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Rapid cooling of magnetized neutron stars

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The neutrino emissivities resulting from direct Urca processes in neutron stars are calculated in a relativistic Dirac-Hartree approach in the presence of a magnetic field. In a quark or a hyperon matter environment, the emissivity due to the nucleon direct Urca process is suppressed relative to that from pure nuclear matter due to the appearance of new degrees of freedom. In all the cases studied, the magnetic field enhances emissivity compared to the field-free cases because of phase space modifications and a shift in the dynamical β-equilibrium condition. [S0556-2821(98)50622-4]

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Neutron stars are born in the aftermath of supernova explosions with interior temperatures $T \gtrsim 10^{11}$ K, but cool rapidly in a few seconds by predominant neutrino emission [1] to $T < 10^{10}$ K. Neutrino cooling then dominates and lasts for $t \sim 10^5 - 10^6$ yr and subsequently photon emission takes over when $T \lesssim 10^8$ K. Since the long term cooling of the young neutron stars ($T \sim 10^8 - 10^{10}$ K) proceeds via emission of neutrinos primarily from matter at supranuclear densities within the core, the study of the cooling of neutron stars by examination of neutrino emissivities may provide considerable insight into their interior structure and composition.

For a long time, the dominant neutrino cooling mechanism has been the so-called standard model based on the modified Urca processes [2]

$$(n,p) + n \rightarrow (n,p) + p + e^- + \overline{\nu}_e,$$

$$(n,p) + p + e^- \rightarrow (n,p) + n + \nu_e. \tag{1}$$

The ROSAT detection [3] of thermal emission from neutron stars indicates the necessity of faster cooling mechanism in some young neutron stars, in particular the Vela pulsar. Faster neutrino emission than the standard model was proposed by invoking pion [4] or kaon [5] condensates which have neutrino emissivities comparable to that from the β decay of quarks [6] in quark matter (consisting of u, d, and s quarks)

$$d(s) \rightarrow u + e^- + \overline{\nu}_e, \quad u + e^- \rightarrow d(s) + \nu_e. \tag{2}$$

The most powerful energy losses, expected to date, are produced by the so-called direct Urca mechanism involving nucleons [7]:

$$n \rightarrow p + e^- + \overline{\nu}_e, \quad p + e^- \rightarrow n + \nu_e. \tag{3}$$

The threshold density for this process is considerably larger than that of the modified Urca process, if it is at all reached depending on the equation of state (EOS).

Observations of pulsars predict large surface magnetic field of $B_m \sim 10^{14}$ G [8]. In the core, the field may be considerably amplified due to flux conservation from the original weak field of the progenitor during its core collapse. In fact, the scalar virial theorem [9] predicts large interior field $B_m \sim 10^{18}$ G or more [10], and these fields are frozen in the highly conducting core. It has been demonstrated [10] that when the field B_m is comparable to or above a critical field $B_m^{(c)}$, the energy of a charged particle changes significantly in the quantum limit.

In this communication, we evaluate the neutrino emissivities for the nucleon and quark direct Urca processes of Eq. (3) in presence of a magnetic field B_m. For this purpose, we consider a npe matter in β-equilibrium within a Dirac-Hartree approach in the linear σ-ω-ρ model [11] and quark matter in the bag model.

At the neutron star core at temperatures well below the typical Fermi temperature of $T_F \sim 10^{12}$ K, the nucleons and electrons participating in neutrino producing processes are all degenerate and have their momenta close to the Fermi momenta p_{F_i}, where $i = n,p,e$. Since neutrino and antineutrino momenta are $\sim kT/c \ll p_{F_i}$, the nucleon direct Urca process is allowed by the momentum conservation when $p_{F_p} + p_{F_e} \geqslant p_{F_n}$. Since matter is very close to β-equilibrium, the chemical potentials of the constituents satisfy the condition $\mu_n = \mu_p + \mu_e$. (Henceforth, we set $\hbar = c = k = 1$.)

