Perturbative expansion around the Gaussian effective potential of the fermion field theory

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We have extended the perturbative expansion method around the Gaussian effective action to fermionic field theory, by taking the two-dimensional Gross-Neveu model as an example. We have computed both the zero-temperature and the finite-temperature effective potentials of the Gross-Neveu model up to the first perturbative correction terms, and have found that the critical temperature, at which dynamically broken symmetry is restored, is significantly improved for the small value of the flavor number. [S0556-2821(98)09920-2]

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I. INTRODUCTION

One of the most important problems in quantum field theory is to develop efficient methods of the nonperturbative approximation. The variational Gaussian approximation method has provided an efficient and convenient device in obtaining nonperturbative information from various quantum field theories [1]. The variational method, however, has a serious shortcoming in that it does not provide a systematic technique to compute the correction terms to the approximation. Efforts to establish a systematic method to improve the variational approximation have been made by the authors of Refs. [2] and [3].

Recently, a systematic method of perturbative expansion around the Gaussian effective action (GEA) [4] has been developed based on the background field method [5]. This method provides an efficient device to compute the generating functionals for the one-particle-irreducible Green's functions in perturbation series, whose zeroth-order term is the GEA. For the effective potentials of time-independent systems, the result of this method is the same as that of the variational perturbation theory developed by Cea and Tedesco [3]. It has been shown, for the quantum mechanical anharmonic oscillator, that the perturbative correction greatly improves the Gaussian approximation even at the first nontrivial correction level [4].

It is the purpose of this paper to extend the perturbative expansion method around GEA to the case of fermionic field theories. In the next section a brief review on background field method is given. We then develop the perturbative expansion method around GEA for the two-dimensional Gross-Neveu model. In Sec. III, we evaluate the finite-temperature effective potential for the Gross-Neveu model [6], and show how the perturbative correction improves the critical temperature, at which dynamically broken symmetry is restored, from that of the Gaussian approximation. We conclude with some discussions in the last section.

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II. PERTURBATIVE EXPANSION AROUND THE GAUSSIAN EFFECTIVE ACTION OF THE GROSS-NEVEU MODEL

To give a brief review on the background field method [5], we start from the action

$$S[\phi]=\int d^d x \mathcal{L}[\phi(x),\partial_\mu\phi(x)], \tag{1}$$

in the d-dimensional space time, where ϕ can be a bosonic or fermionic field variable. The generating functional for Green's functions is defined by

$$\langle 0+|0-\rangle^J \equiv e^{iW}\equiv\int \mathcal{D}\varphi e^{iS[\varphi]+iJ\varphi}, \tag{2}$$

where $W[J]$ is the generating functional for the connected Green's functions, J is the external source, and the integral convention, $J\phi\equiv\int d^d x J(x)\phi(x)$, is used in the exponent. The vacuum expectation value of the field operator in the presence of external source is defined by

$$\hat{\varphi}\equiv\langle\phi(x)\rangle^J=\frac{\delta}{\delta J(x)}W[J], \tag{3}$$

and the effective action is defined by the Legendre transformation,

$$\Gamma[\hat{\varphi}]\equiv W[J]-\hat{\varphi}J. \tag{4}$$

The functional derivative of $\Gamma[\hat{\varphi}]$ with respect to $\hat{\varphi}$ gives

$$\frac{\delta}{\delta\hat{\varphi}}\Gamma[\hat{\varphi}]=-J, \tag{5}$$

which is of the same form as the classical equation of motion.

We now introduce a new action $S[\phi+B]$ obtained by shifting the field ϕ by a background field B. This new action defines a new effective action $\tilde{\Gamma}[\tilde{\varphi},B]$, where $\tilde{\varphi}$ is the vacuum expectation value of ϕ field in the presence of the background field B. One can then show that the effective action (4) can be represented as

$$\Gamma[\hat{\varphi}]=\tilde{\Gamma}[\tilde{\varphi}=0,B]|_{B=\hat{\varphi}}. \tag{6}$$

In other words, the effective action $\Gamma[\hat{\varphi}]$ can be obtained by summing all the one-$\tilde{\varphi}$-particle-irreducible diagrams with no external $\tilde{\varphi}$ lines. This greatly simplifies the perturbative computation of the effective action.

