

U(1)' symmetry breaking in supersymmetric E₆ models

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We study the electroweak and U(1)' symmetry breaking patterns in models with the particle content of supersymmetric E₆, including standard model singlets S and exotic quarks D, \bar{D} . Motivated by free fermionic string models, we do not require E₆-type relations between Yukawa couplings. In particular, we assume that baryon and lepton numbers are conserved, so that the exotic quarks can be light. Gauge invariance allows Yukawa interactions between S and Higgs doublets, and between S and the exotic quarks, allowing radiative U(1)' symmetry breaking and the generation of an effective μ parameter at the electroweak scale. For both the E₆ ψ and η models, universal soft supersymmetry breaking parameters and Yukawa universality at the high (string) scale do not yield acceptable low energy phenomenology. Relaxing universality, we find solutions with phenomenologically acceptable values of $M_{Z'}$, and the $Z-Z'$ mixing angle. In addition, by varying the U(1)' charge assignments due to the mixing of U(1) _{χ} and U(1) _{ψ} of E₆, it is possible to have acceptable low energy phenomenology with universal boundary conditions. [S0556-2821(98)04619-0]

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I. INTRODUCTION

It was argued in [1] that for a class of string motivated models with extra U(1)'s, the U(1)' breaking should be at the electroweak scale, and in [2] a general analysis was done for a model in which the two standard model (SM) Higgs doublets couple to a single SM singlet S which carries a nontrivial U(1)' charge. The breaking of the U(1)' was shown to be at the electroweak scale, with a certain amount of fine tuning of the soft supersymmetry breaking parameters at M_{string} required to suppress the $Z-Z'$ mixing. In addition, the U(1)' symmetry forbids an elementary μ term, but an effective electroweak scale μ term arises when S acquires a vacuum expectation value (VEV). It was argued that the radiative U(1)' breaking is most easily accomplished if there are large Yukawa couplings between the singlet S and exotic particles.

More complicated models involving two or more S fields with opposite signs for their U(1)' charges may lead to U(1)' breaking either at the electroweak scale or an intermediate scale, depending on the details of the superpotential and the supersymmetry breaking terms [3].

One of the major obstacles to analyzing U(1)' breaking in string models is that compactifications which have the SM gauge group and the SM particle content with three generations and two Higgs doublets usually have additional gauge symmetries and many exotic particles. The physics of U(1)' and electroweak breaking can be mingled with other issues such as the decoupling of heavy particles, preserving gauge unification in the presence of exotics, the communication between hidden and observable sectors, breaking of the extra non-Abelian symmetries, etc., that are not well understood. This motivated us to study in more detail the low energy phenomenology of the radiative U(1)' breaking in a simpler model with all the elements that are necessary for a realistic theory.

Such a model should have the SM gauge group plus at least one extra U(1)', three ordinary families, two or more

Higgs doublets (H_1, H_2), a SM singlet (S) with nontrivial U(1)' charge, and exotics (D_i). Phenomenological constraints suggest that the latter should be nonchiral [4]. The model must be anomaly free, and the U(1)' quantum numbers must be such that $Q_{H_1} + Q_{H_2} + Q_S = 0$, $Q_{D_1} + Q_{D_2} + Q_S = 0$ to allow such couplings as $\hat{S}\hat{H}_1 \cdot \hat{H}_2$ and $\hat{S}\hat{D}_1 \cdot \hat{D}_2$ in the superpotential. In addition, the model should have gauge unification at a scale comparable to M_{string} .

A simple model satisfying these constraints is the E₆ model. E₆ is one of the most promising grand unification (GUT) models. Another motivation for E₆ is that Calabi-Yau compactifications of the heterotic superstring model lead to the gauge group E₆ or its subgroups in the observable sector, and also include the standard model representations for the matter multiplets as well as additional exotic fields [5–8]. It is not our intention to assume a full grand unified supersymmetric E₆ model. Strictly speaking, we use the particle content of the E₆ model, which provides acceptable anomaly free U(1)' quantum numbers. Our purpose is to study the electroweak breaking of the model in the presence of an additional U(1)' symmetry. In the full E₆ GUT model, the Yukawa couplings of these exotics would be related by E₆ to the Higgs Yukawa couplings so that the exotics would have to be superheavy to avoid too rapid proton decay [9] (the doublet-triplet problem). In our string motivated model the exotics and Higgs Yukawa couplings do not respect the E₆ symmetries. In particular, the dangerous exotic couplings may be absent, so that the exotics can be light. We also assume Yukawa universality, i.e., that all of the non-zero Yukawa couplings are equal at the string scale.

Bounds from direct searches at the Fermilab Tevatron ($p\bar{p} \rightarrow Z' \rightarrow l^+l^-$) [10] and precision electroweak tests [11] on the Z' mass and the $Z-Z'$ mixing are stringent. The lower limits on the Z' mass are model dependent, but are typically around 500 GeV [10,11] except in the case of suppressed coupling to ordinary particles, such as in leptophobic models [12–15]. Similarly, limits on the mixing angle are around a few $\times 10^{-3}$.

The particle content of the E_6 model we consider includes three E_6 27-plets, each of which includes an ordinary family, two Higgs-type doublets, two standard model singlets, and two exotic $SU(2)$ -singlet quarks with charge $\pm 1/3$. In addition, there can be any number of vector pairs of chiral supermultiplets from one or more $27+27^*$ representations without introducing anomalies. For simplicity and consistency with the desired gauge unification, we assume a single pair of Higgs-type doublets from a $27+27$.¹ Thus, there are eight Higgs doublet candidates, six singlet candidates and three exotic quark pair candidates. We assume that only a subset of these play a direct role in SM and $U(1)'$ symmetry breaking, based on simple (string-motivated) assumptions concerning the nonzero Yukawa terms in the superpotential and the running of soft mass squared parameters being dominated by the largest Yukawa couplings. These are two SM doublets H_1 and H_2 , one SM singlet S , and exotic quarks D and \bar{D} . We assume that all of the soft mass squares are positive at the Planck scale. The soft mass squares of H_2 and S are typically both driven negative at low energy due to the Yukawa couplings $\hat{Q}\hat{u}^c\cdot\hat{H}_2$ and $\hat{S}\hat{D}\cdot\hat{\bar{D}}$ in the superpotential, where Q and u^c represent ordinary quarks and antiquarks. Therefore, the $U(1)'$ breaking as well as the electroweak breaking are radiative. The coupling $\hat{S}\hat{H}_1\cdot\hat{H}_2$ in the superpotential becomes an effective μ term when S acquires a VEV. Kinetic mixing [15,16] between $U(1)_Y$ and $U(1)'$ is necessarily present in the theory from field theoretic loops, and its effects are included in the analysis.

In [2] it was shown that there are two general scenarios to obtain a small $Z-Z'$ mixing angle. In one case, in which the symmetry breaking is driven by large trilinear soft supersymmetry breaking terms, the Z' mass is comparable to M_Z . Such a scenario is only allowed experimentally if the Z' couplings to ordinary fermions are small, such as in leptophobic models [12–15]. We obtain examples of a light Z' and small mixings (in fact, $M_{Z'} < M_Z$). However, the couplings are not leptophobic [1] and these cases are excluded. The other possibility is that $M_{Z'}$ is large, e.g., near the TeV scale, but the electroweak scale (and M_Z) is small due to approximate cancellations. The signs of the $U(1)'$ charges in usual (ψ and η) E_6 models are such that one cannot obtain this cancellation (even in the presence of kinetic mixing, which could in principle change the signs) for universal boundary conditions (universal soft supersymmetry breaking parameters with Yukawa universality). However, with non-universal soft mass square parameters, there exist viable though somewhat fine-tuned solutions for a heavy Z' with small mixing angle. We further utilize the fact that E_6 has two orthogonal $U(1)'$ symmetries, and the surviving $U(1)'$

(see footnote 2)² is in principle a linear combination with charge $Q = Q_\chi \cos\theta_{E_6} + Q_\psi \sin\theta_{E_6}$, where $U(1)_\chi$ and $U(1)_\psi$ refer to the patterns $SO(10) \rightarrow SU(5) \times U(1)_\chi$ and $E_6 \rightarrow SO(10) \times U(1)_\psi$, respectively. With this additional degree of freedom, we find there are interesting solutions.

