Vacua of M theory and string theory

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We argue that supersymmetric higher-dimension operators in the effective actions of M theory and type IIB string theory do not affect the maximally supersymmetric vacua: $AdS_4 \times S^7$ and $AdS_7 \times S^4$ in M theory and $AdS_5 \times S^5$ in type IIB string theory. All these vacua are described in superspace by a fixed point with all components of supertorsion and supercurvature being supercovariantly constant. This follows from 32 unbroken supersymmetries and allows us to prove that such vacua are exact. [S0556-2821(98)10520-9]

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I. INTRODUCTION

There is very limited knowledge of exact solutions in gravitational theories which include higher-dimension operators. An example of such a configuration is given by a *pp* wave. This solves the non-linear equations of motion of pure Einstein theory and can be proved to remain an exact solution in the presence of all possible higher-derivative terms respecting general covariance.

It is interesting to find some solutions in M theory and string theory which can be proved to be exact when all possible corrections to the low-energy supergravity actions are included, which respect not only the general covariance but also the local supersymmetry. It is natural to consider the vacuum solutions and use the power of 32 unbroken supersymmetries.

We shall look at four-dimensional anti–de Sitter space $(AdS_4) \times S^7$ and $AdS_7 \times S^4$ solutions of M theory and $AdS_5 \times S^5$ solution of IIB string theory. There has been a great deal of interest in these solutions lately because of the conjecture [1–3] relating type IIB string theory on $AdS_5 \times S^5$ to $N=4$ Yang-Mills theory. We shall attempt to argue that there are no corrections to the form of this solution from α' corrections. This was already shown for the $AdS_5 \times S^5$ case to order α'^3 in [4]. Similarly, we argue that there are no l_{11} corrections to the form of $AdS_4 \times S^7$ and $AdS_7 \times S^4$ in M theory. The proof in [4] uses essentially the conformal flatness of $AdS_5 \times S^5$ space. Our general proof based on the maximal amount of unbroken supersymmetry will cover the supersymmetric vacua of M theory whose metrics are not conformally flat.

In the case of the M-theory solutions, we still do not have a full formulation of the theory. However, we can study the low energy effective action as an expansion in powers of the Planck length. We expect that the effective action will have $N=1$ supersymmetry in eleven dimensions, which constrains its form. Also, in analogy with string theory, we expect that an exact solution of the effective action is a solution of the full theory.

The strategy will be to write down all possible corrections to the equations of motion consistent with supersymmetry. If all possible corrections to the equations vanish when evaluated in a certain background, then, by definition, this background is an exact solution of the full effective action. We shall show that this situation holds for these solutions.

In general, the possible corrections to the equations of motion could involve the curvature, derivatives of the curvature etc. It is a feature of these solutions that all relevant tensors are covariantly constant. Hence the corrections can only depend on the numerical values of these tensors. We will then show that even these corrections do not affect the solutions.

This is most conveniently done in superspace. We find that in superspace, the equations of motion can be written in a form which have one free spinorial index. It turns out that all nonzero components of the superfields (in this background) have two spinor indices, and it is thus impossible to construct a consistent nonzero correction. (Usually of course, one could have used spinorial derivatives to construct a correction term, but as we have already said, all such terms vanish.) Thus the solution is uncorrected in the full effective action.

We first consider as a warm-up, the cases of *pp* waves in pure gravity, and the $AdS_2 \times S^2$ solution of $N=2$, $d=4$ pure supergravity, where similar considerations allow us to prove the exactness of the solutions. We then turn to the cases of interest i.e. $AdS_4 \times S^7$ and $AdS_7 \times S^4$ in eleven-dimensional supergravity and $AdS_5 \times S^5$ in type IIB string theory. Finally we conclude with discussions.

Recently, quantum corrections to the supersymmetric black hole entropy in string theory [5] and to the minimal value of the central charge in supergravity theory [6] have been calculated. These corrections appear in theories related to $N=2$ supergravity interacting with vector multiplets. Such interaction is not unique. The prepotential in presence of higher dimension operators is modified [6] but the theory is still supersymmetric. It has not been established whether the existence of such corrections is due to the modifications of the solutions or just change of the AdS_2 size in the Bertotti-Robinson throat. In all cases which we will study we will deal only with maximal supersymmetry, 32 in $d=11$, $d=10$ (and 8 in $d=4$ for pure supergravity without extra matter multiplets as a simplest model). These are purely geometric theories in superspace. There are no options in the choice

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of the prepotential. We expect therefore that the superfield structure is not modified in presence of corrections.

