Topology change and causal continuity

Fay Dowker*
Blackett Laboratory, Imperial College of Science, Technology and Medicine, London SW7 2BZ, United Kingdom

Sumati Surya†
IUCAA, Post Bag 4, Ganeshkhind, Pune 411 007, India

(Received 23 January 1998; published 18 November 1998)

The result that, for a scalar quantum field propagating on a ''trousers'' topology in $1+1$ dimensions, the crotch singularity is a source for an infinite burst of energy has been used to argue against the occurrence of topology change in quantum gravity. We draw attention to a conjecture due to Sorkin that it may be the particular type of topology change involved in the trousers transition that is problematic and that other topology changes may not cause the same difficulties. The conjecture links the singular behavior to the existence of ''causal discontinuities'' in the spacetime and relies on a classification of topology changes using Morse theory. We investigate various topology changing transitions, including the pair production of black holes and of topological geons, in the light of these ideas. [S0556-2821(98)07020-9]

PACS number(s): 04.60.Gw, 02.40.Vh, 04.20.Gz

I. INTRODUCTION

It is widely believed that any complete theory of quantum gravity must incorporate topology change. Indeed, within the particle picture of quantum gravity [1] the frozen topology framework for a generic spatial three-manifold leads to the problem of spin-statistics violations and such wild varieties of quantum sectors that it seems that a frozen topology is unmaintainable [2]. There is one result, however, that has been cited as counterevidence for topology change: that of the singular propagation of a quantum field on a trousers spacetime in $1+1$ dimensions [3,4]. We will see how it may be possible to incorporate this result naturally in a framework which nevertheless allows topology change in general.

The most natural way of accommodating topology changing processes in quantum gravity is using the sum-over-histories (SOH) approach, although there has also been some effort in this direction within the Hamiltonian picture [5]. We take a history in quantum gravity to be a pair $(\mathcal{M},g)$, where $\mathcal{M}$ is a smooth n-dimensional manifold and g is a time-oriented Lorentzian metric on $\mathcal{M}$. [Strictly, a history is a geometry and only represented by $(\mathcal{M},g)$.] The amplitude for the transition from an initial space (V_0,q_0) to a final space (V_1,q_1), where the V_i are closed $(n-1)$ manifolds and the q_i are Riemannian $(n-1)$ metrics, receives contributions from all compact interpolating histories $(\mathcal{M},g)$, satisfying the boundary conditions $\partial\mathcal{M}=V_i \amalg V_f$, $g|V_{i,f}=q_{i,f}$ where $\amalg$ denotes a disjoint union and V_0 and V_1 are initial and final spacelike boundaries of $(\mathcal{M},g)$. We call the manifold $\mathcal{M}$ such that $\partial\mathcal{M}=V_i \amalg V_f$, a *topological cobordism*, and $(\mathcal{M},g)$, a *Lorentzian cobordism*. We will say that a topological cobordism or a history is *topology changing* if $\mathcal{M}$ is not a product $V_0\times I$, where I is the unit interval. We will use the terminology *topology changing transition* to refer to the transition from V_0 to V_1 when V_0 and V_1 are not diffeomorphic, without reference to any particular cobordism.

When V_0 and V_1 are not diffeomorphic, the existence of a topological cobordism $\mathcal{M}$ is equivalent to the equality of the Stiefel-Whitney numbers of V_0 and V_1 and is not guaranteed in arbitrary dimensions. If a topological cobordism does not exist we would certainly conclude that the transition is forbidden. In $3+1$ and lower dimensions, however, a topological cobordism always exists. Then, given a topological cobordism $\mathcal{M}$ a Lorentzian cobordism based on $\mathcal{M}$ will exist iff [6,7] (1) n is even and $\chi(\mathcal{M})=0$ or (2) n is odd and $\chi(V_0)=\chi(V_1)$. In $3+1$ dimensions, a topological cobordism with $\chi(\mathcal{M})=0$ always exists and thus any three-dimensional V_0 and V_1 are Lorentz cobordant.

The theorem of Geroch [8], extended to n-spacetime dimensions, tells us that if a time oriented Lorentzian metric exists on a topology changing topological cobordism $\mathcal{M}$ then that metric must contain closed timelike curves. We consider these to be a worse pathology than the alternative which is to allow certain singularities in the metric, i.e., to weaken the restriction that the metric be Lorentzian everywhere, and which, following the proposal of Sorkin [9], is what we will choose to do in this paper. The singularities which we need to admit in order to be able to consider all possible topological cobordisms are rather mild. Given any topological cobordism $(\mathcal{M};V_0,V_1)$, there exists an almost everywhere Lorentzian metric g on $\mathcal{M}$ which has singularities which take the form of degeneracies where the metric vanishes at (finitely many) isolated points. These degeneracies each take one of $(n+1)$ standard forms described by Morse theory as we shall relate. Allowing such singular metrics seems natural in light of the fact that within the path integral formulation, paths are not always required to be smooth; in fact they are known to be distributional. Moreover, such degeneracies are allowed within a vielbien formulation of gravity. For a discussion of these points, see [10].

So, by allowing such mildly singular Lorentz cobordisms in the SOH no topological cobordism is excluded and, in particular, every transition in $3+1$ dimensions is viable at

*Email address: dowker@@ic.ac.uk
†Email address: ssurya@@iucaa.ernet.in

this level of the kinematics. We will refer to these cobordisms as ''Morse cobordisms.'' However it seems that dynamically some Morse cobordisms may be more equal than others. The (1+1)-dimensional case gives us an idea about a possible class of ''physically desirable'' histories. For a massless scalar quantum field on a fixed (flat) metric on the $1+1$ trousers topology there is an infinite burst of energy from the crotch singularity that propagates along the future light cone of the singularity [3,4]. This tends to suggest that such a history would be suppressed in a full SOH for $1+1$ quantum gravity. By contrast, the singular behavior of a quantum field on the background of a flat metric on the $1+1$ ''yarmulke'' cobordism (i.e., a hemisphere representing creation/destruction of an S^1 from/to nothing) is of a significantly different nature, in the sense that when integrated over the future null directions the stress-energy is finite [11]. The singularity in the yarmulke case is therefore effectively ''squelched,'' while it propagates in the trousers. Indeed, in studying $1+1$ models of topology change, the authors of [10] have found that there is a suppression of the trousers cobordism in the SOH and an enhancement by an equal factor of the yarmulke cobordism (over the trivial cylinder) and separate from the suppression due to the backgrounds not being classical solutions.

What features of the trousers and yarmulke might account for the different behaviors of quantum fields in these backgrounds? A closer look shows that in the Morse cobordism on the trousers manifold an observer encounters a discontinuity in the volume of her causal past as she traverses from the leg region into the body. Since such a discontinuity is absent in the yarmulke topology and cylinder topologies, Sorkin has conjectured that there may be an intimate connection between the discontinuity in the volume of the causal past/future of an observer (a *causal discontinuity*) and the physically undesirable infinite burst of energy for a scalar field propagating in such a background. And then further, that this could signal a suppression of the amplitude for a causally discontinuous spacetime in the full SOH in quantum gravity.