In Sec. II, we give the particle content and quantum numbers of the E_6 model. We also describe the explicit superpotential, the scalar potential, the minimization conditions and the two phenomenologically acceptable scenarios for Z' . A brief outline of the RGE analysis is given in Sec. III. In Sec. IV, we show that the universal boundary conditions can result in a light Z' and small mixing angle, in agreement with the large trilinear coupling solutions of [2]. In particular, we find that $M_{Z'} < M_Z$ for both the ψ and η models. Because the effect of kinetic mixing here is too small to make Z' leptophobic in the η model, this scenario is not phenomenologically acceptable. In Sec. V, we show that by deviating from the universal boundary conditions at M_{string} , we can obtain a large VEV for the singlet S , so that $M_{Z'} > 500$ GeV. We argue that certain stringent cancellation conditions have to be satisfied for the large S scenario, which in turn requires fine tuning of the soft supersymmetry breaking parameters at high energy. In Sec. VI, we introduce the mixing angle θ_{E_6} between the two E_6 $U(1)'$ s. We find that with universal boundary conditions there exist acceptable solutions for $M_{Z'}$ and the mixing angle for certain parameter ranges of θ_{E_6} and A_s (the trilinear coupling constant) for a given gaugino mass.

The summary and conclusions are given in Sec. VII. The renormalization group equations used in the analysis are presented in the Appendix.

II. E_6 MODEL AND ELECTROWEAK SYMMETRY BREAKING

Two additional $U(1)'$ symmetries can occur when E_6 is broken,

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi. \quad (1)$$

The decomposition of the fundamental 27 representation under the $SU(5) \times U(1)_\psi$ subgroup is

$$27_L \rightarrow (10,1)_L + (5^*,1)_L + (1,1)_L + (5,-2)_L \\ + (5^*,-2)_L + (1,4)_L \quad (2)$$

where the first and second quantities are the $SU(5)$ representation and $\sqrt{24}Q_\psi$, respectively, and the subscript means that left-chiral fields will be assigned to the multiplets. $(10,1)_L + (5^*,1)_L$ correspond to an ordinary SM family, $(1,1)_L$ and $(1,4)_L$ are SM singlets and $(5,-2)_L + (5^*,-2)_L$ are exotic multiplets which form a vector pair under the standard model gauge group. $(5,-2)_L$ consists of D_L and h_2 , where D_L is a color-triplet quark with charge $-1/3$, and h_2 can be either a Higgs doublet or an exotic lepton doublet. Similarly, $(5^*,-2)_L$ has the exotic antiquark \bar{D}_L , and h_1 , which can be

¹Kolda *et al.* [15] have argued that in a class of E_6 models kinetic mixing can yield leptophobic Z' couplings. We include kinetic mixing in our model, but in contrast to the model considered in [15] (which included additional fields from an extra 78), the kinetic mixing is too small to yield leptophobic couplings.

²We restrict our analysis to the case of one extra surviving $U(1)'$.

TABLE I. Decomposition of E_6 model 27-plet representation.

SO(10)	Su(5)	$2\sqrt{10}Q_\chi$	$2\sqrt{6}Q_\psi$	$2\sqrt{15}Q_\eta$
16	$10(u, d, \bar{u}, e^+)_L$	-1	1	-2
	$5^*(\bar{d}, \nu, e^-)_L$	3	1	1
	$1\bar{N}_L$	-5	1	-5
10	$5(D, h_2^0, h_2^+)_L$	2	-2	4
	$5^*(\bar{D}, h_1^0, h_1^-)_L$	-2	-2	1
1	$1S_L^0$	0	4	-5

either a down-type Higgs or a lepton doublet. Table I lists the $U(1)_\chi$, $U(1)_\psi$, and $U(1)_\eta$ charges of the fields grouped under $SU(5)$, where $U(1)_\eta$ is a particular linear combination of $U(1)_\chi$ and $U(1)_\psi$,

$$U(1)_\eta = \sqrt{\frac{3}{8}}U(1)_\chi - \sqrt{\frac{5}{8}}U(1)_\psi, \quad (3)$$

which occurs in Calabi-Yau compactifications of the heterotic string model if E_6 leads directly to a rank 5 group [17] via the Wilson line (Hosotani) mechanism.

Three 27_L -plets are needed to form the three-family structure for the standard model. Gauge unification can be restored without introducing anomalies by adding a single $27_L + 27_L^*$ pair, assuming that only the Higgs-like doublet h_2 and its conjugate h_3 , associated with the $(5, -2)_L$ (from 27_L) and $(5^*, +2)_L$ (from 27_L^*), remain light, while the other fields from the $27_L + 27_L^*$ pair acquire superheavy masses and decouple. Therefore, the light matter supermultiplets are

$$3 \times 27_L + (27_L + 27_L^*)|_{h_2+h_3}, \quad (4)$$

where the notation indicates that only the $h_2 + h_3$ remain from the $27_L + 27_L^*$. Again, we want to emphasize that it is not our purpose to consider E_6 as a GUT model, but to use its particle content and charge assignments as a concrete example to study $U(1)'$ symmetry breaking. In this model (i) the additional $U(1)'$ is naturally anomaly-free as a result of being embedded in a larger gauge group. (ii) Any of the three h_1 's and four h_2 's can be minimal supersymmetric standard model (MSSM) Higgs doublets. The S_L^0 can be the singlet S which couples to H_1 and H_2 in the superpotential. (iii) There are exotic quarks (antiquarks) D 's (\bar{D} 's), which have the same $U(1)'$ charge as h_2 's and h_1 's, therefore allowing the Yukawa couplings $SD\bar{D}$ in the superpotential.

The left chiral superfields of the model with their $SU(3)_c$, $SU(2)_L$, $\sqrt{5/3}Q_Y$ and extra $U(1)'$ quantum numbers are listed in Table II, where \hat{d}_i^c are the left-handed down-type antiquarks; \hat{D}_i and $\hat{\bar{D}}_i$ are the exotic color triplets.

 TABLE II. The left-handed chiral superfields of the model with their quantum numbers under $SU(3)_c$, $SU(2)_L$, $U(1)_Y$, and $U(1)'$.

Matter multiplets	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
\hat{Q}_i	3	2	1/6	Q_Q
\hat{u}_i^c	3	1	-2/3	Q_u
\hat{d}_i^c	3	1	1/3	Q_d
\hat{L}_i	1	2	-1/2	Q_L
\hat{E}_i^c	1	1	1	Q_E
\hat{H}_{2i}	1	2	1/2	Q_1
\hat{H}_{1i}	1	2	-1/2	Q_2
\hat{D}_i	3	1	-1/3	Q_D
$\hat{\bar{D}}_i$	3	1	1/3	$Q_{\bar{D}}$
\hat{S}_i	1	1	0	Q_S
\hat{N}_i^c	1	1	0	Q_N
\hat{H}_3	1	2	-1/2	$-Q_2$

The gauge couplings are g_3 , g_2 , $g_1 \equiv \sqrt{5/3}g_Y$ and g_1' for $SU(3)_c$, $SU(2)_L$, $U(1)_Y$, and $U(1)'$, respectively.

With our assumptions that the couplings in the superpotential conserve baryon and lepton number, the most general form of the trilinear superpotential allowed by gauge invariance is

$$W = h_Q^{ijk} \hat{u}_i^c \hat{Q}_j \cdot \hat{H}_{2k} + h_d^{ijk} \hat{d}_i^c \hat{Q}_j \cdot \hat{H}_{1k} + h_s^{ijk} \hat{S}_i \hat{H}_{1j} \cdot \hat{H}_{2k} + h_D^{ijk} \hat{S}_i \hat{D}_j \hat{\bar{D}}_k, \quad (5)$$

in which i, j, k are family indices. The large Yukawa couplings dominate the running of soft scalar mass squares and cubic coefficients, so we only take the Yukawa coupling³ of the top quark, $h_Q^{333} \hat{u}_3^c \hat{Q}_3 H_{23}$. Similarly, we assume that only $h_s^{333} \hat{S}_3 H_{13} H_{23}$ and $h_D^{333} \hat{S}_3 \hat{D}_3 \hat{\bar{D}}_3$ are significant. This physical ansatz is motivated by perturbative string models, in which the Yukawa couplings are all zero or of order g_0 (the gauge coupling at the string scale) [6–8]. In our numerical work, we assume Yukawa universality, i.e., the nonzero Yukawa couplings at the string scale are all equal; we take the value g_0 [3] for definiteness. In this case, the superpotential is

$$W = h_Q \hat{u}_3^c \hat{Q}_3 \cdot \hat{H}_2 + h_s \hat{S} \hat{H}_1 \cdot \hat{H}_2 + h_D \hat{S} \hat{D} \hat{\bar{D}}, \quad (6)$$

where family indices have been suppressed. We assume the soft supersymmetry breaking terms

³The b quark Yukawa coupling could be large for large $\tan\beta \equiv \langle H_2^0 \rangle / \langle H_1^0 \rangle$.

$$\begin{aligned}
-L_{SB} = & \left(\sum_i M_i \lambda_i \lambda_i + A_s h_s S H_1 H_2 + A_Q h_Q u_3^c Q_3 H_2 + A_D h_D S D \bar{D} + \text{H.c.} \right) + m_3^2 |H_3|^2 + \sum_i m_{1i}^2 |H_{1i}|^2 + \sum_i m_{2i}^2 |H_{2i}|^2 \\
& + \sum_i m_{Si}^2 |S_i|^2 + \sum_i m_{Qi}^2 |Q_i|^2 + \sum_i m_{ui}^2 |u_i|^2 + \sum_i m_{di}^2 |d_i|^2 + \sum_i m_{Li}^2 |L_i|^2 + \sum_i m_{Ei}^2 |E_i|^2 + \sum_i m_{Di}^2 |D_i|^2 \\
& + \sum_i m_{\bar{D}i}^2 |\bar{D}_i|^2
\end{aligned} \tag{7}$$

where i is the family index, λ_i are gauginos, $A_s h_s$, $A_Q h_Q$, and $A_D h_D$ are coefficients of trilinear scalar couplings, and the rest are mass terms for the scalar components of the chiral supermultiplets.