It has been shown in Ref. [4] that one can rearrange the diagrams in the effective action (6) in such a way that the propagator used in the perturbative expansion becomes that of the Gaussian approximation, and the zeroth-order term of the effective action consists of the GEA.

To extend this method to the case of fermionic field theories, we consider the two-dimensional Gross-Neveu model [6] described by the Langrangian density,

$$\mathcal{L}_0=\bar{\psi}_\alpha i\partial\!\!\!/\psi_\alpha-g\bar{\psi}_\alpha\psi_\alpha\sigma-\frac{1}{2}\sigma^2,\quad \alpha=1,2,3\ldots N, \tag{7}$$

where ψ_α is a two-dimensional Dirac field with α denoting the flavor component. The generating functional for Green's functions is then given by

$$Z[\eta,\bar{\eta},J]=\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\sigma e^{i\int d^2x\mathcal{L}_0+i\bar{\eta}\psi+i\bar{\psi}\eta+i\sigma J}. \tag{8}$$

We are interested in the effective action as a functional of the vacuum expectation value of the composite field operator, $\langle\sigma\rangle=-g\langle\bar{\psi}\psi\rangle$, and thus we shift only the composite operator; $\sigma\rightarrow\sigma+B$. Then the generating functional for the Green's functions in the presence of the background field B is given by

$$\begin{aligned}\tilde{Z}[\eta,\bar{\eta},J,B]&=\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\sigma e^{i\int d^2x\bar{\psi}i\partial\!\!\!/\psi-1/2\,(\sigma+B)^2-g\bar{\psi}\psi(\sigma+B)+i\bar{\eta}\psi+i\bar{\psi}\eta+i\sigma J}\\
&=e^{-(i/2)B^2}e^{-iB(\delta/i\delta J)}e^{ig(\delta/i\delta\eta)(\delta/i\bar{\eta})(\delta/i\delta J)}\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\sigma e^{i\int d^2x\bar{\psi}K^{-1}\psi+i\bar{\eta}\psi+i\bar{\psi}\eta+i\sigma J}\\
&=\det(K^{-1})\det(-iK_B^{(1/2)})e^{-(i/2)B^2}e^{-iB(\delta/i\delta J)}e^{-g(\delta/\delta\eta_a)(\delta/\delta\bar{\eta}_a)(\delta/\delta J)}e^{\bar{\eta}K\eta+(1/2)JK_BJ},\end{aligned} \tag{9}$$

where

$$K^{-1}=-(\partial\!\!\!/+igB),$$
$$K_B^{-1}=-i. \tag{10}$$

The first three factors of the last line of Eq. (9) represent the one-loop effective action, and the remaining factor, upon setting $\tilde{\sigma}=(1/i)(\delta/\delta J)\log\tilde{Z}[\eta,\bar{\eta},J,B]=0$, gives the higher loop contributions to the effective action. Note that the fermion propagator in Eq. (10) already contains the interaction effect through the background field B, which is the reason why the procedure of computing higher loop contributions is simplified.

In order to rearrange the generating functional so that the functional derivative terms in Eq. (9) represent the perturbative expansion around GEA, we follow the procedure of Ref. [4]. To do this, we consider the following relation:

$$\begin{aligned}&\frac{\delta}{\delta\eta_a^x}\frac{\delta}{\delta\bar{\eta}_a^x}\frac{\delta}{\delta J^x}e^{\bar{\eta}G\eta+(1/2)JG_BJ}\\
&\quad=[G_{aa}^{xx}+(G\eta)_a^x(\bar{\eta}G)_a^x](G_BJ)^xe^{\bar{\eta}G\eta+(1/2)JG_BJ},\end{aligned} \tag{11}$$

where the repeated index x implies the integration over x, and G and G_B are arbitrary two-point Green's functions for fermionic and bosonic fields, respectively. Note that this type of functional derivative appears in the expansion of the functional derivative factor in Eq. (9). Equation (11) contains a diagram where an internal line coming out of a point goes back to the same point (cactus-type diagram) which constitues GEA, i.e., the first term in the right-hand side of Eq. (11).

