Model for stars of interacting bosons and fermions

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In this paper we introduce a current-current type interaction term in the Lagrangian density of gravity coupled to complex scalar fields, in the presence of a degenerated Fermi gas. For low transferred momenta, such a term, which might account for the interaction among boson and fermion constituents of compact stellar objects, is subsequently reduced to a quadratic one in the scalar sector. This procedure enforces the use of a complex radial field counterpart in the equations of motion. The real and the imaginary components of the scalar field exhibit different behavior as the interaction increases. The results also suggest that the Bose-Fermi system undergoes a phase transition for a suitable choice of the coupling constant. [S0556-2821(98)07720-0]

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I. INTRODUCTION

In modern approaches to astrophysics and cosmology, concepts that were initially restricted to elementary particle physics acquire a wider meaning, particularly in inflationary models. These models give predictions for the mass density of the present Universe larger than that observed, if one presumes that it is close to the critical value. This suggests that there is a large amount of hidden matter, which has not been detected so far.

Among the possible candidates for the so called dark matter are boson stars [1], which consist of gravitational bound states of scalar particles. These kinds of objects were first predicted theoretically by Ruffini and Bonazzola [5], and there is an increasing interest in the subject since there is the hope that they contribute to the dark matter problem. Moreover, such structures were formed possibly, through gravitational collapse in the early Universe and may appear also in the core of composite objects, whose external envelope is made of standard baryonic matter.

Since in addition to the bosons there were also fermions in the primordial gas, we would expect boson-fermion stars to prevail. This system was studied in detail by Henriques *et al.* [2]. In their work it has been shown that the properties of boson-fermion stars are qualitatively the same, irrespective of the addition of a self-coupling term for bosons. Nevertheless, it seems that there is no modeling for explicitly dealing with the interaction between bosons and fermions in such systems, leading to conclusive results. In Ref. [2] the effects of a direct coupling term between bosons and fermions are briefly considered. Estimative effects are discussed but no numerical computation is performed.

In this work we introduce an effective coupling between bosons and fermions to afford a more realistic description of the system and compare our results with the ones in the current literature. In Sec. II we construct the energy-momentum tensor for interacting fermions and bosons using the Schwarzschild metric. In Sec. III we obtain the evolution equations for the coefficients of the metric and for the fields. In Sec. IV we exhibit the results of numerical simulations for the corresponding dynamical system. Finally, in Sec. V we discuss our results and compare with those obtained without taking the interaction into account.

II. THE BOSON-FERMION INTERACTION

Before introducing the interaction term, we outline the noninteractive boson-fermion model. We assume the metric to be the standard Schwarzschild one:

$$ds^2 = -B(r)d\tau^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2. \quad (1)$$

Our sign and conventions are the same as those used by Liddle and Madsen [3]. The Lagrangian for the complex scalar field with no explicit interaction is given by

$$\mathcal{L} = \frac{R}{16\pi G} - \partial_\mu \Phi^* \partial^\mu \Phi - m^2 \Phi^* \Phi, \quad (2)$$

where

$$\Phi(r,\tau) = \phi(r) e^{-i\omega\tau}, \quad (3)$$

and for our purposes ϕ is a complex scalar field. In the absence of fermion-boson coupling, all the equations for the stationary solutions depend on ω^2 (see Ref. [3], or Eqs. (19)–(25) below). So, the alternative choice $\Phi(r,\tau) = \phi(r)e^{i\omega\tau}$ (the so-called antiboson configuration) leads to the same solutions.

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For the composite boson-fermion star we consider that the fermions are described by a perfect fluid with energy density ρ and pressure p as proposed by Chandrasekhar [4]:

$$\rho = K(\sinh t - t),$$
$$p = \frac{K}{3}\left(\sinh t - 8\sinh\frac{t}{2} + 3t\right), \tag{4}$$

where t is a parameter, $K = m_n^4/32\pi^2$, and m_n is the fermion mass (to be considered as that of neutrons for illustrative purposes). The evolution equation for the fermions is given by an equation of state, namely we have

$$p' = -\frac{1}{2}(\rho + p)\frac{B'}{B}, \tag{5}$$

where the primes stand for derivatives with respect to r.

The corresponding energy-momentum tensor for the bosons and fermions without interaction reads as

$$T^{(0)}_{\mu\nu} = T^B_{\mu\nu} + T^F_{\mu\nu}, \tag{6}$$

with

$$T^B_{\mu\nu} = \partial_\nu\Phi^*\partial_\mu\Phi + \partial_\mu\Phi^*\partial_\nu\Phi - g_{\mu\nu}(\partial_\lambda\Phi^*\partial^\lambda\Phi + m^2\Phi^*\Phi), \tag{7}$$

$$T^F_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \tag{8}$$

where superscripts B and F label bosons and fermions from now on.

At this point we introduce the following interaction term in the Lagrangian density:

$$\mathcal{L}^{int} = \lambda J_\mu(\Phi) j^\mu(\psi), \tag{9}$$

where

$$J_\mu(\Phi) = i(\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*), \tag{10}$$

$$j^\mu(\psi) = \bar{\psi}\gamma^\mu\psi, \tag{11}$$

which represent the boson and fermion currents, respectively, while the γ's are the usual Dirac matrices, which satisfy $\gamma^{0\dagger} = -\gamma^0$ and $\gamma^{i\dagger} = \gamma^i$ $(i = 1,2,3)$. This is a typical contact or current-current interaction between bosons and fermions where the coupling constant has dimension $[\lambda] = M^{-2}$. We emphasize that in dealing with a nonrenormalizable interaction term an energy scale must be introduced when we consider quantum corrections; we find a similar situation in the pure Einstein-Hilbert gravity. Note that this point is of no relevance since we are giving a semiclassical treatment to the problem.

