Unity of forces at the preon level with new gauge symmetries

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In the context of a viable, supersymmetric, preon model, it has been shown by Babu and Pati that the unity of forces can well occur at the level of preons near the Planck scale. This preonic approach to unification is explored further in this paper with the inclusion of threshold effects which arise due to the spreading of masses near the scale of supersymmetry ($M_S = 1$ TeV) and the metacolor scale ($\Lambda_M = 10^{11}$ GeV). These effects, which were ignored in earlier work, are found to have marked consequences on the running and unification of the relevant couplings, leading to new possibilities for flavor color as well as metacolor gauge symmetries. In particular, allowing for seemingly reasonable threshold effects, it is found that the metacolor gauge symmetry, G_M is either $SU(6)_M$ or $SU(4)_M$ [rather than $SU(5)_M$] and the corresponding flavor-color gauge symmetry is either $SU(2)_L\times U(1)_R\times SU(4)^C_{L+R}$ [for $G_M = SU(6)_M$] or even just the standard model symmetry $SU(2)_L\times U(1)_Y\times SU(3)_C$ [for $G_M = SU(6)_M$ or $SU(4)_M$]. The prospects of other preonic gauge symmetries are also investigated. [S0556-2821(98)05915-3]

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I. INTRODUCTION

In the context of grand unification [1–3], it is known that, while the nonsupersymmetric minimal SU(5) model [2] is excluded by proton decay searches [4] and by the recent data from the CERN e^+e^- collider LEP [5], the three coupling constants of the standard model approximately unify at a scale $M_X \approx 2\times 10^{16}$ GeV if one invokes supersymmetry, e.g., into minimal SU(5) [6,7] or SO(10). Despite this success, it seems to us that neither of the two schemes [SU(5) or SO(10)] is likely to be a fundamental theory by itself because each scheme possesses a large number of arbitrary parameters associated with the Higgs sector; the corresponding Higgs exchange force in each case is thus not unified. Furthermore, neither scheme explains the origin of the three families and that of the diverse mass scales which span from the Planck mass ($\equiv M_{\rm Pl}$) to m_ν. These shortcomings are expected to be removed if one of the two schemes, i.e., the minimal supersymmetric (SUSY) SU(5) or the SUSY SO(10), could emerge from superstring theory [8,9], which is such that it yields just the right spectrum of quarks, leptons, and Higgs bosons and just ''the right package'' of Higgs parameters, thereby removing the unwanted arbitrariness. But, so far, this is far from being realized. An alternative possibility is that, instead of a grand unification symmetry, the minimal supersymmetric standard model with the ''right package'' of parameters might emerge directly from a superstring theory. In this case there is, however, the question of mismatch between the unification scale M_X obtained from extrapolation of low-energy LEP data and the expected scale of string unification, which is nearly 20 times higher [10].

For these reasons, it has been suggested in an alternative approach that the unification of forces might occur as well at the level of constituents of quarks and leptons called the ''preons'' [11–15]. On the negative side, the preonic approach needs a few unproven, though not implausible, dynamical assumptions as regards the preferred direction of symmetry breaking and saturation of the composite spectrum [15–17]. On the positive side, it has the advantage that the model is far more economical in field content and especially in parameters than conventional grand unification models. The fundamental forces have a purely gauge origin, as in QCD, with no elementary Higgs bosons, and, therefore, no arbitrary parameters which are commonly associated with the Higgs sector. The most important aspect of the model is that, utilizing primarily the symmetries of the theory and the forbiddenness of SUSY breaking [18], in the absence of gravity, it provides a simple explanation for the protection of composite quark-lepton masses [19]. The model seems capable of addressing successfully the origin of family unification and that of the diverse mass scales [12], including the interfamily mass hierarchy [14]. Finally, it provides several testable predictions [12,14–17].

The question of the unity of forces at the preonic level was explored in a recent work by Babu and Pati [15], where it was shown that the unity occurs near the Planck scale ($\approx 10^{18}$ GeV), in accordance with the LEP data, but with the flavor-color gauge symmetry $G_{fc} = SU(2)_L\times U(1)_R \times SU(4)^C_{L+R}$ and the metacolor gauge symmetry $G_M = SU(5)_M$. Considering that Planck-scale unification, as opposed to unity near 2×10^{16} GeV, goes better with the idea of string unification [8–10], we explore further, in this paper, the preonic approach to unification, with the inclusion of threshold effects, which arise due to the spreading of masses near the scale of supersymmetry ($M_S \approx 1$ TeV) as well as the metacolor scale ($\Lambda_M \approx 10^{11}$ GeV). In particular, allowing for seemingly reasonable threshold effects, it is found that the unity of forces can well occur for certain desirable cases for which the metacolor gauge symmetry G_M is either $SU(6)_M$ or $SU(4)_M$ [rather than $SU(5)_M$] and the corresponding flavor-color gauge symmetry (G_{fc}) is either $SU(2)_L \times U(1)_R\times SU(4)^C_{L+R}$ [for $G_M = SU(6)_M$] or even just the standard model symmetry $SU(2)_L\times U(1)_Y\times SU(3)^C$ [for $G_M = SU(6)_M$ or $SU(4)_M$]. These possibilities were disfavored in earlier work because threshold effects had been ignored altogether. While estimating threshold effects at the supersymmetric and metacolor scales, we have used only bare masses excluding wave-function-renormalization corrections which have been shown by Shifman [20] to be cancelled by two-loop effects. We assure that such a cancellation does not affect the results of this analysis and the threshold effects due to bare masses are enough to establish new gauge symmetries.

An additional new result of this paper is the equality of one-loop β-function coefficients of $SU(2)_L$ and $SU(3)_C$ for $\mu>\Lambda_M$ when these subgroups are embedded in $G_{fc}=SU(2)_L\times U(1)_Y\times SU(3)_C$, $SU(2)_L\times U(1)_R\times U(1)_{B-L}\times SU(3)_C$, or $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C$ as long as the metacolor group is $G_M=SU(6)_M$. This implies one-loop partial unification of the relevant gauge couplings above the metacolor scale.

This paper is organized in the following manner. In Sec. II we present the salient features of the scale-unifying preon model. The spectrum of composites near the electroweak and metacolor scales is given in Sec. III. Threshold effects due to composites are discussed in Sec. IV. The equality of one-loop β-function coefficients for $SU(2)_L$, $SU(2)_R$, and $SU(3)_C$ using $G_M=SU(6)_M$ is proved in Sec. V where the possibilities of different preonic gauge symmetries are also explored. The prospects of $SU(4)_M$ as metacolor gauge symmetry are explored in Sec. VI. Results and conclusions of this work are summarized in Sec. VII.

II. SALIENT FEATURES OF THE SCALE-UNIFYING PREON MODEL

The effective Lagrangian below the Planck mass in the scale-unifying preon model [12] is defined to possess $N=1$ local supersymmetry and a gauge symmetry of the form $G_P=G_{fc}\times G_M$, where $G_M=SU(N)_M$ or $SO(N)_M$ denotes the metacolor gauge symmetry that generates the preon binding force. Although the underlying flavor-color gauge symmetry having preons in the fundamental representation has been suggested [12] to be $G_{fc}=SU(2)_L\times SU(2)_R\times SU(4)^C_{L+R}$ [1], any one of its subgroups could be a candidate for the effective flavor-color symmetry below the Planck scale [15]:

$$\begin{aligned}
G_{213}&=SU(2)_L\times U(1)_Y\times SU(3)_C,\\
G_{2113}&=SU(2)_L\times U(1)_R\times (U)(1)_{B-L}\times SU(3)_C,\\
G_{2213}&=SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C,\\
G_{214}&=SU(2)_L\times U(1)_Y\times SU(4)^C_{L+R},\\
G_{224}&=SU(2)_L\times SU(2)_R\times SU(4)^C_{L+R}. \qquad (1)
\end{aligned}$$

Here G_{2213} and G_{224} are assumed to possess left-right discrete symmetry (=parity), leading to $g_{2L}(\mu)=g_{2R}(\mu)$ for $\mu\gtrsim\Lambda_M$. The gauge symmetry G_P operates on a set of preonic constituents consisting of six positive and six negative chiral superfields, while each of these transforms as the fundamental representation N of $G_M=SU(N)_M$:

$$\Phi^a_\pm=(\phi^a_{L,R},\ \psi^a_{L,R},\ F^a_{L,R}),\quad a=(x,y,r,y,b,l).$$

Here (x,y) denote the two basic flavor attributes (u,d) and, (r,y,b,l), the basic color attributes of a quark lepton family [1]. Thus $\Phi^{X,Y}_+$ and $\Phi^{X,Y}_-$ transform as doublets under $SU(2)_L$ and $SU(2)_R$, respectively, while both $\Phi^{r,y,b,l}_+$ and $\Phi^{r,y,b,l}_-$ transform as quartets under $SU(4)^C_{L+R}$. The effective Lagrangian of this interaction turns out to possess only gauge and gravitational interactions and, as a result, involves only three or four coupling constants of the gauge symmetry $G_{fc}\times G_M$.

The model has a profound interpretation of the hierarchy of mass scales as follows [12]. Corresponding to an input value of the metacolor coupling $\tilde{\alpha}_M=1/20-1/30$ at $M_{\rm Pl}/10$, the asymptotically free metacolor force generated by $SU(N)_M$ becomes strong at scale $\Lambda_M\approx10^{11}$ GeV for $N=4-6$. Thus one small number $(\Lambda_M/M_{\rm Pl})\sim10^{-8}$ arises naturally through renormalization group equations (RGEs) due to a small logarithmic growth of $\tilde{\alpha}_M$ and its perturbative input value at $M_{\rm Pl}/10$. The remaining small scales arise primarily due to the Witten index theorem [18], which would forbid a dynamical breaking of SUSY if there were no gravity. Noting that both the metagaugino condensate $\langle\vec{\lambda}\cdot\vec{\lambda}\rangle$ and the preonic condensate $\langle\overline{\psi^a}\psi^a\rangle$ break SUSY (for massless preons), they must both need a collaboration between the metacolor force and gravity to form. Assuming that they do form, one can argue plausibly that they must each be damped by a factor $\Lambda_M/M_{\rm Pl}$ [20]. Since $\langle\overline{\psi^a}\psi^a\rangle$ breaks not only SUSY, but also $SU(2)_L\times U(1)_Y$ for $a=x,y$, one obtains SUSY-breaking mass splittings $\delta m_S\sim\Lambda_M(\Lambda_M/M_{\rm Pl})\sim1$ TeV and $M_W\sim(1/10)\Lambda_M(\Lambda_M/M_{\rm Pl})\approx100$ GeV. The symmetry of the fermion mass matrix involving three chiral families $q^i_{L,R}$ and two vectorlike families $Q_{L,R}$ and $Q'_{L,R}$, where the chiral families acquire mass almost through their mixings with vectorlike families by the seesaw mechanism [12], explains the interfamily hierarchy $(m)_{u,d,e}\ll(m)_{c,s,\mu}\ll(m)_{t,b,\tau}$, with $m_{u,d,e}\sim O(1)$ MeV and $m_t\sim M_W\sim100$ GeV [14]. Finally, a double-seesaw mechanism with $m(\nu^i_R)\sim\Lambda_M\sim10^{11}$ GeV and $m(\nu)_{\rm Dirac}\sim\Lambda_M(\Lambda_M/M_{\rm Pl})$ yields $m(\nu^i_L)<10^{-3}M_{\rm Pl}(\Lambda_M/M_{\rm Pl})^3\sim10^{-27}M_{\rm Pl}$. In this way the model provides, remarkably enough, a common origin of all the diverse scales from $M_{\rm Pl}$ to m_ν [12].