The tree level scalar potential for H_1 , H_2 , and S has contributions from F -terms, D -terms, and L_{SB} :

$$V = V_F + V_D + V_{\text{soft}}, \tag{8}$$

where

$$V_F = |h_s|^2 [|H_1 H_2|^2 + |S|^2 (|H_1|^2 + |H_2|^2)]; \tag{9}$$

$$\begin{aligned}
V_D = & \frac{g_2^2}{2} |H_1^+ H_2|^2 + \frac{G^2}{8} (|H_2|^2 - |H_1|^2)^2 \\
& + \frac{g_{1'}^2}{2} (Q_1 |H_1|^2 + Q_2 |H_2|^2 + Q_S |S|^2)^2;
\end{aligned} \tag{10}$$

$$\begin{aligned}
V_{\text{soft}} = & m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_S^2 |S|^2 \\
& - (A_s h_s S H_1 H_2 + \text{H.c.})
\end{aligned} \tag{11}$$

where $G^2 = g_2^2 + \frac{3}{5} g_1^2$. The vacuum expectation values of the Higgs fields and the singlet at the minimum of the scalar potential are $\langle H_2^+ \rangle = \langle H_1^- \rangle = 0$, $\langle H_2^0 \rangle = v_2 / \sqrt{2}$, $\langle H_1^0 \rangle = v_1 / \sqrt{2}$ and $\langle S \rangle = s / \sqrt{2}$, where v_1 and v_2 can be chosen to be real and positive; s is real, and $A_s h_s > 0$ at the true minimum.

The minimization conditions for the scalar potential with nonzero VEVs are given by

$$2m_S^2 - \frac{\sqrt{2} A_s h_s v_1 v_2}{s} + |h_s|^2 (v_1^2 + v_2^2) + g_1^2 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S s^2) = 0; \tag{12}$$

$$2m_2^2 - \frac{\sqrt{2} A_s h_s s v_1}{v_2} + |h_s|^2 (v_1^2 + s^2) + g_1^2 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S s^2) = 0; \tag{13}$$

$$2m_1^2 - \frac{\sqrt{2} A_s h_s s v_2}{v_1} + |h_s|^2 (s^2 + v_2^2) + g_1^2 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S s^2) = 0; \tag{14}$$

$v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$ to ensure the correct electroweak breaking scale. $\tan\beta$ is defined as v_2/v_1 . The effective μ -term is $\mu_s = h_s s / \sqrt{2}$.

After the electroweak and $U(1)'$ symmetries are broken, there are two neutral and massive gauge bosons Z and Z' , the $Z-Z'$ mass-squared matrix is given by

$$(M^2)_{Z-Z'} = \begin{pmatrix} M_Z^2 & \Delta^2 \\ \Delta^2 & M_{Z'}^2 \end{pmatrix}, \tag{15}$$

in which

$$M_Z^2 = \frac{1}{4} G^2 (v_1^2 + v_2^2); \tag{16}$$

$$M_{Z'}^2 = g_1^2 (Q_1^2 v_1^2 + Q_2^2 v_2^2 + Q_S^2 s^2); \tag{17}$$

$$\Delta^2 = \frac{1}{2} g_1' G (Q_1 v_1^2 - Q_2 v_2^2). \tag{18}$$

The mass eigenstates are Z_1 and Z_2 with

$$M_{Z_1, Z_2}^2 = \frac{1}{2} [M_Z^2 + M_{Z'}^2 \mp \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4\Delta^4}]. \quad (19)$$

The $Z-Z'$ mixing angle is given by

$$\alpha_{Z-Z'} = \frac{1}{2} \arctan \left(\frac{2\Delta^2}{M_Z^2 - M_{Z'}^2} \right). \quad (20)$$

Present direct searches and precision tests suggest that $M_Z = Gv/2 = 91.2$ GeV, the mixing angle α is less than a few times 10^{-3} and $M_{Z'} > 500$ GeV. It has been argued that for certain (e.g., leptophobic) models, a lighter Z' is allowed.⁴ In principle, there are also bounds on the parameters (e.g., A_i , h_s , etc.) from searches for physical Higgs, exotic particles, gauginos, etc. However, it is not our purpose to construct a fully realistic model or consider the detailed mass spectrum for all of the scalar particles in the theory. Hence, we simplify by taking the limits on the masses of scalar particles at low energy to be > 100 GeV, the gaugino masses to be > 50 GeV and $m_{h_0} > 90$ GeV as rough phenomenological constraints.

It was pointed out in [2] that there are two scenarios which can give desired low energy phenomenology:

(i) The large A_s scenario predicts a small $Z-Z'$ mixing angle and yields a lighter Z' mass. When the A_s term is dominant in the scalar potential, V_D pushes the minimum to take place at $v_1 \sim v_2$ (second term in V_D), and $v_1 \sim v_2 \sim s$ (third term in V_D) since $Q_1 + Q_2 + Q_S = 0$. As a result, $\alpha_{Z-Z'}$, proportional to $Q_1 v_1^2 - Q_2 v_2^2$, is small in the case $Q_1 = Q_2$. This scenario is only acceptable if the Z' has suppressed couplings to ordinary particles, as in leptophobic models.

(ii) The large $\langle S \rangle$ scenario occurs when $s \gg v_1, v_2$ due to cancellations and therefore $M_{Z'} \gg M_Z$, i.e., the $U(1)'$ symmetry breaking occurs at an energy scale higher than the electroweak scale. It was argued in [2] that $M_{Z'}$ could be of order TeV. The first minimization equation gives the singlet VEV,

$$s^2 = -\frac{2m_S^2}{g_1^2 Q_S^2} + \mathcal{O}(M_Z^2). \quad (21)$$

The other two minimization equations can be solved for v_1 and v_2 ,

$$2m_1^2 - \sqrt{2}A_s h_s s \frac{v_1}{v_2} + (|h_s|^2 + g_1^2 Q_1 Q_S) s^2 = \mathcal{O}(M_Z^2); \quad (22)$$

$$2m_1^2 - \sqrt{2}A_s h_s s \frac{v_2}{v_1} + (|h_s|^2 + g_1^2 Q_1 Q_S) s^2 = \mathcal{O}(M_Z^2); \quad (23)$$

where the right-hand side (RHS) of the equations are functions of v_1 and v_2 . As $|m_1^2|$, $|m_2^2|$, and $|m_S^2|$ are expected to be the same order of magnitude $\sim \mathcal{O}(\text{TeV})^2$ at M_Z , cancellations are necessary for the LHS of the equations to have solutions for the Higgs VEVs that yield the correct electroweak scale ($\sqrt{v_1^2 + v_2^2} = 246$ GeV). After symmetry breaking, $M_{Z'} \sim \sqrt{-2m_S^2}$, and $\alpha_{Z-Z'} \sim G(v_1^2 Q_1 - v_2^2 Q_2)/(2g_1^2 Q_S^2 s^2) \ll 1$.

III. RGE ANALYSIS

Before discussing the symmetry breakings, we briefly describe the renormalization group equation analysis of the model. [The complete set of (one-loop) renormalization group equations (RGEs) is presented in the Appendix.]