The plan for this paper is the following. In the next section we include a review of Morse theory and surgery theory, thus setting the stage for our work. We find that whenever a component of the universe is created from nothing, its initial spatial topology must be that of a sphere. In Sec. III we state a conjecture of Borde and Sorkin that relates causal discontinuities to the Morse ''type'' of a cobordism. In order to lend substance to this conjecture, we work out the example of the trousers topology in $1+1$ dimensions which also generalizes to higher dimensions. In Sec. IV we present an argument by Sorkin [12], that *any* topology changing transition in $3+1$ dimensions can be achieved by some causally continuous Morse cobordism, once the Borde-Sorkin conjecture is assumed to hold. We then examine certain specific examples of topology changing topological cobordisms in the following two sections. The first is the $3+1$ black hole pair production instanton studied in [13,14]. We show by direct construction that a causally continuous Morse metric exists on the background manifold of the instanton which is further evidence that that particular topology change is one with a finite amplitude. This result generalizes simply to higher dimensions even though the exact instantons are not known. The second class of cobordisms we analyze is a set of manifolds that describe in a natural way the pair production of topological geons in the particle picture of prime manifolds [1]. We will show that, unfortunately, these manifolds do not support causally continuous Morse metrics. We summarize these results in the last section and discuss their implications.

II. MORSE THEORY AND SURGERY

Suppose $\mathcal{M}$ is an *n*-dimensional, compact, smooth, connected manifold such that $\partial\mathcal{M}$ has two disjoint $(n-1)$-dimensional components V_0 and V_1 that are closed and correspond to the initial and final boundaries of the spacetime, respectively.

Any such $\mathcal{M}$ admits a *Morse function f*: $\mathcal{M}\to[0,1]$, with $f|_{V_i}=0$, $f|_{V_f}=1$ such that *f* possesses a set of critical points $\{p_k\}$ $(\partial_a f(p_k)=0)$ which are nondegenerate (i.e., the Hessian $\partial_a\partial_b f$ at these points is invertible). It follows that the critical points of *f* are isolated and that because $\mathcal{M}$ is compact, there are only a finite number of them.

Using this Morse function and any Riemannian metric h_{ab} on $\mathcal{M}$, we may then construct an almost everywhere Lorentzian metric on $\mathcal{M}$ with a finite number of isolated degeneracies,

$$g_{ab}=h_{ab}(h^{cd}\partial_c f\partial_d f\ )-\zeta\partial_a f\partial_b f, \tag{1}$$

where the constant $\zeta>1$ [10]. Clearly, g_{ab} is degenerate (zero) precisely at the critical points of *f*. We refer to these points as ''Morse singularities.'' Expressing a metric on $\mathcal{M}$ in terms of its Morse functions *f* relates the latter to the causal structure of the spacetime in an intimate manner, as we will see.

We now make the proposal that in the SOH, for the amplitude for a topology changing process, for each topological cobordism, only metrics that can be expressed in the form (1) (i.e., which can be constructed from some Morse function and some Riemannian metric) will be included. We call such metrics ''Morse metrics.'' Note that since a Riemannian metric and Morse function always exist on a given topological cobordism, no cobordism is ruled out of the SOH at this kinematical level.

A comment is in order here to relate this proposal to previous works on Lorentzian topology change and Morse theory. In work by Yodzis [15] the attitude was taken that the Morse singularities should not be considered as part of spacetime, in other words, the Morse points themselves were to be removed by sending them to infinity. In contrast, here we are suggesting that the Morse points should remain as part of the spacetime. Amongst other things, this entails extending the usual gravitational action to Morse metrics. This is discussed in detail for $1+1$ dimensions in [10]. Keeping the Morse points still allows a well-defined causal structure even at the Morse points and hence a well-defined causal ordering of all the spacetime points. This is something which ties in well with the idea that the fundamental underlying structure is a causal set.

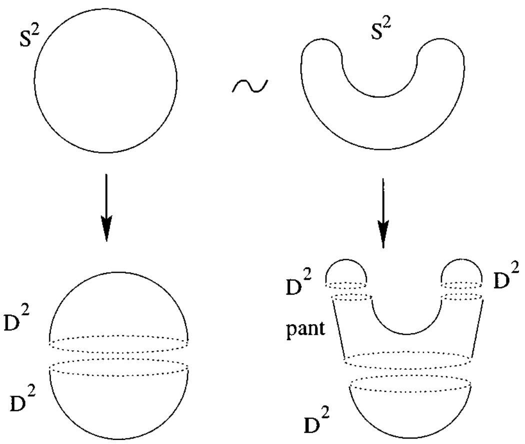

FIG. 1. Two ways of decomposing S^2 into elementary cobordisms.

Before proceeding any further, we briefly review some relevant properties of Morse functions that we will employ later. We have utilized Ref. [16–20], extensively for this purpose.

Lemma 1 (Morse Lemma), If $p \in \mathcal{M}$ *is a critical point of a Morse function* $f:\mathcal{M}\rightarrow[0,1]$, *then there exists local coordinates* $x_1, x_2 \cdots x_n$ *in some neighborhood of p in terms of which f is given, in that neighborhood, by* $f(x_1,\ldots,x_n)=c-x_1^2-x_2^2\cdots-x_\lambda^2+x_{\lambda+1}^2\cdots+x_n^2$ *for* $0\leqslant\lambda\leqslant n$ *and* $c=const.$.

The number of negative signs λ in the above expression is the number of maxima of *f* at the point *p* and is referred to as the *Morse index* of *f* at *p*. For example, the height function on the $1+1$ yarmulke topology has index 0 at the bottom point, while that on its time reversed counterpart has index 2. The height function on the trousers topology on the other hand has a Morse point of index 1 at the crotch as does its time reverse.

The *Morse number* of $\mathcal{M}$ on the other hand is defined to be the minimum over all Morse functions $f:\mathcal{M}\rightarrow[0,1]$ of the number of critical points of *f*. Thus, for example, although the cylinder topology in $1+1$ dimensions *allows* Morse functions with any even number of critical points, its Morse number is nevertheless zero. We then refer to a topological cobordism of Morse number 0 as a *trivial* cobordism and that with Morse number 1 as an *elementary* cobordism.

Lemma 2. Any cobordism can be expressed as a composition of elementary cobordisms [16].

This decomposition is, however, not unique, as can be seen in the case of a two-dimensional closed universe S^2, shown in Fig. 1. Here we see that S^2 could be decomposed into (a) two elementary cobordisms, yarmulke and its time reverse, or (b) into four elementary cobordisms, namely, the yarmulke and an upside down trousers topology with two time reversed yarmulkes, one capping each leg. Clearly, the causal structure of the two resulting histories is very different.

Before introducing surgery we define D^k to be an open *k* ball and B^k to be the closed *k* ball (and $B^1=I$).

A *surgery* of type λ on an $(n-1)$-dimensional manifold *V* is defined to be the following operation: Remove a thickened embedded $(\lambda-1)$ sphere $S^{\lambda-1}\times D^{n-\lambda}$ from *V* and replace it with a thickened $(n-\lambda-1)$ sphere $S^{n-\lambda-1}\times B^{\lambda}$ by identifying the boundaries using a diffeomorphism $d:S^{\lambda-1}\times S^{n-\lambda-1}\rightarrow S^{n-\lambda-1}\times S^{\lambda-1}$.

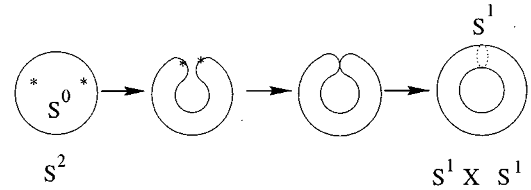

FIG. 2. ''Tracing out'' a type 1 surgery on S^2, whereby an S^0 is destroyed and an S^1 is created to give the torus $S^1\times S^1$.