Owing to the fermion-boson pairing in SUSY, the model also turns out to provide a good reason for family replication and (subject to the saturation at the level of minimum dimensional composite operators) for having just three chiral families $q^i_{L,R}$ [13]. It also predicts two complete vectorlike families $Q_{L,R}=(U,D,N,E)_{L,R}$ and $Q'_{L,R}=(U',D',N',E')_{L,R}$ with masses of the order of 1 TeV where $Q_{L,R}$ couple vectorially to W_L's and $Q'_{L,R}$ to W_R's. The masses of the superpartners of all fermions are predicted to be 0.5–2 TeV.

The model presumes that the preonic condensate Δ_R, transforming under G_{224} as $(1,3_R,10^{*C})$, is formed and its neutral component acquires a vacuum expectation value (VEV), $\langle\Delta^0_R\rangle\simeq\Lambda_M\simeq10^{11}$ GeV, which preserves SUSY, but breaks G_{224} and its subgroups to G_{213}. Finally, the condensate $\langle\overline{\psi^a}\psi^a\rangle$, for $a=x,y$, breaks SUSY as well as the electroweak gauge symmetry, $SU(2)_L\times U(1)_Y$. As a result, the model leads to many consequences common with a two-step breaking of SO(10). Subject to left-right symmetry, the effective Lagrangian has three gauge couplings with $G_{224}\times SU(N)_M$ and four with $G_{2213}\times SU(N)_M$, $G_{214}\times SU(N)_M$, and $G_{213}\times SU(N)_M$, but five with $G_{2113}\times SU(N)_M$. Furthermore, if the gauge symmetry G_P and the associated preon content specified above arise from an underlying superstring theory, in particular, through a four-dimensional construction [9] with $k=1$ Kac-Moody algebra, the few gauge coupling constants of the model would be equal to one coupling at the string unification scale $M_U\sim 10^{18}$ GeV (barring string threshold effects) [10]. It is this possibility of gauge-coupling unification at the preon level, with $G_M=SU(6)_M$ and $SU(4)_M$, which is explored in this paper including threshold effects at M_{SUSY} and Λ_M.

As it is well known that the flavor symmetry near $\mu=100$ GeV is given by the standard gauge symmetry G_{213} with quarks and leptons in the fundamental representation and that at low energies is $U(1)_{\mathrm{em}}\times SU(3)_C$, it might appear that the five flavor-color symmetries given in Eqs. (1) have been arbitrarily chosen for the preonic effective Lagrangian. But realizing that the two important ingredients in the model [12] are left-right symmetry and SU(4) color [1], the flavor-color symmetry G_{224} has been suggested as the natural gauge symmetry near the Planck scale in the presence of $G_M=SU(N)_M$. Thus, below $\mu=M_{\mathrm{Pl}}$, G_{224} itself or any of its four subgroups given in Eqs. (1) could be natural choices for the preonic effective Lagrangian. However, in addition to the assumed saturation of the minimum dimensional operator and the composite spectrum, the model has an arbitrariness in that it does not specify a unique direction of symmetry breaking. This latter feature is also common to the usual SUSY SO(10) with more than one choice for intermediate gauge symmetries. But, nevertheless, the preons combine to form quarks and leptons, and Higgs scalars near $\mu=\Lambda_M$ due to the strong metacolor binding force, and every other G_{fc}, except G_{213}, undergoes spontaneous symmetry breaking, leading to the standard gauge symmetry. In addition to the three standard families of quarks and leptons, the new vectorial fermions are predicted to have masses near 1 TeV which can be testified by accelerator experiments [13–17]. The right-handed neutrinos aquire masses near Λ_M and contribute to the seesaw mechanism.

III. SPECTRUM OF COMPOSITES NEAR ELECTROWEAK AND METACOLOR SCALES

In this section we discuss briefly the spectrum of massive particles near the electroweak scale (M_Z) and the metacolor scale ($\Lambda_M\simeq 10^{11}$ GeV). In the scale-unifying preon model, the left- and right-handed chiral fermions in each of the three families transform as $(2_L,1,4^{*C})$ and $(1,2_R,4^{*C})$, respectively, under $SU(2)_L\times SU(2)_R\times SU(4)^C_{L+R}$ [1]. The two vectorlike families $Q_{L,R}$ and $Q'_{L,R}$ transform as $(2_L,1,4^{*C})$ and $(1,2_R,4^{*C})$, respectively. The members of the five families predicted by the scale-unifying preon model [12,13] are denoted by

$$q^e_{L,R}=(u,d,\nu,e)_{L,R},$$

$$q^\mu_{L,R}=(c,s,\nu_\mu,\mu)_{L,R},$$

$$q^\tau_{L,R}=(t,b,\nu_\tau,\tau)_{L,R},$$

$$Q_{L,R}=(U,D,N,E)_{L,R},$$

$$Q'_{L,R}=(U',D',N',E')_{L,R}. \tag{2}$$

The spectra of light and heavy particles including matter multiplets near the electroweak and the metacolor scales and their quantum numbers under the gauge groups G_{224} and G_{213} are summarized in Table I. In order to compute threshold effects, we present, in Table II, assumed, but plausible values of masses for the Higgs scalars and different members of vectorial families along with the current experimental value for m_t, including their contributions to one-loop β-function coefficients.[1] The corresponding values for all the superpartners of the standard chiral families, gauginos, and Higgsinos are given in Table III. For the sake of simplicity, all the superpartners of two vectorlike families are assumed to be degenerate at the scale $M_S=1.5$ TeV, above which SUSY is assumed to be restored.[2] As usual, there are two Higgs doublets, u type and d type, near the electroweak scale contained in the G_{224} submultiplet $\phi(2,2,1)$, which is a two-body condensate made out of the preons.

As noted in Sec. II, it is essential that the four-body preonic condensate $\Delta_R(1,3,10^{*C})$ is formed with mass near Λ_M to drive the seesaw mechanism, resulting in small values of the left-handed Majorana neutrino mass. The underlying left-right symmetry of the effective Lagrangian then requires the formation of the corresponding composite $\Delta_L(3,1,10^C)$. In fact, preservation of SUSY down to the 1 TeV scale, especially through the D term, may require an additional pair $\overline{\Delta}_L+\overline{\Delta}_R$ having masses the same as their counterparts in the first pair. In what follows we will drop the distinctions between Δ_i and $\overline{\Delta}_i$ $(i=L,R)$ as both have identical contributions to β functions. Thus two sets of Δ_L and Δ_R are the minimal requirements of the scale-unifying preon model. Before Δ^O_R acquires VEV$\simeq\Lambda_M\simeq 10^{11}$ GeV, the massess of $\Delta_L(\overline{\Delta}_L)$ and $\Delta_R(\overline{\Delta}_R)$ are identical. But the VEV splits them, leading to their mass ratio, which could be as large as 3.

In specific cases, we will also assume the formation of composite Higgs supermultiplets of the type $\sigma(1,1,15)$ and $\xi(2,2,15)$ under G_{224} as optional choices. It is to be noted that while the field $\sigma(1,1,15)$ is a two-body composite, $\xi(2,2,15)$ is a four body composite. Since the masses of these composites are not constrained by the VEV of Δ_R, they are allowed

[1]Two vectorlike families have the quantum numbers of a $16+\overline{16}$ of SO(10). Thus their contributions to β functions are the same as those of two standard chiral families.

[2]Changing the superpartner scale from $M_S=1.5$ TeV, used in this analysis, to $M_S=1$ TeV would increase the value of the strong interaction coupling by less than a few percent without any significant change in the results and conclusions.

TABLE I. Light and heavy spectra in the scale-unifying preon model and their quantum numbers under G_{224} and G_{213}.

Particle type and G_{224} quantum nos.	Particle type under the standard model	G_{213} quantum nos.
LH quarks and leptons	$(u,d)_L,(c,s)_L,(t,b)_L$	(2,1/6,3)
$q_L^{e,\mu,\tau}(2_L,1,4_C^*)$	$(\nu_e,e)_L,(\nu_\mu,\mu)_L,(\nu_\tau,\tau)_L$	$(2,-1/2,1)$
RH quarks and leptons	u_R,c_R,t_R	(1,2/3,3)
$q_R^{e,\mu,\tau}(1,2_R,4_C^*)$	d_R,s_R,b_R	$(1,-1/3,3)$
	e_R,μ_R,τ_R	$(1,-1,1)$
	$\nu_{e_R},\nu_{\mu_R},\nu_{\tau_R}$	(1,0,1)
LH vectorial quarks and	$(U,D)_{L,R}$	(2,1/6,3)
Leptons $Q_{L,R}(2_L,1,4_C^*)$	$(N,E)_{L,R}$	$(2,-1/2,1)$
RH vectorial quarks and	$(U',D')_{L,R}$	(1, 2/3, 3)
Leptons $Q'_{L,R}(1,2_R,4_C^*)$	$(N',E')_{L,R}$	$(1,-1/3,1)$
Bidoublet of Higgs	h_u	(2,1/2,1)
scalars $\phi(2,2,1)$	h_d	$(2,-1/2,1)$
Minimal sets of heavy Higgs		
$\Delta_L^{1,2}(3,1,10^C),\Delta_R^{1,2}(1,3,10^{*C})$	See Table V	See Table V
Other sets of heavy Higgs		
$\xi^{1,2}(2,2,15),\sigma^{1,2}(1,1,15)$	See Table IV	See Table IV

to vary over a wider range around Λ_M as compared to the masses of Δ_L and Δ_R. It can be argued that more than one set of σ and ξ fields are allowed to form, but we will confine ourselves to at most two such sets with masses $(1-7)\Lambda_M$ or $(1/7-1)\Lambda_M$ as the case may be. All masses used for the estimation of threshold effects near the metacolor scale as well as the supersymmetry breaking scale are bare masses devoid of wave-function renormalization, which is shown to be cancelled out by two-loop effects [20]. We assure that the threshold effects due to bare masses are enough to establish new gauge symmetries and the observed cancellation [20] does not affect the results of this paper. In Tables IV and V we present the superheavy-particle spectra near the metacolor scale with their respective quantum numbers under G_{224} and G_{213}.