The complete Lagrangian density, including the interaction term, is invariant under global $U(1)$ gauge transformations. For low transferred momenta we can use the Bloch-Nordsieck approximation [7] in Eq. (11) and replace it by

$$j^\mu(\psi) = \bar{\psi}\Gamma^\mu\psi, \tag{12}$$

where, in first approximation,

$$\Gamma^0 \simeq u^0, \quad \Gamma^i \simeq u^i. \tag{13}$$

The four-vector $u^\mu = (u^0, u_r, u_\theta, u_\varphi)$ is the four-velocity of the fermion fluid.

The contribution of the interaction term for the energy-momentum tensor is computed via

$$\frac{1}{2}\sqrt{-g}T_{\alpha\beta} = \partial_\rho\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g^{\alpha\beta}_{,\rho}} - \frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g^{\alpha\beta}} \tag{14}$$

and is given by

$$T^{int}_{\mu\nu} = g_{\mu\nu}\mathcal{L}_{int} - \lambda[J_\mu j_\nu + J_\nu j_\mu], \tag{15}$$

which, together with Eq. (6), gives the total energy-momentum tensor

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{int}_{\mu\nu}.$$

For simplicity and due to the symmetries involved we can consider only radial variations, and so $u^\mu = (u^0, u_r, 0, 0)$. To keep the validity of the approximations used we can consider the boson star in its ground state embedded in a fermion fluid for which the radial velocities are to be understood as a time independent small perturbation, i.e., $v \ll 1$.

Usually the scalar field is taken as $\Phi(r,\tau) = \phi(r)e^{-i\omega\tau}$, with $\phi(r)$ complex, in such a way as to obtain a time independent energy-momentum tensor. Considering $\phi(r)$ as a sum of real and imaginary parts [for instance, $\phi(r) = \phi_1(r) + i\phi_2(r)$] the set of differential equations in Ref. [3] becomes the same for both fields, $\phi_1(r)$ and $\phi_2(r)$, and the equations are just corresponding complex conjugate. This way, one can choose the field definitions such that the imaginary part of $\phi(r)$ vanishes, and, as a consequence, one can consider the real and the imaginary parts overall evolution given by $\phi(r)e^{-i\omega\tau}$, with $\phi(r)$ real.

Meanwhile, dealing with interaction between bosons and fermions there is no reason to exclude the imaginary part related to the scalar field; moreover, the interaction causes real and imaginary counterparts to be distinct from each other, as is shown in this article. Hence, our ansatz is

$$\Phi(r,t) = \phi(r)e^{-i\omega\tau}, \tag{16}$$

with

$$\phi = \phi_1 + i\phi_2, \quad \phi^* = \phi_1 - i\phi_2,$$

where ϕ_1 and ϕ_2 are real fields.

The boson current is purely real, $J^*_\mu = J_\mu$, with components

$$J_0 = i(\Phi^*\partial_0\Phi - \Phi\partial_0\Phi^*) = 2\omega(\phi_1^2 + \phi_2^2),$$

$$J_1 = i(\Phi^*\nabla_1\Phi - \Phi\nabla_1\Phi^*) = 2(\phi_1'\phi_2 - \phi_1\phi_2')$$

and $J_2=J_3=0$. For the fermion current the nonvanishing components are

$$j^0=u^0\bar{\psi}\psi, \quad j^1=u^1\bar{\psi}\psi,$$

for which we have used the low transferred momenta approximation (13). The total energy-momentum tensor appears semiclassically on the right-hand side of the Einstein equations, i.e., $T_{\mu\nu}=\langle 0|:T_{\mu\nu}:|0\rangle$, with $|0\rangle=|N_B;0\rangle\otimes|N_F;k_F\rangle$, where N_B and N_F are the numbers of bosons and fermions, respectively, and k_F is the particles momentum in the Fermi distribution. Henceforward, we consider all quantities within the semiclassical approximation. Thus, instead of the bilinear operator $\bar{\psi}\psi$ we take the ground-state of the fermion system $\langle\bar{\psi}\psi\rangle_F$, in the semiclassical approximation. In this limit we replace this quantity by the average fermion density $\bar{n}_F$, for first analysis purposes.

Thus, the interaction counterpart for the energy-momentum tensor is

$$T_0^{0\,int}=-T_1^{1\,int}$$

$$=-2\lambda\bar{n}_F[(\phi_1\phi_2'-\phi_1'\phi_2)u^1+\omega(\phi_1^2+\phi_2^2)u^0],$$

$$T_2^{2\,int}=T_3^{3\,int}=\lambda J_\alpha j^\alpha,$$

Now, one is led to determine the four-velocities of the fermion fluid. The fermion and the interaction counterparts of the total energy-momentum tensor involves the four-velocities, $u^0=u_\tau$ and $u^1=u_r$, given by $u^\mu=dx^\mu/d\tau$, where $d\tau^2=-ds^2$; hence $u^\mu u_\mu=-1$. Since the fermion fluid is allowed to display only radial displacements, the velocity v of the fermion fluid is given by the relation $dr^2=\mathrm{v}^2d\tau^2$, and

$$u^0=\frac{1}{\sqrt{B-A\mathrm{v}^2}}, \quad u^1=\frac{\mathrm{v}}{\sqrt{B-A\mathrm{v}^2}}.$$