In performing a surgery, effectively, a $(\lambda-1)$ sphere is ''destroyed'' and an $(n-\lambda-1)$ sphere is ''created'' in this process. We then have the following theorem which only depends on surgery type.

Theorem 1. If an $(n-1)$*-dimensional manifold* V_1 *can be obtained from another* $(n-1)$*-dimensional manifold* V_0 *by a surgery of type* λ, *then* $\exists$ *an elementary cobordism* $\mathcal{M}$, *called the trace of a surgery, with boundary* V_0IIV_1 *and a Morse function f on* $\mathcal{M}$, $f:M\rightarrow[0,1]$ *which has exactly one Morse point of index* λ [16].

As an example, consider $V_0=S^2$ and $V_1=S^1\times S^1$ or a wormhole. Performing a type 1 surgery on S^2 can result in the manifold $S^1\times S^1$, where an S^0 is ''destroyed'' and an S^1 is ''created.'' Theorem 1 then says that $\exists$ an elementary cobordism $\mathcal{M}$ with boundary S^2II$S^1\times S^1$ and a Morse function *f* on $\mathcal{M}$ with a single critical point of index $\lambda=1$. The manifold $\mathcal{M}$ may be visualized as shown in Fig. 2. We now explain how to construct the trace of a general surgery.

A λ surgery that turns V_0 into V_1 gives us an embedding $i:S^{\lambda-1}\rightarrow V_0$ and a neighborhood *N* of that embedded sphere whose closure $\bar{N}$ is diffeomorphic to $S^{\lambda-1}\times B^{n-\lambda}$. Indeed, we have a diffeomorphism $d:\partial(\bar{N})\rightarrow S^{\lambda-1}\times S^{n-\lambda-1}$, the ''surgery diffeomorphism.'' Now $S^{\lambda-1}\times S^{n-\lambda-1}$ is the boundary of $\underline{S}^{\lambda-1}\times B^{n-\lambda}$ and we can extend *d* to a diffeomorphism $\tilde{d}:\bar{N}\rightarrow S^{\lambda-1}\times B^{n-\lambda}$ such that $\tilde{d}$ restricts *d* on the boundary. $\tilde{d}$ is unique up to isotopy since $B^{n-\lambda}$ is topologically trivial.

We construct the trace of the surgery by gluing together the two manifolds $M_1=V_0\times I$ and $M_2=B^{\lambda}\times B^{n-\lambda}$ using a diffeomorphism from part of the boundary of one to part of the boundary of the other in the following way. $(\bar{N},1)$ is part of ∂M_1 and is diffeomorphic via $\tilde{d}$ to $S^{\lambda-1}\times B^{n-\lambda}$ which is part of the boundary of M_2. We identify all points $x\in(\bar{N},1)$ and $\tilde{d}(x)$. The resultant manifold clearly has one disjoint boundary component which is V_0. That the other boundary is diffeomorphic to V_1, i.e., the result of the surgery on V_0, takes a little more thought to see. Roughly speaking, in doing the gluing by $\tilde{d}$ we are eliminating *N* from V_0 and replacing it with the rest of the boundary of M_2 [the complement of Im($\tilde{d}$) in ∂M_2], i.e., $B^{\lambda}\times S^{n-\lambda-1}$ exactly as in the original surgery.

Figure 3 is an example of the trace of a type 1 surgery on ™S^1IIS^1, which is just the $1+1$ trousers topology. Here, $\bar{N}$ is the disjoint union of two line segments $\vec{AB}$ and $\vec{CD}$.

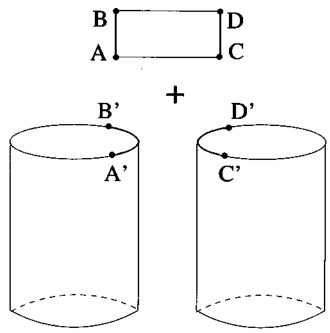

FIG. 3. Construction of the trace of a type 1 surgery on $S^1 \amalg S^1$. The line segments $\vec{AB}$ and $\vec{CD}$ are identified with $\vec{A'B'}$ and $\vec{C'D'}$, respectively.

Thus the trace of a surgery is a manifold with boundary with the property that one part of the boundary is the original manifold and the other part of the boundary is the surgically altered manifold (up to diffeomorphisms).

A. Examples

The n-dimensional yarmulke cobordism and its time reverse hold a special place in our analysis since they are easy to characterize. If $f:\mathcal{M}\to[0,1]$ has a single Morse point of index 0 then $\mathcal{M}$ is the trace of the surgery of type 0 in which an $S^{-1}\equiv\Phi$ is destroyed and an S^{n-1} is created. If $\mathcal{M}$ is connected this implies that $\mathcal{M}\cong B^n$. In other words, a cobordism can have a single index 0 point if and only if it is the yarmulke. This means that when a component of the universe is created from nothing (as opposed to being created by branching off from an already existing universe) its initial topology must be that of a sphere, no matter what the dimension: the big bang always results in an initially spherical universe. This might be thought of as a ''prediction'' of this way of treating topology change. A similar argument for the time reversed case implies that a connected cobordism can have a single Morse point of index n iff it is the time reversed yarmulke and the universe must be topologically spherical before it can finally disappear in a big crunch.

The trousers and its higher dimensional analogues are also important examples. There exists a Morse function on the $1+1$ trousers topology which possesses a single Morse point of index 1 and the trousers is therefore the trace of a surgery of type 1 in which an embedded $S^0\times D^1$ is deleted from the initial $S^1 \amalg S^1$ and replaced with a $B^1\times S^0$ to form a single S^1. In $(n-1)+1$ dimensions, the higher dimensional trousers (the manifold S^n with three open balls removed) for the process $S^{n-1}\amalg S^{n-1}\to S^{n-1}$ has an index 1 point and is the trace of a type 1 surgery in which an $S^0\times D^{n-2}$, i.e., two balls, are removed and an $S^{n-2}\times B^1$, or wormhole, added. In these processes, parts of the universe which were spatially far apart suddenly become close (in these cases the parts of the universe are originally in disconnected components of the universe, but this is not the defining characteristic of index 1 points). An index $n-1$ point is the time reverse of this and corresponds to a type $n-1$ surgery in which a wormhole is removed (or cut) and the ends ''capped off'' with two balls, so that neighboring parts of the universe suddenly become distant. It seems intuitively clear from these examples that there is something causally peculiar about the index 1 and $n-1$ points and in the next section we give a precise statement of a conjecture that encapsulates this.

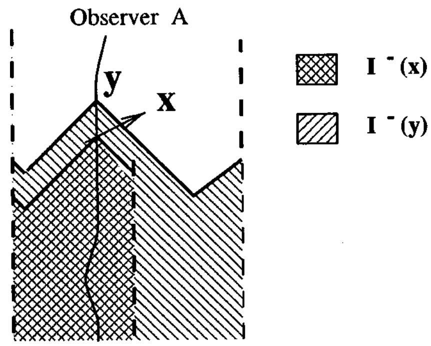

FIG. 4. Discontinuity in the causal past I^- of observer A in the trousers topology (the dashed lines are appropriately identified).

III. CAUSAL DISCONTINUITY

Borde and Sorkin have conjectured that $(\mathcal{M},g_{ab})$ contains a *causal discontinuity* if and only if the Morse function f contains an index 1 or an index $n-1$ Morse point [21]. What do we mean by causal discontinuity? There are many equivalent conditions for a Lorentzian spacetime to be causally discontinuous [22] and we define a Morse metric to be causally discontinuous iff the spacetime minus the Morse points (which is Lorentzian) is causally discontinuous. Roughly speaking, a causal discontinuity results in the causal past or future of a point in spacetime jumping discontinuously as the point is continuously moved around. We see that behavior in the $1+1$ trousers—see Fig. 4. Clearly the same kind of thing will happen in the higher dimensional trousers, but not in the yarmulkes. Furthermore in the cases of index $\lambda\neq 1$, $n-1$, the spheres that are created and destroyed are all connected and so it seems that neighboring parts of the universe remain close and distant ones remain far apart.