(i) Gauge unification. As we have argued in the last section, the supersymmetrized $3 \times 27_L + (27_L + 27_L^*)|_{h_2+h_3}$ of E_6 is consistent with gauge unification. The unification scale M_G is that at which the gauge coupling constants $g_2(\mu)$ for $SU(2)_L$ and $g_1(\mu)$ for $U(1)_Y$ meet, starting with their experimental values at M_Z . For this model, and working at the one-loop level, which is sufficient for our purpose, $M_G \sim 2 \times 10^{16}$ GeV, and $g_3(M_G) = g_2(M_G) = g_1(M_G) = g_1'(M_G) = g_0 = 1.20$. This differs from the value 0.71 found in the MSSM due to the exotic $(5, -2)_L + (5^*, 2)_L$ representations, which do not affect the unification itself (at one-loop) or the value of M_G , but affect the value of the coupling at M_G [4,18]. As a consistency check, the running of g_3 from M_G down to M_Z yields $g_3^2(M_Z)/4\pi = 0.114$, within 5% of the experimental value [4]. (The one-loop β function of g_3 for this model is zero.) The small inconsistency between M_G and the value $\sim 5 \times 10^{17}$ GeV expected in perturbative string models, which is $< 10\%$ in the log, is not significant for the issues considered in this paper.

(ii) Yukawa couplings. Inspired by the predictions of free fermionic constructions of string theory, we assume that the Yukawa couplings are also unified at M_G , and for simplicity, their values are taken to be $h_s(M_G) = h_Q(M_G) = h_D(M_G) = h_0 = g_0$ [3].

(iii) The soft supersymmetry breaking parameters at M_G are written in terms of dimensionless parameters c_i , c_{Ai} , and $c_{1/2i}$

$$m_i^2 = c_i^2 M_0^2; \quad A_i = c_{Ai} M_0; \quad M_i = c_{1/2i} M_0, \quad (24)$$

where $M_0 = \mathcal{O}(\text{TeV})$. In each case, M_0 is rescaled after running everything to low energy to yield $v = 246$ GeV.

(iv) Kinetic mixing. It was shown [16] that the gauge fields of two $U(1)$'s can be mixed through a term in the Lagrangian which is consistent with all the symmetries in the theory. In our model, the pure kinetic energy terms of the $U(1)_Y$ and $U(1)'$ gauge fields can be written as

$$-L = \frac{1}{4} F_Y^{\mu\nu} F_{Y\mu\nu} + \frac{1}{4} F'^{\mu\nu} F'_{\mu\nu} + \frac{\sin \xi}{2} F_Y^{\mu\nu} F'_{\mu\nu}, \quad (25)$$

⁴These include leptophobic models in which Z' couplings to ordinary leptons are absent [12]; fermiophobic models with no couplings to ordinary fermions [13]; and models in which the Z' only couples to the third family [14].

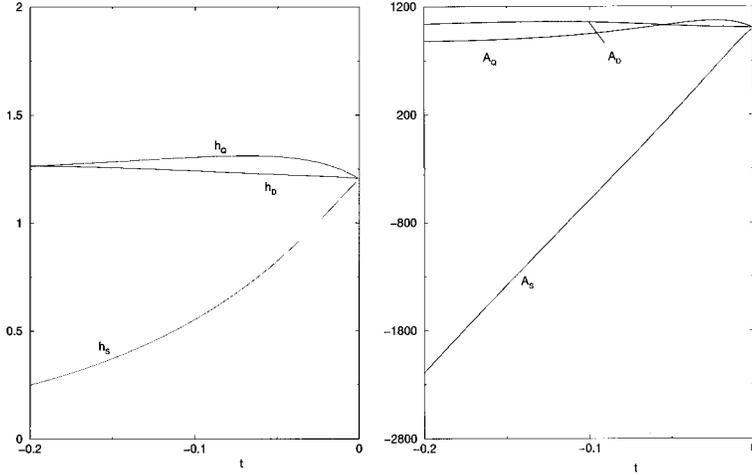


FIG. 1. Running of the Yukawa and trilinear couplings, with universal boundary conditions and $c_A = c_{1/2} = 1.0$.

where $F_Y^{\mu\nu}$ and $F'^{\mu\nu}$ are the field strength for $U(1)_Y$ and $U(1)'$, in the basis of the fields in which the interaction terms have the canonical form. If ξ is initially zero, it can arise from loop effects, when $\text{Tr}Q_Y Q \neq 0$, where the trace is restricted to the states lighter than the energy scale being considered. For the η model, $\text{Tr}Q_Y Q_\eta = 0.8$; for the ψ model, it is $-2/\sqrt{10}$. In both cases, the nonzero value is due to the decoupling of everything but two Higgs doublets in the extra $27_L + 27_L^*$. Therefore, the kinetic mixing must be included in RGE analysis.

A nonunitary transformation on the two $U(1)$ gauge fields can be introduced [15],

$$A_{Y\mu} = A_{1\mu} - \tan \xi A_{2\mu}; \quad A'_\mu = A_{2\mu} / \cos \xi, \quad (26)$$

to diagonalize the kinetic energy terms. In the new basis of the gauge fields, the interaction terms of the chiral fields are

$$L_{\text{int}} = \bar{\psi}_i \gamma^\mu [g_1 Q_{Yi} A_{1\mu} + (g_1' Q_i' + g_{12} Q_{Yi}) A_{2\mu}] \psi_i, \quad (27)$$

where the redefined gauge coupling constants, written in terms of the original ones, are

$$g_1 = g_1^0; \quad g_1' = g_1^0 / \cos \xi; \quad g_{12} = -g_1^0 \tan \xi. \quad (28)$$

Effectively, the kinetic mixing changes the $U(1)'$ charges of the fields to $Q_{\text{eff}} = Q + \delta Q_Y$, where $\delta \equiv g_{12}/g_1'$, while the $U(1)_Y$ charges remain the same. The renormalization group equations for the couplings g_1 , g_1' , and g_{12} are written in the Appendix. As the gauge coupling constants are scale dependent, the effective $U(1)'$ charges are scale dependent as well.

IV. UNIVERSAL BOUNDARY CONDITIONS

We first consider universal soft supersymmetry breaking. The universal boundary conditions are the following:

(i) Universal scalar masses,

$$m_i^2 = M_0^2. \quad (29)$$

(ii) Universal gaugino masses,

$$M_1 = M_2 = M_3 = M_1' = c_{1/2} M_0. \quad (30)$$

(iii) Universal trilinear couplings,

$$A_s = A_Q = A_D = c_A M_0. \quad (31)$$

The RGEs are solved numerically for the running of all of the soft parameters. The running of the Yukawa couplings, trilinear couplings, and the soft mass square parameters are shown in Figs. 1 and 2, with $c_A = c_{1/2} = 1$. These graphs illustrate the general features that h_s is driven down much faster than h_Q and h_D [because h_s is the coupling constant of $SH_1 H_2$, it receives contributions from both h_Q (through H_2) and h_D (through S)]. Thus, $h_s(M_Z) \sim 0.25$, while $h_Q(M_Z) \sim h_D(M_Z) \sim 1.28$. Similarly, $A_s(M_Z)$ is negative and much larger in magnitude than A_Q , A_D . m_s^2 and m_S^2 are both driven to be negative at low energy, due to the Yukawa couplings to the top quark and the exotic quark, while all other soft mass squares remain positive at M_Z . Gaugino masses directly affect the running of the trilinear couplings and the soft masses. In most cases, the competition between A_i 's and M_i 's controls the differences between the soft mass squared parameters at low energy. Only the scalar fields

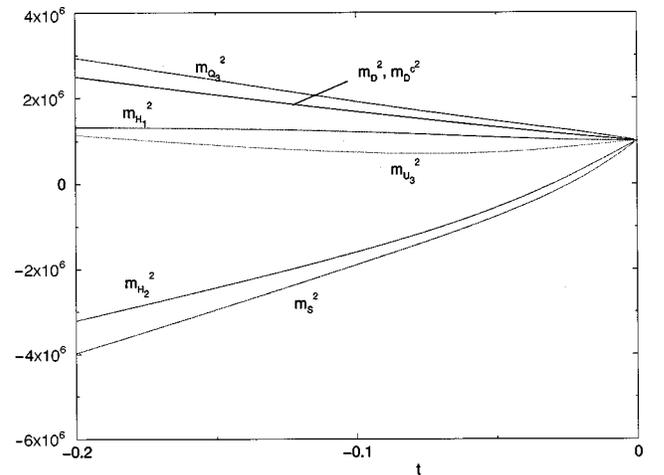


FIG. 2. Running of mass squares, with universal boundary conditions and $c_A = c_{1/2} = 1.0$.

S , H_1 , and H_2 associated with the nonzero terms in the superpotential (6) play a role in low energy symmetry breaking.

