# Higgs-mediated flavor-changing neutral currents in the general framework with two Higgs doublets: An RGE analysis

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We consider the standard model with two Higgs doublets with the most general Yukawa coupling terms (''type III''). In the model, the neutral-Higgs-mediated flavor-changing neutral currents (FCNC's) are allowed, but must be reasonably suppressed at low energies of probes. It has been known that the existing hierarchies of quark masses render this suppression at low energies rather natural. On the other hand, the model has been regarded by many as unnatural because of the absence of any symmetry that would ensure the persistence of this suppression as the energy of the probes increases. The opinion has been based on the expectation that the mentioned FCNC's would increase by large factors at increasing energies. We perform a numerical analysis of the flow of these FCN coupling parameters as governed by the one-loop renormalization group equations (RGE's), in a simplified case when Yukawa couplings of the first quark generation are neglected. The analysis shows a remarkable persistence of the mentioned FCNC suppression and thus indicates that the model is not unnatural in the RGE sense. Further, we point out two mistakes in the Yukawa RGE's of Machacek and Vaughn at the one-loop level. [S0556-2821(98)00621-3]

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## I. INTRODUCTION

The (standard) model with two Higgs doublets whose Yukawa couplings in the quark sector have the most general form was apparently first introduced in 1973 by Lee [1]. His main motivation for introducing the model, later also known as the general two-Higgs-doublet model (G2HDM) or ''type III'' 2HDM, was to study new possible $CP$-violating phenomena. Others [2] continued investigating the phenomenology of the model along these lines.

Subsequently, Glashow and Weinberg [3] in 1977 stressed that only those models with two Higgs doublets whose Yukawa coupling sector possesses specific discrete [or equivalently U(1)-type] family symmetries lead to automatic and full suppression of the effective flavor-changing neutral (FCN) Yukawa parameters, and ensure this suppression at any energy of the probes. They pointed out that there are basically two types of such 2HDM's—the so called ''type I'' and ''type II'' models, in which either one Higgs doublet alone is responsible for all the quark masses [2HDM(I)] or one Higgs doublet is responsible for all the up-type quark masses and the other for the down-type quark masses [2HDM(II)]. This point of Glashow and Weinberg apparently had a great impact on the physics community, especially because most of the mentioned flavor-changing neutral currents (FCNC's) mediated by neutral Higgs boson[1] must be strongly suppressed at low energies of probes ($\lesssim E_{\rm ew}$) due to the firm experimental evidence of FCNC suppression. Consequently, the general 2HDM, which has no such automatic and full suppression of the FCN Yukawa coupling parameters, apparently was not investigated by physicists until the late 1980's.

Since the late 1980's, there has been a moderate resurgence of investigation of the G2HDM [4–7]. These works investigate *low-energy* phenomena ($E \lesssim E_{\rm ew}$) as predicted by G2HDM's, with most of the FCN Yukawa coupling parameters (at low energies of probes) being generally nonzero but reasonably suppressed.[2] Reference [4] investigates predictions of the model mainly for $CP$-violating, and Ref. [7] mainly for FCNC-violating phenomena. The resulting amplitudes then include FCN Yukawa coupling parameters at low energies of probes.

The authors Cheng, Sher, and Yuan (CSY) [5] offered arguments which render the G2HDM reasonably natural from the aspect of *low-energy* physics, thus countering one part of the reservations based on the arguments of Glashow and Weinberg [3]. CSY basically proposed specific *Ansätze*

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[1]A more precise expression would be ''neutral flavor-changing scalar (Yukawa) coupling,'' since these couplings have no four-vector current structure involving $\gamma^{\mu}$.

[2]Low-energy experiments show that those flavor-changing neutral coupling parameters which do not involve a $t$ quark are suppressed in nature at low energies $E \sim m_q$, while for those involving a $t$ quark there is no experimental evidence yet available.

 

for the Yukawa parameters in the G2HDM at low energies of probes, specifically the FCN Yukawa coupling parameters, motivated largely by the existing mass hierarchy of quarks. Therefore, their *Ansätze* are reasonably natural or, more conservatively, not ''unnatural.'' Motivation of their *Ansätze* did not explicitly involve any family symmetries. Moreover, they showed that their *Ansätze* allow the masses of neutral scalars to be as low as $\sim 10^2$ GeV while still not violating the available (low-energy) data on suppressed FCNC phenomena. Later on, Antaramian, Hall, and Rašin (AHR) [6] proposed somewhat similar (but not identical) *Ansätze*, which they motivated by their requirement that the Yukawa interactions have certain approximate flavor symmetries. The CSY and similar *Ansätze* were mainly used by other authors [4] and [7] in their investigations of low-energy phenomenology of the G2HDM.

We wish to reemphasize that the mentioned *Ansätze* countered only *one part* of the arguments (based on Ref. [3]) against the G2HDM. The symmetry arguments of Glashow and Weinberg [3] did not just suggest that a natural 2HDM should have a well motivated suppression of the flavor-changing neutral Yukawa parameters at *low* energies, but that these FCN parameters should remain suppressed also when the energy of probes increases. Based on this latter point of Ref. [3], a large part of physics community has continued regarding the G2HDM as unnatural. The main point against the G2HDM has consisted of the fear, or conjecture, that the FCN Yukawa coupling parameters in the G2HDM, even though suppressed at low energies by reasonably motivated arguments, would behave unnaturally as the energy of probes increases. Stated otherwise, it has been expected that at least some FCN Yukawa coupling parameters would increase by a large factor or even by orders of magnitude at increased energies (well below the Landau pole), due to the absence of explicit discrete [or U(1)-type] family symmetries in the Yukawa sector. The absence of such symmetries, according to the argument, would in general result in a strong ''pull-up'' effect on the small flavor changing by the much larger flavor-conserving Yukawa coupling parameters as the evolving energy increases. In such a case, the model would then generally contain a (thus unnatural) fine-tuning: large ''bare'' FCN Yukawa coupling parameters at high energies would have to be fine-tuned in order to obtain at low energies their phenomenologically acceptable suppression.

Therefore, we investigate this question in the present paper, by performing a numerical analysis of the one-loop renormalization group equations (RGE's) of the G2HDM. In Sec. II we present the model and write down conditions for the suppression of FCN Yukawa coupling parameters at low energies (CSY *Ansatz*). In Sec. III we write down the one-loop RGE's for the Yukawa coupling parameters in the G2HDM in a specific form convenient for numerical analyses. A short derivation of the RGE's is given in the Appendix. Section III contains comparisons with the existing literature on RGE's. In Sec. IV we then numerically investigate the RGE evolution of the Yukawa coupling parameters for quarks, neglecting for simplicity the Yukawa parameters of the light first generation of quarks. We present the resulting evolutions of the FCN Yukawa coupling parameters for various low-energy *Ansätze*, i.e., essentially for variations of the CSY *Ansätze*. We also observe some other interesting properties of the presented evolution. Section V contains a summary and conclusions.

## II. THE MODEL AND LOW ENERGY *ANSÄTZE*

Yukawa interactions for quarks in the G2HDM in any $SU(2)_L$ basis have the most general form

$$\mathcal{L}^{(E)}_{\rm G2HDM} = -\sum_{i,j=1}^{3} \{ \tilde{D}^{(1)}_{ij} (\bar{\tilde{q}}^{(i)}_L \Phi^{(1)}) \tilde{d}^{(j)}_R + \tilde{D}^{(2)}_{ij} (\bar{\tilde{q}}^{(i)}_L \Phi^{(2)}) \tilde{d}^{(j)}_R + \tilde{U}^{(1)}_{ij} (\bar{\tilde{q}}^{(i)}_L \tilde{\Phi}^{(1)}) \tilde{u}^{(j)}_R + \tilde{U}^{(2)}_{ij} (\bar{\tilde{q}}^{(i)}_L \tilde{\Phi}^{(2)}) \tilde{u}^{(j)}_R + \text{H.c.} \} + \{ \bar{\ell} \Phi \ell \ \text{ terms} \}. \tag{1}$$

The tildes above the Yukawa coupling parameters and above the quark fields mean that these quantities are in an arbitrary $SU(2)_L$ basis (i.e., weak basis, not the mass basis). The superscript $(E)$ at the Lagrangian density means that the theory has a finite effective energy cutoff $E$, and the reference to this evolution energy $E$ was omitted at the fields and at the Yukawa coupling parameters in order to have simpler notation ($E \sim 10^2$ GeV for renormalized quantities). The following notation is used:

$$\Phi^{(k)} \equiv \begin{pmatrix} \phi^{(k)+} \\ \phi^{(k)0} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^{(k)}_1 + i\phi^{(k)}_2 \\ \phi^{(k)}_3 + i\phi^{(k)}_4 \end{pmatrix}, \tag{2}$$

$$\tilde{\Phi}^{(k)} \equiv i\tau_2 \Phi^{(k)\dagger T} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^{(k)}_3 - i\phi^{(k)}_4 \\ -\phi^{(k)}_1 + i\phi^{(k)}_2 \end{pmatrix}, \tag{3}$$

$$\tilde{q}^{(i)} = \begin{pmatrix} \tilde{u}^{(i)} \\ \tilde{d}^{(i)} \end{pmatrix}: \ \tilde{q}^{(1)} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}, \ \tilde{q}^{(2)} = \begin{pmatrix} \tilde{c} \\ \tilde{s} \end{pmatrix}, \ \tilde{q}^{(3)} = \begin{pmatrix} \tilde{t} \\ \tilde{b} \end{pmatrix}, \tag{4}$$

$$\langle \Phi^{(1)} \rangle_0 = \frac{e^{i\eta_1}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \ \langle \Phi^{(2)} \rangle_0 = \frac{e^{i\eta_2}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \ v_1^2 + v_2^2 = v^2. \tag{5}$$

In Eq. (5), $v$ $[\equiv v(E)]$ is the usual vacuum expectation value (VEV) needed for the electroweak symmetry breaking, i.e., $v(E_{\rm ew}) \approx 246$ GeV. The phase difference $\eta \equiv \eta_2 - \eta_1$ between the two VEV's in Eq. (5) may be nonzero; it represents $CP$ violation originating at low energies from the scalar 2HD sector (see Ref. [8]). The leptonic sector will be ignored throughout.

We note that the popular ''type I'' and ''type II'' models are special cases (subsets) of this framework, with some of the Yukawa matrices being exactly zero: $\tilde{U}^{(1)} = \tilde{D}^{(1)} = 0$ [2HDM(I)]; $\tilde{U}^{(1)} = \tilde{D}^{(2)} = 0$ [2HDM(II)]. In these two special models, suggested by Glashow and Weinberg [3], the flavor-changing neutral Yukawa coupling parameters are exactly zero. This is so because one of the two nonzero Yukawa matrices is proportional to the mass matrix of the up-type quarks, and the other to the mass matrix of the down-type quarks. Since flavors refer to the physical quarks, and the quark mass matrices in the physical (mass) basis are diagonal by definition, the FCN Yukawa coupling parameters (i.e., off-diagonal elements) are zero. Moreover, this is true at any energy of probes (cutoff energy) $E'$. Stated otherwise, when the original cutoff $E$ is changed to $E'$, no loop-induced ($\ln E'$ cutoff-dependent) FCN Yukawa coupling parameters appear, i.e., the original form of the Lagrangian is preserved under the change of the cutoff. This can be formulated also in terms of explicit U(1)-type family symmetries governing the Yukawa part of the Lagrangian density in the 2HDM(I) and 2HDM(II). These symmetries ensure that, in the course of the change of the cutoff (i.e., evolution energy or the energy of probes), the original form of the Yukawa Lagrangian density is preserved. In the 2HDM(II) the symmetry transformation is $\tilde{d}_R^{(j)} \to e^{i\alpha}\tilde{d}_R^{(j)}$, $\Phi^{(1)} \to e^{-i\alpha}\Phi^{(1)}$ ($j=1,2,3$), the other fields remaining unchanged; in the 2HDM(I), $\tilde{d}_R^{(j)} \to e^{i\alpha}\tilde{d}_R^{(j)}$, $\tilde{u}_R^{(j)} \to e^{-i\alpha}\tilde{u}_R^{(j)}$, $\Phi^{(2)} \to e^{-i\alpha}\Phi^{(2)}$.

In contrast to the 2HDM(I) and 2HDM(II), the G2HDM has no explicit family symmetry enforcing the complete suppression of the FCN Yukawa coupling parameters. There are at least two consequences of this fact.

(1) The FCN Yukawa parameters in the G2HDM are in general nonzero. At low energies of probes ($E \sim E_{\text{ew}}$), those FCN Yukawa parameters which do not involve the top quark must be given quite small values (not necessarily zero) for phenomenological viability of the model.