Employing the Weinberg-Salam theory for weak interactions, the interaction Lagrangian density for the charged current reaction (3) may be expressed as $\mathcal{L}_{\rm int}^{\rm cc} = (G_F/\sqrt{2}) \cos\theta_c l_\mu j_W^\mu$, where $G_F \simeq 1.435 \times 10^{-49}$ erg cm^{-3} is the Fermi weak coupling constant and θ_c the Cabibbo angle. The lepton and nucleon charged weak currents are, respectively, $l_\mu = \overline{\psi}_4 \gamma_\mu (1-\gamma_5) \psi_2$ and $j_W^\mu = \overline{\psi}_3 \gamma^\mu (g_V - g_A \gamma_5) \psi_1$. Here, and in other formulas to follow, the indices $i = 1-4$ refer to n, $\overline{\nu}_e$, p, and e, respectively. The vector and axial-vector coupling constants are $g_V = 1$ and $g_A = 1.226$.

The emissivity due to the antineutrino emission process in presence of a uniform magnetic field B_m along z axis when both the electrons and protons are Landau quantized is given by

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$$\varepsilon_\nu(B_m)=2\int\frac{Vd^3p_1}{(2\pi)^3}\int\frac{Vd^3p_2}{(2\pi)^3}\int_{-qB_mL_x/2}^{qB_mL_x/2}\frac{L_ydp_{3y}}{2\pi}\times\int_{-p_{F_p}}^{p_{F_p}}\frac{L_zdp_{3z}}{2\pi}\int_{-qB_mL_x/2}^{qB_mL_x/2}\frac{L_ydp_{4y}}{2\pi}\int_{-p_{F_e}}^{p_{F_e}}\frac{L_zdp_{4z}}{2\pi}\times\sum_{\eta=0}^{\eta_{\max}}\sum_{\eta'=0}^{\eta'_{\max}}E_2W_{fi}f(\mathbf{p}_1)[1-f(\mathbf{p}_3)][1-f(\mathbf{p}_4)], \tag{4}$$

where $\eta_{\max}$ and $\eta'_{\max}$ are, respectively, the maximum number of Landau levels populated for protons and electrons. The prefactor 2 takes into account the neutron spin degeneracy. The $p_i\equiv(E_i,\mathbf{p}_i)$ are the 4-momenta and E_2 the antineutrino energy. The functions $f(E_i)$ denote the Fermi-Dirac functions for the ith particle. The transition rate per unit volume due to the antineutrino emission process may be derived from Fermi's ''golden rule'' and is given by $W_{fi}=\langle|\mathcal{M}_{fi}|^2\rangle/(tV)$. Here t represents time and $V=L_xL_yL_z$ the normalization volume. $|\mathcal{M}_{fi}|^2$ is the squared matrix element and the symbol $\langle\cdot\rangle$ denotes an averaging over initial spin of n and a sum over spins of final particles (e and p) having spin degrees of freedom one for the ground Landau level and two otherwise. The matrix element for the $V-A$ interaction is given by

$$\mathcal{M}_{fi}=\frac{G_F}{\sqrt{2}}\int d^4X\ \overline{\psi_1}(X)\gamma^\mu(g_V-g_A\gamma_5)\psi_3(X)\ \overline{\psi_2}(X)\times\gamma_\mu(1-\gamma_5)\psi_4(X). \tag{5}$$

In presence of a uniform magnetic field B_m, the normalized proton wave function is $\psi_3(X)=(1/\sqrt{L_yL_z})\exp(-iE_3t+ip_{3y}y+ip_{3z}z)f_{p_{3y},p_{3z}}(x)$, where $f_{p_{3y},p_{3z}}(x)$ is the 4-component spinor solution [10]. The form of the spinor in B_m (see Ref. [10]) restricts the analytical evaluation of the neutrino emissivity to fields strong enough so as to populate only the ground state for electrons and protons, i.e., $\eta=\eta'=0$. The only positive energy spinor for protons in the chiral representation is then [10,12]

$$f^{\eta=0}_{p_{3y},p_{3z}}(x)=N_{\eta=0}\begin{pmatrix}E_3^*+p_{3z}\\0\\-m^*\\0\end{pmatrix}I_{\eta=0;p_{3y}}(x), \tag{6}$$

where $N_{\eta=0}=1/\sqrt{2E_3^*(E_3^*+p_{3z})}$, and $E_3^*=E_3-U^H_{0;p}=(p_{3z}^2+m^{*2})^{1/2}$ is the effective relativistic Hartree energy. The function $I_{\eta=0;p_{3y}}(x)$ is similar in form as in Ref. [10]. The nucleon effective and rest masses are, respectively, m^* and $m=m_n=m_p=939$ MeV. In presence of B_m, the wave functions for free electrons $\psi_4(X)$ have the same form as those for protons, but with m^* and E_3 for protons replaced by the bare mass m_e and kinetic energy for electrons, respectively. The neutrons and neutrinos/antineutrinos being unaffected by B_m, have plane wave functions.