In Fig. 3, $M_{Z'}$ at low energy is plotted as a function of c_A (c_A varies over a wide range for different value of $c_{1/2}$, with the range chosen so that the color symmetry is not broken). Figure 4 is a plot of the $Z-Z'$ mixing angle α vs. c_A for different $c_{1/2}$. Figure 3 shows that with universal boundary conditions, $M_{Z'} < M_Z$, e.g., $M_{Z'} \sim 83$ GeV for the ψ model and $M_{Z'} \sim 65$ GeV for the η model. Figure 4 shows that $\alpha_{Z-Z'}$ is not small in general, e.g., $|\alpha_{Z-Z'}| > 0.4$ for the η model. The ψ model yields a smaller $Z-Z'$ mixing angle, but still larger than is allowed experimentally, unless c_A is large. The scalar potential tends to push v_1 to be close to v_2 at the minimum, while the $Z-Z'$ mixing angle $\alpha \propto (Q_1 v_1^2 - Q_2 v_2^2)$. Compared with the ψ model, which has $Q_1 = Q_2$, the η model with $Q_2 = 4Q_1$ naturally yields a larger mixing angle.

The plot shows that small mixing angles (~ 0.01) are possible for the ψ model when c_A is large (> 10) and the gaugino masses are small. This case is basically the large A_s scenario described in [2], in which the A_s term ($A_s h_s S H_1 H_2$) dominates the scalar potential and pushes the minimum to $v_1 = v_2 = s$. With $Q_1 = Q_2$ for the ψ model, the limiting case has a vanishing $Z-Z'$ mixing angle. However, the Z' mass, controlled by $g_1^2 (Q_1^2 v_1^2 + Q_2^2 v_2^2 + Q_s^2 s^2)$, is not acceptable, because g_1 is smaller than g_2 at electroweak scale and the U(1)' charges are smaller than the U(1) $_Y$ charges. As a result, $M_{Z'} < M_Z$, which is excluded experimentally.

The large $\langle S \rangle$ scenario cannot be realized with universal boundary conditions in either of the E_6 models.⁵ In the ψ model, $Q_1 = Q_2 = -2/\sqrt{24}$, $Q_s = 4/\sqrt{24}$, $h_s(M_Z) \sim 0.25$, and $g_1(M_Z) \sim 0.45$ so that $(|h_s|^2 + g_1^2 Q_2 Q_s) < 0$. Since the $A_s h_s s$ term is always positive at the true minimum, the cancellation conditions (22) and (23) cannot be satisfied with negative m_2^2 . For the same reason, the η model fails to yield large $\langle S \rangle$ solutions with universal boundary conditions.

Finally, we comment on the effect of kinetic mixing. It was argued [15] that for the η model, with an additional pair of Higgs doublets chosen from the 78 representation of the

⁵If Yukawa universality is broken, especially if h_s is much larger than h_U and h_D at the string scale, $h_s(M_Z)$ can be large enough to make $(|h_s|^2 + g_1^2 Q_2 Q_s)$ positive in (22), so that the cancellation equations for the large $\langle S \rangle$ scenario could be satisfied with universal soft supersymmetry breaking parameters. Discussions in [19] belongs to this category. In string models based on the fermionic ($Z_2 \times Z_2$) orbifold construction [6–8], the couplings of the trilinear terms in the superpotential can be $g_0 \sqrt{2}$, g_0 , $g_0/\sqrt{2}$, or $g_0/2$, depending on the number of Ising fermion excitations involved in the string vertex operators [3]. In this case, the maximum possible splitting between h_s and h_U , h_D at the string scale is indeed large enough to make $(|h_s|^2 + g_1^2 Q_2 Q_s)$ positive, for both the η and ψ models, to induce the large s solution at low energy. Even larger splittings are possible for effective Yukawa couplings derived from higher dimensional operators [20].

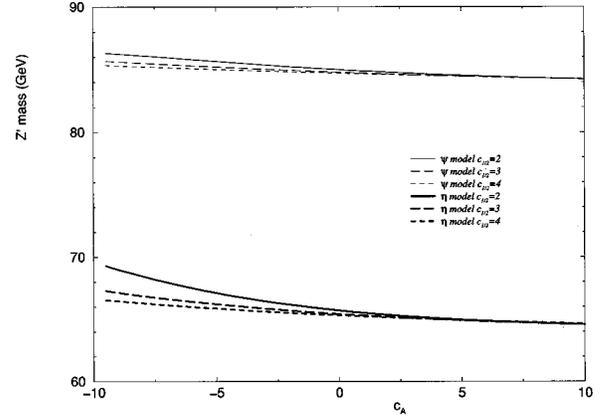


FIG. 3. The Z' mass as a function of c_A for various $c_{1/2}$, with universal boundary conditions.

E_6 group, $\delta(M_Z)$ can be large enough to make the Z' approximately leptophobic. Thus, a lighter Z' could possibly be allowed at low energy. In our model, the additional Higgs doublets come from a $27 + 27^*$. We find that $\delta(M_Z) \sim 0.08$ for the E_6 η model, while $\delta(M_Z)$ needs to be $1/3$ to make the η model leptophobic. Therefore, the kinetic mixing is only a small effect, and Z' is not leptophobic. Similarly, kinetic mixing is too small to reverse the sign of the U(1)' charges so that the cancellation conditions (22) and (23) can be satisfied with negative m_2^2 .

V. NONUNIVERSAL BOUNDARY CONDITIONS

To have acceptable low energy phenomenology for both the η and the ψ model, we invoke nonuniversal boundary conditions to satisfy the conditions (22) and (23) for large $\langle S \rangle$ solutions. We keep universal gaugino masses and universal trilinear couplings: $M_i = c_{1/2} M_0$, $A_i = c_A M_0$, as well as Yukawa universality. Among the soft mass squared parameters, we adjust only m_1^2 , m_2^2 , m_S^2 , $m_{Q_3}^2$, $m_{u_3^c}^2$, m_D^2 , and m_D^2 , i.e., those that dominate the running of the RGEs.

The purpose is to adjust the soft mass squared parameters at M_G so that m_2^2 and m_1^2 are both positive at low energy,

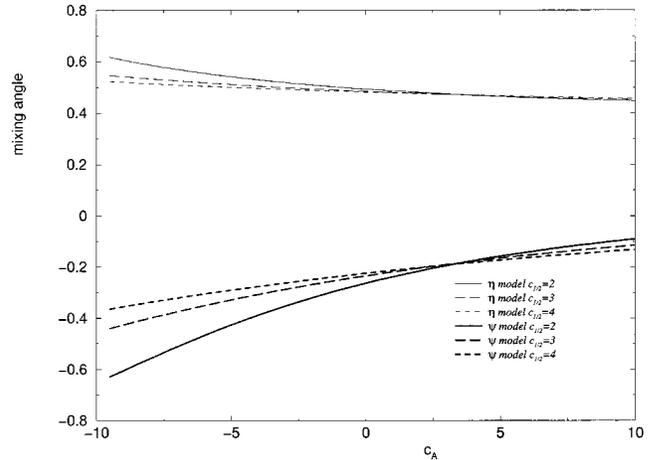


FIG. 4. The $Z'-Z$ mixing angle α as a function of c_A for various $c_{1/2}$, with universal boundary conditions.

TABLE III. Nonuniversal boundary conditions for the large s solution in the η model. The first row for each soft supersymmetry breaking parameter gives its value at the high scale, while the second row gives its value at M_Z . The units are GeV.

	1	2	3	4
m_1^2	(688) ² (102) ²	(1037) ² (167) ²	(814) ² (339) ²	(582) ² (313) ²
m_2^2	(1792) ² (237) ²	(3112) ² (380) ²	(1922) ² (374) ²	(1516) ² (204) ²
m_S^2	(1525) ² -(459) ²	(1922) ² -(685) ²	(1562) ² -(552) ²	(1068) ² -(588) ²
$m_{Q_3}^2$	(805) ² (138) ²	(1620) ² (74) ²	(722) ² (60) ²	(482) ² (48) ²
$m_{u_3^c}^2$	(1220) ² (102) ²	(2299) ² (101) ²	(1216) ² (77) ²	(919) ² (45) ²
m_D^2	(669) ² (386) ²	(898) ² (557) ²	(621) ² (480) ²	(438) ² (462) ²
m_D^2	(669.5) ² (362) ²	(898) ² (489) ²	(621) ² (476) ²	(438) ² (449) ²
(M_4, M_2)	260 (36, 76)	208 (29, 61)	349 (51, 102)	319 (45, 93)
(A_s, A_Q)	-2607 (-23, 193)	-1663 (-105, 158)	-2332 (-275, 269)	-3029 (-61, 238)
$M_{Z'}$	649	974	786	835
$\alpha_{Z-Z'}$	0.003	0.005	0.006	0.007
$\tan\beta$	7.06	3.35	1.92	5.78

while m_S^2 is negative. $|m_1^2|$, $|m_2^2|$, and $|m_S^2|$ should be fairly close to each other to satisfy the cancellation equations, so we must choose the gaugino masses to be small, while ensuring that the low energy values of the chargino and gluino masses satisfy existing experimental bounds. The trilinear coupling A_s must also be small at M_Z to avoid the other extreme, the large A_s scenario. We adjust the squark masses at M_G to ensure that the color symmetry is not broken at the electroweak scale; i.e., all squark masses must be positive (including exotics), and A_Q , A_D must not be too large.⁶ The sparticle masses at M_Z also need to satisfy the phenomenological bounds.