II. STABILITY OF *pp* WAVES

pp-wave geometries are spacetimes admitting a covariantly constant null vector field

$$\nabla_\mu l_\nu = 0, \quad l^\nu l_\nu = 0. \tag{1}$$

Spacetimes with this property were first discovered by Brinkmann [7]. The existence of a covariantly constant null vector field has dramatic consequences [8]. For instance, for the class of d-dimensional pp waves with metrics of the form

$$ds^2 = 2du dv + K(u,x^i)du^2 - dx^i dx^i, \tag{2}$$

where $i,j=1,2,\ldots,d-2$, the Riemann curvature is [8]

$$R_{\mu\nu\rho\sigma} = -2 l_{[\mu}(\partial_{\nu]}\partial_{[\rho}K) l_{\sigma]} . \tag{3}$$

The Ricci tensor vanishes if K is a harmonic function in the transverse space:

$$R_{\mu\sigma} = -\frac{1}{2}(\partial_\nu \partial^\nu K) l_\mu l_\sigma , \quad R = -\frac{1}{2}(\partial_\nu \partial^\nu K) l_\mu l^\mu = 0. \tag{4}$$

The curvature $R_{\mu\nu\rho\sigma}$ is therefore orthogonal to l^μ and to ∇^μ in all its indices. Since K is independent of v, the metric solves Einstein equations $G_{\mu\nu}=0$ if $\partial_T^2 K=0$. Possible corrections to field equations may come from higher dimension operators and depend on the curvature tensors and their covariant derivatives

$$G_{\mu\nu} = F^{corr}_{\mu\nu}(R_{\mu\nu\lambda\sigma}, D_\alpha R_{\mu\nu\lambda\sigma}, \ldots). \tag{5}$$

Corrections to Einstein equations are quadratic or higher order in curvature tensors. However, there is no way to contract two or more of Riemann tensors which will form a two-component tensor to provide the right-hand side (RHS) of the Einstein equation coming from higher dimensions operators. Therefore all higher order corrections vanish for pp-waves solutions. They remain exact solutions of any higher order in derivatives general covariant theory. This includes supergravities and string theory with all possible sigma model and string loop corrections to the effective action, as long as these corrections respect general covariance. Note that supersymmetry played no role in establishing this non-renormalization theorem.

III. SUPERSYMMETRIC BERTOTTI-ROBINSON VACUUM

Our next example is $N=2$, $d=4$ pure supergravity without matter multiplets. A vacuum solution with 8 unbroken supersymmetries is given by the $\mathrm{AdS}_2\times S^2$ metric and a two-form which is a volume form of the AdS_2 space. Before considering $N=2$ theory we will explain our strategy in terms of the more familiar superspace of $N=1$ supergravity. In general the geometric superspace tensors must satisfy some constraints in order to describe correctly the field contents of supergravity theory. When the constraints are imposed, the geometric Bianchi identities are not identities anymore but equations which can be solved. The solutions provide the superspace form of supergravities. In the $N=2$, $d=4$ case the full off-shell superspace solution is available. This is analogous to the well-known $N=1$, $d=4$ supergravity in superspace given in terms of 3 superfields: $W_{\alpha\beta\gamma}, G_{\alpha\dot\beta}, \mathcal{R}$. All components of the constrained geometric tensors like torsion T^C_{AB} and curvature $R_{AB}{}^{cd}$ are expressed in terms of these three superfields and their covariant derivatives. On shell $G_{\alpha\dot\beta}(X,\theta)=0$ and also $\mathcal{R}(X,\theta)=0$. All possible higher dimension operators would modify the form of classical equations of motion as follows:

$$G_{\alpha\dot\beta} = \mathcal{F}^{corr}_{\alpha\dot\beta}(G,\mathcal{R},W,\overline{W},D_A G, D_A R, D_A W, D_A \overline{W}, \ldots). \tag{6}$$

It is expected that the RHS of the quantum corrected equation of motion will depend only on superfields and their covariant derivatives, i.e. on all supertensors of the theory. If one wishes to find out if some particular solution of classical equations remains a solution in the presence of the corrections, one has to study whether

$$\mathcal{F}^{corr}_{\alpha\dot\beta}(G=0,\mathcal{R}=0,W,\overline{W},D_A G=0,D_A R =0,D_A W, D_A \overline{W}, \ldots), \tag{7}$$

vanishes or not. The chiral superfield $W_{\alpha\beta\gamma}$ has in the lowest component θ^0 the gravitino field strength and in the first one θ^1 the Weyl tensor.