(2) Even if the FCN Yukawa parameters are zero at some low energy of probes, they become in general nonzero at higher energies. If some FCN Yukawa parameters are small (but nonzero) at small energies, there exists in principle the possibility that they increase by a large factor, or even by orders of magnitude, when the energy of probes increases (but remains a safe distance away from the Landau pole), because of the absence of an explicit protective family symmetry.

The Lagrangian density[3] (1) can be written in a form more convenient for consideration of the FCN Yukawa coupling parameters, by redefining the scalar isodoublets in the following way:

$$\Phi'^{(1)} = (\cos\beta)\Phi^{(1)} + (\sin\beta)e^{-i\eta}\Phi^{(2)},$$
$$\Phi'^{(2)} = -(\sin\beta)\Phi^{(1)} + (\cos\beta)e^{-i\eta}\Phi^{(2)}, \tag{6}$$

where

$$\eta = \eta_2 - \eta_1, \quad \tan\beta = \frac{v_2}{v_1} \Rightarrow \cos\beta = \frac{v_1}{v}, \quad \sin\beta = \frac{v_2}{v}. \tag{7}$$

Therefore, the VEV's of the redefined scalar isodoublets are

$$e^{-i\eta_1}\langle\Phi'^{(1)}\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad \langle\Phi'^{(2)}\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 0\end{pmatrix}. \tag{8}$$

The isodoublet $\Phi'^{(1)}$ is therefore responsible for the masses of the quarks, and couplings of $\Phi'^{(2)}$ to the quarks lead to the FCN Yukawa couplings, as will be seen below. The original Yukawa Lagrangian density (1) of the G2HDM can then be rewritten in terms of these redefined scalar fields as

$$\mathcal{L}^{(E)}_{\text{G2HDM}} = -\sum_{i,j=1}^{3}\{\tilde{G}_{ij}^{(D)}(\bar{\tilde{q}}_L^{(i)}\Phi'^{(1)})\tilde{d}_R^{(j)} + \tilde{G}_{ij}^{(U)}(\bar{\tilde{q}}_L^{(i)}\tilde{\Phi}'^{(1)})\tilde{u}_R^{(j)} + \text{H.c.}\} - \sum_{i,j=1}^{3}\{\tilde{D}_{ij}(\bar{\tilde{q}}_L^{(i)}\Phi'^{(2)})\tilde{d}_R^{(j)} + \tilde{U}_{ij}(\bar{\tilde{q}}_L^{(i)}\tilde{\Phi}'^{(2)})\tilde{u}_R^{(j)} + \text{H.c.}\}, \tag{9}$$

where the Yukawa matrices $\tilde{G}^{(U)}$ and $\tilde{G}^{(D)}$ are rescaled mass matrices, and $\tilde{U}$ and $\tilde{D}$ the corresponding ''complementary'' Yukawa matrices, in an (arbitrary) $SU(2)_L$ basis (weak basis):

$$\tilde{G}^{(U)} = \sqrt{2}\tilde{M}^{(U)}/v = (\cos\beta)\tilde{U}^{(1)} + (\sin\beta)e^{-i\eta}\tilde{U}^{(2)},$$
$$\tilde{G}^{(D)} = \sqrt{2}\tilde{M}^{(D)}/v = (\cos\beta)\tilde{D}^{(1)} + (\sin\beta)e^{+i\eta}\tilde{D}^{(2)}, \tag{10}$$

$$\tilde{U} = -(\sin\beta)\tilde{U}^{(1)} + (\cos\beta)e^{-i\eta}\tilde{U}^{(2)},$$
$$\tilde{D} = -(\sin\beta)\tilde{D}^{(1)} + (\cos\beta)e^{+i\eta}\tilde{D}^{(2)}. \tag{11}$$

By a biunitary transformation involving unitary matrices $V_L^U$, $V_R^U$, $V_L^D$, and $V_R^D$, the Yukawa parameters can be expressed in the mass basis of the quarks, where the (rescaled) mass matrices $G^{(U)}$ and $G^{(D)}$ are diagonal and real:

$$G^{(U)} = \frac{\sqrt{2}}{v}M^{(U)} = V_L^U\tilde{G}^{(U)}V_R^{U\dagger}[M_{ij}^{(U)} = \delta_{ij}m_i^{(u)}],$$
$$U = V_L^U\tilde{U}V_R^{U\dagger}, \tag{12}$$

$$G^{(D)} = \frac{\sqrt{2}}{v}M^{(D)} = V_L^D\tilde{G}^{(D)}V_R^{D\dagger}[M_{ij}^{(D)} = \delta_{ij}m_i^{(d)}],$$
$$D = V_L^D\tilde{D}V_R^{D\dagger}, \tag{13}$$

$$u_L = V_L^U\tilde{u}_L, \quad u_R = V_R^U\tilde{u}_R, \quad d_L = V_L^D\tilde{d}_L, \quad d_R = V_R^D\tilde{d}_R. \tag{14}$$

The absence of tildes above the Yukawa coupling parameters and above the quark fields means that these quantities are in the quark mass basis (at a given evolution energy $E$). Lagrangian density (9) can be written now in the quark mass basis. The ''neutral current'' part of the Lagrangian density in the quark mass basis is

[3] Throughout this section we omit, for simpler notation, reference to the evolution (cutoff) energy $E$ at the quark fields, at the scalar fields and their VEV's, and at the Yukawa coupling parameters.

$$\mathcal{L}^{(E)\mathrm{nc}}_{\mathrm{G2HDM}} = -\frac{1}{\sqrt{2}} \sum_{i=1}^{3} \{ G^{(D)}_{ii} \overline{d}^{(i)}_L d^{(i)}_R (\phi'^{(1)}_3 + \mathrm{i}\phi'^{(1)}_4) + G^{(U)}_{ii} \overline{u}^{(i)}_L u^{(i)}_R (\phi'^{(1)}_3 - \mathrm{i}\phi'^{(1)}_4) + \mathrm{H.c.} \} - \frac{1}{\sqrt{2}} \sum_{i,j=1}^{3} \{ D_{ij} \overline{d}^{(i)}_L d^{(j)}_R (\phi'^{(2)}_3 + \mathrm{i}\phi'^{(2)}_4) + U_{ij} \overline{u}^{(i)}_L u^{(j)}_R (\phi'^{(2)}_3 - \mathrm{i}\phi'^{(2)}_4) + \mathrm{H.c.} \}. \tag{15}$$

On the other hand, the ''charged current'' part of the Lagrangian density in the quark mass basis is

$$\mathcal{L}^{(E)\mathrm{cc}}_{\mathrm{G2HDM}} = -\frac{1}{\sqrt{2}} \sum_{i,j=1}^{3} \{ (VG^{(D)})_{ij} \overline{u}^{(i)}_L d^{(j)}_R (\phi'^{(1)}_1 + \mathrm{i}\phi'^{(1)}_2) - (V^\dagger G^{(U)})_{ij} \overline{d}^{(i)}_L u^{(j)}_R (\phi'^{(1)}_1 - \mathrm{i}\phi'^{(1)}_2) + \mathrm{H.c.} \} - \frac{1}{\sqrt{2}} \sum_{i,j=1}^{3} \{ (VD)_{ij} \overline{u}^{(i)}_L d^{(j)}_R (\phi'^{(2)}_1 + \mathrm{i}\phi'^{(2)}_2) - (V^\dagger U)_{ij} \overline{d}^{(i)}_L u^{(j)}_R (\phi'^{(2)}_1 - \mathrm{i}\phi'^{(2)}_2) + \mathrm{H.c.} \}. \tag{16}$$

Here, we denoted by $V$ the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V \equiv V_{\mathrm{CKM}} = V^U_L V^{D\dagger}_L. \tag{17}$$

We see from Eq. (15) that the $U$ and $D$ matrices, as defined by Eqs. (11)–(13) through the original Yukawa matrices $\tilde{U}^{(j)}$ and $\tilde{D}^{(j)}$ of the G2HDM Lagrangian density (1), allow the model to possess in general scalar-mediated FCNC's. Namely, in the quark mass basis only the (rescaled) quark mass matrices $G^{(U)}$ and $G^{(D)}$ of Eqs. (12), (13) [see also Eq. (10)] are diagonal, but the matrices $U$ and $D$ in this general framework are in general not diagonal. The off-diagonal elements of the matrices $U$ and $D$ are the FCN Yukawa coupling parameters

$$\mathcal{L}^{(E)\mathrm{FCN}}_{\mathrm{G2HDM}} = -\frac{1}{\sqrt{2}} \sum_{i,j=1 (i\neq j)}^{3} \{ D_{ij} \overline{d}^{(i)}_L d^{(j)}_R (\phi'^{(2)}_3 + \mathrm{i}\phi'^{(2)}_4) + (D^\dagger)_{ij} \overline{d}^{(i)}_R d^{(j)}_L (\phi'^{(2)}_3 - \mathrm{i}\phi'^{(2)}_4) \} - \frac{1}{\sqrt{2}} \sum_{i,j=1 (i\neq j)}^{3} \{ U_{ij} \overline{u}^{(i)}_L u^{(j)}_R (\phi'^{(2)}_3 - \mathrm{i}\phi'^{(2)}_4) + (U^\dagger)_{ij} \overline{u}^{(i)}_R u^{(j)}_L (\phi'^{(2)}_3 + \mathrm{i}\phi'^{(2)}_4) \}. \tag{18}$$

It should be noted that the original four Yukawa matrices $\tilde{U}^{(j)}$ and $\tilde{D}^{(j)}$ $(j=1,2)$ in an $\mathrm{SU}(2)_L$ basis are already somewhat constrained by the requirement that (at low energy) the squares of[4] $M^{(U)}$ and $M^{(D)}$ are diagonalized by unitary transformations involving such unitary matrices $V^U_L$ and $V^D_L$, respectively, which are related to each other by $V^U_L V^{D\dagger}_L = V$. Here, $V$ is the CKM matrix which is, for any specific chosen phase convention, more or less known at low energies.

In order to have at low evolution energies ($E \sim E_{\mathrm{ew}}$) a phenomenologically viable suppression of the scalar-mediated FCNC's, the authors Cheng, Sher, and Yuan (CSY) [5] basically argued that the elements of the $U$ and $D$ matrices (in the quark mass basis and at low evolution energies $E$) should have the form

$$U_{ij}(E) = \xi^{(u)}_{ij} \frac{\sqrt{2}}{v} \sqrt{m^{(u)}_i m^{(u)}_j}, \quad D_{ij}(E) = \xi^{(d)}_{ij} \frac{\sqrt{2}}{v} \sqrt{m^{(d)}_i m^{(d)}_j}, \tag{19}$$

where

$$\xi^{(u)}_{ij}, \xi^{(d)}_{ij} \sim 1 \quad \text{for } E \sim E_{\mathrm{ew}}. \tag{20}$$

This form is in general phenomenologically acceptable. It is strongly motivated by the actual mass hierarchies of the quarks. At least for the diagonal elements, it is suggested by the requirement that (at a given *low* energy $\sim E_{\mathrm{ew}}$) there be no fine-tuning on the right of Eqs. (10), (11) when these equations are written in the quark mass basis (i.e., no tildes over the matrices). For definiteness, consider the up-type sector. The diagonal elements $U^{(i)}_{jj}$ are in general $\sim m^{(u)}_j/v$ unless fine-tuning is involved on the right of Eq. (10). Consequently, also $U_{jj} \sim m^{(u)}_j/v$ unless fine-tuning is involved on the right of Eq. (11). This consideration further suggests (but not necessarily implies) that the off-diagonal elements $U^{(i)}_{jk}$ and $U_{jk}$ have values between those of the corresponding diagonal elements $U^{(i)}_{jj} \sim m^{(u)}_j/v$ and $U^{(i)}_{kk} \sim m^{(u)}_k/v$, for example, roughly the geometrical mean of those, leading thus to the CSY *Ansatz* (19), (20).[5] Therefore, this (CSY) form is considered to be reasonably natural.

From the CSY *Ansatz* (19), (20) we see that the FCN Yukawa vertices involving the heavy top quark are the only ones that are not strongly suppressed (at low evolution energies). As mentioned in the Introduction, scalar-exchange-mediated FCNC processes involving the top quark vertices (not loops with top quarks) are not constrained by present experiments. Later in Sec. IV we will use low-energy conditions (19), (20) for a numerical investigation of the RGE flow of the FCN Yukawa coupling parameters.