To lend further plausibility to the conjecture we will work out an example, the index 1 point in $1+1$ dimensions, in detail. Choose a neighborhood of the Morse point p in which the Morse function has the standard form

$$f(x,y)=f(p)-x^2+y^2 \tag{2}$$

in terms of some local coordinates (x,y). We take the flat Riemannian metric

$$ds_R^2=h_{\mu\nu}dx^\mu dx^\nu=dx^2+dy^2. \tag{3}$$

We define the Morse metric $g_{\mu\nu}$ as in Eq. (1) with $\zeta=2$ and $\partial_\mu f=(-2x,2y)$ to obtain

$$ds_L^2=-4(xdx-ydy)^2+4(xdy+ydx)^2. \tag{4}$$

This metric is actually flat since $2(xdx-ydy)=d(x^2-y^2)$ and $2(xdy+ydx)=2d(xy)$. In Fig. 5 we see that the hyper-

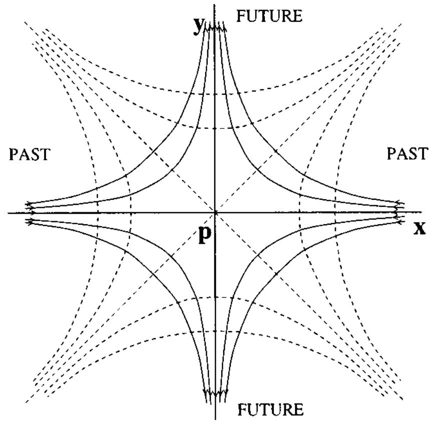

FIG. 5. The behavior of the Morse function f around index 1 point p in $1+1$ dimensions. The solid lines are integral curves of $\xi^{\mu}=h^{\mu\nu}\partial_{\nu}f$ with arrows in the direction of increasing f and the dotted lines are surfaces of constant f.

bolas $xy=c$, c constant, are the integral curves of the vector field $\xi^{\mu}=h^{\mu\nu}\partial_{\nu}f$ and the spatial ''surfaces'' of constant f are the hyperbolas $x^2-y^2=d$, d constant.

What are the null curves in the neighborhood of p? We have $ds_L^2=0$ which implies

$$d(x^2-y^2)=\pm 2d(xy), \tag{5}$$

$$x^2-y^2=\pm 2xy+b. \tag{6}$$

The null curves that pass through p are given by $b=0$ so that there are four solutions: $y=(\pm 1\pm\sqrt{2})x$. These are the straight lines through p at angles $\pi/8$, $3\pi/8$, $5\pi/8$, $7\pi/8$, to the x axis. These are the past and future light ''cones'' of p. The null curves which do not pass through p are given by the hyperbolas $x'y'=c'$ and $x'^2-y'^2=d'$, where (x',y') are rotated coordinates

$$x'=\cos\frac{\pi}{8}\,x+\sin\frac{\pi}{8}\,y, \tag{7}$$

$$y'=-\sin\frac{\pi}{8}\,x+\cos\frac{\pi}{8}\,y. \tag{8}$$

Figure 6 shows a selection of null curves. In particular we see the past and future light cones of point s on the negative x axis and of a point q on the future light cone of p. Using the results of [22] we can see that the spacetime around p is not causally continuous. Indeed consider the point q in Fig. 6. Then $\downarrow I^{+}(q)\neq I^{-}(q)$, where $I^{+}(q)(I^{-}(q))$ is the chronological future (past) of q and $\downarrow(S)$, S, an open set, is the interior of the set of all points x for which there exists a forward directed timelike curve from x to every point in S. The point s is an element of $\downarrow I^{+}(q)$ but not $I^{-}(q)$.

The higher dimensional case can be similarly analyzed. Now we have

$$f(\vec{x},\vec{y})=f(p)-x_1^2-\cdots-x_{\lambda}^2+y_1^2+\cdots+y_{n-\lambda}^2. \tag{9}$$

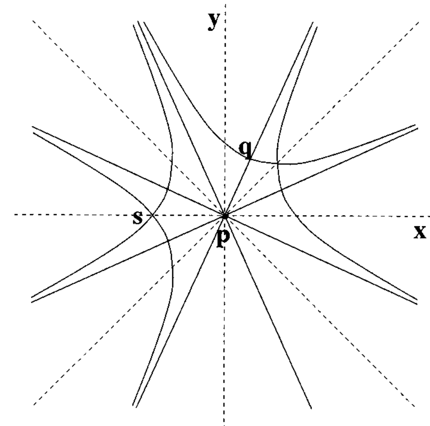

FIG. 6. Examples of null curves in a neighborhood of p, all solid lines. The straight lines are the past and future light cones of p. q is a point on the future null cone of p.

Take the Cartesian metric in the local coordinates and let $r^2=x_1^2+\cdots x_{\lambda}^2$ and $\rho^2=y_1^2+\cdots y_{n-\lambda}^2$ so

$$ds_R^2=dr^2+r^2d\Omega_{\lambda-1}^2+d\rho^2+\rho^2 d\Omega_{n-\lambda-1}^2. \tag{10}$$

The Morse metric we construct from these and $\zeta=2$ is

$$ds_L^2=4(r^2+\rho^2)[r^2d\Omega_{\lambda-1}^2+\rho^2d\Omega_{n-\lambda-1}^2] \tag{11}$$

$$+4(\rho dr+rd\rho)^2-4(rdr-\rho d\rho)^2. \tag{12}$$

This is not flat for $n\geqslant 3$. We can now solve $ds_L^2=0$ for a fixed point on the $(\lambda-1)$ sphere and $(n-\lambda-1)$ sphere and find that the past and future light cones of p have base $S^{\lambda-1}\times S^{n-\lambda-1}$. Note that this base is disconnected for $\lambda=1$ or $n-1$. The light cones of other points are more complicated to calculate but a similar argument to that for the $1+1$ example shows that there is a causal discontinuity for $\lambda=1$ or $n-1$.

From now on we will assume that the Borde-Sorkin conjecture holds. Thus, we can search for causally continuous histories on $\mathcal{M}$ by asking if it admits any Morse function f which has no index 1 or $n-1$ critical points: a history corresponding to such an f would be causally continuous. If on the other hand, such an f does not exist, i.e., all Morse functions on $\mathcal{M}$ have critical points of either index 1 or $n-1$, then $\mathcal{M}$ does not support causally continuous histories.

We should remind ourselves that for a given Morse function f on $\mathcal{M}$ the number of index λ critical points m_{λ} is not a topological invariant; in general different Morse functions will possess different sets of critical points. However there are lower bounds on the m_{λ} depending on the homology type of $\mathcal{M}$. For the topological cobordism $(\mathcal{M},V_0,V_1)$ we have the Morse relation

$$\sum_{\lambda}\,(m_{\lambda}-\beta_{\lambda}(\mathcal{M},V_0))t^{\lambda}=(1+t)R(t), \tag{13}$$

where $\beta_{\lambda}(\mathcal{M},V_0)$ are the Betti numbers of $\mathcal{M}$ relative to V_0 and $R(t)$ is a polynomial in the variable t which has positive coefficients [19,20,18]. Letting $t=-1$, we immediately get the relative Euler characteristic of $\mathcal{M}$ in terms of the Morse numbers

$$\chi(\mathcal{M},V_0)=\sum_{\lambda}(-1)^{\lambda}m_{\lambda}. \tag{14}$$

Another consequence of Eq. (13) is

$$m_{\lambda}\geqslant\beta_{\lambda}(\mathcal{M},V_0)\quad\forall\lambda, \tag{15}$$

which places a lower bound on the m_{λ}.