To extract such cactus-type diagrams out of the perturbative expansion, we define the primed functional derivative as

$$\left(\frac{\delta}{\delta\eta_a^x}\frac{\delta}{\delta\bar{\eta}_a^x}\frac{\delta}{\delta J^x}\right)'\equiv\frac{\delta}{\delta\eta_a^x}\frac{\delta}{\delta\bar{\eta}_a^x}\frac{\delta}{\delta J^x}+G_{aa}^{xx}\frac{\delta}{\delta J^x}. \tag{12}$$

Then the primed derivative acting on the generating functional becomes

$$\begin{aligned}&\left(\frac{\delta}{\delta\eta_a^x}\frac{\delta}{\delta\bar{\eta}_a^x}\frac{\delta}{\delta J^x}\right)'e^{\bar{\eta}G\eta+(1/2)JG_BJ}\\
&\quad=(\bar{\eta}G)_a^x(G\eta)_a^x(G_BJ)^xe^{\bar{\eta}G\eta+(1/2)JG_BJ},\end{aligned} \tag{13}$$

which does not contain any cactus-type diagrams. In order to express the generating functional (9) in terms of the primed derivative, we need to find Green's functions G and G_B which satisfy

$$\begin{aligned}&e^{-g(\delta/\delta\eta_a)(\delta/\delta\bar{\eta}_a)(\delta/\delta J)}e^{\bar{\eta}K\eta+(1/2)JK_BJ}\\
&\quad=Ne^{A(\delta/\delta J)}e^{-g((\delta/\delta\eta_a)(\delta/\delta\bar{\eta}_a)(\delta/\delta J))'}e^{\bar{\eta}G\eta+(1/2)JG_BJ}.\end{aligned} \tag{14}$$

By using the definition of the primed derivative (12), we easily find G, G_B, N, and A that satisfy Eq. (14):

$$N=1, \quad A=-gG_{aa}, \quad G=K, \quad G_B=K_B, \tag{15}$$

where the new Green's functions G and G_B are the same as those of Eq. (10). The reason why the Green's functions G and G_B are simple in this case is that we have introduced the composite operator σ, which turns the four-fermion interaction into the three-particle interaction. From Eqs. (9) and(14) one finally obtains the effective action:

$$e^{i\Gamma[B]}=\tilde{Z}[\eta,\bar{\eta},J,B]|_{\tilde{\sigma}=0} = \det(K^{-1})\det(-iK_B^{1/2})e^{-(i/2)B^2}I[B], \tag{16}$$

where

$$I[B]=e^{-(B+gK_{aa})(\delta/\delta J)}e^{-g((\delta/\delta\eta_a)(\delta/\delta\bar{\eta}_a)(\delta/\delta J))'} \times e^{\bar{\eta}K\eta+(1/2)JK_BJ}|_{\tilde{\sigma}=0}. \tag{17}$$

Since the functional $I[B]$ does not contain any cactus-type diagrams, the coefficient of $I[B]$ in Eq. (16) gives rise to the GEA, as in the case of scalar ϕ^4 theory [4]. Since $I[B]$ can be expanded as a power series in the coupling constant g, we have the perturbative expansion of the effective action around GEA.