## III. ONE-LOOP RGE'S IN A CONVENIENT PARAMETRIZATION

In the Appendix we outlined a derivation of the relevant set of one-loop RGE's for the scalar fields and their VEV's (A10)–(A14), for the quark fields (A16), (A19)–(A21), and for the Yukawa matrices $\tilde{U}^{(k)}$ and $\tilde{D}^{(k)}$ (A25), (A26). One of the reasons for performing an independent derivation is that

[4]Strictly speaking, the following ''squares'': $M^{(U)}M^{(U)\dagger}$ and $M^{(D)}M^{(D)\dagger}$.

[5]For the complete suppression of FCN Yukawa couplings $U_{jk} = 0$ (for $j \neq k$) we would then need fine-tuning on the right of Eq. (11).

we consider the method of finite cutoffs [9], which was used in the derivation, as physically very intuitive. This contrasts with other methods often applied in the literature, which are, however, usually mathematically more efficient at two-loop and higher-loop levels. Another reason is that there is a certain disagreement between the results on one-loop beta functions derived for a general (semi)simple gauge group $G$ in various parts of literature—see comparisons and the discussion toward the end of this section.

We can rewrite all the RGE's derived in the Appendix, now in a a more convenient set of parameters. These are the VEV parameters $v \equiv \sqrt{v_1^2+v_2^2}$, $\tan\beta \equiv v_2/v_1$, and $\eta \equiv \eta_2 - \eta_1$ [see Eq. (5)], and matrices $\tilde{G}^{(U)}$, $\tilde{G}^{(D)}$, $\tilde{U}$, and $\tilde{D}$ [see Eqs. (10), (11)]—this representation is more convenient for discerning the running of the FCN Yukawa coupling parameters. Applying lengthy, but straightforward, algebra to the hitherto obtained RGE's then results in the following RGE's in terms of the mentioned set of parameters

$$16\pi^2 \frac{d(v^2)}{d \ln E} = -2N_c \, \mathrm{Tr}[\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]v^2 + \left[\frac{3}{2}g_1^2+\frac{9}{2}g_2^2\right]v^2, \tag{21}$$

$$16\pi^2 \frac{d(\tan\beta)}{d \ln E} = -\frac{N_c}{2\cos^2\beta} \mathrm{Tr}[\tilde{G}^{(U)}\tilde{U}^\dagger+\tilde{U}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{D}^\dagger+\tilde{D}\tilde{G}^{(D)\dagger}], \tag{22}$$

$$16\pi^2 \frac{d(\eta)}{d \ln E} = \frac{N_c}{\mathrm{i}\sin(2\beta)} \mathrm{Tr}[\tilde{G}^{(U)}\tilde{U}^\dagger-\tilde{U}\tilde{G}^{(U)\dagger}-\tilde{G}^{(D)}\tilde{D}^\dagger+\tilde{D}\tilde{G}^{(D)\dagger}], \tag{23}$$

$$\begin{aligned}16\pi^2 \frac{d}{d \ln E}(\tilde{U}) = {} & N_c\{2\,\mathrm{Tr}[\tilde{U}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{D}^\dagger]\tilde{G}^{(U)}+\mathrm{Tr}[\tilde{U}\tilde{U}^\dagger+\tilde{D}\tilde{D}^\dagger]\tilde{U}\}+\frac{1}{2}N_c(\cot\beta)\tilde{U}\,\mathrm{Tr}[-\tilde{G}^{(U)}\tilde{U}^\dagger+\tilde{U}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{D}^\dagger \\ & -\tilde{D}\tilde{G}^{(D)\dagger}]+\left\{\frac{1}{2}[\tilde{U}\tilde{U}^\dagger+\tilde{D}\tilde{D}^\dagger+\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]\tilde{U}+\tilde{U}[\tilde{U}^\dagger\tilde{U}+\tilde{G}^{(U)\dagger}\tilde{G}^{(U)}]-2\tilde{D}\tilde{D}^\dagger\tilde{U} \right.\\ & \left. -2\tilde{G}^{(D)}\tilde{D}^\dagger\tilde{G}^{(U)}-A_U\tilde{U}\right\},\end{aligned} \tag{24}$$

$$\begin{aligned}16\pi^2 \frac{d}{d \ln E}(\tilde{D}) = {} & N_c\{2\,\mathrm{Tr}[\tilde{D}\tilde{G}^{(D)\dagger}+\tilde{G}^{(U)}\tilde{U}^\dagger]\tilde{G}^{(D)}+\mathrm{Tr}[\tilde{U}\tilde{U}^\dagger+\tilde{D}\tilde{D}^\dagger]\tilde{D}\}+\frac{1}{2}N_c(\cot\beta)\tilde{D}\,\mathrm{Tr}[-\tilde{G}^{(D)}\tilde{D}^\dagger+\tilde{D}\tilde{G}^{(D)\dagger}+\tilde{G}^{(U)}\tilde{U}^\dagger \\ & -\tilde{U}\tilde{G}^{(U)\dagger}]+\left\{\frac{1}{2}[\tilde{U}\tilde{U}^\dagger+\tilde{D}\tilde{D}^\dagger+\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]\tilde{D}+\tilde{D}[\tilde{D}^\dagger\tilde{D}+\tilde{G}^{(D)\dagger}\tilde{G}^{(D)}]-2\tilde{U}\tilde{U}^\dagger\tilde{D} \right.\\ & \left. -2\tilde{G}^{(U)}\tilde{U}^\dagger\tilde{G}^{(D)}-A_D\tilde{D}\right\},\end{aligned} \tag{25}$$

$$\begin{aligned}16\pi^2 \frac{d}{d \ln E}(\tilde{G}^{(U)}) = {} & N_c\,\mathrm{Tr}[\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]\tilde{G}^{(U)}+\frac{1}{2}N_c(\tan\beta)\tilde{G}^{(U)}\,\mathrm{Tr}[-\tilde{G}^{(U)}\tilde{U}^\dagger+\tilde{U}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{D}^\dagger-\tilde{D}\tilde{G}^{(D)\dagger}] \\ & +\frac{1}{2}[\tilde{U}\tilde{U}^\dagger+\tilde{D}\tilde{D}^\dagger+\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]\tilde{G}^{(U)}+\tilde{G}^{(U)}[\tilde{U}^\dagger\tilde{U}+\tilde{G}^{(U)\dagger}\tilde{G}^{(U)}]-2\tilde{D}\tilde{G}^{(D)\dagger}\tilde{U} \\ & -2\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}\tilde{G}^{(U)}-A_U\tilde{G}^{(U)},\end{aligned} \tag{26}$$

$$\begin{aligned}16\pi^2 \frac{d}{d \ln E}(\tilde{G}^{(D)}) = {} & N_c\,\mathrm{Tr}[\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]\tilde{G}^{(D)}+\frac{1}{2}N_c(\tan\beta)\tilde{G}^{(D)}\,\mathrm{Tr}[-\tilde{G}^{(D)}\tilde{D}^\dagger+\tilde{D}\tilde{G}^{(D)\dagger}+\tilde{G}^{(U)}\tilde{U}^\dagger-\tilde{U}\tilde{G}^{(U)\dagger}] \\ & +\frac{1}{2}[\tilde{U}\tilde{U}^\dagger+\tilde{D}\tilde{D}^\dagger+\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}+\tilde{G}^{(D)}\tilde{G}^{(D)\dagger}]\tilde{G}^{(D)}+\tilde{G}^{(D)}[\tilde{D}^\dagger\tilde{D}+\tilde{G}^{(D)\dagger}\tilde{G}^{(D)}]-2\tilde{U}\tilde{G}^{(U)\dagger}\tilde{D} \\ & -2\tilde{G}^{(U)}\tilde{G}^{(U)\dagger}\tilde{G}^{(D)}-A_D\tilde{G}^{(D)}.\end{aligned} \tag{27}$$

Equation (21) is in the Landau gauge, while the other RGE's (22)–(27) are gauge independent.

For a general (semi)simple gauge group $G$, RGE's for various parameters have been derived at one-loop level [10,11], and at two-loop level [12–14]. First we should note that these groups of authors are using conventions which, particularly as to the fermionic sector, differ from each other.

Cheng, Eichten, and Li [10] were using the usual four-component Dirac spinors for quarks. While they allowed an arbitrary number of (real) scalar degrees of freedom, their one-loop RGE's for the Yukawa coupling parameters are directly applicable only when these parameters are real (noncomplex).

The one-loop RGE results of Vaughn [11] and one- and two-loop results of Machacek and Vaughn [12] were written for the general case of complex Yukawa coupling parameters. Their scalar fields $\phi_a$ were real, and for fermions (quarks) they were using two-component spinor fields $\psi_j$ as defined by Sikivie and Gürsey [15] [$j,k=1,\ldots,2n$, where $n$ is the number of fermion (quark) flavors].[6] The Yukawa Lagrangian density was written in Refs. [11,12] in the form

$$\mathcal{L}_{\mathrm{Y.}} = -\mathbf{Y}^a_{jk}\psi_j^T \mathrm{i}\sigma^2 \psi_k \phi_a + \mathrm{H.c.}, \tag{28}$$

with $\sigma^2$ being the second Pauli matrix. Rewriting our RGE's (from the Appendix) for the scalar and quark fields, and for the Yukawa parameters, in terms of the spinor fields of Ref. [15] and of real scalar field components, we notice several differences when compared with one-loop results of Refs. [11, 12]. We can deduce from our RGE's for the Yukawa coupling parameters that RG equation (3.4) of Ref. [12] (second entry), or equivalently, RG Eqs. (2.2), (2.3) of Ref. [11], for $\mathbf{Y}^a$ Yukawa matrices in their language should read

$$\begin{aligned}(4\pi)^2 \frac{d\mathbf{Y}^a}{d\ln E}\bigg|_{1-\mathrm{l.}} &\equiv (4\pi)^2 \boldsymbol{\beta}^a|_{1-\mathrm{l.}} \\ &= 2[\mathbf{Y}^b\mathbf{Y}^{b\dagger}\mathbf{Y}^a + \mathbf{Y}^a\mathbf{Y}^{b\dagger}\mathbf{Y}^b] + 8\mathbf{Y}^b\mathbf{Y}^{a\dagger}\mathbf{Y}^b \\ &\quad + 4\kappa\mathbf{Y}^b\ \mathrm{Tr}(\mathbf{Y}^{b\dagger}\mathbf{Y}^a + \mathbf{Y}^{a\dagger}\mathbf{Y}^b) \\ &\quad - 3g^2\{\mathbf{C}_2(F),\mathbf{Y}^a\},\end{aligned} \tag{29}$$

where $\kappa=1/2$. Stated otherwise, the cubic Yukawa terms on the right of this RGE are effectively those given in Refs. [11, 12], but multiplied[7] by a factor of 4, and the trace there is replaced now by its real (symmetric) part. Similar differences arise when comparing our RGE's for scalar and quark fields with those of Ref. [12]. Instead of Eq. (37) of Ref. [12] (first entry), we get

$$\begin{aligned}(4\pi)^2\gamma^s_{ab}|_{1-\mathrm{l.}} &= 4\kappa\ \mathrm{Tr}(\mathbf{Y}^a\mathbf{Y}^{b\dagger} + \mathbf{Y}^{a\dagger}\mathbf{Y}^b) \\ &\quad - g^2(2+\alpha)\mathbf{C}_2(S)\delta_{ab},\end{aligned} \tag{30}$$

where again $\kappa=1/2$ and $\gamma^s$ is defined via $d\phi_a/d\ln E = -\gamma^s_{ab}\phi_b$. Here, $\phi_a$ are the real scalar fields, and $\alpha$ in Eq. (30) is the gauge parameter ($\alpha\equiv 1-\xi=1$ in the Landau gauge). Instead of Eq. (4.5) of Ref. [12] (first entry), we get

$$(4\pi)^2\gamma^F|_{1-\mathrm{l.}} = 2\mathbf{Y}^{a\dagger}\mathbf{Y}^a + g^2\mathbf{C}_2(F)(1-\alpha), \tag{31}$$

where $\gamma^F$ is defined via $d\psi_j/d\ln E = -\gamma^F_{ji}\psi_i$. Here, $\psi_i$ are left-handed two-component spinors as defined in Ref. [15].