IV. GENERAL TOPOLOGY CHANGE IN $n=4$

As we have noted, in n dimensions critical points of index 0 and n correspond to a big bang and big crunch, which allow causally continuous histories. It is only for $n\geqslant 4$ that other types of causally continuous histories can exist. For example, in four dimensions, elementary cobordisms with index 1 or 3 critical points correspond to causally discontinuous histories while those of index 2 are causally continuous.

For $n=4$, we have already mentioned that any two 3 manifolds V_0 and V_1 are cobordant, i.e., $\exists$ a four-dimensional $\mathcal{M}$ such that $\partial\mathcal{M}=V_0 \amalg V_1$. However, we can ask whether, given a particular pair $\{V_0,V_1\}$, a cobordism $\mathcal{M}$ exists which admits a causally continuous metric. If not, then the Sorkin conjecture would imply that the transition $V_0\rightarrow V_1$ would be suppressed. In other words, does a cobordism $\mathcal{M}$ exist that admits a Morse function with no index 1 or 3 points? The answer to this is supplied by a well known result in three manifold theory, the Lickorish-Wallace theorem, which states that any three manifold V_1 can be obtained from any other V_0 by performing a series of type 2 surgeries on V_0 [12]. Thus, by Theorem 1 there exists an interpolating cobordism $\mathcal{M}$ that is the trace of this sequence of surgeries and that therefore admits a Morse function with only index 2 points, so that $\mathcal{M}$ admits a causally continuous metric.

This result has the immediate consequence that even if the Sorkin and Borde-Sorkin conjectures hold and causally discontinuous histories are suppressed in the SOH, no topological transition $V_0\rightarrow V_1$ would be ruled out in $3+1$ dimensions. Thus, in this sense, there is no ''causal'' obstruction to any transition $V_0\rightarrow V_1$ in $3+1$ dimensions, just as there is no topological (nor Lorentzian) obstruction in $3+1$ dimensions.

This is somewhat disappointing, however, since there are some transitions that we might hope would be suppressed. An important example is the process in which a single prime three manifold is produced. Quantized primes or topological geons occur as particles in canonical quantum gravity similar to the way skyrmions and other kinks appear in quantum field theory (see [1] and Sec. VI). We would therefore not expect single geon production from the vacuum. However, the restriction of causal continuity will not be enough to rule this out and we will have to wait for more dynamical arguments. This situation is in contrast to that for the Kaluza-Klein monopole where there is a purely topological obstruction to the existence of a cobordism for the creation of a single monopole [7] (though that case is strictly not within the regime of our discussion since the topology change involved is not local but changes the boundary conditions at infinity).

This result, however, says nothing about the status of any particular topological cobordism in the SOH. In other words, it may not be true that a given topological cobordism $\mathcal{M}$ admits a causally continuous Morse metric.

V. PAIR PRODUCTION OF BLACK HOLES

The pair creation of black holes has been investigated by studying Euclidean solutions of the equations of motion which satisfy the appropriate boundary conditions for the solution to be an instanton for false vacuum decay. One does not have to subscribe to the Euclidean SOH approach to quantum gravity in order to believe that the instanton calculations are sensible. Indeed, we take the attitude that the instantons are not ''physical'' but only useful machinery for approximately calculating amplitudes [9] and that the functional integral is actually over Morse metrics. The issue of whether quantum fields can propagate in a nonsingular way on these Morse geometries is therefore relevant and the question arises as to whether causally continuous Morse metrics can live on the instanton manifold.

The doubled instanton, or bounce, corresponding to the pair creation and annihilation of nonextremal black holes has the topology $S^2\times S^2-pt$ [13]. Let us compactify this to $S^2\times S^2$. The fact that $S^2\times S^2$ is closed implies that it will include at least one universe creation and one universe destruction, corresponding to Morse index 0 and 4 points, respectively. This can be seen from the Betti numbers, $\beta_0=\beta_4=1$, $\beta_1=\beta_3=0$, and $\beta_2=2$ so the Morse inequalities imply that $m_0\geqslant 1$ and $m_4\geqslant 1$. Although $\beta_1=\beta_3=0$ we cannot conclude that there exists a Morse function that saturates the bounds of the inequalities (see the next section for an example). We will prove that such a Morse function exists (with $m_0=m_4=1$, $m_1=m_3=0$ and $m_2=2$) by an explicit construction on the half-instanton $S^2\times B^2$.

Let (θ,ϕ) be standard polar coordinates on S^2 and (r,ψ) polar coordinates on B^2, where $\theta\in[0,\pi]$, $\phi\in[0,2\pi]$, $0\leqslant r\leqslant 1$ and $\psi\in[0,2\pi]$. The boundary of $S^2\times B^2$ is $S^2\times S^1$ so that $S^2\times B^2$ corresponds to the creation from nothing of an $S^2\times S^1$ wormhole.

We define the function

$$f(\theta,\phi,r,\psi)=\tfrac{1}{3}(1+r^2+\cos(1-r^2)\theta). \tag{16}$$

Now, $f:S^2\times B^2\rightarrow[0,1]$. The level surface $f^{-1}(1)$ satisfies the condition $r=1$. This is easily seen to be the boundary $S^2\times S^1$ of $S^2\times B^2$ (Fig. 7). On the other hand, the level surface $f^{-1}(0)$ satisfies the condition $r=0$, $\theta=\pi$ which is a point on $S^2\times B^2$.

We find the critical points of f by noting that $\partial_r f=(\tfrac{2}{3})r+(\tfrac{2}{3})r\theta\sin(1-r^2)\theta$ and $\partial_\theta f=-\tfrac{1}{3}(1-r^2)\sin(1-r^2)\theta$, while $\partial_\phi f=\partial_\psi f=0$ everywhere. Thus, there are only two (and therefore isolated) critical points of f, i.e., $p_1=(r=0,\theta=\pi)$ and $p_2=(r=0,\theta=0)$ which are not on the boundary. In order to show the critical points are nondegenerate and to determine their indices we make use of the Morse Lemma and rewrite f in suitable local coordinate patches.

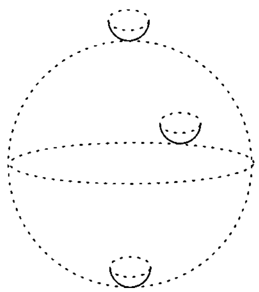

FIG. 7. The level surface $f^{-1}(1)$ is the boundary $S^2\times S^1$ of $S^2\times B^2$.

Near p_1. At p_1, $f=0$. In the neighborhood of p_1, we may write $\theta=\pi-\epsilon$ where ϵ and r are both small and of the same order (note that the topology of this neighborhood is just $B^2\times B^2$). Then,

$$\cos(1-r^2)\theta\approx\cos(\pi-\epsilon) \tag{17}$$

$$\approx -1+\frac{1}{2}\epsilon^2, \tag{18}$$

and putting $x_1=(r/\sqrt{3})\sin\psi$, $x_2=(r/\sqrt{3})\cos\psi$, $x_3=(\epsilon/\sqrt{6})\sin\phi$ and $x_4=(\epsilon/\sqrt{6})\cos\phi$, we see that

$$f\approx x_1^2+x_2^2+x_3^2+x_4^2. \tag{19}$$

Thus, p_1 is an index 0 point.