Since $\mathrm{v}\ll 1$, we can write the four-velocities in the first-order approximation as

$$u^0=\frac{1}{\sqrt{B}}+O(\mathrm{v}^2), \quad u^1=\frac{\mathrm{v}}{\sqrt{B}}+O(\mathrm{v}^2),$$

and the four-vector u^μ obeys the normalization condition up to order v^2, i.e., $u^\mu u_\mu=-1+O(\mathrm{v}^2)$. It is noteworthy that $\mathrm{v}=\mathrm{v}(r)$ imposes difficulties to a proper physical interpretation; actually one should consider convection velocities, with $\mathrm{v}=\mathrm{v}(r,\theta,\phi)$, which would increase the complexity of the equations.

Alternatively, we can model the velocities to avoid approximations in this stage, and we relate the velocity of the fermion fluid to a parameter n and to the metric coefficients by

$$\mathrm{v}^2=n\frac{B}{A} \quad (n\ll 1)$$

and this velocity can be considered, within the scope of this model, as a small perturbation comparable to those used in time-independent perturbation theory. Due to the finiteness of $A(r)$ and $B(r)$, we note that $\mathrm{v}=\mathrm{v}(r)$ is finite. Hence, the four-velocities become

$$u^0=\frac{1}{\sqrt{1-n}}\frac{1}{\sqrt{B}},$$

$$u^1=\sqrt{\frac{n}{1-n}}\frac{1}{\sqrt{A}}, \tag{17}$$

with no approximations. Now, the perturbative velocity of the fermion fluid is ruled by the parameter n, which is the maximum value of $\mathrm{v}^2(r)$.

III. EVOLUTION EQUATIONS

To obtain the set of equations that govern the fields we make use of the equations of Einstein, Klein-Gordon, and of the parametric equation for fermions. To obtain the scalar fields evolution equations, we use the total Lagrangian

$$L_T=r^2\sin\,\theta\sqrt{AB}\Bigg[\frac{R}{16\pi G}+\frac{\omega^2}{B}(\phi_1^2+\phi_2^2)$$
$$-\frac{\phi_1'^2+\phi_2'^2}{A}+m^2(\phi_1^2+\phi_2^2)+\frac{\omega}{\sqrt{1-n}}\frac{\alpha}{\sqrt{B}}(\phi_1^2+\phi_2^2)$$
$$-\sqrt{\frac{n}{1-n}}\frac{\alpha}{\sqrt{A}}(\phi_1\phi_2'-\phi_1'\phi_2)\Bigg], \tag{18}$$

where $\alpha=2\lambda\bar{n}_F$. Using this expression directly in the action principle, we can find the correct Klein-Gordon equation, which now presents a source term. With an appropriate redefinition of the dynamical variables and parameters, namely

$$x=mr,$$

$$\sigma(x)=\sqrt{8\pi G}\phi(r),$$

$$\bar{\rho}(t)=\frac{4\pi G}{m^2}\rho(t),$$

$$\bar{p}(t)=\frac{4\pi G}{m^2}p(t),$$

$$\bar{\alpha}=\frac{\alpha}{m}, \quad \mathrm{w}=\frac{\omega}{m},$$

and choosing Eq. (17), the equations read

$$A' = xA^2\left[2\left(\frac{\bar{\rho}+\bar{p}}{1-n}-\bar{p}\right)+\left(\frac{\mathrm{w}^2}{B}+1\right)\sigma^2+\frac{s^2}{A}+\sqrt{\frac{n}{1-n}}\frac{\bar{\alpha}}{\sqrt{A}}(\sigma_1\sigma_2'-\sigma_1'\sigma_2)+\frac{\mathrm{w}}{\sqrt{1-n}}\frac{\bar{\alpha}}{\sqrt{B}}\sigma^2\right]-\frac{A}{x}(A-1), \tag{19}$$

$$B' = xAB\left[2\left(\frac{n}{1-n}(\bar{\rho}+\bar{p})+\bar{p}\right)+\left(\frac{\mathrm{w}^2}{B}-1\right)\sigma^2+\frac{s^2}{A}+\sqrt{\frac{n}{1-n}}\frac{\bar{\alpha}}{\sqrt{A}}(\sigma_1\sigma_2'-\sigma_1'\sigma_2)+\frac{\mathrm{w}}{\sqrt{1-n}}\frac{\bar{\alpha}}{\sqrt{B}}\sigma^2\right]+\frac{B}{x}(A-1), \tag{20}$$

$$\sigma_1' = s_1, \tag{21}$$

$$\sigma_2' = s_2, \tag{22}$$

$$s_1' = -A\left(\frac{\mathrm{w}^2}{B}-1+\frac{\bar{\alpha}}{\sqrt{1-n}}\frac{\mathrm{w}}{\sqrt{B}}\right)\sigma_1+\sqrt{\frac{n}{1-n}}\,\bar{\alpha}\sqrt{A}s_2+\left[\frac{1}{2}\left(\frac{A'}{A}-\frac{B'}{B}\right)-\frac{2}{x}\right]s_1+\left(\frac{1}{2}\frac{B'}{B}+\frac{2}{x}\right)\sqrt{\frac{n}{1-n}}\frac{\bar{\alpha}\sqrt{A}}{2}\sigma_2, \tag{23}$$