Jack and Osborn [13], on the other hand, worked with Majorana fermions, using the background field method. The Dirac fermions can then be expressed as sums of two Majorana fermions. Their one-loop beta functions for the Yukawa coupling parameters of the (real) scalars with the (Majorana) fermions, can be reexpressed in the notation with left-handed two-component spinors $\psi_j$ as introduced by Ref. [15] and used by Refs. [11,12]. After somewhat lengthy algebra, it can be shown that the one-loop results of Ref. [13] lead precisely to formula (29). Therefore, we finally conclude that our one-loop RGE formulas for the Yukawa coupling parameters, derived in the Appendix and rewritten in Eq. (29) in the language of Ref. [15], *are not in agreement with* those of Vaughn [11] and of Machacek and Vaughn [12], and *are in agreement with* the results of Jack and Osborn [13]. Moreover, the latter authors emphasize that their RGE results agree with those of van Damme [14].[8]

For several reasons, we considered it instructive to perform an independent derivation of the one-loop RGE's for the scalar and quark fields and for the Yukawa matrices in the discussed general 2HDM. One reason is that the one-loop results of Refs. [11] and [12] do not agree entirely with those by other authors [13,14]. Another reason is that the existing works on the one- and two-loop RGE's for general (semi)simple gauge groups $G$ use various conventions for the fermionic fields, and are usually written in a language difficult for nonspecialists in the method used. The third reason is that these works do not apply the method of finite cutoffs [9] which we consider especially appealing and physically intuitive—although, at two-loops, probably not

[6]The usual four-component Dirac spinor field columns $\Psi^{(j)}$ in the chiral basis (i.e., the basis of Ref. [16]) are made up of $\psi_j$ (upper two components) and $-\mathrm{i}\sigma^2\psi^{\dagger T}_{j+n}$ (lower two components of $\Psi^{(j)}$), where $j=1,\ldots,n$.

[7]If Machacek and Vaughn had introduced in the Lagrangian density an additional factor of (1/2) in front of the sum (28) (which they did not), the factor 4 in the cubic Yukawa terms on the right of (29) would not have occurred.

[8]Here we also mention that Fischler and Oliensis [17] have derived RGE's for Yukawa coupling parameters of the minimal SM at two-loop level.

the most efficient one. With our independent cross-check we are confident that the one-loop results of Ref. [13] are correct.

### IV. NUMERICAL EXAMPLES OF EVOLUTION

Here we present a few simple but hopefully typical examples of the RGE evolution of parameters in the G2HDM. Some preliminary numerical results were presented by us in Ref. [18]. For simplicity, we assumed there is no $CP$ violation—all original four Yukawa matrices $\tilde{U}^{(j)}$, $\tilde{D}^{(j)}$ are real, and the VEV phase difference $\eta$ is zero—and the Yukawa parameters of the first quark generation as well as those of the leptonic sector are neglected (the quark Yukawa mass matrices are therefore $2\times 2$).

For the boundary conditions to the RGE's, at the evolution energy $E=M_Z$, we first take the CSY *Ansatz* (19), (20), with $\xi_{ij}^{(u)}=1=\xi_{ij}^{(d)}$ or $\xi_{ij}^{(u)}=2=\xi_{ij}^{(d)}$ for all $i,j=1,2$. We stress that $i=1$ refers now to the second quark family $(c,s)$, and $i=2$ to the third family $(t,b)$. For the $(2\times 2)$ orthogonal CKM mixing matrix $V$ we take $V_{12}(M_Z)=0.045=-V_{21}(M_Z)$. The values of other parameters at $E=M_Z$ are chosen to be $\tan\beta=1.0$, $v\equiv\sqrt{v_1^2+v_2^2}=246.22$ GeV; $\alpha_3=0.118$, $\alpha_2=0.332$, $\alpha_1=0.101$; $m_c=0.77$ GeV, $m_s=0.11$ GeV, $m_b=3.2$ GeV, and $m_t=171.5$ GeV. The latter quark mass values correspond to $m_c(m_c)\approx 1.3$ GeV, $m_s(1\text{ GeV})\approx 0.2$ GeV, $m_b(m_b)\approx 4.3$ GeV, and $m_t^{\rm phys}\approx 174$ GeV $[m_t(m_t)\approx 166$ GeV$]$. For $\alpha_3(E)$ we used two-loop evolution formulas, with threshold effect at $E\approx m_t^{\rm phys}$ taken into account; for $\alpha_j(E)$ $(j=1,2)$ we used one-loop evolution formulas.

The described simplified framework resulted in 18 coupled RGE's (for 18 real parameters: $v^2$, $\tan\beta$, $\tilde{U}_{ij}$, $\tilde{D}_{ij}$, $\tilde{G}_{ij}^{(U)}$, $\tilde{G}_{ij}^{(D)}$), with the mentioned boundary conditions at $E=M_Z$. The system of RGE's was solved numerically, using Runge-Kutta subroutines with adaptive step-size control (given in Ref. [19]). The numerical results were cross checked in several ways, including the following: FORTRAN programs for the RGE evolution and for the biunitary transformations were constructed independently by two of the authors (S.S.H. and G.C.), and they yielded identical numerical results presented in this section.

The results for the FCN Yukawa parameter ratios $X_{ij}(E)/X_{ij}(M_Z)$ $(X=U,D;\ i\neq j)$ are given for the case $\xi_{ij}^{(u)}=1=\xi_{ij}^{(d)}$ in Fig. 1. From the figure we immediately notice that the FCN coupling parameters are remarkably stable as the energy of probes increases. Even those FCN Yukawa coupling parameters which involve $t$ quark remain quite stable. Only very close to the top-quark-dominated Landau pole[9] ($E_{\rm pole}\approx 0.84\times 10^{13}$ GeV) do the coupling parameters start increasing substantially. For example, in the down-type FCN sector $(b\text{-}c)$ the corresponding ratio $D_{21}(E)/D_{21}(M_Z)$ acquires its double initial value (i.e., value 2) at $E\approx 0.7E_{\rm pole}$, which is very near the (Landau) pole. About the same holds also for $U_{21}(E)/U_{21}(M_Z)$. For the ratio $D_{12}(E)/D_{12}(M_Z)$ the corresponding energy is even closer to $E_{\rm pole}$. For the $t$-quark-dominated $U_{12}(E)/U_{12}(M_Z)$ it is somewhat lower.

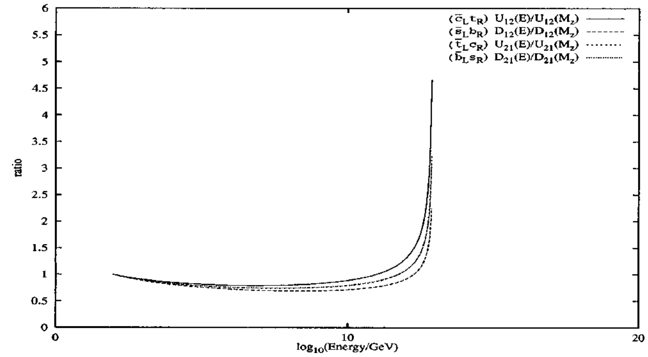


FIG. 1. FCN Yukawa parameter ratios $U_{ij}(E)/U_{ij}(M_Z)$, $D_{ij}(E)/D_{ij}(M_Z)$ $(i\neq j)$ in the G2HDM as the Euclidean energy of probes $E$ increases. These parameters are in the quark mass basis. At $E=M_Z$, the CSY *Ansatz* was taken with $\xi_{ij}^{(u)}=\xi_{ij}^{(d)}=1$ (for all $i,j=1,2$).

In Fig. 2, evolution of the same FCN ratios is depicted for the case of the low-energy CSY parameters $\xi_{ij}^{(u)}=2=\xi_{ij}^{(d)}$. The $t$-quark-dominated Landau pole is now substantially lower ($E_{\rm pole}\sim 10$ TeV), but the behavior of the FCN ratios remains qualitatively the same. Moreover, when some of the CSY parameters $\xi_{ij}$ are varied, the stability of the FCN ratio persists, and the Landau pole is influenced almost entirely by the $t$-quark-dominated CSY parameter $\xi_{22}^{(u)}$. We also looked into cases when the CSY ansatz is effectively abandoned. If we suppress the up-type off-diagonal element at $E=M_Z$ drastically, for example, by taking $\xi_{12}^{(u)}=\xi_{21}^{(u)}\approx 0.0516$ [corresponding to $U_{ij}(M_Z)=D_{ij}(M_Z)$ for $i\neq j$] and all other $\xi_{ij}$ parameters equal to 1, we obtain results depicted in Fig. 3, which are very close to those of Fig. 1.

From all these figures we conclude that the FCN Yukawa coupling parameters in the general 2HDM show remarkable stability when the (Euclidean) energy of probes increases. This stability persists up to the energy regions which are, on the logarithmic scale, quite close to the top-quark-dominated

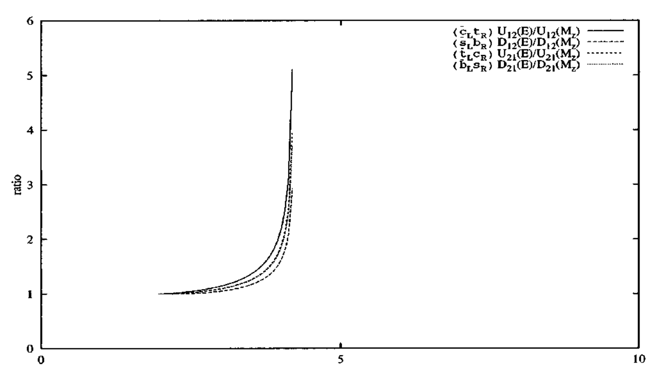


FIG. 2. Same as in Fig. 1, but for the choice $\xi_{ij}^{(u)}=\xi_{ij}^{(d)}=2$ (for all $i,j=1,2$).

[9]The value of $E_{\rm pole}$ is strongly dependent on the given value of parameter $\xi$, as shown later in Fig. 6.

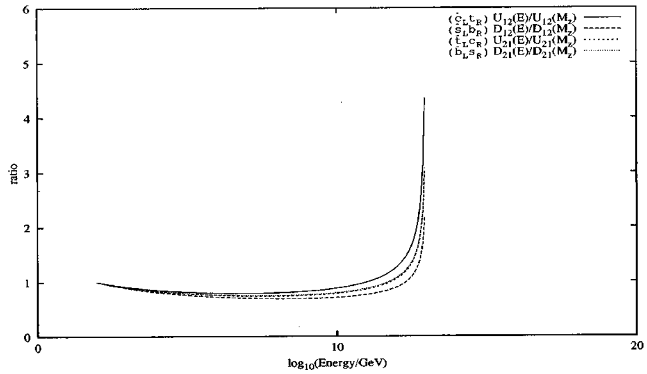


FIG. 3. Same as in Fig. 1, but for the choice $\xi_{12}^{(u)}=\xi_{21}^{(u)}=0.05163$ (other $\xi_{ij}$'s are 1).

Landau pole. The general 2HDM appears to possess this stability even when the off-diagonal low-energy parameters $|\xi_{ij}|$ (19), (20) have values much smaller than 1 (while the diagonal parameters are $\xi_{jj}\sim 1$).

We can also compare the RGE evolution of the Yukawa parameters in the G2HDM with those of the MSM, 2HDM(II), and 2HDM(I). The authors of Refs. [20,21] proposed that (heavy) quark mass and Higgs masses in the MSM could be determined by the infrared fixed points of the RGE's. These questions were numerically investigated also in different variants of the 2HDM(II) and 2HDM(I) [22]. The authors of the latter work found out that relatively unambiguous predictions can be made only if there is a heavy quark generation and the (heavy) quarks couple to both Higgs doublets. It is interesting to note that in the G2HDM, the heaviest ($t$) quark also has an infrared fixed point behavior, as suggested from Figs. 1–3. This is further suggested from Figs. 4, 5 which represent evolution of the ratios $X(E)/X(M_Z)$ involving the flavor-nonchanging neutral Yukawa parameters ($X=U_{jj}$ or $D_{jj}$) or the ''mass'' Yukawa parameters ($X=G_{jj}^{(U)}$ or $G_{jj}^{(D)}$). By the infrared fixed point behavior we mean that for a given approximate Landau pole energy $E_{\rm pole}$, we have very weak sensitivity of $X(M_Z)$ on the otherwise large value $X(E_{\rm pole})\gtrsim 1$. The reason for this similarity with the MSM and 2HDM(II) and 2HDM(I) lies probably in the conjunction of the facts that the CSY *Ansatz* (19), (20) implies dominance of the $t$-related Yukawa coupling parameters and that

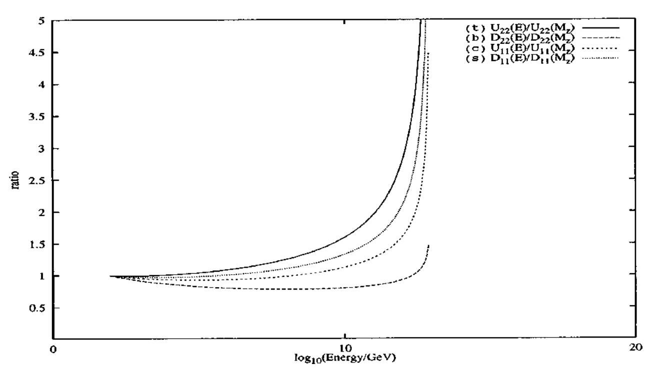


FIG. 4. Same as in Fig. 1, but for the neutral current Yukawa coupling parameters $U_{jj}$ and $D_{jj}$ ($j=1,2$) which do not change flavor.