We proceed to the $N=2$, $d=4$ case to study the supersymmetric Bertotti-Robinson (BR) vacuum. We give below a summary on $N=2$, $d=4$ off shell superspace with 4 bosonic and 8 fermionic coordinates. The supergeometry is given in [9] and we use the two-component spinor notation from there. The structure group consists of Lorentz transformations with $M_{ab}=-M_{ba}$, $a=0,1,2,3$ and central charge transformations $M_{ij}=-M_{ji}$, $i,j=1,2$. The geometric tensors include torsion T^A_{BC}, the Lorentz curvature $R_{ab}{}^{cd}$ and the central charge curvature $F_{AB}{}^{ij}$.

There are two superfields defining the off-shell superspace. There is one spinorial superfield $T^i_\alpha(X,\theta,\overline{\theta})$ which vanishes on shell and therefore represents the superfield equations of motion of the theory. There is also a chiral superfield $W_{\alpha\beta ij}$ satisfying $D_{\dot\gamma k}W_{\alpha\beta ij}=0$. The lowest θ^0 component of the superfield W is the form field, the next one θ^1 is the gravitino field strength and the second one θ^2 is the Weyl tensor

$$W_{\alpha\beta\, ij}(X,\theta)|_{\theta=0} = (\sigma^{ab})_{\alpha\beta} F_{ab\ ij}(X), \tag{8}$$

$$D^i_\alpha W_{\beta\gamma\, ik}(X,\theta)|_{\theta=0} = \psi_{\alpha\beta\gamma\, k}(X), \tag{9}$$

$$D^i_\alpha D^j_\beta W_{\gamma\delta\, ij}(X,\theta)|_{\theta=0} = C_{\alpha\beta\gamma\delta}(X), \tag{10}$$

$$D^{i}_{\dot{\alpha}} D_{\beta\, i} W_{\gamma\delta\, kl}(X,\theta)|_{\theta=0} = D_{\dot{\alpha}\beta} F_{\gamma\delta\, kl}(X). \quad (11)$$

According to our conditions on the corrections to field equations respecting $N=2$ supersymmetry we get the quantum corrected field equation in the form

$$T^{i}_{\alpha}(X,\theta) = \mathcal{F}^{i\; corr}_{\alpha}(T,W,\overline{W},D_A T,D_A W,D_A \overline{W},\ldots). \quad (12)$$

Exactness of flat superspace. Flat superspace has the following properties. There is a non-vanishing constant torsion and central charge curvature:

$$T^{d}_{\alpha i,\dot{\beta} j} = 2i\sigma_{\alpha\dot{\beta}}\delta_{ij}, \quad F_{\alpha i,\beta j}{}^{kl} = C_{\alpha\beta}\delta^{[k}_{i}\delta^{l]}_{j}. \quad (13)$$

The superfields $T^{i}_{\alpha}(X,\theta), W^{ij}_{ab}(X,\theta)$ vanish. If one would try to construct $\mathcal{F}^{i\; corr}_{\alpha}$ out of only constant structures in Eq. (13), one could see that no such structures are available and therefore the flat superspace cannot have quantum corrections.

A superspace form of the near horizon black hole geometry with a 2-form and with enhancement of supersymmetry near the horizon has been studied before [10,11]. It has been found that the supersymmetric branching ratio (BR) vacuum corresponds to a supercovariantly constant superfield W_{ab} [the superfield $T^{i}_{\alpha}(X,\theta,\overline{\theta})=0$ since we consider the solution of the classical field equations]

$$D_A W^{\rm BR}_{ab\; kl} = 0 \Rightarrow D_{\alpha i} W^{\rm BR}_{ab\; kl} = D_{\dot{\alpha} i} W^{\rm BR}_{ab\; kl} = D_c W^{\rm BR}_{ab\; kl} = 0. \quad (14)$$

The integrability condition for the existence of the covariantly constant superfield is verified by checking that the solution admits Killing spinors of the maximal dimension. It can also be simply understood by observing that for the supersymmetric BR the lowest θ^0 component of the superfield is covariantly constant in X space, the next θ^1 component vanishes since the background is bosonic and the second θ^2 component of the superfield vanishes since the Weyl tensor vanishes and the form F is covariantly constant in X space. The higher components of the superfield are not independent and therefore also vanish. The self-dual form is

$$F^{ij} = \epsilon^{ij}(e^0 \wedge e^1 + e^2 \wedge e^3). \quad (15)$$

Therefore all components of the superfield W vanish except the first one which is a constant self-dual form. It breaks the Lorentz part of the structure group $SO(1,3)$ of the superspace with $a=0,1,2,3$ into a product $SO(1,1)\times SO(2)$, with $\hat{a}=0,1$ and $\check{a}=3,4$. The first one is related to the tangent space of AdS_2 and the second one to that of S^2.