IV. THRESHOLD EFFECTS AT LOWER AND INTERMEDIATE SCALES

In this section we discuss renormalization group equations [21] for gauge couplings in the scale-unifying preon model using the gauge symmetry $SU(2)_L\times U(1)_Y \times SU(3)^C(=G_{213})$ for the composite quarks, leptons, and Higgs scalars and their superpartners between M_Z and Λ_M.

TABLE II. One-loop β-function coefficients for particles at lower threshold with their quantum numbers and assigned values of masses used for computation of threshold effects.

Particle type	G_{213} quantum nos.	Mass (GeV)	b_1^α	b_2^α	b_3^α
LH top t_L	(2,1/6,3)	175	1/30	1	1/3
RH top t_R	(1,2/3,3)	175	8/15	0	1/3
u-type Higgs h_u	(2,1/2,1)	120	1/10	1/6	0
d-type Higgs h_d	$(2,-1/2,1)$	250	1/10	1/6	0
LH vectorial quark Doublets $(U,D)_{L,R}$	(2,1/6,3)	500	2/15	2	4/3
RH vectorial u-type Quarks $U'_{L,R}$	(1,2/3,3)	500	16/15	0	2/3
RH vectorial d-type Quarks $D'_{l,R}$	$(1,-1/3,3)$	500	4/15	0	2/3
LH vectorial lepton Doublets $(N,E)_{L,R}$	$(2,-1/2,1)$	100	2/5	2/3	0
RH vectorial charged Leptons $E'_{L,R}$	$(1,-1,1)$	100	4/5	0	0

TABLE III. Same as Table II, but for superpartners only, and the number of squarks and sleptons correspond to summing over three flavors.

Particle type	G_{213} quantum nos.	Mass (GeV)	b_1^α	b_2^α	b_3^α
Gluino	(1,0,8)	150–200	0	0	23
W-ino	(3,0,1)	100–150	0	4/3	0
LH slepton doublets	(2,−1/2,1)	500–1500	3/10	1/2	0
RH charged sleptons	(1,−1,1)	500–1500	3/5	0	0
LH squark doublets	(2,1/6,3)	500–1500	1/10	3/2	1
RH u-type squarks	(1,2/3,3)	500–1500	4/15	0	1/2
RH d-type squarks	(1,1/3,3)	500–1500	1/5	0	1/2
u-type Higgsino	(2,1/2,1)	100–300	1/5	1/3	0
d-type Higgsino	(2,−1/2,1)	100–300	1/5	1/3	0

At first the gauge couplings of G_{213} are evolved from M_Z to Λ_M assuming the SUSY-breaking scale to be $M_S=1.5$ TeV and including threshold effects at M_Z and M_S through the matching functions $\Delta_i^{(Z)}$ and $\Delta_i^{(S)}$, respectively [22,23]. The RGEs for the three gauge couplings of G_{213} $(i=1,2,3)$ are

$$\frac{1}{\alpha_i(M_Z)}=\frac{1}{\alpha_i(\Lambda_M)}+\frac{b_i}{2\pi}\ln\frac{M_S}{M_Z}+\frac{b_i'}{2\pi}\ln\frac{\Lambda_M}{M_S}-\Delta_i^{(L)},$$

$$\Delta_i^{(L)}=\Delta_i^{(Z)}+\Delta_i^{(S)}, \tag{3}$$

where we have neglected two-loop effects. Threshold effects at Λ_M have been included in the second part of this section. The left-hand side (LHS) of Eqs. (3) is extracted using the CERN-LEP data and improved determination of the fine-structure constant at $M_Z=91.18$ GeV [5],

$$\sin^2\theta_W(M_Z)=0.2316,$$

$$\alpha^{-1}(M_Z)=127.9\pm0.2,$$

$$\alpha_S(M_Z)=0.118\pm0.007, \tag{4}$$

leading to the following values of couplings[3] of G_{213} at M_Z:

$$\alpha_1^{-1}(M_Z)=58.96,$$

$$\alpha_2^{-1}(M_Z)=29.62,$$

$$\alpha_3^{-1}(M_Z)=8.33\pm0.63. \tag{5}$$

The matching functions $\Delta_i^{(Z)}$ include threshold effects due to the top-quark coupling to the photon, the electroweak gauge bosons, and gluons, and its Yukawa coupling to the Higgs scalars [23]. The contributions due to the two Higgs doublets, the additional fermions of two vectorlike families (Q,Q') and all superpartners, having specific values of masses within a given range, but below M_S, are included in $\Delta_i^{(S)}$. The one-loop coefficients b_i in Eqs. (3) are computed using three generations of fermions $(n_g=3)$ and excluding the contributions of the Higgs doublets $(n_H=0)$ and vectorlike families. Since the contributions of the Higgs scalars and vectorlike families are included in $\Delta_i^{(S)}$, incorporating the specific assumptions on their masses, the approach adopted here is equivalent to the conventional approach as the contribution due to every particle to the gauge-coupling evolution is accounted for:

$$b_i=-\frac{11}{3}t_2(V)+\frac{2}{3}\sum t_2(F)+\frac{1}{3}\sum t_2(S), \tag{6}$$

where $t_2(V)$, $t_2(F)$, and $t_2(S)$ denote the contributions of gauge bosons, fermions, and Higgs scalars, respectively. For an SU(n) group with matter in the fundamental representation and gauge bosons in the adjoint,

$$t_2(F)=t_2(S)=1/2,\quad t_2(V)=n,$$

whereas $t_2(V)=0$ for any U(1) group. With supersymmetry, Eq. (6) gives

$$b_i'=-3t_2(V)+\sum t_2(F)+\sum t_2(S). \tag{7}$$

TABLE IV. One-loop β-function coefficients for the components of the G_{224} Higgs supermultiplet $\xi(2,2,15)$ near the metacolor scale under the standard gauge group G_{213}.

G_{213} submultiplet	b_1^α	b_2^α	b_3^α	Σb_1^α	Σb_2^α	Σb_3^α
$\xi_1(2,1/2,1)$	3/10	1/2	0			
$\xi_2(2,-1/2,1)$	3/10	1/2	0			
$\xi_3(2,-1/6,3)$	1/10	3/2	1			
$\xi_4(2,-7/6,3)$	49/10	3/2	1			
$\xi_5(2,-7/6,\bar{3})$	49/10	3/2	1	77/5	15	16
$\xi_6(2,1/6,\bar{3})$	1/10	3/2	1			
$\xi_7(2,1/2,8)$	24/10	4	6			
$\xi_8(2,-1/2,8)$	24/10	4	6			

[3]The value of $\sin^2\theta_W(M_Z)=0.2316$ is consistent with a heavy top quark ($m_t=175$ GeV). We ignore negligible threshold effects due to the top-quark mass on electroweak gauge couplings.

TABLE V. Same as Table IV, but for G_{224} multiplets $\Delta_L(3,1,10^C)$, $\Delta_R(1,3,10^{*C})$, and $\sigma(1,1,15)$.

G_{213} submultiplet	b_1^α	b_2^α	b_3^α	Σb_1^α	Σb_2^α	Σb_3^α
$\Delta_{R_1}(1,1,1)$	3/5	0	0			
$\Delta_{R_2}(1,2,1)$	1/5	0	0			
$\Delta_{R_3}(1,1/3,\overline{3})$	1/5	0	1/2			
$\Delta_{R_4}(1,2/3,\overline{3})$	4/5	0	1/2			
$\Delta_{R_5}(1,-4/3,\overline{3})$	16/5	0	1/2	78/5	0	9
$\Delta_{R_6}(1,1/3,\overline{6})$	2/5	0	5/2			
$\Delta_{R_7}(1,-2/3,\overline{6})$	8/5	0	5/2			
$\Delta_{R_8}(1,4/3,\overline{6})$	32/5	0	5/2			
$\Delta_{L_1}(3,1,1)$	9/5	2	0			
$\Delta_{L_2}(3,1/3,\overline{3})$	3/5	6	3/2	18/5	20	9
$\Delta_{L_3}(3,-1/3,6)$	6/5	12	15/2			
$\sigma_1(1,-2/3,3)$	4/5	0	1/2			
$\sigma_2(1,2/3,3)$	4/5	0	1/2	8/5	0	4
$\sigma_3(1,0,8)$	0	0	3			

In region I where $\mu=M_Z$ to $M_S=1.5$ TeV, we evaluate the coefficients by including the contributions of gauge bosons and three standard fermion generations (as all other contributions in this region are included in $\Delta_i^{(S)}$):

$$b_3=-\frac{11}{3}\times 3+\frac{4}{3}\times 3=-7,$$

$$b_2=-\frac{11}{3}\times 2+\frac{4}{3}\times 3=-\frac{10}{3},$$

$$b_1=\frac{4}{3}\times 3=4. \tag{8}$$

In region II where $\mu=M_S$ to Λ_M, the spectrum of particles consists of the gauge bosons of G_{213}, the three normal families of fermions ($n_g=3$), two additional vectorlike families corresponding to $n_g'=n_g+2$, two Higgs doublets, and superpartners of these particles such that SUSY is restored for $\mu>M_S=1.5$ TeV. Using Eq. (7) we evaluate

$$b_3'=-3\times 3+2n_g'=1,$$

$$b_2'=-3\times 2+2n_g'+2\times\frac{1}{2}=5,$$

$$b_1'=2n_g'+\frac{2}{5}=\frac{53}{5}. \tag{9}$$

Now we discuss explicitly how threshold effects at the boundaries M_S and M_Z are evaluated.