Using these wave functions, the transition rate per unit volume is given by

$$W_{fi}=\frac{G_F^2}{E_1^*E_2E_3^*E_4}\frac{1}{V^3L_yL_z}\times\exp\left(-\frac{(p_{1x}-p_{2x})^2+(p_{3y}+p_{4y})^2}{2qB_m}\right)\times[(g_V+g_A)^2(p_1\cdot p_2)(p_3\cdot p_4)+(g_V-g_A)^2(p_1\cdot p_4)(p_3\cdot p_2)-(g_V^2-g_A^2)m^{*2}(p_4\cdot p_2)](2\pi)^3\delta(E_1-E_2-E_3-E_4)\times\delta(p_{1y}-p_{2y}-p_{3y}-p_{4y})\delta(p_{1z}-p_{2z}-p_{3z}-p_{4z}). \tag{7}$$

Substituting Eq. (7) in Eq. (4), and by the change of variable $(p_{3y}+p_{4y})\to p_{3y}$, the integration over dp_{4y} can be performed to yield a factor qB_mL_x. The rest of the integrals of Eq. (4) can then be performed in the standard manner [6]. Electron capture gives the same emissivity as neutron decay, although in neutrinos, and thus the total emissivity (relativistically) for the direct Urca process in nuclear matter (NM) in B_m is $\varepsilon^{\rm NM}_{\rm Urca}(B_m)=2\varepsilon_\nu(B_m)$, i.e.,

$$\varepsilon^{\rm NM}_{\rm Urca}(B_m)=\frac{457\pi}{5040}G_F^2\cos^2\theta_c\ (qB_m)\left[(g_V+g_A)^2\left(1-\frac{p_{F_p}}{\mu_p^*}\right)+(g_V-g_A)^2\left(1-\frac{p_{F_n}}{\mu_n^*}\cos\theta_{14}\right)-(g_V^2-g_A^2)\frac{m^{*2}}{\mu_n^*\mu_p^*}\right]\times\exp\left[\frac{(p_{F_p}+p_{F_e})^2-p_{F_n}^2}{2qB_m}\right]\frac{\mu_n^*\mu_p^*\mu_e}{p_{F_p}p_{F_e}}T^6\Theta, \tag{8}$$

where $\mu_i^*=(p_{F_i}^2+m^{*2})^{1/2}$ and $\cos\theta_{14}=(p_{F_n}^2+p_{F_e}^2-p_{F_p}^2)/2p_{F_n}p_{F_e}$. The threshold factor is $\Theta=\theta(p_{F_p}+p_{F_e}-p_{F_n})$, where $\theta(x)=1$ for $x>0$ and zero otherwise. For $B_m=0$, the relativistic expression for the neutrino emissivity from the nucleon direct Urca process is

$$\varepsilon^{\rm NM}_{\rm Urca}(B_m=0)=\frac{457\pi}{10080}G_F^2\cos^2\theta_c\left[(g_V+g_A)^2\left(1-\frac{p_{F_p}}{\mu_p^*}\cos\theta_{34}\right)+(g_V-g_A)^2\left(1-\frac{p_{F_n}}{\mu_n^*}\cos\theta_{14}\right)-(g_V^2-g_A^2)\frac{m^{*2}}{\mu_n^*\mu_p^*}\right]\mu_n^*\mu_p^*\mu_eT^6\Theta. \tag{9}$$