$$s_2' = -A\left(\frac{\mathrm{w}^2}{B}-1+\frac{\bar{\alpha}}{\sqrt{1-n}}\frac{\mathrm{w}}{\sqrt{B}}\right)\sigma_2-\sqrt{\frac{n}{1-n}}\,\bar{\alpha}\sqrt{A}s_1+\left[\frac{1}{2}\left(\frac{A'}{A}-\frac{B'}{B}\right)-\frac{2}{x}\right]s_2-\left(\frac{1}{2}\frac{B'}{B}+\frac{2}{x}\right)\sqrt{\frac{n}{1-n}}\frac{\bar{\alpha}\sqrt{A}}{2}\sigma_1, \tag{24}$$

$$t' = -2\frac{B'}{B}\frac{\sinh t-2\sinh(t/2)}{\cosh t-4\cosh(t/2)+3}, \tag{25}$$

where

$$\sigma^2=\sigma_1^2+\sigma_2^2, \quad s^2=s_1^2+s_2^2.$$

These equations form a nonautonomous system of nonlinear first-order differential equations, which cannot be linearized due to the quadratic terms involved. Equations (19) and (20) are the Einstein equations; Eqs. (21)–(24) correspond to the Klein-Gordon equation, whereas Eq. (25), which comes from Eq. (5), gives the evolution of the fermion energy density and pressure. In the above set of equations the dynamical variables and parameters are dimensionless. From now on the primes stand for derivatives with respect to x. Note that without the proposed interaction ($\bar{\alpha}=0$, $n=0$) we recover the boson-fermion equations proposed in Ref. [2].

It is noteworthy that these equations are invariant under the scale transformation

$$B \to \eta B, \quad \mathrm{w} \to \sqrt{\eta}\mathrm{w},$$

even after the inclusion of the interaction term. Since the initial value B_0 is undetermined, this permits its redefinition during numerical calculations, in such a way to obtain the asymptotic values for the metric coefficients converging to those of a flat space.

IV. RESULTS

In this section we present the numerical simulation results of the above set of equations. The set of equations were solved by using the fifth-order Runge-Kutta method; we also use an appropriate ‘‘shooting method’’ to infer the value of w, according to an initial value of B_0. We require the metric to be asymptotically flat and the scalar fields as well as their derivatives to vanish at infinity. Note that there is no overlap between σ and σ', which is characteristic of the ground state of the system. Throughout the simulations the initial values are $\sigma_1^0=\sigma_2^0=0.21$, $s_1^0=s_2^0=0$, and $A_0=1$. For further details on the numerical criteria, the interested reader is referred to Ruffini and Bonazzola [5].

In Fig. 1 we show the ground state of a boson-fermion star for $t_0=7$. In this case $B_0=0.067$ and w$=0.964$.

Figure 2 displays the same curves for $t_0=8.7$ which corresponds to $B_0=0.024$ and w$=0.934$. Notice that the peak in the curve for A has increased and has been shifted towards the origin, while σ and σ' approach to zero faster. Larger values of t_0 mean higher fermion energy densities, so that the fermion contribution to the energy-momentum tensor becomes dominant. In this sense we would expect curves like those given in Fig. 3 to be in agreement with the pioneering results of Oppenheimer and Volkoff for neutron stars [6].

Figures 4 and 5 exhibit the results after introducing the interaction term, for the choice $t_0=4.0$, and different values of the dimensionless coupling constant $\bar{\alpha}$. When we switch the interaction on at small values of $\bar{\alpha}$, w increases, as shown in Table I. On the other hand t vanishes at a smaller x. Note also that close to the singularity at the origin we see that the last terms in Eqs. (19) and (20) are dominant. In this region $A\sim x/(x-\mathrm{const})$ and $B\sim(x-\mathrm{const})/x$, so that B and B' diverges and A becomes oscillating. Hence, for numerical purposes it is convenient to start with x slightly shifted from the origin.

If we continue to increase the interaction we observe that, for a certain value of $\bar{\alpha}$, w suddenly decreases, suggesting

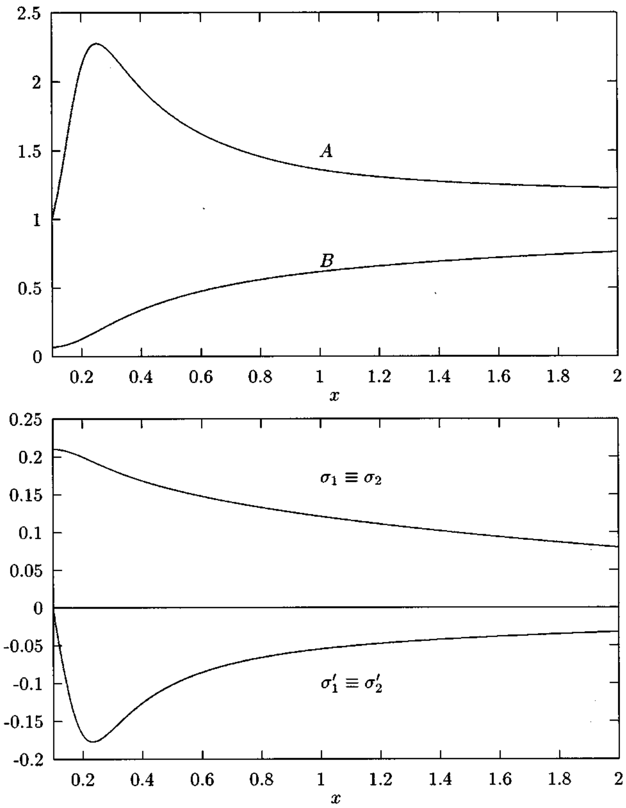

FIG. 1. Fields and metric coefficients for a typical boson-fermion star, with $t_0 = 7$. The corresponding value of w is around 0.96412.