A. Threshold effects at lower scales

The top-quark threshold contribution, which is the same in SUSY and non-SUSY standard models has been discussed in Ref. [23]. Since the value of $\sin^2\theta_W$ in Eqs. (4) is consistent with the experimental value of the top-quark mass, $m_t=175$ GeV, we ignore negligible electroweak threshold corrections due to the heavy top quark, but include those on $\alpha_3^{-1}(M_Z)$ and Yukawa coupling corrections. The coupling of the top quark to gluons gives rise to

$$\Delta_3^{\rm top}=\frac{1}{3\pi}\ln\frac{m_t}{M_Z}=0.07. \tag{10}$$

The top-quark mass $m_t=175$ GeV is consistent with its Higgs-Yukawa coupling $h_t\simeq 1$, leading to threshold corrections at the two-loop level:

$$\Delta_i^{\rm Yuk}=\frac{h_t^2}{32\pi^3}\left(b_i^{\rm top}\ln\frac{M_S}{174\ \mathrm{GeV}}+b_i'^{\,\rm top}\ln\frac{\Lambda_M}{M_S}\right), \tag{11}$$

where $b_i^{\rm top}=(17/10,3/2,2)$ for $i=1,2,3$ in the standard model and $b_i'^{\,\rm top}=(26/5,6,4)$ in the minimal supersymmetric standard model (MSSM). Using $M_S=1.5$ TeV and $\Lambda_M=10^{11}$ GeV gives

$$\Delta_1^{\rm Yuk}=0.10,\quad \Delta_2^{\rm Yuk}=0.12,\quad \Delta_3^{\rm Yuk}=0.08. \tag{12}$$

Adding the contributions in Eqs. (10) and (12) yields

$$\Delta_1^{(Z)}=0.10,\quad \Delta_2^{(Z)}=0.12,\quad \Delta_3^{(Z)}=0.15. \tag{13}$$

It is clear that the corrections are smaller and unlikely to affect our analysis unless the Yukawa couplings of heavy families are much larger,[4] i.e., $h_{Q,Q'}=3-5$.

Threshold effects at M_S due to masses below it are computed explicitly using the second and third terms Eq. (6) depending upon the nature of the particle α:

$$\Delta_i^{(S)}=\sum_\alpha \frac{b_i^\alpha}{2\pi}\ln\frac{M_\alpha}{M_S}. \tag{14}$$

The values of b_i^α and the masses M_α used in this analysis are given in Tables II and III for each particle, which lead to

$$\Delta_1^{(S)}={}^{-1.8}_{-1.0},\quad \Delta_2^{(S)}={}^{-2.3}_{-0.9},\quad \Delta_3^{(S)}={}^{-1.8}_{-1.1}. \tag{15}$$

Combining Eqs. (13) and (15) gives the following threshold corrections at lower scales:

$$\Delta_1^{(L)}={}^{-1.70}_{-0.90},\quad \Delta_2^{(L)}={}^{-2.20}_{-0.80},\quad \Delta_3^{(L)}={}^{-1.65}_{-0.95}. \tag{16}$$

In Eqs. (15) and (16) the upper and lower entries are due to lowest and highest values of M_α given in Tables II and III. The evolution of the gauge couplings up to $\mu=\Lambda_M$, including threshold effects at M_Z and M_S, but excluding those at Λ_M, yields

$$\alpha_1^{-1}(\Lambda_M)=26.6\ \ (25.6),$$

[4]Since the masses of vectorlike families occur as off-diagonal elements, they receive no contributions from the Yukawa couplings of the two Higgs doublets of the standard SUSY model. Hence their Yukawa contributions to threshold effects are likely to be smaller.

$$\alpha_2^{-1}(\Lambda_M)=16.0 \ \ (15.67),$$

$$\alpha_3^{-1}(\Lambda_M)=7.6\pm0.6 \ \ (6.9\pm0.6), \tag{17}$$

where the quantities inside (outside) the parentheses in Eqs. (17) are due to the lowest (highest) values of $\Delta_i^{(L)}$ in Eqs. (16). The gauge couplings at the metacolor scale are then obtained as

$$g_1(\Lambda_M)=0.685 \ \ (0.700),$$

$$g_2(\Lambda_M)=0.833 \ \ (0.894),$$

$$g_3(\Lambda_M)=1.28\pm0.05 \ \ (1.35\pm0.06). \tag{18}$$

B. Threshold effects at the metacolor scale

As explained in Secs. II and III, we will use two sets of the Higgs superfields $\Delta_L(3,1,10^C)$ and $\Delta_R(1,3,10^{*C})$ in all cases and two sets of $\xi(2,2,15)$ and $\sigma(1,1,15)$, wherever necessary. Denoting $\alpha_i'(\Lambda_M)$ for the gauge couplings of G_{213} at Λ_M, including threshold effects through the matching functions δ_i, they are related to $\alpha_i(\Lambda_M)$ of Eqs. (5) and (17) as

$$\frac{1}{\alpha_i(\Lambda_M)}=\frac{1}{\alpha_i'(\Lambda_M)}-\delta_i . \tag{19}$$

In addition to the superheavy-particle-threshold effects, δ_i may have a very small correction due to conversion from the dimensional reduction ($\overline{\mathrm{DR}}$) to modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme [23] in the relevant cases.[5] The matching functions δ_i are evaluated by the one-loop approximation as

$$\delta_i=\sum_\rho \frac{b_i^\rho}{2\pi}\ln\frac{M_\rho}{\Lambda_M}=\sum_\rho \frac{b_i^\rho}{2\pi}\,\eta_\rho , \tag{20}$$

where ρ runs over all the submultiplets of a G_{224} multiplet and we have used the notation $\eta_\rho=\ln(M_\rho/\Lambda_M)$. The decomposition of each G_{224} representation under G_{213} and the contribution to the one-loop β-function coefficient $(=b_i^\rho)$ are presented in Tables IV and V. Since the exact values of the masses of the submultiplets are not predicted by the model, we make the simplifying assumtion that all the submultiplets belonging to the same G_{224} multiplet have a degenerate bare mass [20]. Including all possible contributions due to the G_{213} representations of Tables IV and V, we obtain

$$\delta_1=\frac{1}{10\pi}\,(77\eta_\xi+18\eta_{\Delta_L}+78\eta_{\Delta_R}+8\,\eta_\sigma),$$

$$\delta_2=\frac{1}{2\pi}\,(15\eta_\xi+20\eta_{\Delta_L}),$$

$$\delta_3=\frac{1}{2\pi}\,(16\eta_\xi+9\,\eta_{\Delta_L}+9\,\eta_{\Delta_R}+4\,\eta_\sigma). \tag{21}$$

There are slight variations from Eqs. (21) in specific cases depending upon the preonic gauge symmetry given in Eqs. (1). In the case of $G_{fc}=G_{2213}$, certain components of $\Delta_R(1,3,10^{*C})$ are absorbed as longitudinal modes of $\mathrm{SU}(2)_R$ gauge bosons, leading to

$$\delta_1=\frac{1}{10\pi}\,(77\eta_\xi+18\eta_{\Delta_L}+75\eta_{\Delta_R}+8\,\eta_\sigma), \tag{22}$$

but the expressions for δ_2 and δ_3 are the same as in Eqs. (21). Similarly, when $G_{fc}=G_{224}$, the submultiplet having the G_{213} quantum numbers (1,2/3,3) is absorbed as a longitudinal mode of massive $\mathrm{SU}(4)_C$ gauge bosons and does not contribute to δ_1 and δ_3:

$$\delta_1=\frac{1}{10\pi}\,(77\eta_\xi+18\eta_{\Delta_L}+71\eta_{\Delta_R}+8\,\eta_\sigma),$$

$$\delta_3=\frac{1}{2\pi}\left(16\eta_\xi+9\,\eta_{\Delta_L}+\frac{17}{2}\,\eta_{\Delta_R}+4\,\eta_\sigma\right)-\frac{1}{4\pi}, \tag{23}$$

where the term $-(4\pi)^{-1}$ arises due to conversion from the $\overline{\mathrm{DR}}$ to $\overline{\mathrm{MS}}$ scheme [23]. The expression for δ_2 in this case is the same as in Eqs. (21). In the case of $G_{fc}=G_{214}$,

$$\delta_1=\frac{1}{10\pi}\,(77\eta_\xi+18\eta_{\Delta_L}+74\eta_{\Delta_R}+8\,\eta_\sigma), \tag{24}$$

but the expressions for δ_2 [δ_3] are given by Eqs. (21) [Eqs. (23)].

V. PREONIC GAUGE SYMMETRIES AND UNIFICATION OF GAUGE COUPLINGS

In this section we explore possible gauge symmetries of the preonic effective Lagrangian that operates from $\mu=\Lambda_M \simeq 10^{11}$ GeV to M_U $(=M_{\mathrm{Pl}}/10=10^{18}$ GeV). In Ref. [15] it has been successfully demonstrated that unity of fundamental forces occurs with preons as fermion representations of the gauge group $G_P=\mathrm{SU}(2)_L\times\mathrm{U}(1)_R\times\mathrm{SU}(4)_{L+R}^C\times\mathrm{SU}(5)_M$. In this section we confine ourselves to prospects of $\mathrm{SU}(6)_M$. In what follows we search for converging solutions to gauge couplings as we approach M_{Pl}. We prefer approximate to exact unification of the gauge couplings as the gravitational effects are to make substantial contributions which might compensate for the remaining small differences.

The RGEs for the gauge couplings $[\tilde{\alpha}_i(\mu)=\tilde{g}_i^2(\mu)/4\pi]$ of the preonic effective Lagrangian for $\mu=\Lambda_M$ to M_U can be written at the one-loop level as [21,23]

$$\frac{1}{\tilde{\alpha}_i(\Lambda_M)}=\frac{1}{\tilde{\alpha}_i(\mu)}+\frac{b_i''}{2\pi}\ln\frac{\mu}{\Lambda_M}, \tag{25}$$

where b_i'' is the one-loop coefficient of the β function with preons in the fundamental representation, which are separately evaluated in each case. For the computation of thresh-

[5] The term $\Delta_i^{CNV}=-C_2(G_i)/12\pi$, where $C_2(G_i)=N$ for SU(N), but $C_2(G_i)=0$ for U(1), appears from the necessity to use the $\overline{\mathrm{DR}}$ scheme.

old effects, while a mass ratio $\rho = M_{\Delta_L}/M_{\Delta_R} = 2-3$ could be considered natural, we also keep an open mind to explore unification possibilities with such values of the inverse mass ratio. We adopt the strategy of examining approximate unification starting from smaller values of the $\Delta_L - \Delta_R$ mass difference within 10%–20% and then increasing the mass difference corresponding to higher values of ρ. When we find that approximate unification is not achievable with the minimal two sets of Δ_L and Δ_R fields, we introduce threshold effects due to the two optional sets of fields $\xi(2,2,15)$ and $\sigma(1,1,15)$. We report our investigations in different cases.