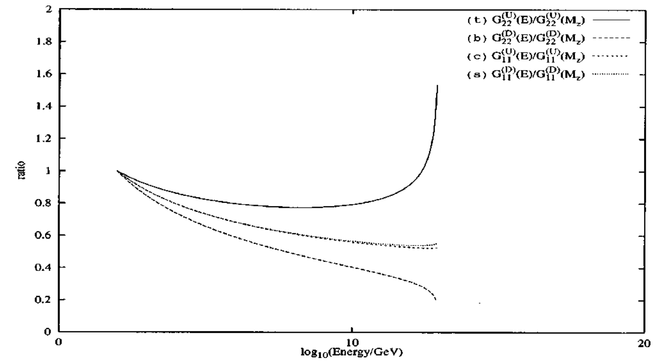


FIG. 5. Same as in Fig. 1, but for the ''mass'' Yukawa parameters $G_{jj}^{(U)}$ and $G_{jj}^{(D)}$ ($j=1,2$) instead. Since $G^{(U)}(E)$ and $G^{(D)}\times(E)$ matrices are diagonal by definition (quark mass basis), these neutral current Yukawa matrices have zero FCN components automatically.

the QCD contribution to the evolution of these parameters has the sign which is in general opposite to that of the ($t$-related) Yukawa parameter contributions. Inspecting the work [22] further, we note that it would also be interesting to investigate the RGE behavior of the quartic Higgs coupling parameters in the G2HDM. This would tell us when these parameters have infrared fixed point behavior and thus when the physical Higgs boson mass spectrum can be determined.

In this context, we mention that the idea of RGE fixed points had been introduced earlier by Chang [23]. He, and subsequently others [24], investigated connection of RGE fixed points with asymptotic freedom in massive gauge theories. Cabibbo *et al.* [25] were apparently the first to investigate mass constraints in the minimal SM by imposing boundary conditions on (perturbative) RGE's at the unification energy of grand unified theories [SU(5) or O(10)]. A somewhat related analysis was performed later by the authors of Ref. [26] who used SU(5) fixed-point conditions.

It should be stressed that the presented numerical results are independent of the chosen value of the VEV ratio, tan $\beta$, at $E=M_Z$. This is connected with our choice of the CSY boundary conditions (19), (20) at $E=M_Z$ for the Yukawa matrices in the quark mass basis (all $\xi_{ij}$'s taken real) and the reality of the chosen Cabibbo-Kobayashi-Maskawa (CKM) matrix at $E=M_Z$. These boundary conditions result in real and $\beta$-independent Yukawa matrices $\tilde{U}$, $\tilde{D}$, $\tilde{G}^{(U)}$, $\tilde{G}^{(D)}$ in a weak $[SU(2)_L]$ basis[10] at $E=M_Z$. The RGE's (24)–(27) then imply that these matrices remain real and independent of $\beta$ at any evolution energy $E$, and that also their counter-

[10]We chose at $E=M_Z$ the following weak basis: $\tilde{U}=U$, $\tilde{G}^{(U)}=G^{(U)}$, $\tilde{D}=VD$, $\tilde{G}^{(D)}=VG^{(D)}$, where $V$ is the CKM matrix (at $E=M_Z$). According to relations (10), (11), the reality of the Yukawa matrices $\tilde{U}$, $\tilde{D}$, $\tilde{G}^{(U)}$, and $\tilde{G}^{(D)}$ at low energy $E=M_Z$ would follow, for example, from the requirement of no $CP$ violation in the Yukawa sector (i.e., the original Yukawa matrices $\tilde{U}^{(j)}$ and $\tilde{D}^{(j)}$ are all real) *together with* the requirement of no $CP$ violation in the scalar sector (i.e., the VEV phase difference $\eta=0$) at that low energy.

parts $U$, $D$, $G^{(U)}$, and $G^{(D)}$ in the quark mass basis, as well as the CKM matrix $V$, remain real and independent of $\beta$ at any energy $E$. Stated otherwise, if there is $\beta$-independence and no $CP$ violation (neither in the original Yukawa matrices nor in the scalar sector) at low energy ($E=M_Z$), then these properties persist at all higher energies of evolution.[11]

This feature is in stark contrast with the situation in the 2HDM(II) where the Yukawa matrices strongly depend on $\beta$ already at low energies, e.g., $g_t(M_Z)=m_t(M_Z)\sqrt{2}/v_u =m_t(M_Z)\sqrt{2}/[v \sin\beta(M_Z)]$. Also the location of the Landau pole in the 2HDM(II) then crucially depends on $\beta(M_Z)$—smaller $\beta(M_Z)$ implies larger $g_t(M_Z)$ and hence a drastically lower Landau pole.

On the other hand, the G2HDM treats the up-type and the down-type sectors of quarks (the two VEV's $v_1$ and $v_2$) nondiscriminatorily. Therefore, it should be expected that any reasonable boundary conditions for Yukawa coupling parameters at low energies should also be independent of $\beta$ in such frameworks, and this independence then persists to a large degree also at higher energies. Also the locations of the Landau poles (i.e., of the approximate scales of the onset of new physics) should then be expected to be largely $\beta$ independent. In this sense, the G2HDM has more similarity to the minimal SM (MSM) than to the 2HDM(II). The persistence of complete $\beta$ independence of the Yukawa coupling parameters at high energies and of the Landau poles, however, can then be ''perturbed'' by $CP$ violation because RGE's (24)–(27) are somewhat $\beta$ dependent when the Yukawa matrices $\tilde{U}$, etc., are not real.

In addition to the connection between (low-energy) $CP$ violation and $\beta$ dependence of high-energy results, there is yet another feature that distinguishes the G2HDM from the MSM—the Landau pole of the G2HDM is in general much lower than that of the MSM. We can see that in the following way: let us consider that only the Yukawa parameters connected with the top quark degree of freedom are substantial, i.e., $G_{22}^{(U)}=g_t\sim 1$ and $U_{22}=g_t'\sim 1$. We have $g_t(E) =m_t(E)\sqrt{2}/v(E)$, as in the MSM, and $g_t'(E)$ is an additional large Yukawa parameter—both crucially influence location of the Landau pole. Inspecting RGE's (24) and (26) for this special approximation of two variables $g_t$ and $g_t'$, we see that RGE for $g_t$ is similar to that in the MSM, but with an additional large positive term on the right $(3/2)(g_t')^2 g_t$. The RGE for $g_t'$ has a similar structure as the RGE for $g_t$, but with substantially larger coefficients at the positive terms on the right. As a result, $g_t'(E)$ is in general larger than $g_t(E)$. Our specific numerical example $\xi_{ij}^{(u)}=\xi_{ij}^{(d)}=1$ shows that $g_t'(E)$ is on average (average over the whole evolution energy range) almost twice as large as $g_t(E)$. If we then simply replace in the mentioned additional term $(3/2)(g_t')^2 g_t$ the parameter $(g_t')^2$ by $3.5 g_t^2$, we obtain from the resulting

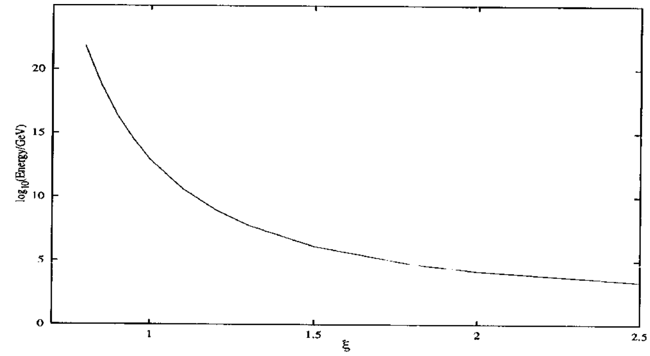


FIG. 6. Variation of the Landau pole energy when the low-energy parameters $\xi_{ij}^{(u)}=\xi_{ij}^{(d)}\equiv\xi$ of the CSY *Ansatz* (19), (20) are varied. For $\xi=2.5$, the onset scale of new physics is already quite low: $E_{\rm pole}\approx 2$ TeV.

''modified'' MSM RGE for $g_t$ a value for the Landau pole in the region of $10^{12}$–$10^{13}$ GeV, which is roughly in agreement with the actual value of the Landau pole of our numerical example $E_{\rm pole}\approx 0.84\times 10^{13}$ GeV. And this value is much lower than $E_{\rm pole}$ in the MSM which is above the Planck scale. As already mentioned, the value of $E_{\rm pole}$ is largely influenced by the value of the top-quark-dominated parameter $\xi_{22}^{(u)}$. Of course, when we allow the $\xi_{ij}$ ($\xi_{22}^{(u)}$) parameters of the CSY *Ansatz* (19), (20) at $E=M_Z$ to deviate from 1, we obtain larger $\log(E_{\rm pole})$ for smaller $\xi_{ij}$, and smaller $\log(E_{\rm pole})$ for larger $\xi_{ij}$. In Fig. 6 we depicted this variation of the Landau pole energy when the CSY low-energy parameters $\xi_{ij}$ are varied.

It is also interesting to note that RGE (23) for the evolution of the difference of the VEV phases ($\eta$) implies in the G2HDM that $\eta$ can change when the energy of probes changes. This would generally occur when we have $CP$ violation in the Yukawa sector (i.e., complex Yukawa matrices). Even when $\eta=0$ at some low energy, it may become nonzero at some higher energy due to the $CP$ violation in the Yukawa sector. This contrasts with the 2HDM(II) or 2HDM(I) where the right side of Eq. (23) is zero always and thus the $CP$ violation in the Yukawa sector does not influence $\eta$.

## V. SUMMARY AND CONCLUSIONS

We performed a numerical analysis of the one-loop RGE's in the general two-Higgs-doublet model (G2HDM). In the analysis, we neglected the Yukawa coupling parameters of the light first quark generation, as well as the contributions of the leptonic sector. At low energies of probes, we first adopted the CSY *Ansatz* (19), (20) which is largely motivated by the existing quark mass hierarchies. We found out that the flavor-changing neutral (FCN) Yukawa parameters remain remarkably stable when the energy of probes increases all the way to the vicinity of the ($t$-quark-dominated) Landau pole. This conclusion survives even when the CSY *Ansatz* is effectively abandoned, i.e., when the off-diagonal low-energy parameters are additionally suppressed: $|\xi_{ij}^{(u)}| \ll 1$ ($i\neq j$). This behavior indicates that the G2HDM does not behave unnaturally with respect to the RGE evolution of

[11] CP conservation in the pure scalar sector at a low energy $E=M_Z$ (i.e., $\eta=0$) also persists then at all higher energies of evolution, since $d\eta/d\ln E=0$ by the reality of the Yukawa matrices, according to RGE (23).

the vertices of the Higgs-exchanged flavor-changing neutral currents. Since the G2HDM, in contrast to the 2HDM(II) and 2HDM(I), has no explicit and exact discrete [or U(1)] family symmetries which would ensure persistence of the FCN Yukawa suppression at increasing energies of probes, the behavior of FCN Yukawa parameters found numerically in the present paper may be somewhat surprising. The general suspicion about the G2HDM in the past had centered on the fact that absence of the mentioned family symmetries in the Lagrangian density would in general not keep FCN Yukawa parameters suppressed under the RGE evolution and would thus render the model unnatural and fraught with fine-tuning of ''bare'' FCN Yukawa parameters. In other words, the RGE's of the G2HDM would in general allow a ''pull-up'' effect by the diagonal Yukawa parameters on the much smaller off-diagonal (FCN) ones. This would increase the values of the latter by a large factor or even by orders of magnitude when the energy of probes increases by one or several orders of magnitude. Our numerical analysis shows that this does not happen, at least as long as the low-energy parameters of the CSY *Ansatz* (19), (20) satisfy $|\xi_{ij}|\lesssim 1$ for $i\neq j$ and $\xi_{jj}\sim 1$. Perhaps it is the latter condition (for the diagonal $\xi$ parameters) which causes the mentioned persistence of the FCN Yukawa suppression. The latter condition, together with the known form of the CKM matrix, effectively represents an approximate symmetry in which only the third quark generation has substantially nonzero Yukawa parameters (and almost no CKM mixing). This can also be called approximate flavor democracy.