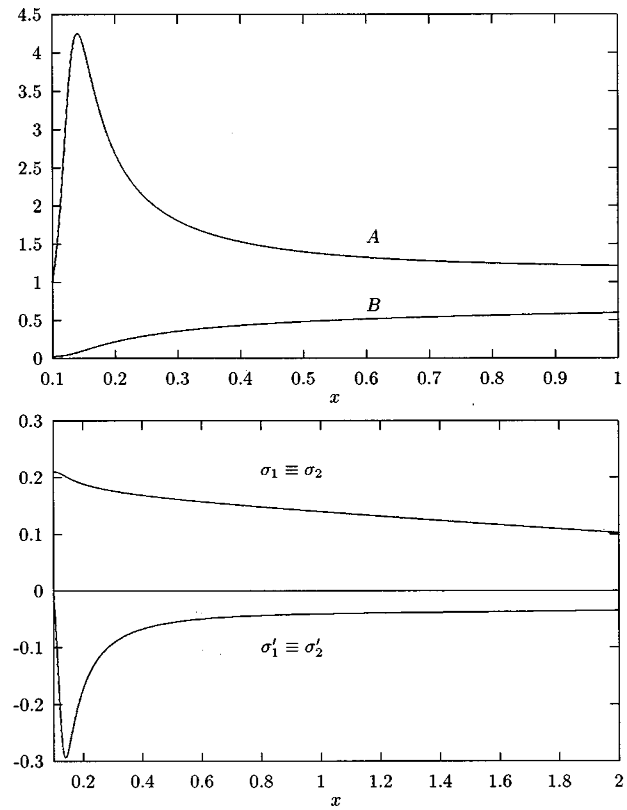

FIG. 2. The same curves as those of Fig. 1, for $t_0 = 8.7$.

that there is a critical value of $\overline{\alpha}$, in the range $e^{-3} - e^{-2}$, in which the system experiences a second-order phase transition, corresponding to boson-fermion pair formation. In Figs. 5(b) and (c) we can observe the splitting of the real and imaginary parts of the scalar field that also occurs in such interval. For completeness, we also exhibit the phase space diagrams for the metric coefficients and for the scalar fields at $\overline{\alpha} = e^{-2}$ in Fig. 6.

Notice that, differently from the case with no boson-fermion interaction, Eqs. (19)–(25) are not symmetric under the sign change $\omega \to -\omega$. So, we can expect different results for the two possible choices $\Phi(r,\tau) = \phi(r) e^{-i\omega\tau}$ and $\Phi(r,\tau) = \phi(r) e^{i\omega\tau}$. Hence, from now on we refer to the $-i\omega\tau$ solution as *type I* star and $+i\omega\tau$ solution as *type II* star.

The results for a type II star are presented in Table I, for which one can also observe a transition when $\overline{\alpha}$ is in the range $e^{-2} - e^{-1}$, where a similar scalar fields splitting can be obtained. The results presented in Table I show that there is a clear distinction between the type I and type II cases, and a stability analysis is in order.

Although not sufficient, the binding energy (E_b) can give a necessary condition for the stability of the star:

$$E_b = M - (m|N_B| + m_n N_F), \tag{26}$$

where M is the total mass of the star, m and m_n are the boson and the fermion particle masses, respectively, $|N_B|$ is the number of scalars and N_F is the number of fermions. If $E_b > 0$ the configuration is unstable. If $E_b < 0$ it has the possibility of being stable; in this case, a complete analysis would involve time dependence of the fields [9]. In Eq. (26) we have introduced the absolute value for the scalar particles to give a proper interpretation for the binding energy, since the number of scalar particles in the type II star ($+i\omega\tau$ ansatz) gives $N_B < 0$. The quantities listed above are given by

$$M = \int \rho d^3 r = \frac{M_{\mathrm{Pl}}^2}{m} \mathcal{M}(\infty), \tag{27}$$

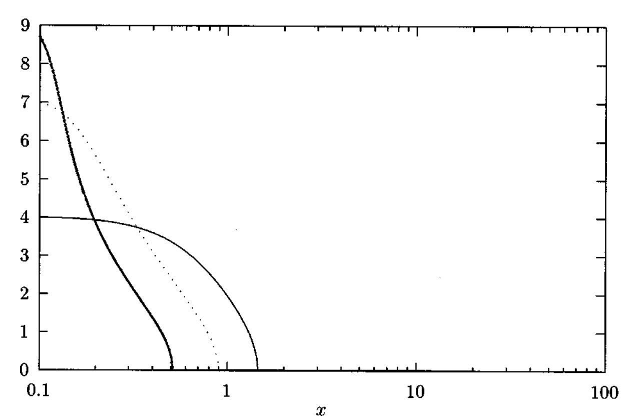

FIG. 3. The evolution of t for different initial values ($t_0 = 4.0$, $t_0 = 7.0$ and $t_0 = 8.7$). All curves on this t vs x graphic uses $\sigma_1^0 = \sigma_2^0 = 0.21$, $s_1^0 = s_2^0 = 0$, and $A_0 = 1$.