A. $G_P = SU(2)_L \times U(1)_R \times SU(4)^C_{L+R} \times SU(6)_M$

Corresponding to $G_P = G_{214} \times SU(6)_M$, $i = 1R,\ 2L,\ 4C$, and 6 in Eq. (25), and the one-loop coefficients are

$$b''_{1R} = 3, \quad b''_{2L} = -3, \quad b''_{4C} = -6, \quad b''_6 = -12.$$

The matching conditions between the gauge couplings of elementary preons $[\tilde{g}_i(\mu)]$ and composite fields $[g_i(\mu)]$ at $\mu = \Lambda_M$ are written as

$$\alpha_2^{-1}(\Lambda_M) + \delta_2 = \tilde{\alpha}_{2L}^{-1}(\Lambda_M), \tag{26a}$$

$$\alpha_3^{-1}(\Lambda_M) + \delta_3 = \tilde{\alpha}_{4C}^{-1}(\Lambda_M), \tag{26b}$$

$$\alpha_1^{-1}(\Lambda_M) + \delta_1 = \frac{3}{5}\,\tilde{\alpha}_{1R}^{-1}(\Lambda_M) + \frac{2}{5}\,\tilde{\alpha}_{4C}^{-1}(\Lambda_M), \tag{26c}$$

where the LHS in Eqs. (26a)–(26c) are $\alpha_i'^{-1}(\mu = \Lambda_M)$ $(i = 1,2,3)$ of Eq. (19). Using Eqs. (26b) and (17) in Eq. (26c) gives

$$\frac{3}{5}\,\tilde{\alpha}_{1R}^{-1}(\Lambda_M) = \delta_1 - \frac{2}{5}\,\delta_3 + \frac{2}{5}\,\delta_3 + 23.66 \pm 0.38, \tag{27}$$

which yields $\tilde{\alpha}_{1R}^{-1}(\Lambda_M)$ once δ_1 and δ_3 are specified. Excluding threshold effects at $\mu = \Lambda_M(\delta_i = 0)$ and extrapolating the gauge couplings to $\mu = 10^{18}$ GeV gives $\tilde{\alpha}_{2L}^{-1}(M_U) = 23.8$ and $\tilde{\alpha}_{4C}^{-1}(M_U) = 23.0$. This implies that when threshold effects are included, $\delta_2 = \delta_3 = 7$ for an approximate unification of gauge couplings with $SU(6)_M$ corresponding to $\tilde{\alpha}_6^{-1}(M_U) = 30$ provided the matching condition (27) is satisfied with suitable values of δ_1 and $\tilde{\alpha}_{1R}^{-1}(\Lambda_M)$. It is found that these threshold corrections are significantly less compared to other models with $SU(6)_M$ investigated in this paper.

To see how unification is achieved, we start with $\delta_2 = 8$ and $\delta_3 = 7$. Then using Eqs. (21) and (23) and setting $\eta_\sigma = \eta_\xi = 0$, we obtain

$$\eta_{\Delta_L} = 2.5, \quad \eta_{\Delta_R} = 2.4. \tag{28}$$

TABLE VI. Gauge couplings at different mass scales in the presence of two sets of relevant Higgs superfields for the preonic symmetry $G_P = SU(2)_L \times U(1)_R \times SU(4)^C_{L+R} \times SU(6)_M$, with $M_{\Delta_L} = 3.5 \times 10^{11}$ GeV and $\rho = M_{\Delta_L}/M_{\Delta_R} = 1.1$. Note that the four gauge couplings converge within 4% as μ approaches 10^{19} GeV.

Mass scale (μ) (GeV)	$\tilde{g}_{1R}(\mu)$	$\tilde{g}_{2L}(\mu)$	$\tilde{g}_{4C}(\mu)$	$\tilde{g}_6(\mu)$
10^{19}	0.580	0.617	0.624 ± 0.007	0.613
10^{18}	0.570	0.628	0.646 ± 0.007	0.658
10^{11}	0.527	0.722	0.927 ± 0.007	

In the presence of only the minimal number of two sets of fields, $\Delta_L + \Delta_R$ and $\bar{\Delta}_L + \bar{\Delta}_R$, as mentioned in Sec. III, Eqs. (28) imply[6]

$$M_{\Delta_L} = M_{\bar{\Delta}_L} = 3.5 \times 10^{11}\ \text{GeV},$$

$$M_{\Delta_R} = M_{\bar{\Delta}_R} = 3.2 \times 10^{11}\ \text{GeV}, \tag{29}$$

which differ by only 10%. It is to be noted that these are bare masses including splitting due to the VEV of Δ_R^0, since the wave-function renormalization effects have been shown to be cancelled by two-loop contributions [20]. The values of $\tilde{\alpha}_{1R}^{-1}(\Lambda_M)$ are obtained from Eq. (27) as δ_1 is determined using Eqs. (28) and $\eta_\sigma = \eta_\xi = 0$ in Eqs. (23). Then $\tilde{\alpha}_{1R}^{-1}(M_U)$ is known through its RGE. With $\tilde{\alpha}_6^{-1}(10^{18})$ GeV$=29$, the gauge couplings at three different scales $\mu = 10^{11}$, 10^{18}, and 10^{19} GeV are presented in Table VI. It is clear that the least difference between the gauge couplings, which is 2%–3%, occurs near 10^{19} GeV; i.e., the unification appears to occur at a scale one order higher than expected. The evolution of gauge couplings in this model is shown in Fig. 1. Even though the mass difference between $\Delta_L(\bar{\Delta}_L)$ and $\Delta_R(\bar{\Delta}_R)$ is small, the strong interaction coupling $[g_{3C}(\Lambda_M)]$ of composite fields and the $SU(4)^C_{L+R}$ coupling of preons $[\tilde{g}_{4C}(\Lambda_M)]$ exhibit a nearly 35% difference due to the threshold effect at $\mu = \Lambda_M$. Similarly, $g_{2L}(\Lambda_M)$ and $\tilde{g}_{2L}(\Lambda_M)$ show a nearly 20% difference. These are due to the fact that the individual masses of the two sets of fields given Eqs. (29) deviate from Λ_M by a factor of 3.2–3.5 which contribute to such significant threshold corrections. The remaining small differences among the gauge couplings at $\mu = 10^{19}$ GeV are expected to be compensated by gravitational effects.

B. $G_P = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times SU(6)_M$

In this case we assume the gauge group to possess left-right discrete symmetry starting from $\mu = \Lambda_M$ to $M_{\rm Pl}$ with

[6]In our notation $\Delta_{L,R} \equiv \Delta^1_{L,R}$ and $\bar{\Delta}_{L,R} \equiv \Delta^2_{L,R}$ for the minimal two sets of fields needed from considerations of left-right symmetry, spontaneous breaking of gauge symmetry, or generation of RH Majorana neutrino mass and preservation of SUSY down to the TeV scale (see also Tables I and V).

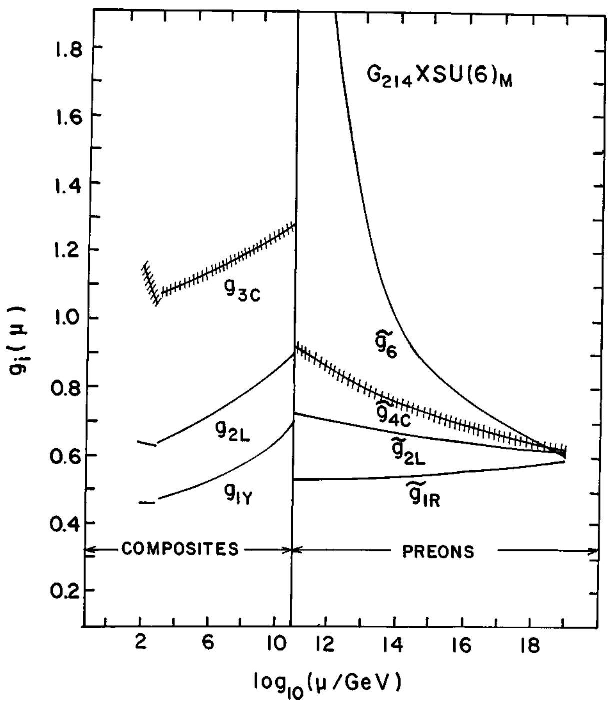

FIG. 1. Unification of gauge couplings and their evolution from $M_{\rm Pl}=10^{19}$ GeV to M_Z including threshold effects at lower and intermediate (metacolor) scales for the preonic gauge symmetry $SU(2)_L\times U(1)_R\times SU(4)^C_{L+R}\times SU(6)_M$ with a minimal two sets of Δ_L and Δ_R fields and 10% mass difference between them.

$\tilde{g}_{2L}(\mu)=\tilde{g}_{2R}(\mu)$. Denoting $i=1_{BL}$, 2_L, 2_R, 3_C, and 6 in Eq. (25), the one-loop coefficients are

$$b''_{BL}=6,\quad b''_{2L}=b''_{2R}=-6+\frac{6}{2}=-3,$$

$$b''_{3C}=-9+6=-3,\quad b''_6=-3\times 6+6=-12. \tag{30}$$

The equality of the coefficients $b''_{2L}=b''_{2R}=b''_{3C}$ signifies unification of $SU(2)_L$ and $SU(3)_C$ gauge couplings from $\mu=\Lambda_M$ to $M_{\rm Pl}$ at the one-loop level when preons are in the fundamental representation and the metacolor symmetry is $SU(6)_M$. This is a common feature for $G_{fc}=G_{213}$, G_{2113}, and G_{2213} when $G_M=SU(6)_M$ as can be seen in the following.

Suppose that $G=SU(N)_M$ for all three types of G_{fc}. Then the one-loop coefficients for $SU(2)_L$ and $SU(3)_C$ are

$$b''_{2L}=-6+\frac{N}{2},\quad b''_{3C}=-9+N. \tag{31}$$

The one-loop unification for all values of μ starting from $\mu=\Lambda_M$ to $\mu=M_U$ is guaranteed by the RGEs provided,

$$b''_{2L}=b''_{3C}=b''_i, \tag{32}$$

with

$$\frac{1}{\tilde{\alpha}_i(\mu)}=\frac{1}{\tilde{\alpha}_i(M_U)}+\frac{b''_i}{2\pi}\ln\frac{M_U}{\mu},\quad i=2L,3C, \tag{33}$$

since $\tilde{\alpha}_{2L}(M_U)=\tilde{\alpha}_{3C}(M_U)$. But Eqs. (31)–(33) imply

$$N=6, \tag{34}$$

proving that the metacolor gauge group is $SU(6)_M$ to achieve such one-loop unification from $\mu=\Lambda_M$ to M_U.

