PHYSICAL REVIEW D, VOLUME 58, 117301

Decay $\Omega^- \to \Xi^- \gamma$ and the $\Omega^-\Omega^-\gamma$ vertex

Alfonso Queijeiro
Departamento de Física, Escuela Superior de Física y Matemáticas del Instituto Politécnico Nacional, U. P. Adolfo López Mateos, Edif. 9, México D. F. 07738, Mexico

(Received 29 April 1998; revised manuscript received 30 June 1998; published 30 October 1998)

We review the radiative decay $\Omega^- \to \Xi^- \gamma$ assuming a general Lorentz covariant $\Omega^-\Omega^-\gamma$ electromagnetic vertex, beyond minimal coupling. We also consider the dependence on a parameter, related to the invariance under contact transformations, of the Feynman rules.
[S0556-2821(98)04221-0]

PACS number(s): 13.40.Hq, 13.30.−a, 13.40.Em

I. INTRODUCTION

The magnetic dipole moment of the spin-3/2 Ω^- hyperon is predicted to be $\mu_\Omega = -1.83\ \mu_p$ in the nonrelativistic quark model [1], and $\mu_\Omega = -\frac{2}{3}(M_p/M_\Omega)\mu_p = -0.36\ \mu_p$ from minimal coupling in the Rarita-Schwinger equation, where μ_p is the proton magnetic moment, and M_p and M_Ω are the masses of the proton and of the omega particles. The (central) experimental value [2] is $\mu_\Omega = -1.94\ \mu_p$, showing that the minimal coupling is far from giving the correct value. In fact, not even for an elementary spin-3/2 particle does minimal coupling give the correct gyromagnetic ratio $g=2$, making it necessary to add nonminimal couplings to render $g=2$, as has been emphasized in Ref. [3]. In this Brief Report we include two nonminimal terms, linear in the photon momentum and gauge invariant, in the $\Omega\Omega\gamma$ vertex to adjust the theoretical expression for μ_Ω to satisfy the experimental value. This vertex is used to calculate the branching ratio for the decay $\Omega^- \to \Xi^- \gamma$ in order to determine the two parameters introduced; however, we end with two possible sets of values, requiring another observable to properly define the vertex.

In Sec. II we give the most general Lorentz and gauge invariant electromagnetic vertex for a spin-3/2 charged particle, along with other related Feynman rules. We also discuss the dependence of this set of rules in the parameter associated with the invariance of any spin $\geqslant 3/2$ theory under contact transformations. With this in hand we calculate in Sec. III the radiative decay $\Omega^- \to \Xi^- \gamma$.

II. THE MAGNETIC DIPOLE MOMENT OF A SPIN-3/2 CHARGED PARTICLE

A spin-3/2 particle can be formulated [4] by the Rarita-Schwinger vector-spinor field ψ_μ. The most general form of the free particle Rarita-Schwinger Lagrangian is given by

$$L_0 = \overline{\psi^\mu}(x)\Lambda_{\mu\alpha}(D)\left[g^{\alpha\beta}(i\partial\!\!\!/ - M_\Omega) + \frac{1}{3}i(\gamma^\alpha \partial\!\!\!/ \gamma^\beta - \gamma^\alpha\partial^\beta - \gamma^\beta\partial^\alpha) + \frac{1}{3}M_\Omega\gamma^\alpha\gamma^\beta\right]\Lambda_{\beta\nu}(D)\psi^\nu(x), \tag{1}$$

where

$$\Lambda_{\mu\alpha}(D) = g_{\mu\alpha} + \frac{1+3D}{2}\gamma_\mu\gamma_\alpha \tag{2}$$

is the contact transformation, depending on the parameter D, which is arbitrary except that $D \neq -\frac{1}{2}$. The invariance under Eq. (2) is a consequence that the spin-1/2 component of the field ψ_μ must not mix with the spin-3/2 component. Furthermore, the field ψ_μ is constrained to satisfy

$$\partial^\mu\psi_\mu = \gamma^\mu\psi_\mu = 0. \tag{3}$$

The propagator from Eq. (1) is given by

$$i\Delta_{\mu\nu}(p,D) = \frac{\Sigma_{\mu\nu}(p,D)}{p^2 - M_\Omega^2} \tag{4}$$

with

$$\Sigma_{\mu\nu}(p,D) = \left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_\Omega}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) + \frac{2}{3M_\Omega^2}p_\mu p_\nu\right](p\!\!\!/ + M_\Omega) - \frac{1+D}{6(1+2D)}\frac{p^2 - M_\Omega^2}{M_\Omega^2}\left[2\gamma_\mu p_\nu + 2\gamma_\nu p_\mu - 2M_\Omega\gamma_\mu\gamma_\nu - \frac{1+D}{1+2D}\gamma_\mu(p\!\!\!/ - 2M_\Omega)\gamma_\nu\right]. \tag{5}$$