Thus our *BR vacuum in the superspace* can be described by a covariantly constant superfield $W^{\rm BR}_{ab}$ which consist of two parts:

$$W^{\rm BR}_{\hat{a}\hat{b}} = \epsilon_{\hat{a}\hat{b}}, \quad W^{\rm BR}_{\check{a}\check{b}} = \epsilon_{\check{a}\check{b}}. \quad (16)$$

All non-vanishing components of torsion and curvature are constant and given by Eq. (13) as in the flat superspace as well as new constant torsions and curvatures:

$$T_{a,\beta j,\dot{\gamma} k} = -i\sigma^{b}_{\beta,\dot{\gamma}} W^{\rm BR}_{ab}, \quad F^{kl}_{ab} = \epsilon^{kl} W^{\rm BR}_{ab}, \quad (17)$$

$$R_{\alpha i,\beta j}{}^{cd} = -2iC_{\alpha\beta}(\overline{\sigma}^{cd}\overline{W}^{\rm BR})^{\dot{\delta}}_{\dot{\delta}}, \quad \text{etc.} \quad (18)$$

Now we can look what will happen with corrections to the equation of motion with account of Eqs. (14) and (16).

$$T^{i}_{\alpha}(X,\theta) = \mathcal{F}^{i\; corr}_{\alpha}(W^{\rm BR}_{\hat{a}\hat{b}}, W^{\rm BR}_{\check{a}\check{b}}). \quad (19)$$

It is not possible to build the object $\mathcal{F}^{i\; corr}_{\alpha}$ with one fermionic index from the available supercovariantly constant superfields. Therefore we do not see any possibility for the supersymmetric BR vacuum to be corrected by higher dimension supersymmetric operators.

IV. $AdS_4 \times S^7$ AND $AdS_7 \times S^4$ VACUA OF M THEORY

The background is in the AdS_4 case

$$F^{\rm (AdS)}_{mnps} = e\,\epsilon_{mnps}, \quad (20)$$

$$R^{\rm (AdS)}{}_{mn}{}^{ps} = -\frac{4e^2}{9}(\eta^{p}_{m}\eta^{s}_{n} - \eta^{s}_{m}\eta^{p}_{n}), \quad (21)$$

$$R^{(Sph)}{}_{mn}{}^{ps} = \frac{e^2}{9}(\eta^{p}_{m}\eta^{s}_{n} - \eta^{s}_{m}\eta^{p}_{n}), \quad (22)$$

and for the AdS_7 case

$$F^{(Sph)}_{mnps} = e\,\epsilon_{mnps}, \quad (23)$$

$$R^{\rm (AdS)}{}_{mn}{}^{ps} = -\frac{e^2}{9}(\eta^{p}_{m}\eta^{s}_{n} - \eta^{s}_{m}\eta^{p}_{n}), \quad (24)$$

$$R^{(Sph)}{}_{mn}{}^{ps} = \frac{4e^2}{9}(\eta^{p}_{m}\eta^{s}_{n} - \eta^{s}_{m}\eta^{p}_{n}). \quad (25)$$

The relevant on-shell superspace was constructed in [13,14]. There is a single superfield[1] $W_{rstu}(X,\theta)$. The field content of this superfield follows from that of eleven-dimensional supergravity.

The first few components of the superfield are

[1]We follow the notation of [13] with the exception of renaming spinorial indices in tangent space from a to α to be in agreement with other sections of this paper.

$$W_{rstu}(X,\theta)|_{\theta=0}=F_{rstu}(X), \tag{26}$$

$$(D_{\alpha}W_{rstu}(X,\theta))|_{\theta=0}=6(\gamma_{[rs}\hat{D}_{t}\psi_{s})_{\alpha}(X), \tag{27}$$

$$(D_{\alpha}(\hat{D}_{[r}\psi_{s]})_{\beta})|_{\theta=0}=\left(\frac{1}{8}\hat{R}_{rsmn}(X)\gamma^{mn}+\frac{1}{2}[T_{r}^{tuvw},T_{s}^{xyzp}]\hat{F}_{tuvw}(X)\hat{F}_{xyzp}+T_{[s}^{tuvw}\hat{D}_{r]}\hat{F}_{tuvw}(X)\right)_{\alpha\beta}. \tag{28}$$

Here T^{rstuv} is a product of γ matrices defined in [13].