Further, the high-energy Yukawa coupling parameters in the model have in general little dependence on the VEV ratio tan $\beta$ as long as the $CP$ violation is weak. Moreover, we found out that the G2HDM has an interesting behavior of the Landau pole energies. They can become quite low ($\sim 1$ TeV) already at not very high $\xi_{ij}$ parameters ($\xi_{22}^{(u)}\leqslant 3$), as shown in Fig. 6. These energies, signaling the breakdown of the perturbative approach in the G2HDM, can be interpreted as possible scales of the onset of a new physics and/or a strong coupling regime.

## ACKNOWLEDGMENTS

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## APPENDIX: ONE-LOOP RGE'S IN THE GENERAL 2HDM

We outline here a derivation of the one-loop RGE's for the scalar and quark fields and for the Yukawa coupling matrices $\tilde{D}^{(k)}$ and $\tilde{U}^{(k)}$ $(k=1,2)$ in the general two-Higgs doublet model (G2HDM) whose Lagrangian density in the Yukawa sector is represented by Eq. (1). The derivation follows the finite cutoff interpretation of RGE's as presented, for example, in Ref. [19]. We consider it useful to present the derivation because the approach used here is physically very intuitive, and because the existing relevant literature on RGE's in general (semi)simple Lie gauge groups is written in a rather cryptic manner and not all the works agree completely with each other. In Sec. III we also compared the results obtained here with those implied by the existing literature.

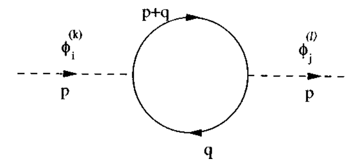


FIG. 7. The diagram leading to the two-point Green function $-i\Sigma_{ij}^{(k,\ell)}(p^2;E^2)$. Full lines represent quark propagators.

### 1. One-loop RGE's for the scalar fields

To obtain the evolution of the scalar fields $\phi_i^{(k)}(E)$ with ''cutoff'' energy $E$, the truncated (one-loop) two-point Green functions $-i\Sigma_{ij}^{(k,\ell)}(p^2;E^2)$, represented diagrammatically in Fig. 7, have to be calculated first. More specifically, it suffices to calculate only their cutoff-dependent parts $\propto p^2 \ln E^2$ which are responsible for effective[12] kinetic-energy-type terms $\sim \partial_\nu \phi_i^{(k)}(E)\partial^\nu \phi_j^{(\ell)}(E)$. In the course of the calculations, all the masses $m\sim E_{\rm ew}$ of the relevant particles in the diagrams are ignored. This would correspond to the picture with a finite but large ultraviolet energy cutoff $E\gg E_{\rm ew}$. Therefore, calculations need not be performed in the mass basis of the relevant particles. These particles are regarded as effectively massless in the approximation, the transformations between the original bases of the relevant fields and their mass bases are unitary, and therefore the (mass-independent parts of the) calculated Green functions are the same in both bases.

Calculation of the Green functions $-i\Sigma_{i,j}^{(k,\ell)}(p^2;E^2)$ is then straightforward. The relevant massless integrals over internal quark-loop momenta $q$ can be carried out in the Euclidean metric $[\overline{q}=(-iq^0,-q^j),\quad \overline{p}=(-ip^0,-p^j)]$, where the upper bound in the loop integral is $\overline{q}^2\leqslant E^2$. After rotating back into Minkowski metric $(\overline{p}^2\mapsto -p^2)$, we obtain the following.

(1) Green functions whose external legs $\phi_i^{(k)}$ and $\phi_j^{(\ell)}$ have the same scalar indices $(i=j)$:

$$-i\Sigma_{j,j}^{(k,\ell)}(p^2;E^2) = i\,\frac{N_c}{32\pi^2}p^2\ln\left(\frac{E^2}{m^2}\right)\mathrm{Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}+\tilde{U}^{(\ell)}\tilde{U}^{(k)\dagger} + \tilde{D}^{(k)}\tilde{D}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}](E), \tag{A1}$$

[12]For clearer notation, we denote in this section the evolving (UV cutoff) energy $E$ at the fields not as a superscript, but rather as an argument.

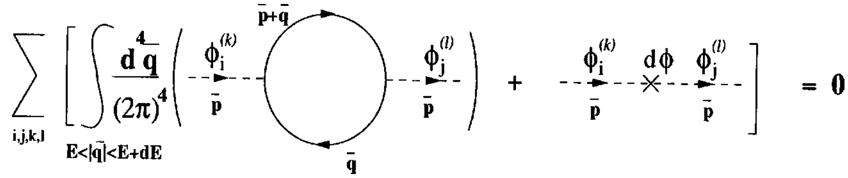


FIG. 8. Diagrammatic illustration of the RGE relation (A3) leading to the evolution of the scalar fields. $\phi_i^{(k)}$ stands for $\phi_i^{(k)}(E)$ and $d\phi$ stands for $\phi(E+dE)-\phi(E)$ ($\phi$ is a generic notation for $\phi_j^{(\ell)}$'s). The cross represents the contribution of the change of the kinetic energy terms originating from the changes $d\phi$ of scalar fields.

where $j=1,2,3,4$ (no running over $j$), $k,\ell=1,2$, and $m$ is an arbitrary but fixed mass ($m\sim E_{\rm ew}$).

(2) Green functions whose external legs $\phi_j^{(k)}$ and $\phi_{j'}^{(\ell)}$ have complementary scalar indices $(jj')=(12)$, (21), (34), (43):

$$-i\Sigma^{k,\ell}_{(j,j')}(p^2;E^2)=(-1)^j\frac{N_{\rm c}}{32\pi^2}p^2\ln\left(\frac{E^2}{m^2}\right){\rm Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}-\tilde{U}^{(\ell)}\tilde{U}^{(k)\dagger}-\tilde{D}^{(k)}\tilde{D}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}]\times(E). \tag{A2}$$

(3) Green functions $-i\Sigma_{i,j}^{(k,\ell)}$ where other indices are zero.

All these Green functions can be induced alternatively at the tree level by kinetic energy terms. For example, in the theory with UV cutoff $E$, the kinetic energy term $\partial_\nu\phi_i^{(k)}(E)\partial^\nu\phi_j^{(\ell)}(E)$ induces (at the tree level) the two-point Green function value $-i\Sigma_{i,j}^{(k,\ell)}(p^2;E^2)=ip^2$ if $\phi_i^{(k)}(E)\neq\phi_j^{(\ell)}(E)$, and the value $2ip^2$ if $\phi_i^{(k)}(E)\equiv\phi_j^{(\ell)}(E)$. Now, following the finite-cutoff interpretation of RGE's as described, for example, in Ref. [9], we compare the kinetic energy terms in the theory with the UV cutoff $E$ and in the equivalent theory with the slightly different cutoff ($E+dE$). The two-point Green functions in these two equivalent theories must be identical. Imposition of this requirement in the tree plus one-loop approximation then leads to the following relation:

$$\begin{aligned}\frac{1}{2}\sum_{j=1}^{4}\sum_{k=1}^{2}\partial_\nu\phi_j^{(k)}(E)\partial^\nu\phi_j^{(k)}(E) &=\frac{1}{2}\sum_{j=1}^{4}\sum_{k=1}^{2}\partial_\nu\phi_j^{(k)}(E+dE)\partial^\nu\phi_j^{(k)}(E+dE)\\ &+\frac{N_{\rm c}}{32\pi^2}(d\ln E^2)\\ &\times\Bigg\{\sum_{j=1}^{4}\sum_{k,\ell=1}^{2}A_{k\ell}(E)\partial_\nu\phi_j^{(k)}(E)\partial^\nu\phi_j^{(\ell)}(E)\\ &+\sum_{(j,j')}\sum_{k,\ell=1}^{2}(-1)^{j'+\ell}B_{k\ell}(E)\\ &\times\partial_\nu\phi_j^{(k)}(E)\partial^\nu\phi_{j'}^{(\ell)}(E)\Bigg\},\end{aligned}\tag{A3}$$

where the summation in the last sum runs over the already mentioned complementary indices $(jj')=(12)$, (21), (34), (43), $d(\ln E^2)\equiv\ln(E+dE)^2-\ln E^2=2dE/E$, and elements of the real symmetric matrices $A(E)$ and $B(E)$ are related to the Green function expressions (A1) and (A2), respectively,

$$A_{k\ell}(E)=\frac{1}{2}{\rm Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}+\tilde{U}^{(\ell)}\tilde{U}^{(k)\dagger}+\tilde{D}^{(k)}\tilde{D}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}](E), \tag{A4}$$

$$B_{k\ell}(E)=(-1)^\ell\frac{i}{2}{\rm Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}-\tilde{U}^{(\ell)}\tilde{U}^{(k)\dagger}-\tilde{D}^{(k)}\tilde{D}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}](E). \tag{A5}$$

Equation (A3) is described in the following way: the double sum on the left and the first double sum on the right represent the kinetic energy terms of the scalars in the formulation with UV cutoff $E$ and $(E+dE)$, respectively. The one-loop contributions of Fig. 7 with the loop momentum $|\bar{q}|$ in the Euclidean energy interval $E\leqslant|\bar{q}|\leqslant\Lambda$ are already contained in the kinetic energy terms of the left effectively at the tree level ($\Lambda$ is a large cutoff where the theory is presumed to break down). On the other hand, the kinetic energy terms of the $(E+dE)$ cutoff formulation [the first double sum on the right of Eq. (A3)] effectively contain, at the tree level, the one-loop effects of Fig. 7 for the slightly smaller energy interval $(E+dE)\leqslant|\bar{q}|\leqslant\Lambda$. Therefore, the Green function contributions[13] $-{\rm i}d\Sigma_{i,j}^{(k,\ell)}(p^2;E^2)$ of Fig. 7 from the loop-momentum interval $E\leqslant|\bar{q}|\leqslant(E+dE)$ had to be included on the right of Eq. (A3)—these are the terms in the last two double sums there. This is illustrated in Fig. 8.

In order to find RGE's for the scalar fields $\phi_j^{(k)}(E)$, we make the following *Ansatz* for the solution of Eq. (A3):

$$\vec{\phi}_j(E+dE)=\vec{\phi}_j(E)+d\alpha^{(j)}(E)\vec{\phi}_j(E)+d\beta^{(j)}(E)\vec{\phi}_{j'}(E), \tag{A6}$$

where $\vec{\phi}_j$ is two-component column made up of $\phi_j^{(1)}$ and $\phi_j^{(2)}$, $d\alpha^{(j)}(E)$ and $d\beta^{(j)}(E)$ are infinitesimally small $2\times2$ matrices and $d\beta^{(j)}$ has zero diagonal elements, and scalar

[13]More precisely: the corresponding effective kinetic energy terms.

indices $(jj')$ are again complementary. Inserting *Ansatz* (A6) into RGE relation (A3), we obtain relations

$$d\alpha_{k\ell}^{(j)}(E)+d\alpha_{\ell k}^{(j)}(E)=-N_{\rm c}(d\ \ln E^2)A_{k\ell}(E)/(16\pi^2), \tag{A7}$$

$$d\beta_{k\ell}^{(j)}(E)+d\beta_{\ell k}^{(j')}(E)=(-1)^{j+\ell}N_{\rm c}(d\ \ln E^2)\times B_{k\ell}(E)/(16\pi^2). \tag{A8}$$

In principle, these relations alone do not define the elements $d\alpha_{k\ell}^{(j)}(E)$ and $d\beta_{k\ell}^{(j)}(E)$. However, RGE evolution of the isodoublet fields $\Phi^{(1)}(E)$ and $\Phi^{(2)}(E)$ should be invariant under the exchange of Higgs generation indices $1\leftrightarrow 2$, because these two Higgs doublets appear in the original Lagrangian density (1) in a completely $1\leftrightarrow 2$ symmetric manner. We will see in retrospect that this exchange symmetry is respected once we impose the conditions

$$d\alpha_{k\ell}^{(j)}(E)=d\alpha_{\ell k}^{(j)}(E),\quad d\beta_{k\ell}^{(j)}(E)=d\beta_{\ell k}^{(j')}(E). \tag{A9}$$