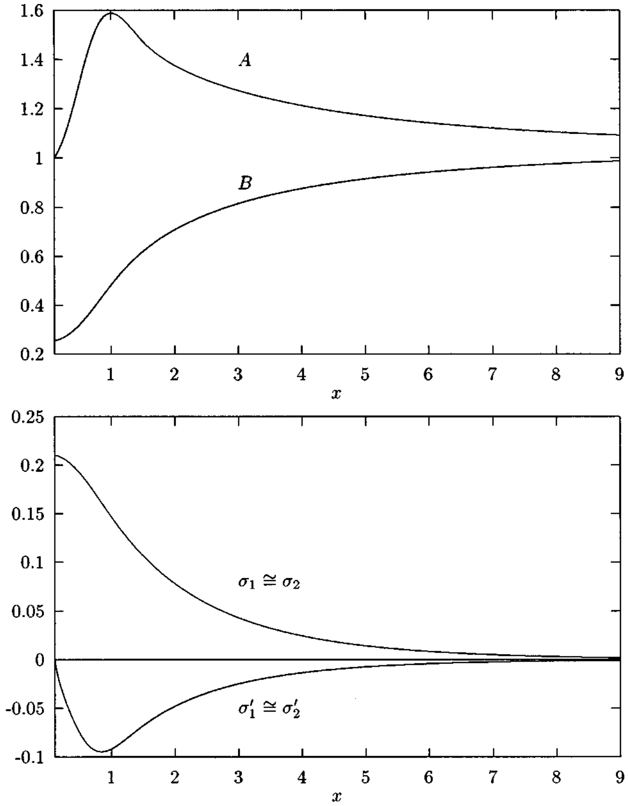

FIG. 4. Fields and metric coefficients for a boson-fermion star when we switch on the interaction at $\overline{\alpha}=e^{-15}$. Here $t_0=4.0$ and $n=0.01$.

where $\rho=-(T_0^{0\mathrm{B}}+T_0^{0\mathrm{F}})$ is the total energy density and

$$\mathcal{M}(\infty)=\lim_{x\to\infty}\left(1-\frac{1}{A(x)}\right),$$

$$R=\frac{\int \rho r d^3 r}{\int \rho d^3 r}=\frac{M_{\mathrm{Pl}}^2}{m^2}\int \overline{\rho} x^3 dx, \tag{28}$$

where $\overline{\rho}=(8\pi G/m^2)\rho$,

$$N_B=\int d^3 r\sqrt{-g}J^0=\frac{M_{\mathrm{Pl}}^2}{m^2}\,\mathrm{w}\int_0^\infty \sqrt{\frac{A}{B}}\,x^2\sigma^2 dx, \tag{29}$$

$$N_F=\int d^3 r\sqrt{-g}j^0=\frac{M_{\mathrm{Pl}}^2}{m_n m}\frac{8}{3\pi}\int_0^\infty \sqrt{A}\,\sinh^3(t/4)x^2 dx, \tag{30}$$

where, in the last expression we have used $j^0=n_F/\sqrt{B}$ and $n_F(r)=(m_n^3/3\pi^2)\sinh^4(t/4)$, which is the Fermi fluid particles density.

The results for type I stars are listed in Table II, and those for type II stars are on Table III.

For the type I star case, when $\overline{\alpha}$ is small, we obtain stability as expected in Ref. [2]. Meanwhile, we observe that the increasing of the interaction causes the star to become unstable, which is not observed for the type II case. The

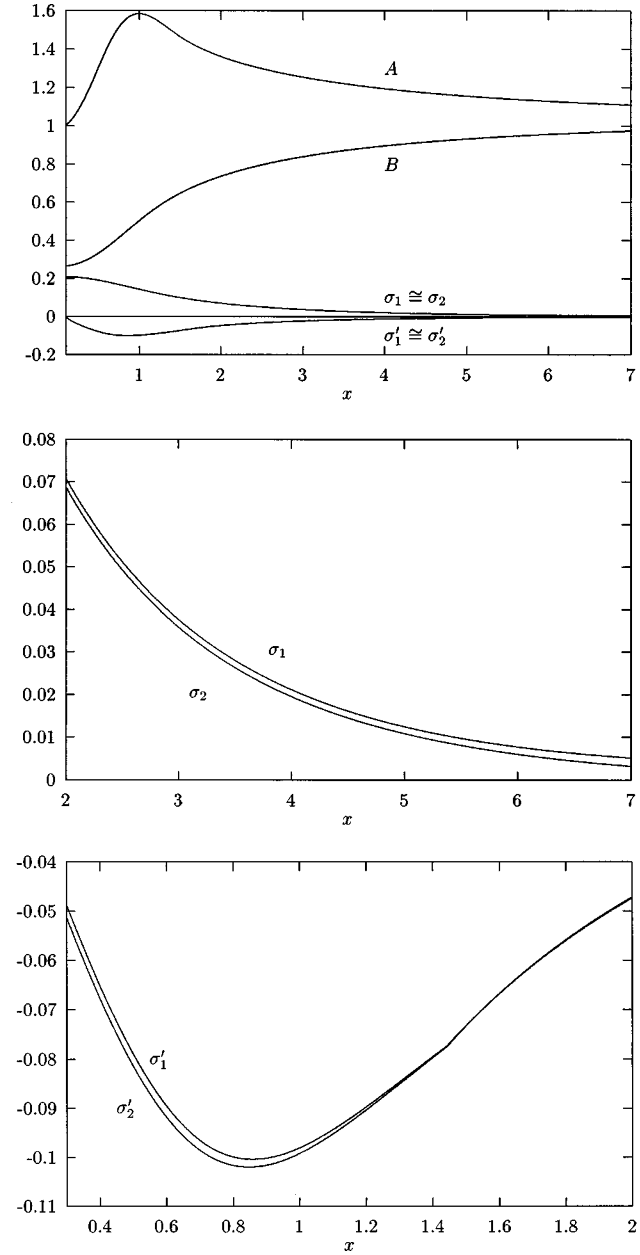

FIG. 5. The same curves as those shown in the Fig. 4, for $\overline{\alpha}=e^{-2}$.