In Tables III and IV, we give a few examples of nonuniversal boundary conditions that induce the large $\langle S \rangle$ scenario at M_Z , for both the ψ and the η model. For each example, the first row for each soft supersymmetry breaking parameter gives its value at the high scale M_G , while the second row

gives its value at M_Z . Only the gaugino masses M_2 and M_1' are listed, because M_3 is a constant due to the vanishing one-loop β function for $SU(3)_c$, and M_1 is always only slightly larger than M_1' . A_s and A_Q are given, and $A_D \sim A_Q$. The Z' mass, $Z-Z'$ mixing angle, and $\tan\beta$ are given for each case.

Some general patterns can be seen from these examples. For example, m_2^2 has to be larger than the other mass squared parameters at M_G so that it can be positive at the electroweak scale. m_S^2 has to be increased to decrease the difference between $|m_2^2|$ and $|m_S^2|$. Squark masses have to be adjusted accordingly to avoid color symmetry breaking at M_Z . Generally, the trilinear couplings are large and negative, so that A_s can be small at low energy.

Compared with the ψ model, the η model yields solutions with large $\tan\beta$ (>1) and a slightly larger $Z-Z'$ mixing angle. The $U(1)'$ charge assignments of the particles in the theory affect the low energy phenomenology in a nontrivial way.

VI. MIXING BETWEEN THE TWO $U(1)$ 'S IN E_6

There is another way to satisfy the cancellation equations (22) and (23) for the large $\langle S \rangle$ minimum. As we have seen,

⁶A large A_Q , which implies that the global minimum of the potential breaks charge and color, may be acceptable if the charge/color conserving minimum is populated first cosmologically and is sufficiently long-lived [21].

TABLE IV. Same as Table III, but for the ψ model.

	1	2	3	4
m_1^2	(1087) ² (489) ²	(1590) ² (644) ²	(1383) ² (354) ²	(848) ² (237) ²
m_2^2	(3225) ² (498) ²	(4940) ² (689) ²	(3651) ² (372) ²	(2241) ² (240) ²
m_S^2	(1609) ² -(767) ²	(2317) ² -(1461) ²	(3142) ² -(796) ²	(1857) ² -(570) ²
$m_{Q_3}^2$	(1609) ² (96) ²	(2498) ² (216) ²	(1777) ² (370) ²	(1013) ² (250) ²
$m_{u_3^c}^2$	(2336) ² (114) ²	(2572) ² (204) ²	(1216) ² (185) ²	(1536) ² (185) ²
m_D^2	(61) ² (654) ²	(884) ² (1142) ²	(1257) ² (712) ²	(678) ² (498) ²
$m_{\bar{D}}^2$	(61) ² (532) ²	(884) ² (1004) ²	(1257) ² (579) ²	(678) ² (430) ²
(M_4, M_2)	375 (55, 110)	544 (80, 159)	471 (69, 138)	363 (53, 106)
(A_s, A_Q)	-2281 (-331, 293)	-5081 (-104, 407)	-5106 (58, 347)	-3625 (-19, 269)
$M_{Z'}$	1087	1561	1120	809
$\alpha_{Z-Z'}$	0.001	0.0005	0.0007	0.002
$\tan\beta$	0.97	0.74	0.80	0.70

negative m_2^2 and m_S^2 with positive m_1^2 at low energy are generic to our models with quarks that couple to H_2 and exotics that couple to S through large Yukawa couplings with universal boundary conditions, while m_1^2 remains positive. Yet, Eqs. (22) and (23) could be satisfied if $Q_1 Q_S < 0$ and $Q_2 Q_S > 0$. These conditions do not hold for the ψ and η models even with kinetic mixing, illustrating the strong U(1)' charge influence on the final Z' mass and $\alpha_{Z-Z'}$. The U(1)' charges affect the running of the RGEs only slightly; $\alpha_{Z-Z'}$ is affected through the U(1)' charge dependence of the $Z-Z'$ mass-squared matrix. However, the largest effect of the U(1)' charges comes from the minimization of the scalar potential.

In a general E_6 model with a single extra U(1)', the U(1)' charge can be combination of $U(1)_\psi$ and $U(1)_\chi$ with

$$Q = Q_\chi \cos \theta_{E_6} + Q_\psi \sin \theta_{E_6}, \quad (32)$$

where θ_{E_6} is a mixing angle which can be chosen to be in the range $0-\pi$. If $\cos \theta_{E_6} > \sqrt{3/5} \sim 0.79$, one has $Q_1 Q_S < 0$ and $Q_2 Q_S > 0$.

With universal soft supersymmetry breaking parameters at the large scale, this model has four free parameters: θ_{E_6} , M_0 , c_A , and $c_{1/2}$. s is fixed by Eq. (21), and the other

two equations, taken as cancellation conditions for a large s minimum, are to be satisfied. Our strategy is to take a particular gaugino mass, vary both c_A and θ_{E_6} , search for large $s \gg v_1, v_2$ at low energy, then rescale M_0 .

As an example, we show the parameter ranges of (c_A, θ_{E_6}) for $M_{Z'} > 500$ GeV as a two-dimensional (2D) contour plot in Fig. 5, taking $c_{1/2} = 0.4$ for the gaugino mass. As we have argued, gaugino masses need to be small to avoid huge splitting between the mass squares at low energy, and for smaller $c_{1/2}$, smaller $Q_i Q_S$ is needed for the cancellation so that large s occurs with smaller $\cos \theta_{E_6}$. If $|A_s|$ is large [$A_s(M_Z)$ should be small to avoid a large A_s minimum], a large value of $\cos \theta_{E_6}$ is necessary to increase the Q_i/Q_S ratio for the cancellation. The curves are terminated at the ends to preserve the color symmetry.

The curves are approximately symmetric around $c_A = -4$. In the limit in which the gaugino masses are neglected in the RGEs, the symmetry can be seen as the follows: consider the cases 1 and 2 with $c_A = \pm c_0$ at the large scale. The RGEs of A_i tell us that at $A_s(t) = 0$,

$$\frac{d}{dt} A_s = 6A_Q h_Q^2 + 6A_D h_D^2 \sim 12A_Q h_Q^2, \quad (33)$$

where we have assumed that $h_Q \sim h_D$, $A_Q \sim A_D$. Also, A_Q

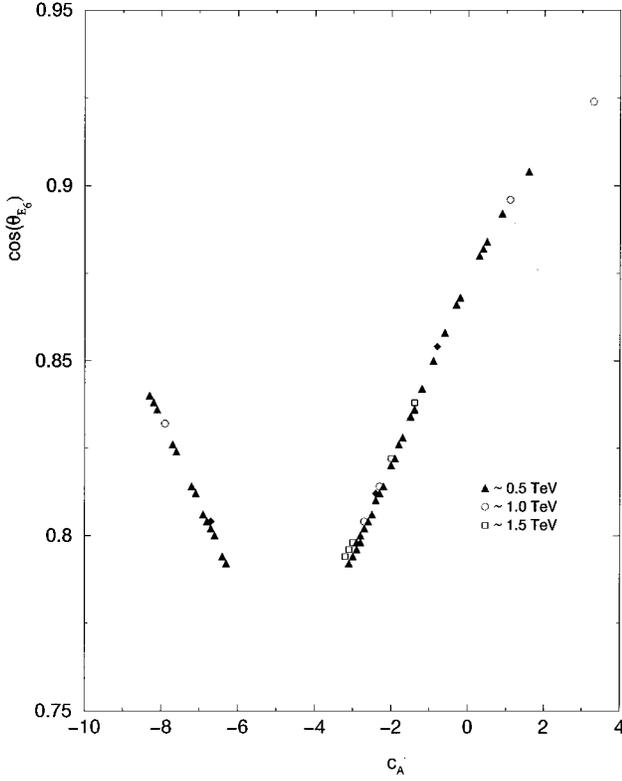


FIG. 5. Contour plot of the $(c_A, \cos\theta_{E_6})$ parameter ranges for $M_{Z'} > 500$ GeV, $c_{1/2} = 0.4$, where c_A , $c_{1/2}$ are the dimensionless parameters for the trilinear couplings A_i and the gaugino mass; θ_{E_6} is the mixing angle between the $U(1)_\chi$ and $U(1)_\psi$ of E_6 .

and A_D stay close to their initial values at M_G . Hence, at the point that A_s crosses zero, the slopes of A_s for cases 1 and 2 are approximately equal and opposite, which induces the equal and opposite A_s for cases 1 and 2 at M_Z . The running of the soft mass squared parameters only depend on A_i^2 , so that $m_i^2(1) \sim m_i^2(2)$ at the electroweak scale. Hence, equal and opposite initial conditions of A_i result in the same low energy phenomenology. However, the contribution of the gaugino masses shift the center of the symmetry from $c_A = 0$.