It was shown [6] that quark matter (QM), if present, the β-decay (i.e., direct Urca process) of d quarks is kinematically allowed through reaction (2) if finite mass (and/or quark-quark interaction) is incorporated. The relativistic expression of the neutrino emissivity for the direct Urca process involving u and d quarks for $B_m=0$ and without quark-quark interaction is given by [6]

$$\varepsilon_{\rm Urca}^{\rm QM}(B_m=0)=\frac{457\pi}{840}G_F^2\cos^2\theta_c(1-\cos\theta_{34})\mu_d\mu_u\mu_e T^6, \tag{10}$$

in the usual notation [6]. The emissivity for the β decay of s quark for $B_m=0$ is similar to Eq. (10) with $\cos\theta_c$ replaced by $\sin\theta_c$. For values of B_m used here [see Eq. (12)], only the ground states of e and u quark are populated, while for d (or s) quark large number of Landau levels [$\eta\propto 1/(|q|B_m)$; see Ref. [10]] are populated, and hence, behaves as a field-free particle with plane wave function as for neutrons. The emissivity for the β decay of free d quark in B_m may then be obtained from Eq. (8) by substituting $g_V=g_A=1$ with $\mu_n^*\to\mu_d$, $\mu_p^*\to\mu_u$, and multiplying a color factor 3 for d quark:

$$\varepsilon_{\rm Urca}^{\rm QM}(B_m)=\frac{457\pi}{420}G_F^2\cos^2\theta_c\,(qB_m)\left(1-\frac{p_{F_u}}{\mu_u}\right)\times\exp\left[\frac{(p_{F_u}+p_{F_e})^2-p_{F_d}^2}{2qB_m}\right]\frac{\mu_d\mu_u\mu_e}{p_{F_u}p_{F_e}}T^6. \tag{11}$$

Similar expression is obtained for s quark, but is Cabibbo suppressed. The kinematical conditions for the decay of d and s quarks are $p_{F_u}-p_{F_e}\leqslant p_{F_d}\leqslant p_{F_u}+p_{F_e}$ and $p_{F_u}-p_{F_e}\leqslant p_{F_s}\leqslant p_{F_u}+p_{F_e}$.

To estimate numerically the various neutrino emissivities for the direct Urca processes with and without magnetic field in a neutron star, we describe the nuclear matter and electrons within the relativistic Hartree approach in the linear σ-ω-ρ model [10,12] which provides a good description of normal nuclear matter and neutron star properties at high densities. The values for the dimensionless coupling constants for the σ, ω and ρ mesons are adopted from Ref. [13] which are determined by reproducing the nuclear matter properties at a saturation density of $n_0=0.16$ fm^{-3}. The variation of B_m with density n_b from surface to center of the star is parametrized by the form [12]

$$B_m(n_b/n_0)=B_m^{\rm surf}+B_0[1-\exp\{-\beta(n_b/n_0)^\gamma\}], \tag{12}$$

where the parameters are chosen to be $\beta=10^{-4}$ and $\gamma=6$. The maximum field prevailing at the center is taken as $B_0=5\times10^{18}$ G and the surface field is $B_m^{\rm surf}\simeq10^8$ G. The number of Landau levels populated for a given species is determined by the B_m and n_b [10].

In Fig. 1, we show the neutrino emissivity as a function of baryon density at $B_m=0$ [see Eq. (9)] for the direct Urca process in nuclear matter (NM) at an interior temperature $T=10^9$ K. Due to momentum conservation, the threshold density at which this process occurs, is at $n_t=0.346$ fm^{-3}. The variation of emissivity with n_b in B_m [see Eq. (8)] as seen in the figure may be explained as follows: At very low densities $n_b\sim0.35$–0.73 fm^{-3}, the field B_m [given in Eq. (12)] is rather small $\lesssim10^{18}$ G, and consequently a large number of Landau levels are populated. This gives essentially field-free results, i.e., $\varepsilon(B_m)$ and threshold density same as that for $B_m=0$. At densities $n_b\geqslant0.75$ fm^{-3}, the field is strong enough to populate only the ground levels of both electrons and protons [10], the critical field for electron is $B_m^{(e)(c)}=4.414\times10^{13}$ G. The emissivity then rapidly increases with density and could have values as high as ~2 orders of magnitude larger than $B_m=0$ case at $n_b\approx1.2$ fm^{-3}. This enhancement may be attributed to the phase space modifications and to the change in the dynamical β-equilibrium condition [10,12]. Comparing Eqs. (8) and (9), we have found that the factor $(qB_m)\exp[\{(p_{F_p}+p_{F_e})^2-p_{F_n}^2\}/2qB_m]$ is mainly responsible for this dramatic enhancement at this density. Hereafter, B_m saturates to a maximum of 5×10^{18} G so that for $n_b>1.2$ fm^{-3}, higher level states start to populate, and, as in the low density situation, results in field-free emissivity values. The central densities n_c of neutron stars with maximum masses are also shown in Fig. 1 with (open circles) and without (solid circles) the magnetic field. For $B_m\neq0$ star, $n_c=1.448$ fm^{-3} and thus falls above the kernel of enhanced emissivity leading to faster cooling compared to the field-free case.

