those induced by fermionic zero modes. There is of course the possibility of spontaneous symmetry breaking, in which fields acquire a non-zero expectation value. In such a scenario, the fermion-boson cancellation may not occur, and we are left with the (difficult) task of deriving an effective Hamiltonian. However, our observations suggest that for theories with enough supersymmetry, the zero-mode degrees of freedom may be ignored. As a result, as long as one is interested only in the dynamics of $k^+ \neq 0$ modes in such massive supersymmetric theories, zero modes can be discarded. This implies that for such theories DLCQ (in the continuum $K \rightarrow \infty$ limit, and with the zero modes discarded) leads to the same Green's functions for $k^+ \neq 0$ modes as a covariant formulation. Clearly, it would be interesting to understand the precise connection between the decoupling of zero modes in supersymmetric theories and various non-renormalization theorems that are known to exist. We leave this for future work.

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APPENDIX: SPECTRAL DENSITIES

In this appendix, we will derive some results which are useful to relate spectral densities to eigenstates of a LF Hamiltonian.

We start by expressing the spectral density for a scalar field, which has a simple expression in terms of the eigenstates of the LF Hamiltonian [15]:

$$\rho(p) = 2\pi \sum_n \int dp_n^+ \delta(p_n^+ - p^+) \times \delta(p_n^- - p^-) |\langle 0 | \phi(0) | n, p_n^+ \rangle|^2, \quad (\text{A1})$$

where we split up the sum over states into a sum over states at fixed momentum p_n^+ and a sum (i.e. integral) over p_n^- . In the next step we integrate over p_n^- where we make use of both of the $\delta(p^+ - p_n^+)$ and the relation between the LF energy of the state and its invariant mass $p_n^- = M_n^2/2p_n^+$, yielding

$$\begin{aligned} \rho(p) &= 2\pi \sum_n \delta\left(\frac{M_n^2}{2p^+} - p^-\right) |\langle 0 | \phi(0) | n, p^+ \rangle|^2 \\ &= 4\pi p^+ \sum_n \delta(M_n^2 - 2p^- p^+) |\langle 0 | \phi(0) | n, p^+ \rangle|^2. \end{aligned} \quad (\text{A2})$$

In order to relate Eq. (A2) to the Fock expansion of the eigenstates $|n, p^+\rangle$ we use the expansion of $\phi(0)$ in terms of

elementary raising and lowering operators. For a real scalar field, the canonical commutation relations at equal LF time,

$$[\phi(x^-), \partial_- \phi(y^-)] = \frac{i}{2} \delta(x^- - y^-), \quad (\text{A3})$$

are satisfied if one expands

$$\phi(x^-, x^+ = 0) = \int_0^\infty \frac{dk^+}{\sqrt{4\pi k^+}} [a_{k^+} e^{-ik^+ x^-} + a_{k^+}^\dagger e^{ik^+ x^-}], \quad (\text{A4})$$

where $[a_{k^+}, a_{q^+}^\dagger] = \delta(k^+ - q^+)$ with all other commutators vanishing. Inserting Eq. (A4) into Eq. (A2) one thus finds

$$\rho(p) = \sum_n \delta(M_n^2 - 2p^- p^+) b_n, \quad (\text{A5})$$

where

$$b_n \equiv 4\pi p^+ |\langle 0 | \phi(0) | n, p^+ \rangle|^2 = \left| \langle 0 | \int_0^\infty dk^+ a_k | n, p^+ \rangle \right|^2 \quad (\text{A6})$$

is the probability to find the state $|n, p^+\rangle$ in the one boson Fock component (note that b_n is p^+ independent).

For the spectral density ρ_1 entering the full fermion propagator a similar result can be derived. Using the representation

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad (\text{A7})$$

i.e.

$$\gamma^+ \equiv \frac{\gamma^0 + \gamma^1}{\sqrt{2}} = \sqrt{2} \begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix}, \quad (\text{A8})$$

one finds, for the ‘‘kinetic energy’’ of a canonical Dirac field,

$$\mathcal{L} = \Psi^\dagger i \gamma^+ \partial_+ \Psi + \dots = \sqrt{2} \Psi_-^\dagger i \partial_+ \Psi_- + \dots \quad (\text{A9})$$

where

$$\Psi = \begin{pmatrix} \Psi_- \\ \Psi_+ \end{pmatrix}. \quad (\text{A10})$$

Equation (A9) implies

$$\sqrt{2} \{\Psi_-, \Psi_-^\dagger\} = \delta(x^- - y^-) \quad (\text{A11})$$

and hence ($x^+ = 0$)

$$\Psi_{-}(x^{-})=2^{-1/4}\int_0^{\infty}\frac{dk^{+}}{\sqrt{2\pi}}[b_k e^{-ik^{+}x^{-}}+d_k^{\dagger}e^{ik^{+}x^{-}}], \quad (\text{A12})$$

where $b_{k^{+}}$ and $d_{k^{+}}$ satisfy the anti-commutation relations

$$\{b_{k^{+}}, d_{q^{+}}^{\dagger}\}=\{d_{k^{+}}, d_{q^{+}}^{\dagger}\}=\delta(k^{+}-q^{+}). \quad (\text{A13})$$

In order to use this result to obtain a representation of spectral densities in terms of the LF eigenstates, we start from the definition [15]

$$(\rho_1 p_{\mu} \gamma^{\mu} + \rho_2)_{\alpha\beta} = 2\pi \sum_n \int dp_n^{+} \delta(p_n^{+} - p^{+}) \delta(p_n^{-} - p^{-}) \\ \times \langle 0 | \Psi_{\alpha}(0) | n, p_n^{+} \rangle \langle n, p_n^{+} | \bar{\Psi}_{\beta}(0) | 0 \rangle, \quad (\text{A14})$$

multiply by γ^{+} and take the (Dirac) trace in order to project out ρ_1 , yielding

$$2p^{+} \rho_1(p) = 2\pi \sum_n \int dp_n^{+} \delta(p_n^{+} - p^{+}) \delta(p_n^{-} - p^{-}) \\ \times \sqrt{2} \langle 0 | \Psi_{-}(0) | n, p_n^{+} \rangle \langle n, p_n^{+} | \Psi_{-}^{\dagger}(0) | 0 \rangle \\ = 4\pi p^{+} \sum_n \delta(M_n^2 - 2p^{-} p^{+}) |\langle 0 | \Psi_{-}(0) | n, p_n^{+} \rangle|^2 \quad (\text{A15})$$

and therefore

$$\rho_1(p) = \sum_n \delta(M_n^2 - 2p^{-} p^{+}) f_n, \quad (\text{A16})$$

where

$$f_n \equiv \left| \langle 0 | \int_0^{\infty} dk^{+} b_k | n, p_n^{+} \rangle \right|^2 \quad (\text{A17})$$

is the (p^{+} independent) probability for the state n to be in its one fermion Fock component.

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