TABLE I. Eigenvalue w for type I stars (and values of w_A for type II star), and corresponding values of B_0, for different coupling constants $\overline{\alpha}$, with $n=0.01$, $\sigma_0=0{,}29698$, and $t_0=4{,}0$.

	Type I		Type II	
$\overline{\alpha}$	w	B_0	w_A	B_0
e^{-15}	0.910943	0.2472	-0.910943	0.2472
e^{-6}	0.917884	0.2512	-0.917704	0.2507
e^{-3}	0.927375	0.2598	-0.924390	0.2511
e^{-2}	0.926882	0.2661	-0.925015	0.2456
e^{-1}	0.922895	0.2840	-0.917396	0.2261

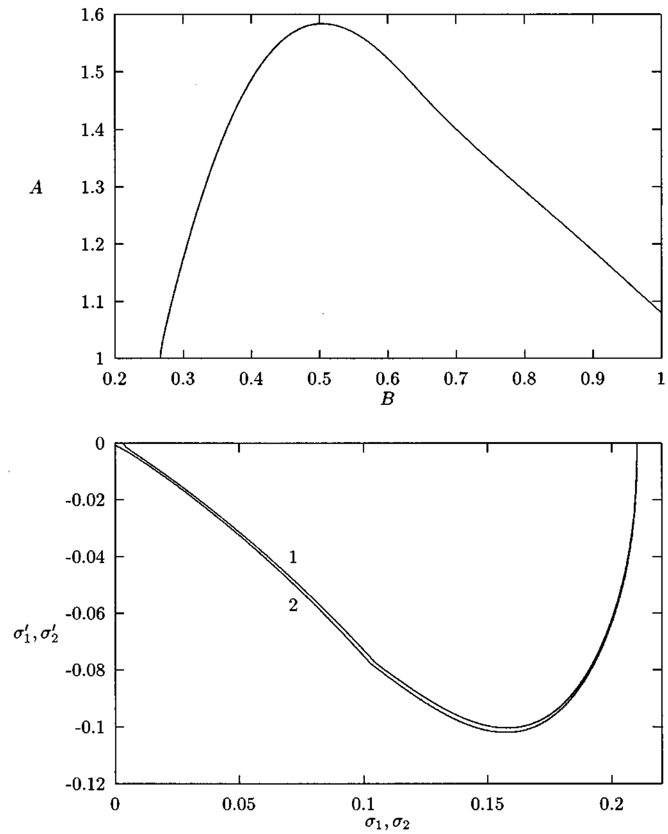

FIG. 6. Phase space diagrams (a) for the metric coefficients and (b) for the scalar fields.

increasing of the interaction, given by $\bar{\alpha}$, causes the type I star radius to decrease, reaching values that are smaller than that for the fermion star radius. The opposite occurs for the type II star radius; the increasing of the interaction causes the fermion star radius to be overcome by that of the type II star, which, under these conditions, makes an envelope of type II external to the fermion star.

As suggested by Liddle and Madsen [3], the stability can also be established by finding the asymptotic solutions analytically. Considering that s_1, s_2, and t vanish as $x \to \infty$, and $A \to 1$ and $B \to 1$ as $x \to \infty$, Eqs. (19)–(25) give

$$\sigma_i'' = \left(-\mathrm{w}^2 + 1 - \frac{\bar{\alpha}\mathrm{w}}{\sqrt{1-n}} \right) \sigma_i \quad (i = 1,2). \tag{31}$$

and the solutions are

$$\sigma_i \sim e^{-\theta x}, \tag{32}$$

where $\theta = \sqrt{\mathrm{w}^2 + a\mathrm{w} - 1}$ and $a = \bar{\alpha}/\sqrt{1-n}$. If θ is complex, σ_i oscillates and the stability is doubtful. Hence, w is restricted to the range

$$-\frac{-a - \sqrt{a^2+4}}{2} < \mathrm{w} < \frac{-a + \sqrt{a^2+4}}{2}. \tag{33}$$

When $a \ll 1$, the eigenvalues are restricted to the interval $-1 < \mathrm{w} < 1$, as obtained in Ref. [3]. In this case, for type I stars the ground state and the excited eigenvalues are presented as $\mathrm{w}_0 < \mathrm{w}_1 < \mathrm{w}_2 < \cdots < 1$. Meanwhile, when $a \sim e^{-3}$,

$$\mathrm{w}_0 < \mathrm{w}_1 < \mathrm{w}_2 < \cdots < 1 - \delta\mathrm{w},$$

where $\delta\mathrm{w}$ measures the peripheral shift of the positive root of the function $\theta(\mathrm{w})$. Thus, for positive energies, corresponding to the type n star case

$$\frac{-a + \sqrt{a^2+4}}{2} + \delta\mathrm{w} = 1$$

and, since we are considering $a < 1$,

$$\delta\mathrm{w} = \frac{a}{2} - \frac{a^2}{16} + O(a^4).$$

We can observe that $\delta\mathrm{w}$ is larger in the region where the interaction causes the splitting between σ_1 and σ_2 to be larger. Now, one can see that a qualitative analysis about the effects of the velocity of the fermion fluid on the splitting of the fields is available: the greater is the parameter n, the smaller are the values necessary for $\bar{\alpha}$ to cause the peripheral shift pronounced. This eliminates the necessity of plotting different curves for different values of the fermion fluid velocities, for comparision purposes.