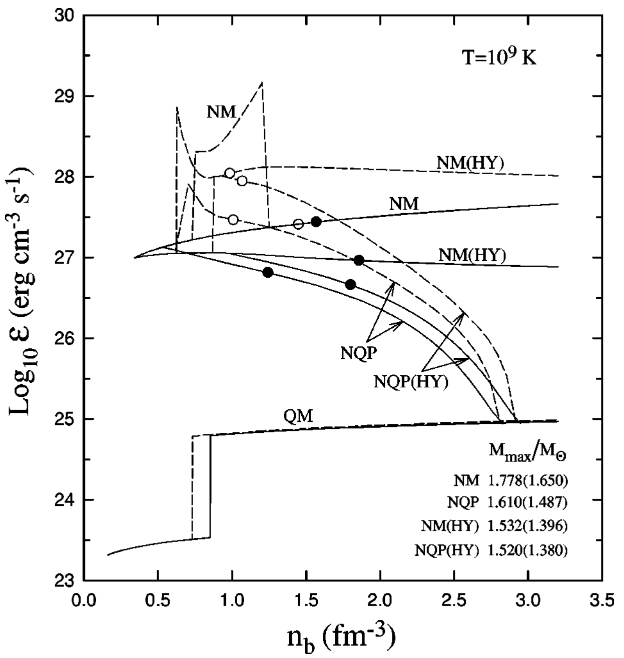

FIG. 1. The neutrino emissivities as a function of baryon density from the direct Urca process for a magnetic field $B_m=0$ (solid line) and for $B_m=10^8-5\times10^{18}$ G (dashed line) for: nucleons in nuclear matter (NM); quarks in quark matter (QM); a nucleon-quark phase transition (NQP); nucleons in nuclear matter with hyperons [NM(HY)]; a nuclear matter with hyperons to quark phase transition [NQP(HY)]. The maximum masses of the stars $M_{\rm max}$ with these various compositions are given for $B_m=0$ and those in the parentheses are for $B_m=10^8$–5×10^{18} G. The corresponding central densities are indicated by solid and open circles, respectively.

The neutrino energy losses from direct Urca processes of quark matter composed of free u, d, and s quarks and e are estimated in the bag model. The current masses of the quarks are taken as $m_u = 5$ MeV, $m_d = 10$ MeV, and $m_s = 150$ MeV, and the bag constant as $B = 250\ \mathrm{MeV\,fm^{-3}}$. In Fig. 1, we display the neutrino emissivity from the β-decay of d and s quarks at $B_m = 0$ [see Eq. (10)] in quark matter (QM). The d quark β-decay reactions are kinematically allowed if $n_b \gtrsim n_0$. At densities $n_b \simeq 0.85\ \mathrm{fm}^{-3}$ and above when s quark decay is allowed, the emissivity is increased to about an order of magnitude in spite of Cabibbo suppression. This is caused by the large s quark mass which allows the momenta of the free particles to deviate appreciably from collinearity which tends to increase the matrix element $[\sim(1-\cos\theta_{34})]$. It was, however, shown [6] that by the inclusion of quark-quark interaction, the neutrino emissivities from d and s quark β-decay are comparable in magnitude. The emissivities for the quark direct Urca processes in presence of the magnetic field [see Eq. (11)], remain virtually unaltered from the field-free case due to the population of a large number of levels in all the quark species. In either case, it is found that $\varepsilon^{\mathrm{QM}}_{\mathrm{Urca}}/\varepsilon^{\mathrm{NM}}_{\mathrm{Urca}} \lesssim 10^{-3}$. It may however be noted that the threshold density for s quark in B_m shifts to a lower value which depends sensitively on the parameters β and γ of Eq. (12) (see also Ref. [12]). This shift is caused by the large electron abundances, in particular, which alter the kinematical condition for s quark decay at this density and field.