The minimal coupling receipt $\partial_\mu \to \partial_\mu - ieA_\mu$ in Eq. (1) gives the electromagnetic interaction vertex

$$ie\Gamma_{\nu\mu\alpha}(D) = \Lambda_{\nu\lambda}(D)\Gamma_\mu^{\lambda\sigma}\Lambda_{\sigma\alpha}(D), \tag{6}$$

where

$$\Gamma_\mu^{\lambda\sigma} = g^{\lambda\sigma}\gamma_\mu + \frac{1}{3}\gamma^\lambda\gamma_\mu\gamma^\sigma - \frac{1}{3}(\gamma^\lambda g_\mu^\sigma + \gamma^\sigma g_\mu^\lambda). \tag{7}$$

In Eq. (6) the index μ is related to the photon A_μ, and the indexes ν and α are related to the incoming and outgoing spin-3/2 particles in the vertex. As has been shown in Ref. [3] the magnetic dipole moment obtained from Eq. (7) is

$$\mu_\Omega = -\frac{2}{3}\frac{e}{2M_\Omega}$$
$$= -\frac{2}{3}\left(\frac{M_p}{M_\Omega}\right)\mu_p . \quad (8)$$

Thus, we need to add nonminimal terms to get the correct value for μ_Ω. This can be done considering terms linear in the photon momentum, consistent with gauge invariance

$$\Gamma^{\lambda\sigma}_\mu = g^{\lambda\sigma}\gamma_\mu + \frac{1}{3}\gamma^\lambda\gamma_\mu\gamma^\sigma - \frac{1}{3}(\gamma^\lambda g^\sigma_\mu + \gamma^\sigma g^\lambda_\mu) + \frac{\kappa}{M_\Omega}(k^\lambda g^\sigma_\mu - k^\sigma g^\lambda_\mu) + \frac{\kappa'}{M_\Omega} g^{\lambda\sigma}\not{k}\gamma_\mu . \quad (9)$$

In this way, we obtain for the magnetic dipole moment the expression

$$\mu_\Omega = -\left(\frac{M_p}{M_\Omega}\right)\mu_p\left(\frac{2}{3}+\kappa+\kappa'\right) \quad (10a)$$

and, using the experimental value

$$\kappa+\kappa' = 2.79. \quad (10b)$$

To calculate the decay $\Omega^-\to\Xi^-\gamma$ we also need the Feynman rule for the vertex $\Omega^-\Xi^-$. This was done in Ref. [4], which after taking into account Eq. (2) this results in

$$W_\nu(p,D) = \frac{i}{M_\Omega}(A+B\gamma_5)p^\lambda\Lambda_{\nu\lambda}(D). \quad (11)$$

In Ref. [4] the factors A and B are named parity-violating and parity-conserving amplitudes, while in Ref. [5] they are named p- and d-wave amplitudes, which are parity conserving and parity violating, respectively. (This apparent contradictory naming of these parameters resides in that in Ref. [4] the pion odd parity has been explicitly taken into account.) The values of A and B can be obtained from the experimental decay rate and the asymmetry parameter of the decay $\Omega^-\to\Xi^-\pi^0$. The asymmetry parameter for this decay is consistent [6] with zero, implying that A or B are zero. Then the experimental value for $\Gamma(\Omega^-\to\Xi^-\pi^0)$ gives

$$A = 1.17\times10^{-6} \quad \text{if } B=0 \quad (12)$$

and

$$B = 1.27\times10^{-5} \quad \text{if } A=0, \quad (13)$$

where A and B have been assumed real.

III. THE DECAY $\Omega^-\to\Xi^-\gamma$

As was shown in Ref. [4] there are four possible Feynman diagrams contributing to the radiative decay $\Omega^-\to\Xi^-\gamma$, one where the photon is emitted by the Ω^-, one from Ξ^- photon emission, one from emission of the photon from the weak vertex, and one where the photon is emitted from the Ξ^{-*} pole. However, due to Eq. (3) the second one vanishes, the third one can be computed from gauge invariance, and the last one vanishes due to conservation of U spin. Thus, we have for the first one the amplitude

$$\mathcal{M}_1 = \overline{u}(p_2)W_\alpha(p_2,D)\Delta^{\alpha\beta} \times(p_1-k,D)\Gamma_{\beta\mu\nu}(D)u^\nu(p_1)\varepsilon^\mu(k), \quad (14)$$

where $p_1=p_2+k$, p_2, and k are the momenta of Ω^-, Ξ^-, and the photon, respectively. After substitution of the tensors previously defined, and using Eq. (3), one obtains [7]

$$\mathcal{M}_1 = \frac{ie}{M_\Omega}\overline{u}(p_2)(A+B\gamma_5)p_{2\alpha}\Delta^{\alpha\beta} \times(p_1-k)\Gamma_{\beta\mu\nu}u^\nu(p_1)\varepsilon^\mu(k) \quad (15)$$

and the parameter D has been canceled out. For the other contribution one can write

$$\mathcal{M}_2 = ie\overline{u}(p_2)G_{\nu\mu}(p_1-k)u^\nu(p_1)\varepsilon^\mu(k). \quad (16)$$