The equation of motion of classical supergravity in superspace is

$$(\gamma^{rst}D)_{\alpha}W_{rstu}(X,\theta)=0. \tag{29}$$

In a generic background one can write down corrections to the RHS of the superfield equations involving the superfields, derivatives of the superfield etc. There is no reason to expect that such corrections will vanish in general.

We now claim that the supersymmetric $\mathrm{AdS}_4\times S^7$ and $\mathrm{AdS}_7\times S^4$ vacua of M theory are described by a fixed point in superspace, where all components of torsion, curvature and 4-form are covariantly constant. To prove this it is sufficient to prove that the superfield $W_{rstu}(X,\theta)$ is supercovariantly constant (since all other superfields can be derived from it).

The lowest component of the superfield W according to Eq. (26), is given by the form field strength. In the AdS_7 case, we have $F_{0123}=\epsilon_{0123}$, and in the AdS_4 case, we have $F_{45678910}=\epsilon_{45678910}$. These are manifestly covariantly constant.

The next component of the superfield, as shown in Eq. (27), is the gravitino field strength and this vanishes since our vacua are purely bosonic.

The next component of the superfield is bosonic and is shown in Eq. (28). Remarkably, it vanishes as well (as can be verified by explicit computation).

The remaining higher components are given by some derivatives of the previous ones and therefore all vanish. Putting these facts together, we see that the superfield $W_{rstu}(X,\theta)$ is supercovariantly constant.

The vanishing of the θ^2 component of the superfield is related to the fact that these vacua have maximal supersymmetry. The integrability condition for the requirement that the bosonic configuration admits maximal unbroken 32-dimensional supersymmetry is

$$\delta_{SUSY}\psi_{r}=D_{r}\epsilon+T_{r}^{tuvw}\epsilon F_{tuvw}=0. \tag{30}$$

It was shown in [15] (in the context of the study of the near horizon Killing spinors of M2 and M5 branes) that this equation yields

$$\delta_{SUSY}(\hat{D}_{[r}\psi_{s]})=\frac{1}{8}\hat{R}_{rsmn}\gamma^{mn}\epsilon+\frac{1}{2}[T_{r}^{tuvw},T_{s}^{xyzp}]\epsilon F_{tuvw}F_{xyzp}+T_{[s}^{tuvw}\hat{D}_{r]}\epsilon F_{tuvw})=0, \tag{31}$$

which is exactly the statement that the θ^2 component vanishes.

Thus we have shown that the integrability condition for the 32 Killing spinors of the vacua provides the proof that the superfield is covariantly constant:

$$D_{A}W_{rstu}=0\Rightarrow D_{a}W_{rstu}=D_{v}W_{rstu}=0. \tag{32}$$

Let us look now at the corrected equations of motion. Since $D_AW_{rstu}=0$ the corrections can depend only on W_{rstu} and other constant tensors like γ matrices etc. Again we observe that it is impossible to get one spinorial index without using spinorial derivatives, but such derivatives are zero on all the terms. Hence there is no possible correction we can write down. This shows that the $\mathrm{AdS}_4\times S^7$ and $\mathrm{AdS}_7\times S^4$ solutions are exact.

V. $\mathrm{AdS}_5\times S^5$ VACUUM OF STRING THEORY

We have, in this case, to consider the superspace formulation of type IIB supergravity. This was constructed in [12].

The background has a nonzero five-form field strength and a nonzero curvature. These split into the AdS part and the sphere part. For the AdS part, we have

$$g_{mnpst}^{(\mathrm{AdS})}=e\,\epsilon_{mnpst}, \tag{33}$$

$$R^{(\mathrm{AdS})}{}_{mn}{}^{ps}=-\frac{e^2}{16}(\eta_m^p\eta_n^s-\eta_m^s\eta_n^p), \tag{34}$$

where the indices run over the AdS indices (0 to 4), and for the sphere part, we have

$$g_{mnpst}^{(Sph)}=e\,\epsilon_{mnpst}, \tag{35}$$

$$R^{(Sph)}{}_{mn}{}^{ps}=\frac{e^2}{16}(\eta_m^p\eta_n^s-\eta_m^s\eta_n^p), \tag{36}$$

where the indices now run over the sphere indices (5 to 9). The important point about these values is that again, all the tensors are covariantly constant in X-space.