In a realistic situation, if quarks at all exist, a star with increasing density from the surface to the center would have a pure nucleon phase at the inner crust and core with a possible pure quark phase at the center and a mixed nucleon-quark phase (NQP) in between. The mixed phase of nucleons and quarks is described following Glendenning [13]. The conditions of global charge neutrality and baryon number conservation are $\chi Q^n + (1-\chi)Q^q = 0$ and $n_b = \chi n_b^n + (1-\chi)n_b^q$, where χ represents the fractional volume occupied by the hadron phase. Furthermore, the mixed phase satisfies the Gibbs' phase rules: $\mu_p = 2\mu_u + \mu_d$ and $P^n = P^q$. The neutrino energy loss rate in this phase is given by $\varepsilon^{\mathrm{NQP}}_{\mathrm{Urca}} = \chi\varepsilon^{\mathrm{NM}}_{\mathrm{Urca}} + (1-\chi)\varepsilon^{\mathrm{QM}}_{\mathrm{Urca}}$. The neutrino emissivities for the nucleon-quark phase transition are shown in Fig. 1 (denoted by NQP). For $B_m = 0$ case, with the appearance of the quarks at $n_b = 0.533\ \mathrm{fm}^{-3}$, the emissivity decreases from the corresponding NM case. Apart from the reduced emissivity of the quark phase (which being, however small at large χ), the reduction in the chemical potentials of the nucleons and electrons resulting from the requirement of the global charge neutrality and baryon number conservation conditions in the mixed phase, primarily causes the decrease in emissivity in the NM sector and thereby the total emissivity $\varepsilon^{\mathrm{NQP}}_{\mathrm{Urca}}$. For stars with $B_m \neq 0$, the emissivities in the mixed phase are increased due to its enhancement in the NM sector in B_m and also for the kinematical change in the threshold density for s quark decay as described above. The central densities of maximum mass stars fall within the mixed phase, and consequently such stars would have faster cooling than pure quark stars. The maximum mass NQP stars with and without magnetic field, however, have much smaller emissivities than that of the corresponding NM stars, while NQP stars with B_m have nearly identical cooling as that of field-free NM stars even though their maximum masses are very distinct.

We now explore the effect of strange baryons, namely hyperons (Λ's, Σ's and Ξ's) on the nucleon direct Urca process. The β-equilibrium conditions then generalize to $\mu_i = b_i\mu_n - q_i\mu_e$, where b_i and q_i are the baryon number and charge for the ith particle. Since the hyperons are more massive than the protons, the effect of the magnetic field on their direct Urca processes is negligible. Because of the large uncertainties in the hyperon-nucleon interactions even at nuclear density, for a conservative estimate of the emissivities, we set the nucleon-meson and hyperon-meson coupling constants equal. Furthermore, the critical density for nucleon direct Urca process is nearly identical to the hyperon threshold density in the relativistic mean field model, and the emissivities from the hyperon direct Urca processes are about 5–100 times less than that from the nucleons [14]. Therefore, we shall present ε vs n_b results only for the nucleon direct Urca process in presence of hyperons. This is shown in Fig. 1 and denoted by NM(HY). With the appearance of hyperons, the reduction in the chemical potentials of the nucleons and electrons required by the baryon number conservation and charge neutrality condition causes a substantial reduction of the emissivity compared to that from NM. In fact, with increasing density when hyperon abundances grow rapidly, the emissivities gradually decrease. For $B_m \neq 0$, only the ground Landau levels for e and p are populated over a considerable density range in this matter. Consequently, the emissivities with B_m in NM(HY) stars are significantly larger than that for the corresponding field-free stars.

Allowing now baryon to quark phase transition, the emissivity displayed in Fig. 1 [denoted by NQP(HY)] for $B_m = 0$ is larger than that from the NQP matter. This is caused by the delayed appearance of quarks in hyperon rich matter, so that the the total emissivity is primarily dominated by the nucleons. In presence of the field, the total emissivity of NQP(HY) matter is about an order of magnitude larger for the maximum mass star and therefore leads to faster cooling compared to the corresponding field-free star.