Near p_2. At p_2, $f=\frac{2}{3}$. In the neighborhood of p_2, r and θ are small and of the same order. Then

$$f\approx\tfrac{2}{3}+\tfrac{1}{3}r^2-\tfrac{1}{6}\theta^2, \tag{20}$$

and using $y_1=(\theta/\sqrt{6})\sin\phi$ and $y_2=(\theta/\sqrt{6})\cos\phi$, $y_3=(r/\sqrt{3})\sin\psi$, $y_4=(r/\sqrt{3})\cos\psi$, we see that

$$f\approx\tfrac{2}{3}-y_1^2-y_2^2+y_3^2+y_4^2. \tag{21}$$

So p_2 is an index 2 point.

The existence of such a Morse function with two critical points, one of index 0 and the other of index 2, shows that the black hole pair production topology can support histories that are causally continuous. The index 0 point is the creation of an S^3 from nothing and the index 2 point is the transition from S^3 to $S^2\times S^1$. That this is means that a Morse function with the same Morse points exists on the original noncompact cobordism, half of $S^2\times S^2$-{point} was later shown in [23]. This result is evidence of consistency between the conclusion that the existence of an instanton implies that the process has a finite rate (approximated by $\tilde{e}^{-I}$ where I is the Euclidean action) and the idea that only causally continuous Morse histories contribute to the SOH.

We note that a simple generalization of the above Morse function shows that the higher dimensional black hole pair creation-annihilation topological cobordism $S^{n-2}\times B^2$ admits a Morse function with one index 0 point and an index $(n-2)$ point and thus supports histories that are causally continuous for any dimension $n>4$ (though the actual instanton solution is unknown). It is also interesting that there is another simple cobordism for the transition from S^3 to $S^2\times S^1$ which is $B^3\times S^1$ with an embedded open four-ball deleted. This, however, by virtue of the Morse inequalities, admits no Morse function without an index 1 point and so is causally discontinuous. In some sense, this second causally discontinuous process is the way one might naturally imagine a wormhole forming: two distant regions of space coming ‘‘close in hyperspace’’ and touching to form the wormhole. The index 2 cobordism for creation of a wormhole is harder to visualise.

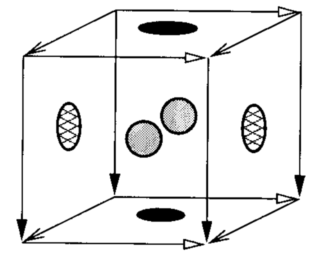

FIG. 8. A T^3 prime: the opposite sides of the cube are identified as are the opposite edges.

VI. PAIR PRODUCTION OF TOPOLOGICAL GEONS

Topological geons are particles that exist because of the nontrivial topology of space. A geon is based on a prime three manifold, one which cannot be divided further into nontrivial pieces by embedded two spheres. One can build a kinematical particle picture in quantum gravity whereby the geons can be endowed with spin and statistics [1,24,25,2]. Every prime can be constructed from a solid polyhedron by identifying its boundary in some way—it is helpful in what follows to imagine the prime as a torus T^3, so the polyhedron is a solid cube and opposite faces are identified (Fig. 8). To take the connected sum of a prime P with any three manifold V, denoted $P\#V$, the (open) solid polyhedron is deleted from V and the same identifications made on the resultant boundary. (The connected sum is also formed by removing open balls from each of two three-manifolds and identifying the resulting S^2 boundaries.)

A rather natural cobordism for pair-production of topological geons, inspired by its Feynman-diagram likeness, is the ‘‘U tube’’ [26,27]. Figure 9 is a $2+1$ sketch of this manifold which is formed by removing a U tube of solid polyhedral cross section out of $\mathbb{R}^3\times I$ as shown and identifying the resulting boundaries in a manner appropriate to the

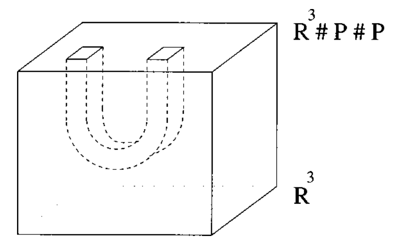

FIG. 9. A $(2+1)$-dimensional representation of the U-tube cobordism for $\mathbb{R}^3\rightarrow\mathbb{R}^3\#P\#P^*$.

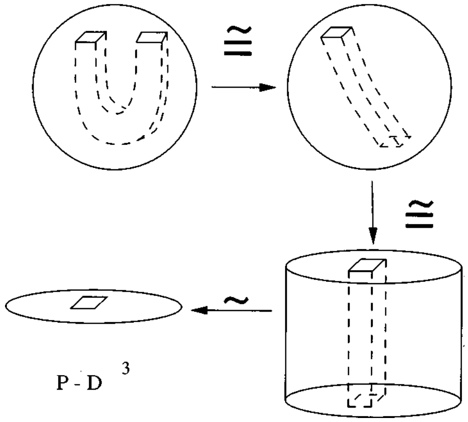

FIG. 10. The U-tube pair creation of the prime P is homotopic to $P-D^3$.

prime P. The initial boundary is $\mathbb{R}^3$ and the final boundary is $\mathbb{R}^3 \# P \# P^*$ where P^* denotes the chiral conjugate (mirror image) of P. (In our example, T^3 is self-conjugate.) Such a U-tube cobordism was used to prove a spin-statistics correlation for certain lens space topological geons (all of which are self-conjugate) [27]. Moreover, the argument that certain proposed rules for assigning quantum phases to different cobordisms would give a completely general spin-statistics correlation for geons also relies on the U tube [24]. In the present context, then, it seems important to test the causal continuity of the U tube.

In order to use our Morse technology we compactify the cobordism by adding a point at spatial infinity at every spatial hypersurface. This creates a cobordism between S^3 and $P\#P^*$. Then we close off the initial boundary by capping it with B^4. This produces a cobordism $\mathcal{M}$ between $\emptyset$ and $P\#P^*$ which is B^4 with a U tube of prime P.

The question we ask is whether the U-tube cobordism $\mathcal{M}$, admits Morse functions with $m_1 = m_3 = 0$. In order to do this we first calculate the Euler characteristic $\chi(\mathcal{M})$ and then employ Eq. (14) which relates it to the m_λ's.

