

Dynamical Wilson fermions and the problem of the chiral limit in compact lattice QED

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We study the approach to the chiral transition line $\kappa_c(\beta)$ in quenched and full compact lattice QED with Wilson fermions within the confinement phase especially in the pseudoscalar sector of the theory. We show that in the strong coupling limit ($\beta=0$) both the quenched and the full theory behave partial-conservation-of-axial-vector-current-like. However, at larger β in contrast with the quenched theory the full one exhibits a chiral transition most likely of first order, such that the pseudo-scalar mass has no zero-mass limit. [S0556-2821(98)06721-6]

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I. INTRODUCTION

Chiral symmetry as a major concept in continuum quantum field theory has remained a problematic topic in lattice gauge theories over the years. It is well known that for Wilson fermions chiral symmetry is explicitly broken in QCD and QED on the lattice [1,2]. Hopefully, it can be recovered by fine tuning the parameters in the continuum limit. Then some line $\kappa_c(\beta)$ in the phase diagram is associated with the chiral limit of the theory. On the other hand, for nonvanishing lattice spacing only a partial restoration of chiral symmetry at $\kappa = \kappa_c(\beta)$ is possible with Wilson fermions [3,4]. How this mechanism of partial symmetry restoration should eventually be integrated into the general conception of spontaneously broken chiral symmetry is still an open question. One cannot exclude that the breakdown of some other symmetry group governs the dynamics of the transitions at $\kappa_c(\beta)$ (e.g., Ref. [5]). Viewed in this light the vanishing of the pseudo-scalar “pion” mass m_π for $\kappa \rightarrow \kappa_c(\beta)$ is a necessary but not sufficient condition for probing the chiral limit. Another point which sharpened the look on the chiral limit in QCD [6,7] is the discussion of “enhanced logs” due to quenching, demonstrating the role of dynamical fermions in chiral properties of the theory.

In this paper we are concerned with the behavior of fermionic observables close to $\kappa_c(\beta)$ in the confinement phase of compact QED with Wilson fermions. Many similarities of this phase with the QCD confinement make compact QED a valuable test ground for lattice QCD with Wilson fermions. We shall confront full QED with its valence fermion approximation.

In spite of the fact, that Monte Carlo studies of compact U(1) lattice gauge theory started a long time ago, the phase transition between the confinement and Coulomb phases remained an interesting subject and provided new room for

speculations (see, e.g., Refs. [8–10]). The corresponding phase transition line $\beta_c(m)$ in the (β, m) plane has thoroughly been studied for staggered fermions within the quenched approximation and for the full theory [11,12]. The latter investigations have shown the transition becoming stronger (first order) in the zero-mass limit $m \rightarrow 0$ taken along the line $\beta_c(m)$. However, the limit $m \rightarrow 0$ at fixed strong coupling within the confinement phase ($\beta < \beta_c$) has not been investigated in great detail even for staggered fermions. To our knowledge, only the quenched case has been considered very recently [12], showing that the pseudoscalar mass m_π tends to zero in accordance with PCAC (partial conservation of axial vector current).

It is the analogon of the latter limit for Wilson fermions [$\kappa \rightarrow \kappa_c(\beta)$ at fixed $\beta < \beta_c$] which we are going to discuss in this paper. We shall see that the inclusion of the fermionic determinant can change the behavior of the theory drastically in this case.

The outline of the paper is as follows. In Sec. II we introduce the model and discuss its phase diagram. Section III will present the chiral limit in the confinement phase and discuss the effects of the fermionic determinant. The conclusions are drawn in Sec. IV.