TABLE II. Values for mass, number of particles and radius for equilibrium configuration of type I stars in Table I. Notice that not all configurations are stable.

$\bar{\alpha}$	M	$m\|N_B\| + m_n N_F$	mR	E_b
e^{-15}	0,38300	0,38939	1,62674	<0
e^{-6}	0,38238	0,38849	1,62446	<0
e^{-3}	0,37094	0,37139	1,57159	<0
e^{-2}	0,35299	0,34508	1,49618	>0
e^{-1}	0,31299	0,29113	1,25427	>0

V. CONCLUDING REMARKS

In this work we studied a model for boson-fermion stars with interaction, in the region of low frequencies, and we observed the behavior of the system for increasing values of the coupling constant $\bar{\alpha}$. At small values of $\bar{\alpha}$ there is no significant change in the system; as $\bar{\alpha}$ becomes larger and larger the ground-state energy of the system increases and

TABLE III. The same results as those in Table II for type II stars. All configurations are possibly stable.

$\bar{\alpha}$	M	$m\|N_B\| + m_n N_F$	mR	E_b
e^{-15}	0,38297	0,38939	1,62677	<0
e^{-6}	0,38378	0,39057	1,63229	<0
e^{-3}	0,39647	0,41008	1,68681	<0
e^{-2}	0,42106	0,44982	1,78725	<0
e^{-1}	0,48669	0,57737	1,95772	<0

the fermion energy density and pressure vanish at smaller distances. As a result, the fermion star is confined to a smaller region, after which the scalar fields are still present. We would also expect an enhancement in the emission rate of gravitational waves, thus increasing the probability of detecting such stellar objects.

When $\bar{\alpha}$ reaches a critical value, the ground-state energy of the system suddenly decreases, indicating the possible occurrence of a second-order phase transition. In this case, we might have supplied enough energy to bind bosons and fermions together. We have also observed that this occurs for different values of $\bar{\alpha}$ when considering separately the cases of type I or type II stars.

The presence of the interaction, in addition to the splitting between the scalar fields, makes type I ($-i\omega\tau$ ansatz) and type II ($+i\omega\tau$ ansatz) cases distinct.

There is a different increase for the ground-state energy for these cases when the interaction grows and the configurations exhibit distinct mass, radius and number of particles. This structure difference leads us to conclude that in a configuration where type I and type II are present, there is an asymmetry resulting from the interaction with fermions; in particular we have shown that type II stars may be stable in a larger range of values for $\bar{\alpha}$.

It is interesting to point out that, although the interaction studied in this work is different from the interaction proposed in Ref. [2], the numerical results do agree with the estimates made by Henriques, Liddle, and Moorhouse. We obtain non-negligible effects by the introduction of the interaction term in the case of light bosons, in which the Compton wavelength turns out to be of the order of the fermion star radius as discussed in Ref. [2].

Besides, as pointed out by Liddle and Madsen [3], the complex scalar field theory is described by both particles and antiparticles, and similarly one should expect the formation of an object composed by bosons and antibosons. The ''effective'' charge is proportional to the number of particles and antiparticles

$$N = N_{\phi} - N_{\bar{\phi}}, \tag{34}$$

where N_{ϕ} and $N_{\bar{\phi}}$ refers to bosons and antibosons, respectively. In this case, an asymmetry between the number of particles in type I and type II stars might contribute for the formation of the object [10]. In fact, for type I stars we get $N>0$ and for type II stars $N<0$, but equivalent configurations (same $\bar{\alpha}$ and initial values) of these types of stars do not have the same value of $|N|$. From the results outlined above it is possible to estimate a similar asymmetry; for instance, in the critical case when $\bar{\alpha} \sim e^{-2}$ the asymmetry between type I and type II gives $N_{\mathrm{I}}/N_{\mathrm{II}} = 1.63$.

It is also of importance knowing if $\bar{\alpha}$ is experimentally expected to be in the range to which the asymmetry is related. Pisano and Tomazelli [8] have reinterpreted the boson-fermion contact interaction in the context of the minimal supersymmetric standard model. They have determined that $\bar{\alpha} \lesssim 10^{-19}$, and it is beneath the expectations for the transition. Nevertheless, they have considered flat space-time values and the experimental expected value for the boson mass, $m \sim 45$ GeV; the present work considers light bosons as proposed in Ref. [2], $m \sim 5\times 10^{-11}$ eV, which gives the bound $\bar{\alpha} \lesssim 90$, using the same prescription as in Ref. [8].

Another interesting matter is to develop a qualitative approach to our evolution equations with Schwarzschild metric, in order to compare the phase diagrams with our corresponding numerical results. It may be also of interest to search for similar effects in the realm of solid state physics, since many theories use slave bosons in contact with fermions.

A more general treatment of boson-fermion gravitationally bounded systems should incorporate spin effects by considering the full Dirac Lagrangian. However, it would be necessary to extend the system of differential equations from seven to fifteen, which might demand a great effort.

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