Relations (A7)–(A9) lead to specific expressions for the evolution coefficients $d\alpha_{k\ell}^{(j)}(E)$ and $d\beta_{k\ell}^{(j)}(E)$. When inserting these coefficients back into *Ansatz* (A6), we obtain the one-loop RGE's for the evolution of the scalar fields:

$$\frac{16\pi^2}{N_{\rm c}}\frac{d}{d\ \ln E}\phi_j^{(1)}(E)=-{\rm Tr}[\tilde U^{(1)}\tilde U^{(1)\dagger}+\tilde D^{(1)}\tilde D^{(1)\dagger}]\phi_j^{(1)}-\frac{1}{2}\,{\rm Tr}[\tilde U^{(1)}\tilde U^{(2)\dagger}+\tilde U^{(2)}\tilde U^{(1)\dagger}+\tilde D^{(1)}\tilde D^{(2)\dagger}+\tilde D^{(2)}\tilde D^{(1)\dagger}]\phi_j^{(2)}$$
$$+i(-1)^j\frac{1}{2}\,{\rm Tr}[\tilde U^{(1)}\tilde U^{(2)\dagger}-\tilde U^{(2)}\tilde U^{(1)\dagger}-\tilde D^{(1)}\tilde D^{(2)\dagger}+\tilde D^{(2)}\tilde D^{(1)\dagger}]\phi_{j'}^{(2)}, \tag{A10}$$

$$\frac{16\pi^2}{N_{\rm c}}\frac{d}{d\ \ln E}\phi_j^{(2)}(E)=-{\rm Tr}[\tilde U^{(2)}\tilde U^{(2)\dagger}+\tilde D^{(2)}\tilde D^{(2)\dagger}]\phi_j^{(2)}-\frac{1}{2}\,{\rm Tr}[\tilde U^{(1)}\tilde U^{(2)\dagger}+\tilde U^{(2)}\tilde U^{(1)\dagger}+\tilde D^{(1)}\tilde D^{(2)\dagger}+\tilde D^{(2)}\tilde D^{(1)\dagger}]\phi_j^{(1)}$$
$$+i(-1)^{j+1}\frac{1}{2}\,{\rm Tr}[\tilde U^{(1)}\tilde U^{(2)\dagger}-\tilde U^{(2)}\tilde U^{(1)\dagger}-\tilde D^{(1)}\tilde D^{(2)\dagger}+\tilde D^{(2)}\tilde D^{(1)\dagger}]\phi_{j'}^{(1)}, \tag{A11}$$

where again $j'$ is the scalar index complementary to index $j$: $(jj')=(12)$, (21), (34), (43). These RGE's lead to RGE's for scalar isodoublets $\Phi^{(k)}$:

$$\frac{16\pi^2}{N_{\rm c}}\frac{d}{d\ \ln E}\Phi^{(k)}(E)=-\sum_{\ell=1}^{2}{\rm Tr}[\tilde U^{(k)}\tilde U^{(\ell)\dagger}+\tilde D^{(\ell)}\tilde D^{(k)\dagger}]\Phi^{(\ell)}. \tag{A12}$$

We really see that this set of one-loop RGE's is invariant under the exchange $1\leftrightarrow 2$, as required by the form of the Yukawa Lagrangian density (1).

In addition to quark loops, there are also loops of the electroweak gauge bosons contributing to one-loop two-point Green functions of the scalars. However, since these gauge bosons couple to the Higgs isodoublets identically as in the minimal standard model (MSM), their contributions[14] to the right of RGE's (A10)–(A12) are the same as in the MSM.[15] Hence, the full one-loop RGE's for the the scalar isodoublets in the G2HDM, in the Landau gauge, are

$$16\pi^2\frac{d}{d\ \ln E}\Phi^{(k)}(E)=-N_{\rm c}\sum_{\ell=1}^{2}{\rm Tr}[\tilde U^{(k)}\tilde U^{(\ell)\dagger}+\tilde D^{(\ell)}\tilde D^{(k)\dagger}]\Phi^{(\ell)}+\left[\frac{3}{4}g_1^2(E)+\frac{9}{4}g_2^2(E)\right]\Phi^{(k)}(E), \tag{A13}$$

and completely analogous gauge boson contributions also have to be added on the right of Eqs. (A10), (A11). These RGE's are simultaneously also RGE's for the corresponding VEV's (5):

$$16\pi^2\frac{d}{d\ \ln E}(e^{i\eta_k}v_k)=-N_{\rm c}\sum_{\ell=1}^{2}{\rm Tr}[\tilde U^{(k)}\tilde U^{(\ell)\dagger}+\tilde D^{(\ell)}\tilde D^{(k)\dagger}](e^{i\eta_\ell}v_\ell)+\left[\frac{3}{4}g_1^2(E)+\frac{9}{4}g_2^2(E)\right](e^{i\eta_k}v_k). \tag{A14}$$

In this paper we do not discuss the question of quadratic cutoff terms $\Lambda^2$ which appear in the radiative corrections to

[14]They are gauge dependent.

[15]For these contributions of EW gauge bosons in the MSM, see, for example, Arason *et al.* [27], Appendix A. However, note that they use for the $U(1)_Y$ gauge coupling $g_1$ a different, GUT-motivated convention: $(g_1^2)_{\text{Arason }et\ al.}=(5/3)(g_1^2)_{\text{here}}$.

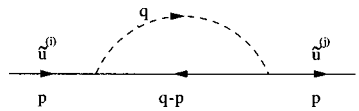


FIG. 9. The diagram leading to the two-point Green function $-i\Sigma(p;E;\tilde{u}^{(i)},\tilde{u}^{(j)})$. Dashed and full lines represent scalar and quark propagators, respectively.

VEV's in any SM framework. In the MSM, their consideration—under the assumption of the top quark dominance of the radiative corrections in the scalar sector—leads to severe upper bounds on the ultraviolet cutoff $\Lambda$ for a substantial subset of values of the bare doublet mass and of the bare scalar self-interaction parameters $M^2(\Lambda)$ and $\lambda(\Lambda)$, see Ref. [28].

In order to derive one-loop RGE's for the Yukawa matrices $\tilde{U}^{(k)}$ and $\tilde{D}^{(k)}$, the results (A10)–(A13) are needed. In addition, RGE's for evolution of the quark fields $\tilde{u}^{(j)}_{L,R}$ and $\tilde{d}^{(j)}_{L,R}$ are also needed.

#### 2. One-loop RGE's for the quark fields

These RGE's can be derived in close analogy with the derivation of the evolution of scalar fields of the previous section. Now, the diagrams (Green functions) of Figs. 7 and 8 are replaced by those of Figs. 9 and 10, and the scalar field kinetic energy terms in Eq. (A3) are replaced by those of the quark fields. The Green function of Fig. 9, with the incoming $\tilde{u}^{(i)}$ and outgoing $\tilde{u}^{(j)}$ of momentum $p$, in the framework with UV cutoff $E$, is

$$-i\Sigma(p;E;\tilde{u}^{(i)},\tilde{u}^{(j)}) = \frac{i}{64\pi^2}\ln\left(\frac{E^2}{m^2}\right)\not{p}\Bigg\{2(1+\gamma_5)\sum_{\ell=1}^{2}[\tilde{U}^{(\ell)\dagger}\tilde{U}^{(\ell)}]_{ji} + (1-\gamma_5)\sum_{\ell=1}^{2}[\tilde{U}^{(\ell)}\tilde{U}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(\ell)\dagger}]_{ji}\Bigg\}. \tag{A15}$$

The Green function with the incoming $\tilde{d}^{(i)}$ and outgoing $\tilde{d}^{(j)}$ of momentum $p$ is obtained from the above expression by simply exchanging $\tilde{U}^{(\ell)}\leftrightarrow\tilde{D}^{(\ell)}$ and $\tilde{U}^{(\ell)\dagger}\leftrightarrow\tilde{D}^{(\ell)\dagger}$. The quark fields evolve according to the *Ansatz*

$$d\tilde{q}^{(k)}(E)_{L,R}=df_q(E)^{(L,R)}_{k\ell}\tilde{q}^{(\ell)}(E)_{L,R}, \tag{A16}$$

where $d\tilde{q}(E)$ generically stands for $\tilde{q}(E+dE)-\tilde{q}(E)$ $[\tilde{q}^{(k)}=\tilde{u}^{(k)},\tilde{d}^{(k)}]$ and subscripts $L,R$ denote the handedness of the quark fields: $\tilde{q}_L\equiv(1-\gamma_5)\tilde{q}/2$, $\tilde{q}_R\equiv(1+\gamma_5)\tilde{q}/2$. In complete analogy with the previous section, we obtain from this *Ansatz* and from the RGE relation[16] illustrated in Fig. 10 the following relations for the quark field evolution matrices $df_u$:

$$df_u(E)^{(L)*}_{ij}+df_u(E)^{(L)}_{ji} = -\frac{(d\ \ln E^2)}{32\pi^2}\sum_{k=1}^{2}[\tilde{U}^{(k)}\tilde{U}^{(k)\dagger}+\tilde{D}^{(k)}\tilde{D}^{(k)\dagger}]_{ji}(E), \tag{A17}$$

$$df_u(E)^{(R)*}_{ij}+df_u(E)^{(R)}_{ji} = -\frac{2(d\ \ln E^2)}{32\pi^2}\sum_{k=1}^{2}[\tilde{U}^{(k)\dagger}\tilde{U}^{(k)}]_{ji}(E). \tag{A18}$$

The relations for the $df_d$ evolution matrices of the down-type sector are obtained from the above by simple exchanges $\tilde{U}^{(\ell)}\leftrightarrow\tilde{D}^{(\ell)}$ and $\tilde{U}^{(\ell)\dagger}\leftrightarrow\tilde{D}^{(\ell)\dagger}$. A Hermitean solution to these relations is

$$df_u(E)^{(L)}_{ij}=-\frac{(d\ \ln E^2)}{64\pi^2}\sum_{k=1}^{2}[\tilde{U}^{(k)}\tilde{U}^{(k)\dagger}+\tilde{D}^{(k)}\tilde{D}^{(k)\dagger}]_{ij}(E) = df_d(E)^{(L)}_{ij}, \tag{A19}$$

$$df_u(E)^{(R)}_{ij}=-\frac{2(d\ \ln E^2)}{64\pi^2}\sum_{k=1}^{2}[\tilde{U}^{(k)\dagger}\tilde{U}^{(k)}]_{ij}(E), \tag{A20}$$

$$df_d(E)^{(R)}_{ij}=-\frac{2(d\ \ln E^2)}{64\pi^2}\sum_{k=1}^{2}[\tilde{D}^{(k)\dagger}\tilde{D}^{(k)}]_{ij}(E). \tag{A21}$$

The results (A19)–(A21), in conjunction with Eq. (A16), represent one-loop RGE's for evolution of the quark fields in

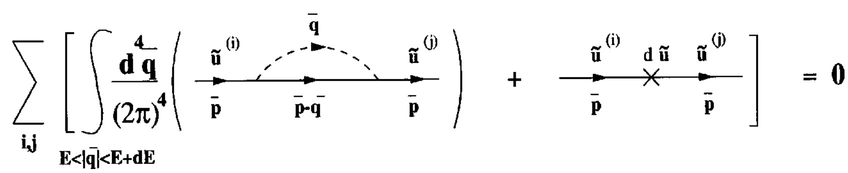


FIG. 10. Diagrammatic illustration of the RGE relation leading to the evolution of quark fields. This relation means that the two-point Green functions with truncated external quark legs, at one-loop level, are the same in the theory with $E$ cutoff and in the theory with $E+dE$ cutoff. Conventions are the same as in previous figures.

[16]RGE relation represented by Fig. 10 is analogous to relation (A3) represented by Fig. 8.

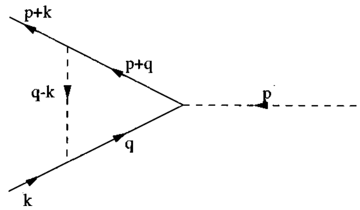


FIG. 11. One-particle-irreducible (1PI) diagram contributing to the evolution of the Yukawa coupling parameters. Conventions are the same is in previous figures.

the G2HDM, but without the gauge boson contributions. The latter contributions are the same as in the MSM and can be included in Eqs. (A19)–(A21).