The linear term in the exponent of Eq. (17) generates tadpole diagrams, which do not contribute to the effective action [4]. Note that Eq. (17) has the same structure as the higher-order contribution part of Eq. (9) except that Eq. (17) involves only the primed derivative. We can therefore compute the perturbative correction terms to GEA using the same procedure as the conventional background field method, by using the Feynman rule (in momentum space),

$$\text{propagators:}\quad K=\frac{i}{\gamma\cdot p+m}, \quad m=gB, \quad K_B=i,$$

$$\text{vertex:}\quad -g,$$

$$\text{loop integral:}\quad \int\frac{d^2p}{(2\pi)^2}. \tag{18}$$

Thus the perturbative correction, $I[B]$ to GEA, consists of one-$\tilde{\sigma}$-particle-irreducible bubble diagrams with no external lines and without cactus-type diagrams, as in the case of ϕ^4 theory [4].

Up to the first nontrivial contribution from $I[B]$, the effective action for the Gross-Neveu model can easily be shown to be

$$\Gamma=-iTr\,\ln(K^{-1})-\frac{1}{2}B^2+ig^2(K_B)_{xy}(K_{ca})_{xy}(K_{ac})_{xy}, \tag{19}$$

where the first two terms are the GEA and the last term is the first-order perturbative correction to GEA.

The effective potential is defined by

$$V_{eff}[B]\equiv-\frac{\Gamma[B]}{\int d^2x}, \tag{20}$$

where B is the space-time–independent background field. Thus the effective potential of the Gross-Neveu model, up to the first-order perturbative correction term, is given by

$$V_{eff}(m)=V_G(m)+V_P(m), \tag{21}$$

where V_G is the Gaussian effective potential,

$$V_G=\frac{1}{2}B^2-N\int\frac{dp}{2\pi}\sqrt{p^2+m^2}=\frac{m^2}{2g^2}+\frac{Nm^2}{4\pi^2}\left(\ln\frac{m^2}{\Lambda^2}-1\right), \tag{22}$$

and V_P is the perturbative correction,

$$V_P=\frac{N}{2}g^2m^2\left[\int\frac{dp}{2\pi}\frac{1}{\sqrt{p^2+m^2}}\right]^2=\frac{Ng^2}{8\pi^2}m^2\left(\ln\frac{m^2}{\Lambda^2}\right)^2, \tag{23}$$

with Λ being the ultraviolet momentum cutoff.

To extract physical information from the effective potentials V_G or V_{eff}, we need to renormalize them. We can renormalize the effective potentials by requiring the renormalization conditions,

$$\left.\frac{d^2V}{dm^2}\right|_{m=m_0}=\frac{1}{g_R^2}, \tag{24}$$

$$g\sigma=g_R\sigma_R, \tag{25}$$

where m_0 represents a renormalization point.

For the Gaussian effective potential, V_G, the renormalization condition becomes

$$\left.\frac{d^2V_G}{dm^2}\right|_{m=m_0}=\frac{1}{g_R^2}=\frac{1}{g^2}+\frac{N}{2\pi}\left(\ln\frac{m_0^2}{\Lambda^2}+2\right). \tag{26}$$

Substituting Eq. (26) into Eq. (22), one obtains the renormalized Gaussian effective potential,

$$V_G=\frac{1}{2}\frac{m^2}{g_R^2}+\frac{Nm^2}{4\pi}\left(\ln\frac{m^2}{m_0^2}-3\right), \tag{27}$$

which is equivalent to that of the large-N approximation [6].

For the perturbatively corrected Gaussian effective potential, V_{eff} of Eq. (21), the renormalization condition (24) becomes

$$\left.\frac{d^2V}{dm^2}\right|_{m=m_0}=\frac{1}{g_R^2}=\frac{1}{g^2}+\frac{N}{2\pi}\left(\ln\frac{m_0^2}{\Lambda^2}+2\right)+\frac{Ng^2}{4\pi^2}\left[\ln\frac{m_0^2}{\Lambda^2}{}_2+6\ln\frac{m_0^2}{\Lambda^2}+4\right]. \tag{28}$$

Requiring that the renormalized coupling constant g_R be finite, we find the condition

$$g^2=\pi\left(-1\pm\sqrt{1-\frac{4}{N}}\right)\Big/\ln\frac{m_0^2}{\Lambda^2} \tag{29}$$

for which the effective potential can be made finite. The case of the negative sign in Eq. (29) leads to unphysical theory. Taking the case of the positive sign in Eq. (29), we finally obtain the renormalized effective potential,

$$V=\frac{1}{2}\frac{m^2}{g_R^2}+N\sqrt{1-\frac{4}{N}}\frac{m^2}{4\pi}\left(\ln\frac{m^2}{m_0^2}-3\right), \tag{30}$$

which clearly shows that the higher-order corrections to the Gaussian or the large-N approximation. This shows that our method is meaningful when N is larger than 4 for the Gross-Neveu model.