Now, we can unbend the U tube until it is straight (Fig. 10) to see that $\mathcal{M} \cong I \times (P\#B^3) \sim P\#B^3 \cong P-D^3$ (where $\cong$ implies diffeomorphic and $\sim$ homotopy equivalence) and so $\chi(\mathcal{M} = \chi(P-D^3)$. We now use the Mayer-Vietoris sequence for homology groups [28].

$$\cdots \to H_k(X_1 \cap X_2) \to H_k(X_1) \oplus H_k(X_2) \to H_k(X) \to H_{k-1}(X_1 \cap X_2) \to \cdots, \tag{22}$$

where X_1 and X_2 are subspaces of X with $X = \mathrm{int}(X_1) \cup \mathrm{int}(X_2)$. Choose $X_1 \cong P-D^3$ and $X_2 \cong B^3$ such that $X = \mathrm{int}(X_1) \cup \mathrm{int}(X_2) = P$ and $X_1 \cap X_2 \cong S^2 \times I \sim S^2$. On substitution, the above sequence breaks up into the two long exact sequences

$$0 \to H_3(P-D^3) \xrightarrow{\alpha} H_3(P) \xrightarrow{D} H_2(S^2) \xrightarrow{\beta} H_2(P-D^3) \xrightarrow{\delta} H_2(P) \to 0, \tag{23}$$

and

$$0 \xrightarrow{a} H_1(P-D^3) \xrightarrow{b} H_1(P) \xrightarrow{c} H_0(S^2) \xrightarrow{d} H_0(P-D^3) \oplus H_0(D^3) \xrightarrow{e} H_0(P) \xrightarrow{f} 0. \tag{24}$$

Let us first examine the map $D{:}H_3(P) \to H_2(S^2)$ in Eq. (23). For an n-dimensional space $X = X_1 \cup X_2$, each n cycle z in X is homologous to a cycle of the form $\gamma_1 + \gamma_2$ where γ_i is an n cycle in X_i. Moreover, if $D{:}H_\lambda(X_1 \cup X_2) \to H_{\lambda-1}(X_\perp \cap X_2)$ is the connecting homomorphism in the Mayer-Vietoris sequence, then $D(clsz) = D(cls(\gamma_1 + \gamma_2)) = cls(\partial\gamma_1)$ (Lemma 6.19 in [28]).

Now, $H_3(P) = \mathbb{Z}$ and $H_2(S^2) = \mathbb{Z}$. Let $clsz$ be the generator of $H_3(P)$. From the above, $D(clsz) = D(cls(\gamma_1 + \gamma_2)) = cls(\partial\gamma_1)$, where γ_1 is a three cycle in $P-D^3$ and γ_2 one in B^3. Remembering that $P = (P-D^3) \cup B^3$ is a closed manifold, the only nontrivial three cycle is one that fully triangulates P. This means that $\partial\gamma_1$ is a nontrivial two cycle in $\partial(P-D^3) \sim (P-D^3) \cap B^3 \sim S^2$ and hence $cls\partial\gamma_1$ is the generator of $H_2(S^2)$. Thus D maps the generator of $H_3(P) = \mathbb{Z}$ to the generator of $H_2(S^2) = \mathbb{Z}$ which implies that it is an isomorphism.

Since D is an isomorphism, $ker(D) = 0 = im(\alpha)$. α being a $1-1$ map, $H_3(P-D^3) = 0$. Next, $ker(\beta) = im(D) = H_2(S^2)$. Hence $im(\beta) = 0 = ker(\delta)$. Thus, δ which is an onto map is also $1-1 \Rightarrow \delta$, an isomorphism, or $H_2(P-D^3) = H_2(P)$.

From Eq. (24), using $H_0(X) = \mathbb{Z}$ for X connected, we see that $ker(e) = \mathbb{Z} = im(d) \Rightarrow d$ is onto and hence $1-1$. Thus, $ker(d) = 0 = im(c) \Rightarrow ker(c) = H_1(P) = im(b)$. This implies that b is onto and also being $1-1$, an isomorphism. Thus $H_1(P-D^3) \cong H_1(P)$.

Summarizing, we have

$$H_\lambda(P-D^3) = H_\lambda(P) \quad \text{for } \lambda = 0,1,2 \tag{25}$$

$$= 0 \quad \text{for } \lambda \geq 3. \tag{26}$$

Thus, the first three Betti numbers of $\mathcal{M}$: $\beta_0(\mathcal{M})$, $\beta_1(\mathcal{M})$, $\beta_2(\mathcal{M})$ are the same as those for P. Since P is a closed three-manifold, $\chi(P) = 0$, and $\beta_3(P) = 1$ and therefore $\chi(\mathcal{M}) = \chi(P) + 1 = 1$.

From the Morse inequalities we have $m_0 \geq 1$ and $m_4 \geq 0$. Using this along with relation (14) we see that

$$m_1+m_3-m_2\geqslant 0. \tag{27}$$

Equation (27) implies that either (a) m_1 or m_3 (or both) are nonzero or (b) $m_1=m_2=m_3=0$.

From our earlier comments on the special role played by the big bang and big crunch topologies it seems that (b) must be ruled out since there would otherwise be no topology change apart from the big bang creation of an S^3 from nothing. A systematic argument leading to this conclusion employs the following theorem due to Reeb [29]:

Theorem 2. If $\mathcal{M}$ is a compact n-dimensional manifold without boundary, admitting a Morse function $f:\mathcal{M}\to[0,1]$ with only two critical points, then $\mathcal{M}$ is homeomorphic to S^n.

Using this, we now show that (b) leads to a contradiction. First, this implies that $m_0=1$ and $m_4=0$. Then, consider the double of $\mathcal{M}$, the manifold $\mathcal{N}=\mathcal{M}\cup\bar{\mathcal{M}}$ where $\bar{\mathcal{M}}$ is a time-reversed copy of $\mathcal{M}$ and the union is taken by identifying the boundaries in the obvious way. If $\bar{f}$ is the time-reversed Morse function on $\bar{\mathcal{M}}$ then the number of index λ critical points $\bar{m}_\lambda$ of $\bar{f}$ are related to the m_λ by $m_\lambda=\bar{m}_{n-\lambda}$. We can extend the Morse function f on $\mathcal{M}$ to some $\mathcal{F}$ on $\mathcal{N}$ as follows:

$$\mathcal{F}|_{\mathcal{M}}=f, \tag{28}$$

$$\mathcal{F}|_{\bar{\mathcal{M}}}=\bar{f}. \tag{29}$$

$\mathcal{F}$ will therefore have exactly twice the total number of critical points that f has, and the number of index λ points of $\mathcal{F}$ are given by

$$\mu_\lambda=m_\lambda+\bar{m}_\lambda=m_\lambda+m_{n-\lambda} \tag{30}$$

so that $\mu_\lambda=\mu_{n-\lambda}$. Then $\mu_0=\mu_4=1$, $\mu_1=\mu_2=\mu_3=0$ and so $\mathcal{F}$ possesses only two critical points, one of index 0 and the other of index 4. Since $\mathcal{N}$ is a closed manifold, theorem 2 implies that $\mathcal{N}$ is homeomorphic to S^4, which is clearly false, i.e., (b) is incorrect.

Thus, from (a) we see that *any* Morse function f on $\mathcal{M}$ must possess critical points of index 1 or 3. This means therefore that any spacetime $(\mathcal{M},g_{ab})$ where $\mathcal{M}$ is a *generic U*-tube cobordism in which an arbitrary prime P is pair-produced will have causal discontinuities. Notice that we can choose the prime manifold P to be such that the Betti numbers of the cobordism are zero, except for β_0 and β_4. For example, $P=RP^3$. This, then, is an example where the bounds of the Morse inequalities cannot be realized.

The implications of this result are not very favorable to the particle picture of primes. It seems that either the picture we have been building here in which causally discontinuous histories are suppressed in the SOH fails in some way or the restoration of the spin-statistics correlation for geons in an illusion (the kinematical calculations of [27] would remain true but the more dynamical considerations of causal continuity would reveal the amplitudes considered to be negligible.) We discuss some possible ways out in the final section.

VII. CONCLUSIONS

We have described a rather natural framework for considering topology change within the SOH for quantum gravity based on Morse theory. Two key conjectures lead to the proposal that only causally continuous cobordisms be included in the sum and that these are identified with Morse metrics with no index 1 or $n-1$ points. The Lickorish-Wallace theorem on surgery on three-manifolds together with the Borde-Sorkin conjecture means that any topology changing transition in $3+1$ dimensions is achievable by a causally continuous cobordism. The higher dimensional statement is not known.