The on-shell superspace description of type IIB string theory is related to $N=2$, $d=10$ chiral supergravity [12]. The superspace has some constrained torsion T_{AB}^{C},, Lorentz curvature R_{AB}^{cd} and $U(1)$ curvature M_{AB}. Besides, there are the 3-form F_{ABC}, the 5-form G_{ABCD} and the scalar field strength P_A.

In the full non-linear theory there are two superfields, $\Lambda_{\alpha}(X,\theta,\bar{\theta})$ and $Z^{+}_{abcde}(X,\theta,\bar{\theta})$. All geometric tensors are functionals of these superfields and their covariant derivatives. $\Lambda_\alpha(X,\theta,\overline{\theta})$ starts with the dilatino and $Z^+_{abcde} = 1/192\, G_{abcde}$ starts with the self-dual 5-form $1/192 g_{abcde}(x)$. Even though there is only one supermultiplet, the second superfield is not a derivative of the first. The scalars of this theory belong to the coset space of $SU(1,1)/U(1)$. The construction in fact starts with the superfield $V(X,\theta,\overline{\theta})$ which is an element of $SU(1,1)$. From this a $SU(1,1)$ singlet P_A is built where the scalars appear. In this form scalars can be found in derivatives of $\Lambda_\alpha(X,\theta,\overline{\theta})$.

In the linear approximation one can also consider an analytic superfield A with $\overline{D}_\alpha A=0$ and the constraint $D^4A = \overline{D}^4\overline{A}$. This superfield in the proper basis depends only on half of the components of the superspace. The superinvariants of the type R^4 can be analyzed as superspace integrals over 16 θ. The θ^4 component of this linear superfield is a Weyl tensor. This automatically proves that the higher dimension operator with four powers of the Weyl tensor will not change the background, which is conformally invariant [4]. In what follows we will not use the linearized approximation and study the full theory.

The first step, as before, is to prove that all the superfields are supercovariantly constant in this background. For the superfield $\Lambda_\alpha(X,\theta,\overline{\theta})$, the lowest component is the dilatino, which automatically vanishes in this background. The next component involves the 3-form field strength, which is also automatically zero. The following component is the gravitino field strength which is also zero. However, at order θ^3 in the superfield, we have a non-trivial expression involving the curvature. We must show that this expression is zero.

The story is similar for the second superfield Z^+_{abcde}. The lowest (bosonic) component is the 5-form field strength, which, as mentioned before, is covariantly constant in our vacuum. The next component is the gravitino field strength, which vanishes. However, at order θ^2, we obtain a non-trivial expression involving the curvature. Again, we must show that this expression is zero.

It is also sufficient to prove that these two problematic expressions vanish. All higher components of these superfields are related to derivatives of the components already referred to. Hence, if we can show that these problematic expressions vanish, we will have shown that the superfield Λ_α is identically zero, and that the superfield Z^+_{abcde} is supercovariantly constant.

Actually, since both these problematic expressions are preceded in the superfield by the gravitino field strength, they are related to each other and to the variation of the gravitino field strength under supersymmetry transformations.

We will again use the existence of maximal supersymmetry in this background to help us analyze this situation. The Killing spinor equation is

$$\delta_{SUSY}\psi_r = \nabla_r\epsilon - i\,\frac{1}{192}\, g_{rabcd}\sigma^{abcd}\epsilon = 0. \tag{37}$$

As in the previous case of M2 and M5 branes near the horizon, the integrability condition for the existence of such 32 spinors for the D3 branes near the horizon was established in [15]. This transfers to the statement that for the supersymmetric $AdS_5\times S^5$ vacuum we have

$$\delta_{SUSY}\hat{\nabla}_{[r}\psi_{s]} = 0. \tag{38}$$

This is the integrability condition for the requirement that the bosonic configuration admits maximal unbroken 32-dimensional supersymmetry.

What we see is that the variation of the gravitino field strength vanishes. This also implies that the problematic expressions in the two superfields also vanish. This then implies that the superfields are supercovariantly constant.