Using the nonrelativistic limit of the present EOS for the specific heat c_v and emissivity, the time for the center of a NM star to cool by the direct Urca process to a temperature T_9 at $B_m = 0$ may be estimated to be $\Delta t = -\int(c_v/\varepsilon^{\mathrm{NM}}_{\mathrm{Urca}})dT \sim 10T_9^{-4}$ s. In contrast, for $B_m \neq 0$, the NM star's center cools faster with $\Delta t \sim 0.5T_9^{-4}$ s. By invoking quarks and/or hyperons, the decrease in emissivity is much more compared to that of the specific heat resulting in slow cooling of the star's center. The typical time scale associated with the propagation of thermal signals through the outer core and the crust to the surface before the sudden temperature drop is quite high ~ 1 to 100 yr, depending on the crustal composition and relative sizes of the crust and the core and thus upon the EOS. Therefore, it seems to be quite difficult to distinguish observationally from the effects of direct Urca process, the interior constitution of a star.

Throughout our discussion, we have assumed that the electron is the only lepton. If the triangle inequality $p_{F_p} + p_{F_\mu} \geqslant p_{F_n}$ is satisfied, then nucleon direct Urca process with muons will occur; the threshold density for this process is higher than electrons since $m_\mu > m_e$. The β-equilibrium condition $\mu_e = \mu_\mu$ moreover implies that the emissivity for Urca process with muons is same as that for the corresponding process with electrons. In the present model, the nucleon direct Urca processes are not permitted at densities $n_b < 0.34\ \mathrm{fm}^{-3}$.

At certain densities and temperature $T < T_c \approx 10^8 - 10^{10}$ K, the nucleon superfluidity may set in. The specific heat and direct Urca rate are then reduced by a factor $\sim \exp(-\Delta/T)$, where Δ is the larger of the neutron and proton gaps. The modified Urca rates are, however, reduced by a factor $\sim \exp(-2\Delta/T)$ for neutron superfluidity and by $\sim \exp(-\Delta/T)$ for superconductivity [15]. In presence of a magnetic field, the superfluid protons are believed to form a type II superconductor in the outer core within the density range $0.7 n_0 < n_b < 2 n_0$, and the estimated lower and upper critical magnetic fields are respectively, $H_{c1} \sim 10^{15}$ G and $H_{c2} \sim 3 \times 10^{16}$ G [16]. With the choice of variation of B_m with n_b [see Eq. (12)], the field is $10^{14} - 10^{16}$ G at the bulk of the outer core and could form a superconducting region at $T < T_c$, while the inner core and center with $B_m \sim 10^{18}$ G is in the normal state without superconductivity. The superconducting outer core region can, in principle, trap the much stronger field at the inner core and center preventing its decay to the surface [17]; the field at the surface is then the estimated value of $\sim 10^{14}$ G originating from crustal decay only. The variation of B_m for the parameter values β and γ used in Ref. [12] however does not provide such a mechanism.

In conclusion, the neutrino emissivities for all the cases studied here are found to be dramatically enhanced in a magnetic field compared to that from the nonmagnetized stars. However, for certain stars unambiguous determination of the interior constituents may be difficult. There can be stars with the same composition, as for example the NM stars in a magnetic field, but with slightly different masses of $1.55 M_\odot$ and $1.60 M_\odot$ having their central densities at $0.72\ \mathrm{fm}^{-3}$ and $0.88\ \mathrm{fm}^{-3}$ residing below and within the kernel of fast cooling respectively, and thus have completely different emissivities. On the other hand, a NM star with $B_m = 0$ and a NQP star in a magnetic field, though possessing different interior compositions, have nearly identical emissivities. It is also found that when pure nuclear matter is injected with nonleptonic negative charges, namely hyperons and quarks, the emissivities turn out to be smaller than that from the nuclear matter. It has been already demonstrated [18] that nonleptonic negative charges cause a softening of the equation of state. We thus arrive at a general result that when matter contains nonleptonic negative charges, the maximum masses of the stars are smaller with a suppression of the neutrino emissivity than that of the pure nuclear matter with and without a magnetic field.

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