We have shown that the black hole pair production instanton $S^2\times S^2$ admits causally continuous Morse metrics whereas the ''*U*-tube'' cobordism for pair production of topological geons of any sort is necessarily causally discontinuous.

The result on the black hole pair production instanton cobordism fits in well with the conjectures. However, the topological geon *U*-tube pair production cobordism calculation is a serious setback. It is hard to see how to rescue the spin-statistics theorem for lens spaces if the *U*-tube cobordism is indeed suppressed because it cannot support causally continuous histories. It seems to be the canonical pair-creation cobordism and the proof of the theorem rests heavily on its properties. Moreover the more general rules proposed by Sorkin [24] that would lead to a spin-statistics correlation for all geons also rely on cobordisms that contain *U*-tubes and these would also be in jeopardy.

This might mean that the notion of primes as particles does not survive with topology change. The causal continuity of the single prime creation and the causal discontinuity of the *U*-tube cobordism can then be regarded as a manifestation of this problem. However, since an important and physically appealing motivation for topology change comes from the study of primes as particles [1,24], we suggest here that this is not the case.

A possible resolution that might save the geon spin-statistics result, is that there must be a weakness in the sequence of conjectures to which we have drawn attention and which form the framework in which causal continuity becomes so central. The Borde-Sorkin conjecture—that a Morse metric is causally continuous iff it contains no index 1 or $(n-1)$ points—seems to be the most solid. Work on a proof is currently underway [30]. The Sorkin conjecture that an infinite energy/particle production would occur in a Morse spacetime iff it contained a causal discontinuity seems plausible but would need to be verified by more examples than the $(1+1)$-dimensional trousers and yarmulke studied so far. In particular, the first example of a causally continuous spacetime that is not the yarmulke occurs in $3+1$ dimensions. Work on this second conjecture will be easier once the first is proved since then simple examples of causally continuous metrics can be written down using the Morse construction. Then finally, there is the idea that the singular behavior of quantum fields on a causally discontinuous background is a signal that it is infinitely suppressed in the SOH. What one means by this is the following. Consider a scalar field minimally coupled to gravity. The path integral is

$$\sum_{\text{all topologies}} \int [dg][d\phi]e^{i\int\sqrt{-g}R+i\int\sqrt{-g}(\partial\phi)^2} \tag{31}$$

(where we have omitted the explicit and important statement about boundary conditions). We may integrate out the scalar field degrees of freedom, i.e.,

$$\int [d\phi]e^{i\int\sqrt{-g}(\partial\phi)^2}=F[g]. \tag{32}$$

The functional $F[g]$ which is the path integral for a scalar field in a fixed background can now be regarded as an overall weight in the path integral over metrics,

$$\sum_{\text{all topologies}} \int [dg]F[g]e^{i\int\sqrt{-g}R}. \tag{33}$$

The idea is that $F[g]$ is zero if g is causally discontinuous.

Perhaps, however, all the conjectures do hold at the continuum level and the simplest loophole of all is that the SOH should be defined fundamentally as a sum over whatever discrete structure will prove to underly the differentiable manifold of general relativity. If it is a causal set then all quantities calculated will be regulated. The elimination altogether of the causally discontinuous cobordisms would then be too severe a truncation, and even if they are still suppressed, they might give a nontrivial contribution.

ACKNOWLEDGMENTS

We would like to thank Rafael Sorkin for motivating this work and for several clarifying discussions. We also thank A. Chamblin, G. Gibbons, R. Penrose, Siddhartha Sen, P. Tod, and N. Woodhouse for helpful discussions. S. Surya would also like to thank S. M. Vaidya for help with procuring references. F. Dowker is supported in part by the EPSRC and thanks the Department of Applied Maths and Theoretical Physics at University of Cambridge for hospitality during part of this work. Sumati Surya was supported in part by Syracuse University and by Inter University Centre for Astronomy and Astrophysics, Pune.

[1] R. D. Sorkin, in *Proceedings of Topological Properties and Global Structure of Space-Time*, Erice, Italy, 1985, edited by P. G. Bergmann and V. de Sabbata (Plenum, New York, 1986), pp. 249–270.
[2] Rafael D. Sorkin and Sumati Surya, Int. J. Mod. Phys. A **13**, 3749 (1998).
[3] A. Anderson and B. DeWitt, Found. Phys. **16**, 91 (1986).
[4] S. G. Harris and T. Dray, Class. Quantum Grav. **7**, 149 (1990).
[5] A. P. Balachandran, G. Bimonte, G. Marmo, and A. Simoni, Nucl. Phys. **B446**, 299 (1995).
[6] B. L. Reinhart, Topology **2**, 173 (1963).
[7] R. D. Sorkin, Phys. Rev. D **33**, 978 (1986).
[8] R. P. Geroch, J. Math. Phys. **8**, 782 (1967).
[9] R. D. Sorkin, Int. J. Theor. Phys. **36**, 2759 (1997).
[10] Jorma Louko and Rafael D. Sorkin, Class. Quantum Grav. **14**, 179 (1997).
[11] A. Daughton, Zh-Ch Gu, D. J. O'Connor, and R. D. Sorkin, "Topology change and field fluctuations in two dimensions" (in preparation).
[12] R. D. Sorkin (private communication).
[13] D. Garfinkle and A. Strominger, Phys. Lett. B **256**, 146 (1991).
[14] Fay Dowker, Jerome P. Gauntlett, David A. Kastor, and Jennie Traschen, Phys. Rev. D **49**, 2909 (1994).
[15] P. Yodzis, Gen. Relativ. Gravit. **4**, 299 (1973).
[16] John Willard Milnor, *Lectures on the h-cobordism Theorem* (Princeton University Press, Princeton, NJ 1965).
[17] David B. Gauld, *Differential Topology: An Introduction* (Dekker, New York, 1982).
[18] B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov, *Modern Geometry—Methods and Applications—III* (Springer-Verlag, New York, 1992).
[19] Charles Nash, *Differential Topology and Quantum Field Theory* (Academic, New York, 1991).
[20] Charles Nash and Siddhartha Sen, *Topology and Geometry for Physicists* (Academic, New York, 1983).
[21] Arvind Borde and Rafael D. Sorkin, Causal cobordism: Topology change without causal anomalies (in preparation).
[22] S. Hawking, and Hawking Sachs, Commun. Math. Phys. **35**, 287 (1974).
[23] H. F. Dowker and R. Garcia, Class. Quantum Grav. **15**, 1859 (1998).
[24] R. D. Sorkin, in *Geometrical and Algebraic Aspects of Nonlinear Field Theories*, Proceedings, Amalfi, Italy, 1988, edited by M. Marinaro (Elsevier, Amsterdam, 1989), pp. 201–218.
[25] J. L. Friedman and R. D. Sorkin, Phys. Rev. Lett. **44**, 1100 (1980).
[26] J. L. Friedman and A. Higuchi, Nucl. Phys. **B339**, 491 (1990).
[27] H. F. Dowker and R. D. Sorkin, Class. Quantum Grav. **15**, 1153 (1998).
[28] Joseph J. Rotman, *An Introduction to Algebraic Topology* (Springer-Verlag, New York, 1993).
[29] Morris W. Hirsch, *Differential Topology* (Springer-Verlag, New York, 1976).
[30] Arvind Borde, Fay Dowker, Raquel Garcia, Rafael Sorkin, and Sumati Surya (in progress).