Let us look at this from the superspace perspective. The gravitino field strength forms a T^δ_{ab} component of the torsion tensor and $T^\delta_{b\gamma}$ is a function of the form field. The superspace Bianchi identity defines the fermionic derivative of the torsion through

$$\begin{aligned} R_{ab,\gamma}{}^{\delta} &= D_\gamma T^\delta_{ab} + \{D_a T^\delta_{b\gamma} + T_{a\gamma}{}^{\epsilon}T_{b\epsilon}{}^{\delta} \\ &\quad - T_{a\gamma}{}^{\overline{\epsilon}}T_{b\overline{\epsilon}}{}^{\delta} - (a-b)\} - i\,\delta^\delta_\gamma M_{ab} \\ &= \frac{1}{4}(\sigma^{cd})^\delta_\gamma R_{ab,cd}\,. \end{aligned} \tag{39}$$

The term $D_\gamma T^\delta_{ab}$ vanishes due to the Killing spinor equation, the term $D_a T^\delta_{b\gamma}$ vanishes since our form is covariantly constant in X space. Finally M_{ab} vanishes for our background. We are left with

$$R_{ab,\gamma}{}^{\delta} = T_{a\gamma}{}^{\epsilon}T_{b\epsilon}{}^{\delta} - T_{a\gamma}{}^{\overline{\epsilon}}T_{b\overline{\epsilon}}{}^{\delta} - (a-b) = \frac{1}{4}(\sigma^{cd})^\delta_\gamma R_{ab,cd}\,. \tag{40}$$

This coincides with the integrability condition for the existence of 32 unbroken supersymmetries and proves that the superfield $Z^+_{abcde}(X,\theta,\overline{\theta})$ is covariantly constant and that all components of the superfield $\Lambda_\alpha(X,\theta,\overline{\theta})$ vanish.

To prove that the $AdS_5\times S^5$ vacuum is exact we have to study the possibilities to modify the equations of motion in this vacuum.

The equations of motion are those for the dilatino superfield and the one for the gravitino as in previous cases. The equations of motion for bosonic fields come out as some higher components of these fermionic equations. Following the same reasoning as in previous cases we may conclude that higher dimension supersymmetric operators cannot modify this vacuum defined by a covariantly constant superfield.

VI. NEW SUPERGEOMETRIES

In this section, we will present a description of the $AdS_7\times S_4$, $AdS_4\times S_7$ and $AdS_5\times S_5$ geometries in superspace. This provides an invariant description of these geometries, much as the equation $R_{rstu} = -k^2(\eta_{rt}\eta_{su} - \eta_{ru}\eta_{st})$ provides an invariant description of anti–de-Sitter geometry. We begin with the two M-theory solutions.

In the coordinate system in which the lowest component is also independent of X the superfield is given by a constant completely antisymmetric tensor, for $p=2$,

$$W^{el.vac}_{\hat{r}\hat{s}\hat{t}\hat{u}}=\epsilon_{\hat{r}\hat{s}\hat{t}\hat{u}}, \quad \hat{r},\hat{s}=0,1,2,3. \tag{41}$$

and for $p=5$ by a dual one:

$$W^{ma.vac}_{\hat{r}\hat{s}\hat{t}\hat{u}}=i\epsilon_{\hat{r}\hat{s}\hat{t}\hat{u}}. \tag{42}$$

These tensors break the structure group of the superspace $SO(1,10)$ to the product $SO(1,3)\times SO(7)$ and $SO(1,6)\times SO(4)$, respectively. Now we can give a superspace definition of the $AdS_4\times S^7$ and $AdS_7\times S^4$ vacua of M theory where all components of torsion, curvature and forms are covariantly constant. In addition to the flat superspace structures, which are independent on W, we have a few more *X,θ-independent components of supercurvature and supertorsion* (we only give the nonzero values)

$$T^r_{\alpha\beta}=-\frac{i}{2}(\gamma^0\gamma^r)_{\alpha\beta}, \quad F_{rs\alpha\beta}=-\frac{1}{2}(\gamma^0\gamma_{rs})_{\alpha\beta}, \tag{43}$$

$$T^{\gamma}_{\alpha r}=\frac{1}{2}W^{vac}_{pstu}(T^{pstu}_r)^{\gamma}_{\alpha}, \quad R^{mn}_{\alpha\beta}=(\gamma^0 S)^{mnuvzw}_{\alpha\beta}W^{vac}_{uvzw}, \tag{44}$$

$$R_{rs}{}^{\beta\gamma}=\frac{1}{4}R^{mn}_{rs}(\gamma_{mn})^{\beta\gamma}=-[T^{tuvw}_r,T^{xyzp}_s]W^{vac}_{tuvw}W^{vac}_{xyzp}, \tag{45}$$

where for the constant tensors W^{vac}_{rstu} we have to substitute their values (41) or (42) for each vacuum. The value of the spacetime curvature in Eq. (45) precisely shows that the Killing spinor integrability equation (31) is satisfied since $D_r F_{tuvw}=0$ for both vacua.