VII. SUMMARY

In this paper, we presented a detailed analysis of the electroweak and $U(1)'$ symmetry breaking for a supersymmetric model with the matter content $3 \times 27_L + (27_L + 27_L^*)|_{h_2+h_3}$ of E_6 . It has an anomaly free $U(1)'$ symmetry, is consistent with gauge unification, and is meant to be a concrete example of the more general scenarios discussed in [2]. It includes the three family standard model particle structure and a pair of Higgs doublets; in addition, there are three exotic quark singlet pairs D and \bar{D} , two extra Higgs doublets H_1 , three extra Higgs doublets H_2 , and one Higgs doublet H_3 , which is the conjugate of H_2 . The extra $U(1)'$ symmetry is taken to be $U(1)_\psi$ or $U(1)_\eta$ of the E_6 model, or a combination of $U(1)_\psi$ and $U(1)_\chi$. Couplings between the singlet S

and Higgs doublets H_1 , H_2 and couplings between the singlet and the exotic quarks D , \bar{D} are naturally allowed by gauge invariance. The model is string (rather than GUT) motivated. Thus, some terms in the superpotential that would be allowed by gauge invariance are assumed to be absent due to string selection rules. In particular, we assume the absence of baryon and lepton number violating couplings of the exotic D , \bar{D} quarks, so that D and \bar{D} can be light. Similarly, we assume that most of the possible Yukawa couplings of the Higgs doublets and singlets vanish, so that only m_t is large and two Higgs doublets and one singlet participate in the symmetry breaking. We make the string motivated assumption that the nonzero Yukawa couplings are equal at the string scale (Yukawa universality), and use the value $\sim \sqrt{2}g_0$ for definiteness.

With the specific particle content and $U(1)'$ charge assignments, we use the numerical results of the RGE analysis to explore features of the radiative symmetry breaking at the electroweak scale. Our results are summarized as follows.

- (i) Only the Higgs doublets and singlet with nonzero Yukawa couplings play a role in symmetry breaking.
- (ii) The symmetry breaking scenarios yield an effective μ term at low energy, $\mu_{\text{eff}} = h_s \langle S \rangle$.
- (iii) The universal soft supersymmetry breaking mass parameters at the large scale fail to give phenomenologically acceptable scenarios at low energy for the E_6 ψ and η models.

(iv) To achieve the desired low energy phenomenology for the supersymmetric E_6 model, one has to look for solutions with large S VEVs, i.e., $\langle S \rangle \gg \langle H_1 \rangle, \langle H_2 \rangle$. There are two ways to reach such solutions while maintaining Yukawa universality.

(i) For both the E_6 ψ and η models, by changing the soft supersymmetry breaking mass squared parameters at the unification scale, there are parameter regions that yield large s at the electroweak scale, with a heavy Z' and small $Z-Z'$ mixing angle.

(ii) By introducing a mixing angle θ_{E_6} between the two extra $U(1)$ symmetries of the E_6 model, one can vary the $U(1)'$ charge assignments of the particles in the model, while keeping the model anomaly free. With universal boundary conditions at the unification scale, for given gaugino masses, there exist parameter ranges of c_A (trilinear couplings) and θ_{E_6} at M_G that yield the large s minimum at low energy.

We confirm the two scenarios that may induce acceptable low energy phenomenology, the large A_s scenario and large $\langle S \rangle$ scenario, proposed in Ref. [2]. The large A_s scenario fails in this case to give phenomenologically acceptable Z' masses in the E_6 ψ and η models because the couplings are not leptophobic, even including kinetic mixing. It is nevertheless a useful example of this scenario, which maybe viable in other string derived models with suppressed couplings to ordinary fermions. The large $\langle S \rangle$ scenario can be realized, either through nonuniversal boundary conditions for either the ψ or η model or by varying the $U(1)'$ quantum numbers of the particles.

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APPENDIX: RENORMALIZATION GROUP EQUATIONS

We present here the complete one loop renormalization group equations of the anomaly-free E_6 model: $3 \times 27_L + (27_L + 27_L^*)|_{h_2+h_3}$, based on the particle content as listed in Table II. In these equations, the scale variable is defined as

$$t = \frac{1}{16\pi^2} \ln \frac{\mu}{M_G}, \quad (\text{A1})$$

where M_G is determined to be 2×10^{16} GeV by the running of the $SU(2)_L$ gauge coupling constant $g_2(\mu)$ and the $U(1)_Y$ gauge coupling constant $g_1(\mu)$, with their inputs from $\mu = M_Z$.

1. Gauge couplings

$$\frac{d}{dt} g_i = \beta_i g_i^3, \quad (\text{A2})$$

where $i=1, 2, 3$ for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, respectively. The β functions are

$$\beta_1 = \text{Tr} Q_Y^2 = \frac{48}{5}; \quad \beta_2 = 4; \quad \beta_3 = 0. \quad (\text{A3})$$

3. Yukawa couplings

$$\frac{d}{dt} h_Q = h_Q \left[6h_Q^2 + h_s^2 - \left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 + 2g_1^2 (Q_u^2 + Q_Q^2 + Q_2^2) \right) \right]; \quad (\text{A8})$$

$$\frac{d}{dt} h_s = h_s \left[3h_Q^2 + 4h_s^2 + 3h_D^2 - \left(3g_2^2 + \frac{3}{5} g_1^2 + 2g_1^2 (Q_s^2 + Q_1^2 + Q_2^2) \right) \right]; \quad (\text{A9})$$

$$\frac{d}{dt} h_D = h_D \left[5h_D^2 + 2h_s^2 - \left(\frac{16}{3} g_3^2 + \frac{4}{15} g_1^2 + 2g_1^2 (Q_D^2 + Q_D^2 + Q_s^2) \right) \right]. \quad (\text{A10})$$

4. Trilinear couplings

$$\frac{d}{dt} A_Q M = 12A_Q h_Q^2 + 2A_s h_s^2 - 2 \left[\frac{16}{3} g_3^2 M_3 + 3g_2^2 M_2 + \frac{13}{15} g_1^2 M_1 + 2g_1^2 (Q_u^2 + Q_Q^2 + Q_2^2) M_{1'} \right]; \quad (\text{A11})$$

$$\frac{d}{dt} A_s = 6A_Q h_Q^2 + 8A_s h_s^2 + 6A_D h_D^2 - 2 \left[3g_2^2 M_2 + \frac{3}{5} g_1^2 M_1 + 2g_1^2 (Q_s^2 + Q_1^2 + Q_2^2) M_{1'} \right]; \quad (\text{A12})$$

$$\frac{d}{dt} A_D = 10A_D h_Q^2 + 4A_s h_s^2 - 2 \left[\frac{16}{3} g_3^2 M_3 + \frac{4}{15} g_1^2 M_1 + 2g_1^2 (Q_D^2 + Q_D^2 + Q_s^2) M_{1'} \right]. \quad (\text{A13})$$

With kinetic mixing, the equations for the $U(1)'$ gauge coupling constant $g_{1'}$ and the $U(1)_Y$ - $U(1)'$ -mixing constant $g_{11'}$ [15] (see Sec. III for explanations) are

$$\frac{d}{dt} g_{1'} = (\beta_1 g_{1'}^2 + \beta_1 g_{11'} + 2\beta_{11'} g_{1'} g_{11'}) g_{1'}, \quad (\text{A4})$$

$$\begin{aligned} \frac{d}{dt} g_{11'} = & (\beta_1 g_{1'}^2 + \beta_1 g_{11'} + 2\beta_{11'} g_{1'} g_{11'}) g_{11'} \\ & + 2g_1^2 (g_{11'} \beta_1 + g_{1'} \beta_{11'}), \end{aligned} \quad (\text{A5})$$

where

$$\beta_1 = \text{Tr} Q^2; \quad \beta_{11'} = \text{Tr} Q Q_Y. \quad (\text{A6})$$

The $U(1)'$ charges of the particles Q are Q_ψ in the ψ model, Q_η in the η model and $(Q_\chi \cos \theta_{E_6} + Q_\psi \sin \theta_{E_6})$ in the model with mixing between $U(1)_\chi$ and $U(1)_\psi$. With the kinetic mixing, the effective $U(1)'$ charge is $Q_{\text{eff}} = Q + \delta Q_Y$, with $\delta = g_{11'}/g_{1'}$, so that the $U(1)'$ charges are scale dependent.