Adding Eqs. (15) and (16) and imposing gauge invariance one obtains for the tensor $G_{\nu\mu}(p_1-k)$ the expression [8]

$$G_{\nu\mu}(p_1-k) = -\frac{1}{M_\Omega}(A+B\gamma_5)g_{\nu\mu}. \quad (17)$$

Finally, we get for the total amplitude

$$\mathcal{M} = \frac{ie}{M_\Omega}\overline{u}(p_2)(A+B\gamma_5)\Bigg[M_\Omega g_{\mu\nu} - \gamma_\mu k_\nu - \not{p}_2 g_{\mu\nu} + \frac{\kappa}{M_\Omega}(p_2^\alpha - \not{p}_2\gamma^\alpha)(g_{\alpha\mu}k_\nu - g_{\nu\mu}k_\alpha) + \frac{\kappa'}{M_\Omega}(p_2^\alpha - \not{p}_2\gamma^\alpha)g_{\alpha\nu}\not{k}\gamma_\mu\Bigg]u^\nu(p_1)\varepsilon^\mu(k). \quad (18)$$

The decay rate from Eq. (18) is

$$\Gamma(\Omega^-\to\Xi^-\gamma) = \frac{\alpha}{12}M_\Omega R(1-R^2)^3\Bigg\{2(A^2+B^2)(3+R^2) + \kappa[A^2(1+8R-R^2)+B^2(1-8R-R^2)] + \frac{\kappa^2}{2}[A^2(1-R)(2-R+3R^2-2R^3) + B^2(1+R)(2+R+3R^2-2R^3)]\Bigg\}, \quad (19)$$

where $R=M_\Xi/M_\Omega$, and no contribution from κ' is obtained. Using the physical masses, the branching ratio can be written as

$$B(\Omega^-\to\Xi^-\gamma) = 0.067 - 0.0619\kappa + 0.0117\kappa^2 \quad (20)$$

for $B=0$, and

$$B(\Omega^-\to\Xi^-\gamma) = 7.869 + 6.453\kappa + 3.577\kappa^2 \quad (21)$$

for $A=0$. If one assumes $B(\Omega^- \to \Xi^- \gamma)=0.002$ to be the upper experimental value, Eq. (21) leads to a complex value for κ, while Eq. (20) leads to

$$\kappa=1.44 \quad \text{or} \quad \kappa=4.10 \tag{22}$$

which in turn leads to

$$\kappa'=1.35 \quad \text{for} \quad \kappa=1.44 \tag{23}$$

or

$$\kappa'=-1.31 \quad \text{for} \quad \kappa=4.10 \tag{24}$$

consistent with Eq. (10b). Then, we have two possible sets of values for κ and κ', requiring another observable, like the electric quadrupole moment, to pick up the correct one.

ACKNOWLEDGMENTS

The author wishes to thank Dr. Gabriel López for a very illuminating discussion and to Comisión de Operación y Fomento de las Actividades Académicas del I. P. N. for partial support.

[1] L. Brekke and J. Rosner, Comments Nucl. Part. Phys. **18**, 83 (1988).

[2] H. T. Diehl *et al.*, Phys. Rev. Lett. **67**, 804 (1991).

[3] S. Ferrara, M. Porrati, and V. L. Telegdi, Phys. Rev. D **46**, 3529 (1992).

[4] L. M. Ram Mohan, Phys. Rev. D **3**, 2148 (1971).

[5] Y. Hara, Phys. Rev. **150**, 1175 (1966).

[6] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).

[7] Equation (15) is the amplitude for $D=-\frac{1}{3}$. This is so because to get Eq. (1) we started from the Rarita-Schwinger Lagrangian and then we performed the transformation in Eq. (2).

[8] The usual way to get Eq. (17) is via the principle of minimal coupling in Eq. (11) of the vertex $\Omega^-\Xi^-$, giving the result $G_{\nu\mu}(p_1-k,D)=-(1/M_\Omega)\,(A+B\gamma_5)\Lambda_{\nu\mu}(D)$. This procedure involves a little more algebra to get the result in Eq. (17) after taking into account the constraints in Eq. (3).