The matching conditions with $G_{fc}=G_{2213}$ at $\mu=\Lambda_M$ are

$$\alpha_2^{-1}(\Lambda_M)+\delta_2=\tilde{\alpha}_{2L}^{-1}(\Lambda_M)=\tilde{\alpha}_{2R}^{-1}(\Lambda_M),$$

$$\alpha_3^{-1}(\Lambda_M)+\delta_3=\tilde{\alpha}_3^{-1}(\Lambda_M),$$

$$\alpha_1^{-1}(\Lambda_M)+\delta_1=\frac{3}{5}\,\tilde{\alpha}_{2R}^{-1}(\Lambda_M)+\frac{2}{5}\,\tilde{\alpha}_{BL}^{-1}(\Lambda_M). \tag{35}$$

Combining the first and third equations in Eqs. (35) and using Eqs. (17), we have the following matching constraint:

$$\tilde{\alpha}_{BL}^{-1}(\Lambda_M)=\frac{5}{2}\,\delta_1-\frac{3}{2}\,\delta_2+42.5. \tag{36}$$

Approximate unification of gauge couplings at $M_U=10^{18}$ GeV with two sets of four fields is found to be possible when the $\Delta_L-\Delta_R$ mass difference is enhanced, but remains within an acceptable limit corresponding to $\rho=M_{\Delta_L}/M_{\Delta_R}=1.6$. The individual masses and values of coupling constants at $\mu=M_U=10^{18}$ GeV and $\mu=\Lambda_M=10^{11}$ GeV are found to be

$$M_{\Delta_L}=7.8\times 10^{11}\text{ GeV},\quad M_{\Delta_R}=4.7\times 10^{11}\text{ GeV},$$

$$M_\xi=5.7\times 10^{11}\text{ GeV},\quad M_\sigma=3.3\times 10^{12}\text{ GeV}, \tag{37a}$$

$$\tilde{g}_{2L}(M_U)=\tilde{g}_{2R}(M_U)=0.640,\quad \tilde{g}_{BL}(M_U)=0.616,$$

$$\tilde{g}_{3C}(M_U)=0.643\pm 0.007,\quad \tilde{g}_6(M_U)=0.636,$$

$$\tilde{g}_{2L}(\Lambda_M)=\tilde{g}_{2R}(\Lambda_M)=0.740,\quad \tilde{g}_{BL}(\Lambda_M)=0.508,$$

$$\tilde{g}_{3C}(\Lambda_M)=0.745\pm 0.007. \tag{37b}$$

It is to be noted that the masses of Δ_L and Δ_R are constrained by spontaneous breaking of the left-right discrete symmetry and the $SU(2)_R\times U(1)_{B-L}$ gauge symmetry in G_{2213}, but there are no such constraints on the masses of ξ and σ fields. In no case should the mass of any of the four fields be widely different from Λ_M. From such considerations the mass $M_\sigma=33\Lambda_M$ in Eq. (37a) may be near the maximally permitted value. However, if there are more than two sets of degenerate σ condensates in the model, its mass is likely to decrease. The evolution of gauge couplings from M_Z to M_U through Λ_M is presented in Fig. 2 where threshold effects at lower and intermediate scales are also exhibited.

C. $G_P=SU(2)_L\times U(1)_Y\times SU(3)_C\times SU(6)_M$

Corresponding to this symmetry, $i=1Y$, $2L$, $3C$, and 6 in Eq. (25), and the one-loop coefficients are

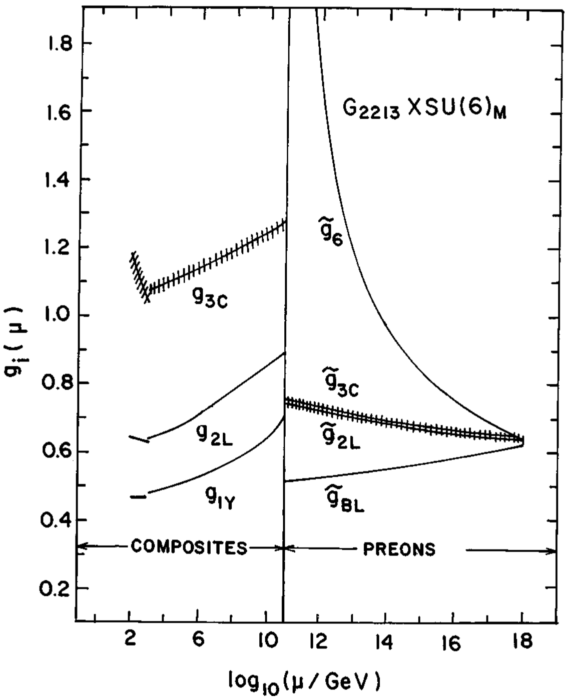

FIG. 2. Same as Fig. 1, but for the left-right symmetric preonic gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C\times SU(6)_M$ with two sets of four fields Δ_L, Δ_L, ξ, and σ, as described in the text, and for 60% of mass difference between Δ_L and Δ_R. The $SU(3)_C$ and $SU(2)_L$ couplings follow almost the same trajectory from $\mu=10^{11}$ to 10^{18} GeV because of one-loop unification in this range in the presence of $SU(6)_M$.

$$b''_{1Y}=\frac{7\times 6}{10}=\frac{21}{5},\quad b''_{2L}=-3\times 2+\frac{6}{2}=-3,$$

$$b''_{3C}=-3\times 3+6=-3,\quad b''_6=-3\times 6+6=-12. \quad (38)$$

It is interesting to note that

$$b''_{2L}=b''_{3C}=-3,$$

which implies unification of the preonic gauge couplings of $SU(2)_L$ and $SU(3)_C$ at the one-loop level for all values of μ from Λ_M to M_U as explained in Sec. V B, i.e.,

$$\tilde g_{2L}(\mu)=\tilde g_{3C}(\mu),\quad \mu=\Lambda_M \text{ to } M_U.$$

In order to achieve approximate unification of the gauge couplings at $M_U\simeq 10^{18}$ GeV, we need $\tilde\alpha_6^{-1}(M_U)\simeq 27-30$. Neglecting threshold effects at Λ_M gives

$$\tilde\alpha_{1Y}^{-1}(\Lambda_M)=\alpha_1^{-1}(\Lambda_M)=26.7,$$

$$\tilde\alpha_{2L}^{-1}(\Lambda_M)=\alpha_2^{-1}(\Lambda_M)=16.1,$$

$$\tilde\alpha_{3C}^{-1}(\Lambda_M)=\alpha_3^{-1}(\Lambda_M)=7.6\pm 0.6. \quad (39)$$

Since

$$\frac{21}{10\pi}\ln\frac{M_U}{\Lambda_M}=10.8,\quad \frac{3}{2\pi}\ln\frac{M_U}{\Lambda_M}=7.7, \quad (40)$$

Eqs. (25) and (38)–(40) have the predictions

$$\tilde\alpha_{1Y}^{-1}(M_U)=26.7-10.8=15.9,$$

$$\tilde\alpha_{2L}^{-1}(M_U)=16.1-7.7=23.8,$$

$$\tilde\alpha_{3C}^{-1}(M_U)=7.6+7.7\pm 0.6=15.3\pm 0.6. \quad (41)$$

Thus, starting from the CERN-LEP data at $\mu=M_Z$, including SUSY threshold effects, but ignoring the intermediate-scale-threshold corrections at $\mu=\Lambda_M$, there is no possibility of the unification of gauge couplings at the preonic level with $G_{fc}=G_{213}$. When attempt is made to unify the gauge couplings including intermediate-scale-threshold effects, we note from Eqs. (41) that the corrections on each of $\alpha_1^{-1}(\Lambda_M)$ and $\alpha_3^{-1}(\Lambda_M)$ must be nearly 2 times as large as that on $\alpha_2^{-1}(\Lambda_M)$. Including threshold effects, the matching conditions at Λ_M are

$$\alpha_i^{-1}(\Lambda_M)+\delta_i=\tilde\alpha_i^{-1}(\Lambda_M),\quad i=1Y,2L,3C. \quad (42)$$

We have observed that a good approximate unification of gauge couplings is possible with two sets of four fields if the $\Delta_L-\Delta_R$ mass difference is enhanced to correspond to the ratio $M_{\Delta_R}/M_{\Delta_L}=3.8$ for the following values of the individual masses:

$$M_{\Delta_L}=1.3\times 10^{11}\ \text{GeV},\quad M_{\Delta_R}=5\times 10^{11}\ \text{GeV},$$

$$M_\xi=3.4\times 10^{11}\ \text{GeV},\quad M_\sigma=5.7\times 10^{11}\ \text{GeV}. \quad (43)$$

The values of the couplings at M_U and Λ_M are

$$\tilde g_{1Y}(M_U)=0.633,\quad \tilde g_{2L}(M_U)=0.633,$$

$$\tilde g_{3C}(M_U)=0.655\pm 0.007,\quad \tilde g_6(M_U)=0.633,$$

$$\tilde g_{1Y}(\Lambda_M)=0.546,\quad \tilde g_{2L}(\Lambda_M)=0.735,$$

$$\tilde g_{3C}(\Lambda_M)=0.758\pm 0.007. \quad (44)$$

The evolution of gauge couplings from M_Z to M_U is shown in Fig. 3 where the approximate unification at M_U and the one-loop unification of $\tilde g_{2L}(\mu)$ and $\tilde g_{3C}(\mu)$ for $\mu=\Lambda_M$ to M_U are clearly exhibited. A nearly 70% difference between the $SU(3)_C$ gauge couplings of composites and preons compensated by threshold effects at Λ_M is found to exist in this model. The corresponding differences between the $SU(2)_L$ and $U(1)_Y$ gauge couplings are noted to be nearly 20% and 27%, respectively.

D. $G_P=SU(2)_L\times U(1)_R\times U(1)_{B-L}\times SU(3)_{3C}\times SU(6)_M$

In this case $i=1R$, BL, $2L$, $3C$, and 6 and the one-loop coefficients are

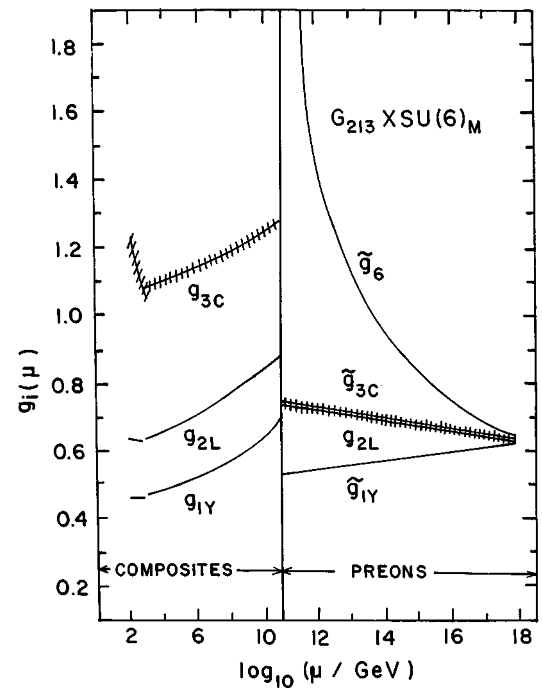

FIG. 3. Same as Fig. 2, but for the preonic gauge symmetry $SU(2)_L\times U(1)_Y\times SU(3)_C\times SU(6)_M$ and $M_{\Delta_R}/M_{\Delta_L}=3.8$ and two sets of four fields.