III. EFFECTIVE POTENTIAL AT FINITE TEMPERATURE

To illustrate how our method improves the Gaussian approximation, we compute the first nontrivial perturbative correction term to the Gaussian effective potential at finite temperature and evaluate the critical temperature, by using the imaginary time formulation of the finite-temperature quantum field theory. The generating functional for the thermal Green's functions is given by

$$Z_\beta[J]=N\int_{periodic}\mathcal{D}\varphi e^{\int_0^\beta dt\int dx\mathcal{L}(\varphi,\dot{\varphi})+\varphi J}, \tag{31}$$

where τ is the imaginary time with period β, the inverse temperature, $\dot{\varphi}=\partial\varphi/\partial\tau$, and the functional integration is performed only for the periodic field φ. Since the finite-temperature generating functional (31) is of the same form as the zero-temperature generating functional (2) except that it is defined in Euclidean space with periodic boundary conditions, we can evaluate the perturbative expansion by following the same procedure as in the last section, except that the Feynman rule is now modified by

$$\text{propagator:}\quad \frac{i}{\gamma\cdot p-m},\quad p_\mu=(p_0=i\omega_n,p_1),\quad \omega_n=(2n+1)\pi\beta^{-1},$$

$$\text{loop integration:}\quad \frac{i}{\beta}\sum_{i=-\infty}^{\infty}\int\frac{dp}{2\pi}. \tag{32}$$

One can then find the finite-temperature effective potential, up to the first perturbative correction,

$$V_{eff}^\beta(\sigma)=\frac{1}{2}\sigma^2-2N\int\frac{dp}{2\pi}\left[\frac{\omega}{2}+\frac{1}{\beta}\ln(1+e^{-\beta\omega})\right]+\frac{Ng^4\sigma^2}{2}\left[\int\frac{dp}{2\pi}\frac{1}{\omega}\left(1-\frac{2}{e^{\beta\omega}+1}\right)\right]^2, \tag{33}$$

where $\omega=\sqrt{p^2+g^2\sigma^2}$.

To renormalize the effective potential (33), it is convenient to separate the zero-temperature and the finite-temperature parts of the effective potential:

$$V_{eff}^\beta=V_0+V_\beta, \tag{34}$$

where

$$V_0=\frac{1}{2}\sigma^2-2N\int\frac{dp}{2\pi}\frac{\omega}{2}+\frac{g^2m^2N}{2}\left[\int\frac{dp}{2\pi}\frac{1}{\omega}\right]^2, \tag{35}$$

$$V_\beta=-2N\frac{1}{\beta}\int\frac{dp}{2\pi}\ln(1+e^{-\beta\omega})-g^2m^2N\left[\int\frac{dp}{2\pi}\frac{1}{\omega}\right]F(\beta,m)+\frac{g^2m^2N}{2}[F(\beta,m)]^2, \tag{36}$$

with $m=g\sigma$ and $F(\beta,m)$ are defined by

$$F(\beta,m)\equiv\int\frac{dp}{2\pi}\frac{1}{\omega}\frac{2}{e^{\beta\omega}+1}. \tag{37}$$