2. Gaugino masses

$$\frac{d}{dt} M_i = 2\beta_i g_i^2 M_i, \quad (\text{A7})$$

where $i=1, 2, 3, 1'$ for $U(1)_Y$, $SU(2)_L$, $SU(3)_c$, and $U(1)'$, respectively. Based on the observation that the kinetic mixing effect is small (less than 8%) in our model, we neglect the contributions from kinetic mixing in the runnings of soft supersymmetry breaking parameters.

5. Soft scalar mass-squared parameters

$$\begin{aligned} \frac{d}{dt}m_2^2 = & 6(m_2^2 + m_{Q_3}^2 + m_{u_3}^2 + A_Q^2)h_Q^2 + 2(m_2^2 + m_1^2 + m_S^2 + A_S^2)h_s^2 - 8\left(\frac{3}{4}g_2^2M_2^2 + \frac{3}{20}g_1^2M_1^2 + Q_2^2g_1^2M_1^2\right) + \frac{3}{5}g_1^2S_1 \\ & + 2Q_2g_1^2S_{1'}; \end{aligned} \quad (\text{A14})$$

$$\frac{d}{dt}m_1^2 = 2(m_2^2 + m_1^2 + m_S^2 + A_S^2)h_s^2 - 8\left(\frac{3}{4}g_2^2M_2^2 + \frac{3}{20}g_1^2M_1^2 + Q_1^2g_1^2M_1^2\right) - \frac{3}{5}g_1^2S_1 + 2Q_1g_1^2S_{1'}; \quad (\text{A15})$$

$$\frac{d}{dt}m_S^2 = 6(m_D^2 + m_{\bar{D}}^2 + m_S^2 + A_D^2)h_D^2 + 4(m_2^2 + m_1^2 + m_S^2 + A_S^2)h_s^2 - 8Q_S^2g_1^2M_1^2 + 2Q_Sg_1^2S_{1'}; \quad (\text{A16})$$

$$\frac{d}{dt}m_{Q_3}^2 = 2(m_2^2 + m_{Q_3}^2 + m_{u_3}^2 + A_Q^2)h_Q^2 - 8\left(\frac{4}{3}g_3^2M_3^2 + \frac{3}{4}g_2^2M_2^2 + \frac{1}{60}g_1^2M_1^2 + Q_Q^2g_1^2M_1^2\right) + \frac{1}{5}g_1^2S_1 + 2Q_Qg_1^2S_{1'}; \quad (\text{A17})$$

$$\frac{d}{dt}m_{u_3}^2 = 4(m_2^2 + m_{Q_3}^2 + m_{u_3}^2 + A_Q^2)h_Q^2 - 8\left(\frac{4}{3}g_3^2M_3^2 + \frac{4}{15}g_1^2M_1^2 + Q_u^2g_1^2M_1^2\right) - \frac{4}{5}g_1^2S_1 + 2Q_u g_1^2S_{1'}; \quad (\text{A18})$$

$$\frac{d}{dt}m_D^2 = 2(m_D^2 + m_{\bar{D}}^2 + m_S^2 + A_D^2)h_D^2 - 8\left(\frac{4}{3}g_3^2M_3^2 + \frac{1}{15}g_1^2M_1^2 + Q_D^2g_1^2M_1^2\right) - \frac{2}{5}g_1^2S_1 + 2Q_Dg_1^2S_{1'}; \quad (\text{A19})$$

$$\frac{d}{dt}m_{\bar{D}}^2 = 2(m_D^2 + m_{\bar{D}}^2 + m_S^2 + A_D^2)h_D^2 - 8\left(\frac{4}{3}g_3^2M_3^2 + \frac{1}{15}g_1^2M_1^2 + Q_{\bar{D}}^2g_1^2M_1^2\right) + \frac{2}{5}g_1^2S_1 + 2Q_{\bar{D}}g_1^2S_{1'}; \quad (\text{A20})$$

$$\frac{d}{dt}m_L^2 = -8\left(\frac{3}{4}g_2^2M_2^2 + \frac{3}{20}g_1^2M_1^2 + Q_L^2g_1^2M_1^2\right) + \frac{3}{5}g_1^2S_1 + 2Q_Lg_1^2S_{1'}; \quad (\text{A21})$$

$$\frac{d}{dt}m_E^2 = -8\left(\frac{3}{5}g_1^2M_1^2 + Q_E^2g_1^2M_1^2\right) + \frac{6}{5}g_1^2S_1 + 2Q_Eg_1^2S_{1'}; \quad (\text{A22})$$

$$\frac{d}{dt}m_N^2 = -8Q_N^2g_1^2M_1^2 + 2Q_Ng_1^2S_{1'}; \quad (\text{A23})$$

$$\frac{d}{dt}m_{Q_{1,2}}^2 = -8\left(\frac{4}{3}g_3^2M_3^2 + \frac{3}{4}g_2^2M_2^2 + \frac{1}{60}g_1^2M_1^2 + Q_Q^2g_1^2M_1^2\right) + \frac{1}{5}g_1^2S_1 + 2Q_Qg_1^2S_{1'}; \quad (\text{A24})$$

$$\frac{d}{dt}m_{u_{1,2}}^2 = -8\left(\frac{4}{3}g_3^2M_3^2 + \frac{4}{15}g_1^2M_1^2 + Q_u^2g_1^2M_1^2\right) - \frac{4}{5}g_1^2S_1 + 2Q_u g_1^2S_{1'}; \quad (\text{A25})$$

$$\frac{d}{dt}m_{Hu}^2 = -8\left(\frac{3}{4}g_2^2M_2^2 + \frac{3}{20}g_1^2M_1^2 + Q_H^2g_1^2M_1^2\right) + \frac{3}{5}g_1^2S_1 + 2Q_Hg_1^2S_{1'}; \quad (\text{A26})$$

$$\frac{d}{dt}m_{Hd}^2 = 8\left(\frac{3}{4}g_2^2M_2^2 + \frac{3}{20}g_1^2M_1^2 + Q_d^2g_1^2M_1^2\right) - \frac{3}{5}g_1^2S_1 + 2Q_dg_1^2S_{1'}; \quad (\text{A27})$$

$$\frac{d}{dt}m_{H_3}^2 = -8\left(\frac{3}{4}g_2^2M_2^2 + \frac{3}{20}g_1^2M_1^2 + Q_3^2g_1^2M_1^2\right) - \frac{3}{5}g_1^2S_1 - 2Q_3g_1^2S_{1'}; \quad (\text{A28})$$

$$\frac{d}{dt}m_{S_{1,2}}^2 = -8Q_S^2g_1^2M_1^2 + 2Q_Sg_1^2S_{1'}; \quad (\text{A29})$$

$$\frac{d}{dt}m_{D_{1,2}}^2 = -8\left(\frac{4}{3}g_3^2M_3^2 + \frac{1}{15}g_1^2M_1^2 + Q_D^2g_1^2M_1^2\right) - \frac{2}{5}g_1^2S_1 + 2Q_Dg_1^2S_{1'}; \quad (\text{A30})$$

$$\frac{d}{dt} m_{\bar{D}_{1,2}}^2 = -8 \left(\frac{4}{3} g_3^2 M_3^2 + \frac{1}{15} g_1^2 M_1^2 + Q_{\bar{D}}^2 g_1^2 M_1^2 \right) + \frac{2}{5} g_1^2 S_1 + 2 Q_{\bar{D}} g_1^2 S_1', \quad (\text{A31})$$

where m_{H_u} and m_{H_d} are the masses of the Higgs doublets that do not have Yukawa couplings. S_1 and S_1' in these equations are defined as

$$S_1 = \sum_{i=1}^3 (m_{Q_i}^2 - 2m_{u_i}^2 + m_{d_i}^2 - m_{L_i}^2 + m_{E_i}^2 + m_{D_i}^2 + m_{\bar{D}_i}^2) + m_2^2 - m_1^2 + 3m_{H_u}^2 - 2m_{H_d}^2 - m_{H_3}^2; \quad (\text{A32})$$

$$S_1' = \sum_{i=1}^3 (6Q_Q^2 m_{Q_i}^2 + 3Q_u^2 m_{u_i}^2 + 3Q_d^2 m_{d_i}^2 + 2Q_L^2 m_{L_i}^2 + Q_E^2 m_{E_i}^2 + 3Q_D^2 m_{D_i}^2 + 3Q_{\bar{D}}^2 m_{\bar{D}_i}^2 + Q_S^2 m_{S_i}^2 + Q_N^2 m_{N_i}^2) + 2Q_1(m_1^2 + 2m_{H_d}^2) + 2Q_2(m_2^2 + 3m_{H_u}^2) - 2Q_3 m_{H_3}^2. \quad (\text{A33})$$

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