$$b''_{1R}=\frac{6}{2}=3,\quad b''_{BL}=6,$$

$$b''_{2L}=-3\times2+\frac{6}{2}=-3,\quad b''_{3C}=-3\times3+6=-3,$$

$$b''_6=-3\times6+6=-12. \tag{45}$$

As in the cases of $G_{fc}=G_{213}$ and G_{2213}, we find $b''_{2L}=b''_{3C}=-3$ in Eqs. (45), signifying one-loop unification of preonic gauge couplings of $SU(2)_L$ and $SU(3)_C$ over the mass range $\mu=\Lambda_M$ to M_U. The matching conditions for gauge couplings at Λ_M are

$$\frac{1}{\alpha_1(\Lambda_M)}+\delta_1=\frac{3}{5}\,\frac{1}{\tilde{\alpha}_{1R}(\Lambda_M)}+\frac{2}{5}\,\frac{1}{\tilde{\alpha}_{BL}(\Lambda_M)}, \tag{46a}$$

$$\frac{1}{\alpha_2(\Lambda_M)}+\delta_2=\frac{1}{\tilde{\alpha}_{2L}(\Lambda_M)}, \tag{46b}$$

$$\frac{1}{\alpha_3(\Lambda_M)}+\delta_3=\frac{1}{\tilde{\alpha}_{3C}(\Lambda_M)}. \tag{46c}$$

It is to be noted that one of the gauge couplings on the RHS of Eq. (46a), namely, $\tilde{\alpha}_{1R}(\Lambda_M)$ or $\tilde{\alpha}_{BL}(\Lambda_M)$, appears to remain undetermined. But in unified theories, once any of the coupling constants is known at M_U, the unification constraint gives other gauge couplings at that scale:

$$\tilde{\alpha}_{2L}(M_U)=\tilde{\alpha}_{1R}(M_U)=\tilde{\alpha}_{BL}(M_U)=\tilde{\alpha}_{3C}(M_U).$$

The knowledge of RGEs then determines the values of hitherto unknown couplings at lower scales $\mu<M_U$. With two sets of four fields, we obtain $\delta_1=16.1$, $\delta_2=7.7$, and $\delta_3=15.1$ and all the four gauge couplings close to one another while satisfying approximate one-loop unification, $g_2(\mu)=g_3(\mu)$, for all μ from M_U to Λ_M. The masses of the four fields are

$$M_{\Delta_L}=10^{11}\ \mathrm{GeV},\quad M_{\Delta_R}=4.4\times10^{11}\ \mathrm{GeV},$$

$$M_\xi=5\times10^{11}\ \mathrm{GeV},\quad M_\sigma=7.3\times10^{11}\ \mathrm{GeV}. \tag{47a}$$

The gauge couplings at M_U and Λ_M are computed as

$$\tilde{g}_{1R}(M_U)=\tilde{g}_{BL}(M_U)=0.630,\quad \tilde{g}_{2L}(M_U)=0.631,$$

$$\tilde{g}_{3C}(M_U)=0.642\pm0.007,\quad \tilde{g}_6(M_U)=0.641,$$

$$\tilde{g}_{1R}(\Lambda_M)=0.563,\quad \tilde{g}_{BL}(\Lambda_M)=0.515,$$

$$\tilde{g}_{2L}(\Lambda_M)=0.726,\quad \tilde{g}_{3C}(\Lambda_M)=0.743\pm0.01. \tag{47b}$$

Apart from requiring $\rho^{-1}=M_{\Delta_R}/M_{\Delta_L}=4.4$, the model also needs about 70% threshold corrections for the $SU(3)_C$ coupling and nearly 20% for the $SU(2)_L$ coupling of composite fields that are introduced by these masses.

E. $G_P=SU(2)_L\times SU(2)_R\times SU(4)^C_{L+R}\times SU(6)_M$

In this case the model possesses left-right discrete symmetry with $\tilde{g}_{2L}(\mu)=\tilde{g}_{2R}(\mu)$ for $\mu=\Lambda_M$ to M_U. The one-loop coefficients are $b''_{2L}=b''_{2R}=-3$, $b''_{4C}=-6$, and $b''_6=-12$. The coupling constants at Λ_M are matched using

$$\alpha_2^{-1}(\Lambda_M)+\delta_2=\tilde{\alpha}^{-1}_{2L}(\Lambda_M)=\tilde{\alpha}^{-1}_{2R}(\Lambda_M),$$

$$\alpha_3^{-1}(\Lambda_M)+\delta_3=\tilde{\alpha}^{-1}_{4C}(\Lambda_M),$$

$$\alpha_1^{-1}(\Lambda_M)+\delta_1=\frac{3}{5}\,\tilde{\alpha}^{-1}_{2R}(\Lambda_M)+\frac{2}{5}\,\tilde{\alpha}^{-1}_{4C}(\Lambda_M).$$

We have noted that it is impossible to achieve even a roughly approximate unification of gauge couplings with the above matching conditions unless the number of Δ_L, Δ_R, ξ, and σ fields is unusually large and their masses are widely different from Λ_M. Thus the flavor-color symmetric gauge group $G_{fc}=G_{224}$ is unrealistic.

VI. PROSPECTS OF SU(4) METACOLOR

In this section, assuming the metacolor gauge symmetry to be $SU(4)_M$, we explore possible forms of flavor-color gauge symmetry G_{fc} which could unify the relevant gauge couplings at M_U or near the Planck scale. We follow strategies similar to those explained in Sec. V.

A. $G_P=SU(2)_L\times U(1)_R\times U(1)_{B-L}\times SU(3)_C\times SU(4)_M$

With $G_{fc}=G_{2113}$ and $G_M=SU(4)_M$, the one-loop coefficients in the RGEs of Eq. (25) are $b''_{1R}=2$, $b''_{BL}=4$, $b''_{2L}=-4$, $b''_{3C}=-5$, and $b''_4=-6$. The matching conditions at

TABLE VII. Values of gauge couplings at different mass scales obtained using two sets of relevant Higgs superfields for the preonic gauge symmetry $G_P = SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \times SU(3)_C \times SU(4)_M$ with $M_{\Delta_L} = 5.3\times 10^{10}$ GeV, $M_{\Delta_R} = 6.4\times 10^{10}$ GeV, $M_\sigma = 7.3\times 10^{11}$ GeV, and $\rho^{-1} = M_{\Delta_R}/M_{\Delta_L} = 1.2$.

Mass scale (μ) (GeV)	$\tilde{g}_{2L}(\mu)$	$\tilde{g}_{3C}(\mu)$	$\tilde{g}_{1R}(\mu)$	$\tilde{g}_{BL}(\mu)$	$\tilde{g}_4(\mu)$
10^{19}	0.731	0.753 ± 0.013	0.720	0.732	0.764
10^{18}	0.755	0.792 ± 0.013	0.710	0.710	0.806
10^{11}	1.034	1.32 ± 0.05	0.647	0.598	1.80

$\mu = \Lambda_M$ are given by Eqs. (46a)–(46c). Although one of the gauge couplings, $\tilde{\alpha}_{1R}(\Lambda_M)$ or $\tilde{\alpha}_{BL}(\Lambda_M)$, is not determined by the matching conditions, this does not pose a problem in studying unification as explained in Sec. V D. For the sake of simplicity we use $\tilde{\alpha}_{1R}(M_U) = \tilde{\alpha}_{BL}(M_U)$ at $M_U = 10^{18}$ GeV. Unlike the case of $SU(6)_M$, where approximate unification was impossible under a small mass difference of 20% between M_{Δ_L} and M_{Δ_R}, we find that with $SU(4)_M$ the gauge group achieves a good approximate unification with gaps between the gauge couplings closing in gradually as we approach $\mu = M_{\rm Pl}$. The values of masses of the two sets of four fields, needed for approximate unification, are

$$M_{\Delta_L} = 5.37\times 10^{10}\ \text{GeV}, \quad M_{\Delta_R} = 6.44\times 10^{10}\ \text{GeV},$$

$$M_\sigma = 7.37\times 10^{11}\ \text{GeV}, \quad M_\xi = 10^{11}\ \text{GeV}, \tag{48}$$

where $M_{\Delta_R}/M_{\Delta_L} = 1.2$. In Table VII we present values of the gauge couplings at three different mass scales $\mu = 10^{11}$, 10^{18}, and 10^{19} GeV. The evolution of the gauge couplings of the effective gauge theories for preons, quarks, and leptons are presented in Fig. 4, which exhibits a clear tendency of the preonic gauge couplings to converge near $\mu = M_{\rm Pl}$. The remaining small differences among the couplings at $M_{\rm Pl}$ are expected to be filled up by gravitational corrections. One remarkable feature of this model is that the difference between the $SU(3)_C$ couplings of composite fields and preons is negligible, whereas that between the $SU(2)_L$ couplings is only 14%.

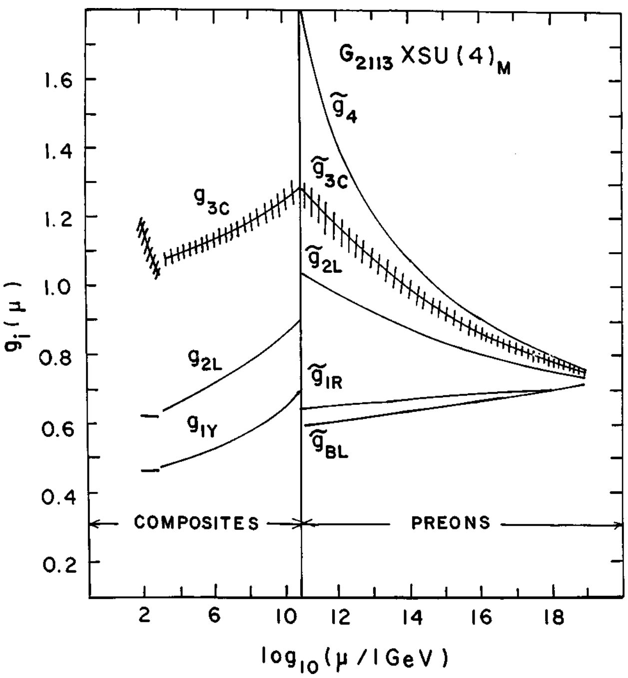

FIG. 4. Same as Fig. 1, but for the preonic gauge symmetry $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \times SU(4)_M$ with two sets of three fields Δ_L, Δ_R, and σ and $M_{\Delta_R}/M_{\Delta_L} = 1.2$.