We note that the zero-temperature part of the effective potential (35) is the same as Eq. (34) of the last section, and we need only to renormalize this part of the effective potential. Thus we have the renormalized effective potential at finite temperature,

$$V_{eff}^\beta=\frac{1}{2}\frac{m^2}{g_R^2}+N\sqrt{1-\frac{4}{N}}\frac{m^2}{4\pi}\left(\ln\frac{m^2}{m_0^2}-3\right)+V_\beta, \tag{38}$$

where $m=g_R\sigma_R$. Since $F(\beta,m)$ is finite, the renormalization condition (29) reduces the finite-temperature part of the effective potential, V_β, to

$$V_\beta=-2N\frac{1}{\beta}\int\frac{dp}{2\pi}\ln(1+e^{-\beta\omega})-\frac{N}{2\pi}\left[1-\sqrt{1-\frac{4}{N}}\right]m^2F(\beta,m). \tag{39}$$

The symmetry that is broken at zero temperature is restored as temperature increases beyond the critical temperature. It is well known that the critical temperature should be zero in two-dimensional space time [7], while the large-N [8] and the Gaussian approximations of the Gross-Neveu model imply the nonvanishing critical temperature. To see how the perturbatively improved Gaussian approximation improves this result, we now evaluate the critical temperature from the effective potential (39).

Dynamical symmetry breaking in the Gross-Neveu model is manifested by the fact that the minimum of the zero-temperature effective potential occurs at the nonvanishing value of the composite field, $\sigma_R=\sigma_m$, which breaks the symmetry of the classical potential, i.e., the symmetry under $\sigma\to-\sigma$. σ_m is determined by

$$\left.\frac{dV_0}{d\sigma}\right|_{\sigma=\sigma_m}=1+N\sqrt{1-\frac{4}{N}}\frac{g^2}{2\pi}\left(\ln\frac{\sigma_m^2}{\sigma_0^2}-2\right)=0. \tag{40}$$

As the temperature increases, the value of $\sigma_m(\beta)$ at which V_{eff}^β is minimized decreases. At the critical temperature, $T_c=1/\beta_c$, $\sigma_m(\beta)$ vanishes, which implies symmetry restoration. Thus the critical temperature is determined by

$$\left.\frac{dV}{d\sigma}\right|_{\sigma_R=\sigma_m(\beta_c)=0}=0, \tag{41}$$

which, together with Eqs. (38) and (39), gives the value of the critical temperature,

$$T_c=0.57g_R\sigma_m. \tag{42}$$

Due to Eq. (40), this can be written as

$$T_c=0.57g_R\sigma_0\exp\left[1-\frac{\pi}{g_R^2}\frac{1}{N\sqrt{1-\frac{4}{N}}}\right], \tag{43}$$

where σ_0 represents the renormalization point. The critical temperature in the Gaussian approximation can similarly be obtained:

$$T_c^{gaussian}=0.57g_R\sigma_m^{gaussian}=0.57g_R\sigma_0\exp\left[1-\frac{\pi}{g_R^2}\frac{1}{N}\right]. \tag{44}$$

Although the critical temperature T_c from the perturbatively corrected Gaussian approxmation does not vanish, it is smaller than that of the Gaussian approximation. This shows that the perturbatively improved Gaussian approximation significantly improves the critical temperature for small N, the flavor number of the fermion field even at the first-order correction level.

IV. DISCUSSION

We have extended the method of the perturbative expansion around GEA developed in Ref. [4] to the fermionic field theory, taking the Gross-Neveu model as an example. This method is based on the observation that the Gaussian effective action consists of cactus-type diagrams, which are extracted out of the functional derivative part of the effective action, i.e., the last two factors of the last line of Eq. (9), by introducing the primed functional derivative defined in Eq. (12). This procedure effectively rearranges the diagrams in such a way that the zeroth-order term of the effective action consists of the GEA. Due to the introduction of the composite field, $\sigma=-g\bar{\psi}\psi$, as an order parameter, the expansion works only for the flavor number N larger than 4.

In the last section the finite-temperature effective potential of the Gross-Neveu model is obtained up to the first nontrivial perturbative correction terms, and the critical temperature, at which the dynamically broken symmetry is restored, is evaluated. The result shows a significant improvement of the critical temperature compared to the Gaussian and the large-N results for small values of the flavor number.

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