For the $AdS_5\times S_5$ background, we have

$$T^c_{\alpha\bar{\beta}}=-i(\sigma^c)_{\alpha\beta} \quad F_{a\beta\gamma}=-i(\sigma_a)_{\beta\gamma}, \tag{46}$$

$$F_{a\bar{\beta}\bar{\gamma}}=-i(\sigma_a)_{\beta\gamma} \quad G_{abc\alpha\beta}=(\sigma_{abc})_{\alpha\beta}, \tag{47}$$

$$T^{\gamma}_{a\beta}=\frac{i}{192}(\sigma^{bcde})^{\gamma}_{\beta}g_{abcde}, \tag{48}$$

$$T^{\bar{\gamma}}_{a\bar{\beta}}=\frac{i}{192}(\bar{\sigma}^{bcde})^{\gamma}_{\beta}g_{abcde}, \tag{49}$$

$$R_{\alpha\bar{\beta},ab}=-\frac{1}{24}(\sigma^{cde})_{\alpha\beta}g_{abcde}, \tag{50}$$

$$R_{ab,\gamma}{}^{\delta}=T_{a\gamma}{}^{\epsilon}T_{b\epsilon}{}^{\delta}-T_{a\gamma}{}^{\bar{\epsilon}}T_{b\bar{\epsilon}}{}^{\delta}-(a-b). \tag{51}$$

VII. DISCUSSION

We have established that the $AdS_{p+2}\times S^{d-p-2}$ vacua of M theory and string theory are uncorrected by higher-dimension supersymmetric operators. Thus we have three distinct vacua in M theory, flat superspace, that of the near horizon M2 brane and that of the near horizon M5 brane. In the string case we have two vacua, the flat superspace and that of the near horizon D3 brane.[2] The X space geometry of these configurations, $AdS_{p+2}\times S^{d-p-2}$ with forms was found in [17]. Here we found the supergeometry of these three vacua of M theory and two vacua of string theory. Since all the components of torsion and curvature in superspace for all these vacua are found to be supercovariantly constant (and actually constant in the coordinate system related to the near horizon geometry of branes) we concluded that there are no corrections modifying such vacua.

Although we have established that the form of the geometry is unchanged, we cannot *a priori* exclude a change in the values of the parameters. We believe, however, that in these cases, the Dirac quantization condition fixes the flux of the field strength through the sphere to be an integer, and thus the flux should not be affected by small deformations. This fixes the parameters of the solution in terms of the Planck length. In addition, the Planck length may itself be renormalized from its bare value, because we cannot exclude, via this analysis, the appearance in the effective action of terms proportional to the original equations of motion (which vanish on shell).

In *even dimensions for the self-dual vacua* $AdS_5\times S^5$ *and* $AdS_2\times S^2$ the transformation of the gravitino field strength can be brought to a form which depends on the Weyl tensor and derivatives of the form field. In particular it means that Eq. (40) can be rewritten using Einstein's equation and one finds that it is equivalent to the vanishing of the Weyl tensor. It is then simple to observe that it is *the conformal flatness of these vacua* and the fact that the form is constant, which force the superfields to be supercovariant. This was the argument used in [10,11] with respect to Bertotti-Robinson vacuum and for the analysis of R^4 terms in [4]. Now however we see that this is only a part of a larger picture: *in odd dimension where there are both electric as well as magnetic supersymmetric vacua which are dual to each other, the metric of $AdS_{p+2}\times S^{d-p-2}$ is not conformally flat* [18]. Still the integrability condition for the existence of the *maximal unbroken supersymmetry* as shown e.g. in the M-theory case in Eq. (28) provides the crucial vanishing of the component of the basic superfield depending on the curvature.

Given the strong argument for the exactness of both the maximally supersymmetric flat superspace $SO(1,d-1)$-symmetric vacuum and the compactified ones with $SO(1,p+1)\times SO(d-p-3)$ symmetry, it is tempting to speculate that the branes which according to [17] interpolate between these vacua may also be proven to be exact. This however may be more difficult to establish since only 1/2 of unbroken supersymmetry is available. The second half of supersymmetries which are broken generate ultrashort multiplets, and all relevant superfields are not covariantly constant but ultrashort (depend on half of θ's). Recently an absence of corrections from R^4 terms to equations for

[2]It has been anticipated in [16] that the exactness of $AdS_5\times S^5$ may be derived using 32 unbroken supersymmetries.

Reissner-Nordstrom black holes in $N=2$, $d=4$ supergravity without matter multiplets was demonstrated in [19], using the relation between the $W_{\alpha\beta}$ superfield of Poincaré supergravity and the unconstrained superfield V of conformal supergravity.

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