### 3. One-loop RGE's for the Yukawa coupling matrices

To derive these RGE's, we need, in addition to the results of the two previous sections, also another Green function. It is represented by the diagram of Fig. 11. When the external legs there are $\tilde{u}^{(i)}$ (incoming, with momentum $k$), $\tilde{u}^{(j)}$ (outgoing, with momentum $p+k$), and $\phi_3^{(\ell)}$ [or $\phi_4^{(\ell)}$], it turns out that only the diagram with the *charged* scalar exchange contributes, and the resulting truncated three-point Green function, in the framework with the UV cutoff $E$, is

$$G^{(3)}(k,p;E;\tilde{u}^{(i)},\tilde{u}^{(j)};\phi_3^{(\ell)}) = -\frac{i}{32\pi^2\sqrt{2}}\ln\left(\frac{E^2}{m^2}\right)\sum_{r=1}^{2}\{(1+\gamma_5)[\tilde{D}^{(r)}\tilde{D}^{(\ell)\dagger}\tilde{U}^{(r)}]_{ji} + (1-\gamma_5)[\tilde{U}^{(r)\dagger}\tilde{D}^{(\ell)}\tilde{D}^{(r)\dagger}]_{ji}\}. \tag{A22}$$

The corresponding Green function with the down-type quark external legs is obtained from the above by the exchanges $\tilde{U}^{(s)} \leftrightarrow \tilde{D}^{(s)}$ and $\tilde{U}^{(s)\dagger} \leftrightarrow \tilde{D}^{(s)\dagger}$.

Now, the one-loop RGE's for the Yukawa matrices are obtained in analogy with the reasoning leading, in the case of two-point scalar Green functions, to the RGE relation (A3) (see Fig. 8). It is straightforward to check that the contribution of the quark loops in the scalar external leg cancel the contributions coming from the renormalizations of the scalar fields in the kinetic energy terms of the scalars—this is illustrated in Fig. 12. Furthermore, it can be checked that the contributions of the scalar exchanges on the external quark legs cancel the contributions coming from the renormalizations of the quark fields in the kinetic energy terms of the quarks—illustrated in Fig. 13. All in all, the one particle reducible (1PR) one-loop contributions are canceled by the contributions of field renormalizations in the kinetic energy terms. Therefore, the only one-loop terms contributing to evolution of the $\tilde{U}^{(k)}$ Yukawa matrices are those depicted in Fig. 14. The three diagrams with crosses there correspond to contributions of the following changes *in the Yukawa coupling* terms: Yukawa matrix change (renormalization) $d\tilde{U}^{(k)}$ $[\equiv \tilde{U}^{(k)}(E+dE) - \tilde{U}^{(k)}(E)]$—Fig. 14(b); the scalar field renormalization $d\tilde{\phi}_s^{(k)}$ $[\equiv \tilde{\phi}_s^{(k)}(E+dE) - \tilde{\phi}_s^{(k)}(E)]$—Fig. 14(c); the quark field renormalization $d\tilde{u}^{(i)}$ and $d\tilde{u}^{(j)}$—Fig. 14(d). Figure 14 is a diagrammatical representation of the physical requirement that the three-point (quark-antiquark-scalar) Green function, at one-loop level, in the theory with the cutoff $E+dE$ [left side of Fig. 14: (a)$+\cdots+$(e)] be the same as in the theory with the slightly lower cutoff $E$ (right side).

Using the results of this and the previous sections of the Appendix, we can then write down the one-loop RGE for $\tilde{U}^{(k)}$ corresponding to Fig. 14, at the right-handed component $[\propto(1+\gamma_5)]$ of the three-point Green function

$$\tilde{U}_{ji}^{(k)} + d\tilde{U}_{ji}^{(k)} + \frac{1}{32\pi^2}(d\ \ln E^2)\Bigg\{-N_c\sum_{\ell=1}^{2}\ \mathrm{Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger} + \tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}]\tilde{U}^{(\ell)} - \frac{1}{2}\sum_{\ell=1}^{2}[(\tilde{U}^{(\ell)}\tilde{U}^{(\ell)\dagger} + \tilde{D}^{(\ell)}\tilde{D}^{(\ell)\dagger})\tilde{U}^{(k)} + 2\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}\tilde{U}^{(\ell)}] + 2\sum_{\ell=1}^{2}[\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}\tilde{U}^{(\ell)}]\Bigg\}_{ji} = \tilde{U}_{ji}^{(k)}. \tag{A23}$$

Taking the left-handed component of the three-point Green function results in the Hermitean conjugate of Eq. (A23), i.e., in an equivalent relation. The first sum on the left ($\propto N_c$) of Eq. (A23) corresponds to Fig. 14(c) [cf. Eqs. (A10)–(A12)], the second sum to Fig. 14(d) [cf. Eqs. (A19), (A20)], and the third sum to Fig. 14(e) [cf. Eq. (A22)]. The left-handed part of the Green function yields just the Hermitean conjugate of the above matrix relation. The analogous consideration of the three-point

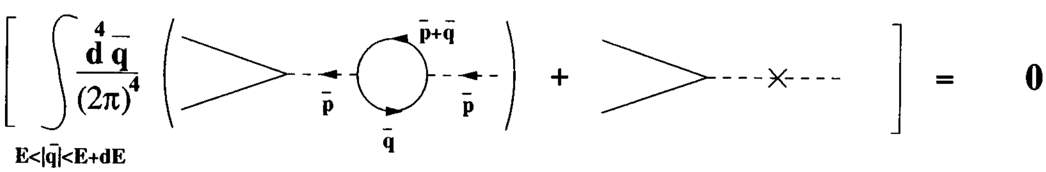


FIG. 12. Cancellation of contributions from the quark loop (one-particle-reducible) with those of the scalar field renormalizations in the kinetic energy term of the scalars, for the energy cutoff interval $(E,E+dE)$.

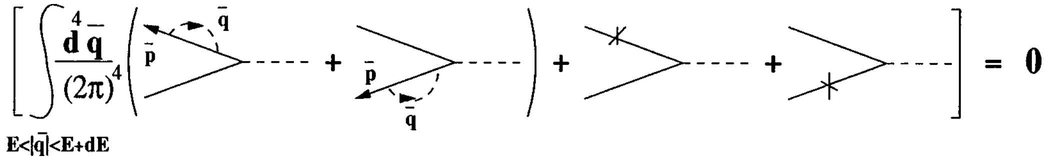


FIG. 13. Cancellation of contributions from the scalar exchange on the quark legs (1PR) with those of the quark field renormalizations in the kinetic energy term of the quarks, for the energy cutoff interval $(E,E+dE)$.

Green functions with the down-type external quark legs $\tilde{d}^{(i)}$ and $\tilde{d}^{(j)}$ gives relations which can be obtained from the above relation again by the exchanges $\tilde{U}^{(s)}\leftrightarrow\tilde{D}^{(s)}$ and $\tilde{U}^{(s)\dagger}\leftrightarrow\tilde{D}^{(s)\dagger}$. These relations can be rewritten in a more conventional form

$$16\pi^2\frac{d}{d\ \ln E}\ \tilde{U}^{(k)}(E)=\Bigg\{N_c\sum_{\ell=1}^{2}\ \mathrm{Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}]\tilde{U}^{(\ell)}+\frac{1}{2}\sum_{\ell=1}^{2}\ [\tilde{U}^{(\ell)}\tilde{U}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(\ell)\dagger}]\tilde{U}^{(k)} + \tilde{U}^{(k)}\sum_{\ell=1}^{2}\ \tilde{U}^{(\ell)\dagger}\tilde{U}^{(\ell)}-2\sum_{\ell=1}^{2}\ [\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}\tilde{U}^{(\ell)}]\Bigg\}, \tag{A24}$$

and an analogous RGE for $\tilde{D}^{(k)}$. These RGE's still do not contain one-loop effects of exchanges of gauge bosons. However, since the couplings of quarks and the Higgs doublets to the gauge bosons are identical to those in the usual MSM, 2HDM(I), and 2HDM(II), their contributions on the right of the above RGE's are identical to those in these theories. Therefore, the final form of the one-loop RGE's for the Yukawa matrices in the general 2HDM now reads

$$16\pi^2\frac{d}{d\ \ln E}\ \tilde{U}^{(k)}(E)=\Bigg\{N_c\sum_{\ell=1}^{2}\ \mathrm{Tr}[\tilde{U}^{(k)}\tilde{U}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}]\tilde{U}^{(\ell)}+\frac{1}{2}\sum_{\ell=1}^{2}\ [\tilde{U}^{(\ell)}\tilde{U}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(\ell)\dagger}]\tilde{U}^{(k)}+\tilde{U}^{(k)}\sum_{\ell=1}^{2}\ \tilde{U}^{(\ell)\dagger}\tilde{U}^{(\ell)} -2\sum_{\ell=1}^{2}\ [\tilde{D}^{(\ell)}\tilde{D}^{(k)\dagger}\tilde{U}^{(\ell)}]-A_U\tilde{U}^{(k)}\Bigg\}, \tag{A25}$$

$$16\pi^2\frac{d}{d\ \ln E}\ \tilde{D}^{(k)}(E)=\Bigg\{N_c\sum_{\ell=1}^{2}\ \mathrm{Tr}[\tilde{D}^{(k)}\tilde{D}^{(\ell)\dagger}+\tilde{U}^{(\ell)}\tilde{U}^{(k)\dagger}]\tilde{D}^{(\ell)}+\frac{1}{2}\sum_{\ell=1}^{2}\ [\tilde{U}^{(\ell)}\tilde{U}^{(\ell)\dagger}+\tilde{D}^{(\ell)}\tilde{D}^{(\ell)\dagger}]\tilde{D}^{(k)}+\tilde{D}^{(k)}\sum_{\ell=1}^{2}\ \tilde{D}^{(\ell)\dagger}\tilde{D}^{(\ell)} -2\sum_{\ell=1}^{2}\ [\tilde{U}^{(\ell)}\tilde{U}^{(k)\dagger}\tilde{D}^{(\ell)}]-A_D\tilde{D}^{(k)}\Bigg\}, \tag{A26}$$

where the functions $A_U$ and $A_D$, characterizing the contributions of the gauge boson exchanges, are gauge independent and are the same as in the MSM, 2HDM(I), and 2HDM(II):

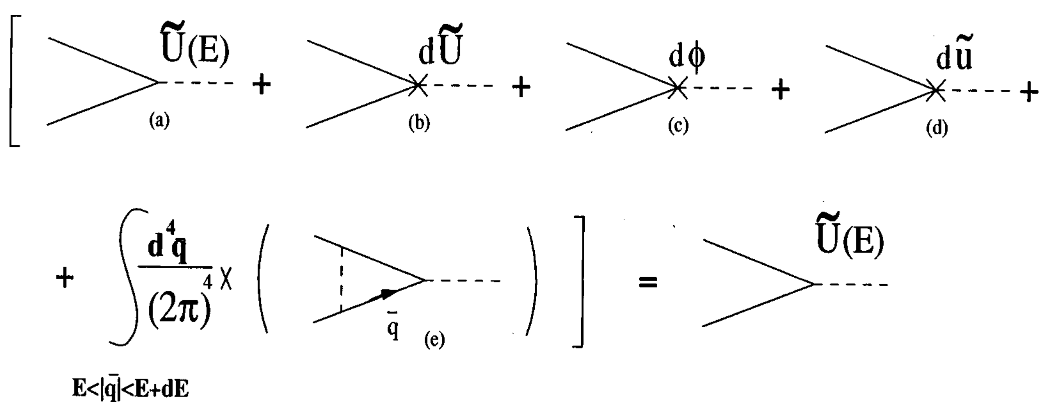


FIG. 14. Diagrammatic representation of the RGE for the up-type Yukawa matrix $\tilde{U}$. Only the 1PI scalar exchange (e) and the effects of the renormalizations of the Yukawa matrix, of the scalar fields and the quark fields in the Yukawa couplings [(b), (c), (d), respectively] contribute when the cutoff is changed from $E$ (RHS) to $E+dE$ (LHS). Note that $d\tilde{U}$ stands for $\tilde{U}(E+dE)-\tilde{U}(E)$, etc. The contributions of the gauge boson exchanges were not considered in the figure.

$$A_U = 3\,\frac{(N_c^2-1)}{N_c}g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}g_1^2, \quad A_D = A_U - g_1^2, \tag{A27}$$

and the gauge coupling parameters $g_j$ satisfy the one-loop RGE's

$$16\pi^2 \frac{d}{d\ln E} g_j = -C_j g_j^3, \tag{A28}$$

with the coefficients $C_j$ being those for the 2HDM's ($N_H = 2$)

$$C_3 = \frac{1}{3}(11N_c - 2n_q), \quad C_2 = 7 - \frac{2}{3}n_q, \quad C_1 = -\frac{1}{3} - \frac{10}{9}n_q. \tag{A29}$$

Here, $n_q$ is the number of effective quark flavors, e.g., for $E > m_t$ we have $n_q \approx 6$; for $m_b < E < m_t$ we have $n_q \approx 5$, etc. This completes the derivation of the one-loop RGE's.


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