B. $G_P = SU(2)_L \times U(1)_Y \times SU(3)_C \times SU(4)_M$

In the notation of Eq. (25), the one-loop coefficients are $b''_{1Y} = 14/5$, $b''_{2L} = -4$, $b''_{3C} = -5$, and $b''_4 = -6$. The matching conditions are given by Eq. (42). Restricting the difference between M_{Δ_L} and M_{Δ_R} to at most 20%, we find that approximate unification of gauge couplings at $\mu = 10^{18}$–10^{19} GeV is impossible. In Table VIII we present values of the gauge couplings $\mu = 10^{19}$, 10^{18}, and 10^{11} GeV. Rather larger differences between the gauge couplings are found to be contradicting the idea of unification. However, we note that the coupling constants can unify at $M_U = 10^{18}$ GeV only if the $\Delta_L - \Delta_R$ mass difference is allowed to be larger with $\rho^{-1} = M_{\Delta_R}/M_{\Delta_L} \cong 2.9$ corresponding to the following values of individual masses:

$$M_{\Delta_L} = 1.7\times 10^{10}\ \text{GeV}, \quad M_{\Delta_R} = 5.1\times 10^{10}\ \text{GeV},$$

$$M_\xi = 1.96\times 10^{11}\ \text{GeV}, \quad M_\sigma = 7.37\times 10^{11}\ \text{GeV}. \tag{49}$$

TABLE VIII. Gauge couplings at different mass scales but for $G_P = SU(2)_L \times U(1)_Y \times SU(3)_C \times SU(4)_M$. Here $M_\sigma = 7.3\times 10^{11}$ GeV throughout. For case (a), $M_{\Delta_L} = 1.3\times 10^{10}$ GeV, $M_{\Delta_R} = 1.6\times 10^{10}$ GeV, $M_{\Delta_R}/M_{\Delta_L} = 1.2$, and $M_\xi = 4\times 10^{11}$ GeV, but $M_{\Delta_L} = 1.7\times 10^{10}$ GeV, $M_{\Delta_R} = 5.1\times 10^{10}$ GeV, $M_{\Delta_R}/M_{\Delta_L} = 2.9$, and $M_\xi = 1.96\times 10^{11}$ GeV for case (b).

	Mass scale (μ) (GeV)	$\tilde{g}_1(\mu)$	$\tilde{g}_{2L}(\mu)$	$\tilde{g}_{3C}(\mu)$	$\tilde{g}_4(\mu)$
(a)	10^{18}	0.872 ± 0.005	0.798 ± 0.008	0.808 ± 0.06	0.859
	10^{11}	0.707 ± 0.004	1.149 ± 0.01	1.4 ± 0.07	2.5
(b)	10^{18}	0.802 ± 0.004	0.806 ± 0.007	0.784 ± 0.015	0.806
	10^{11}	0.686 ± 0.002	1.175 ± 0.01	1.287 ± 0.015	1.80

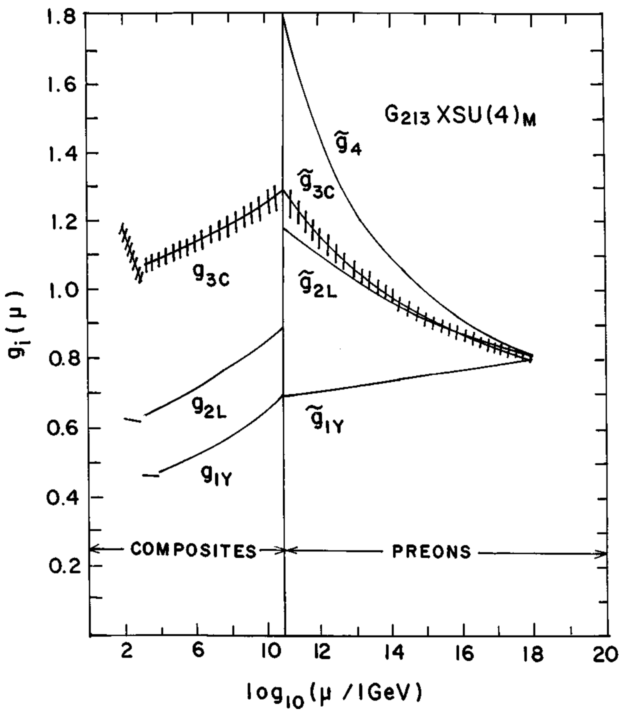

FIG. 5. Same as Fig. 3, but for the preonic gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C \times SU(4)_M$ and $M_{\Delta_R}/M_{\Delta_L}=2.9$.

Such a unification of gauge couplings and their evolution down to the Z mass are presented in Fig. 5. A very attractive feature of this model is that it needs an almost negligible difference between $\tilde{g}_{3C}(\Lambda_M)$ and $g_{3C}(\Lambda_M)$ and also between $\tilde{g}_{1Y}(\Lambda_M)$ and $g_{1Y}(\Lambda_M)$. The model is found to require a nearly 25% threshold correction on $SU(2)_L$ coupling of composites which is provided by the masses of two sets of four fields given in Eqs. (49).

C. Difficulties with other flavor-color symmetries

The flavor-color groups investigated in Secs. VI A–VI B are the most successful ones in the presence of $SU(4)_M$. The difficulties faced with other symmetries are summarized as mentioned here. For $G_P = G_{214} \times SU(4)_M$, $b''_{2L}=-4$, $b''_{1R}=2$, $b''_{4C}=-8$, and $b''_4=-6$. Although the masses of two sets of all four fields needed for unification near M_{Pl} are reasonable, with $M_{\Delta_L}/M_{\Delta_R}=1.2$, we find

$$\tilde{g}_{4C}(\mu) < \tilde{g}_4(\mu), \quad \mu = 10^{11}\text{–}10^{14}\ \mathrm{GeV},$$

showing that $\tilde{g}_4(\mu)$ is no longer the highest coupling near $\mu = \Lambda_M$ responsible for binding the preons. This is against the basic assumption of the model. For $G_P = G_{2213} \times SU(4)_M$, $b''_{BL}=4$, $b''_{2L}=b''_{2R}=-4$, $b''_{3C}=-5$, and $b''_4=-6$. With two sets of four fields, the masses of Δ_L and Δ_R needed for approximate unification are nearly two orders lighter and those of ξ and σ are one order heavier than Λ_M. For $G_P = G_{224} \times SU(4)_M$, either the number of some of the four types of fields is unusually large or some of the masses are five to six orders different from Λ_M. Because of such undesirable features, $G_{fc}=G_{214}$, G_{2213}, or G_{224} are unacceptable in the presence of $SU(4)_M$.

VII. SUMMARY AND CONCLUSION

We have used the CERN-LEP measurements at M_Z to study unity of forces and preonic gauge symmetries of the type $G_P = G_{fc} \times G_M$ in the scale-unifying preon model [12], which serves to provide a unified origin of the diverse mass scales and an explanation of family replication. Threshold effects form an important and essential part of gauge-coupling renormalization. Neglecting these effects has led to $G_P = G_{214} \times SU(5)_M$ as the only successful gauge symmetry of the preonic effective Lagrangian [15]. In this analysis, threshold effects are found to play a crucial role in determining the unification of forces near the Planck scale and, consequently, the gauge symmetry G_P with new possibilities for G_{fc} and $G_M = SU(6)_M$ or $SU(4)_M$.

With $SU(6)_M$ as the metacolor gauge group, the most attractive possibility of flavor-color symmetry is found to be $G_{fc}=G_{214}$ for which a good approximate unification of gauge couplings occurs at $\mu = M_{\mathrm{Pl}} = 10^{19}$ GeV with only a 10%–20% mass difference between $\Delta_L(3,1,10^C)$ and $\Delta_R(1,3,10^{*C})$ and the model needs just the minimal set of fields, $\Delta_L + \Delta_R$ and $\bar{\Delta}_L + \bar{\Delta}_R$, which are essential from considerations of left-right symmetry, preservation of SUSY down to the TeV scale, and spontaneous symmetry breaking of G_{214} to the standard model gauge group at Λ_M.

For the next attractive possibility with $SU(6)_M$ corresponding to the left-right symmetric gauge group $G_{fc} = G_{2213}$, two sets of all four fields Δ_L, Δ_R, ξ, and σ are needed and an approximate unification of gauge couplings is possible for acceptable value of the mass ratio $\rho = M_{\Delta_L}/M_{\Delta_R} = 1.6$ and $M_\xi = 5.7 \times 10^{11}$ GeV provided $M_\sigma = 3.3 \times 10^{12}$ GeV. Unification of gauge couplings is also observed with the standard model gauge group $G_{fc}=G_{213}$; similar threshold effects with two sets of four fields provided the mass ratio $M_{\Delta_R}/M_{\Delta_L}=3.8$, and the individual masses of these fields are between 1.3×10^{11} and 5.7×10^{11} GeV.

With $SU(4)_M$ as the metacolor gauge symmetry, two of the flavor-color gauge symmetries, G_{213} and G_{2113}, appear to be quite successful in achieving good approximate unification of the relevant gauge couplings at $M_U = 10^{18}$ and 10^{19} GeV, respectively. For $G_{fc}=G_{2113}$, the model needs the $\Delta_L - \Delta_R$ mass difference within 20% and two sets of three fields with reasonable values of masses near Λ_M. With the standard model gauge group $G_{fc}=G_{213}$ and $G_M = SU(4)_M$, the $\Delta_L - \Delta_R$ mass ratio needed is found to be such that $M_{\Delta_R}/M_{\Delta_L}=3$ and the other masses are $M_\xi = 2 \times 10^{11}$ GeV and $M_\sigma = 7.4 \times 10^{11}$ GeV. In this case two sets of all four fields are needed.

All heavy and superheavy masses used in this paper for threshold effects refer to bare masses. They are devoid of wave-function renormalization effects, which have been shown to be cancelled out by two-loop effects [20]. We assure that the bare masses are enough to produce threshold effects needed for new gauge symmetries. The cancellation observed in Ref. [20] does not affect the results and conclusions of this analysis.

One of the most challenging problems is to derive the preonic model with one of the choices for the metacolor and flavor-color gauge symmetry, mentioned above, from a string theory. Also one of the major issues is to address some of the dynamical assumptions of the model as regards the preferred directions of symmetry breaking and the saturation of the composite spectrum, mentioned in the Introduction [14,15]. In the absence of a derivation of the model from a deeper theory, apart from a number of unproven assumptions, the possible presence of more than one flavor-color symmetry group above $\mu=\Lambda_M$ has an arbitrariness similar to SUSY SO(10) with different possibilities for intermediate gauge symmetries. In spite of present theoretical limitations, the preonic approach seems promising because it is most economical and explains certain basic issues [12–15], by utilizing primarily symmetries of the underlying theory and general results such as the Witten index theorem, rather than detailed dynamics. A crucial test of the model hinges on the detection of vectorial quarks and leptons with masses near 1–2 TeV. At present, there is no compelling evidence that quarks and leptons are composite as proposed in the model, although some possible signature has been investigated [16].

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