II. MODEL DESCRIPTION AND PHASE DIAGRAM

The partition function of 4d compact QED reads as follows:

$$Z_{\text{QED}} = \int [dU][d\bar{\psi}d\psi] e^{-S_W(U, \bar{\psi}, \psi)}, \quad (1)$$

where $S_W(U, \bar{\psi}, \psi)$ denotes the standard Wilson lattice action

$$S_W = S_G(U) + S_F(U, \bar{\psi}, \psi) \quad (2)$$

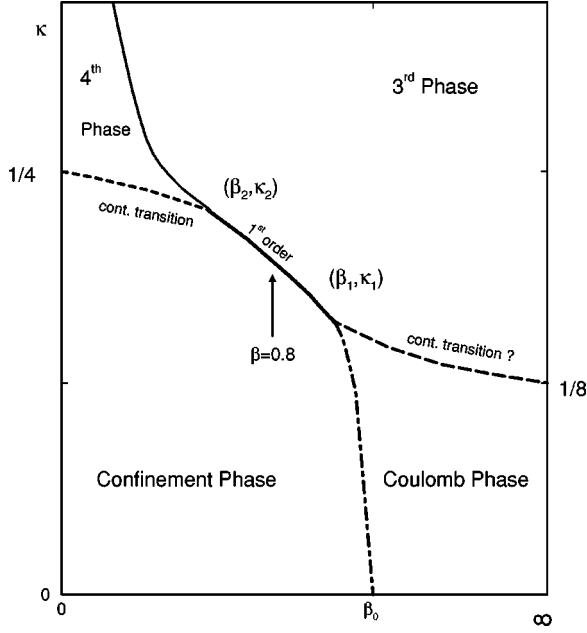


FIG. 1. The phase diagram of the compact lattice QED with Wilson fermions.

consisting of the plaquette action

$$S_G(U) = \beta \cdot \sum_{x;\mu>\nu} (1 - \cos \theta_{x;\mu\nu}), \quad (3)$$

and the fermionic part $S_F(U, \bar{\psi}, \psi)$

$$S_F = \sum_{f=1}^2 \sum_{x,y} \bar{\psi}_x^f \mathcal{M}_{xy} \psi_y^f, \quad (4)$$

$$\mathcal{M}_{xy} \equiv \hat{1} - \kappa \cdot [\delta_{y,x+\hat{\mu}} \cdot (\hat{1} - \gamma_\mu) \cdot U_{x\mu} + \delta_{y,x-\hat{\mu}} \cdot (\hat{1} + \gamma_\mu) \cdot U_{x-\hat{\mu},\mu}^\dagger],$$

with $\beta = 1/g_{\text{bare}}^2$, and $U_{x\mu} = \exp(i\theta_{x\mu})$, $\theta_{x\mu} \in (-\pi, \pi]$ represent the link fields. The plaquette angles $\theta_{x;\mu\nu}$ in Eq. (3) are given by $\theta_{x;\mu\nu} = \theta_{x;\mu} + \theta_{x+\hat{\mu};\nu} - \theta_{x+\hat{\nu};\mu} - \theta_{x;\nu}$. In the fermionic part of the action \mathcal{M}_{xy} denotes Wilson's fermionic matrix with the hopping parameter κ and the flavor-index f .

The phase diagram of this model in the presence of dynamical fermions has been studied in Ref. [13] and its conjectured form is shown qualitatively in Fig. 1. Within the region of values $0 \leq \kappa \leq 0.30$ the existence of four phases has been argued for. The transition region between the third and the fourth phase has not yet been studied in great detail. A thorough study of this area is left for the future. The line $\kappa_c(\beta)$ separates both the confinement phase [$0 \leq \beta < \beta_0(\kappa)$ with $\beta_0(0) \approx 1.01$] and the Coulomb phase ($\beta > \beta_0$) from two upper phases (which we called the third phase at weak and the fourth phase at strong coupling). In Ref. [13] we already have reported on a discontinuity of the scalar condensate $\langle \bar{\psi}\psi \rangle$ on the line between the points (β_1, κ_1) and (β_2, κ_2) (see Fig. 1) giving a first indication that the transition there might be of first order. But the pseudoscalar mass

was not studied at all, a task which will be completed in this paper. It is interesting to compare this phase diagram with phase diagrams of other lattice models with Wilson fermions, in particular with that of QCD.

The existence of a strong coupling upper phase (which corresponds to the fourth phase in Fig. 1) was numerically established for lattice QCD [5], the Nambu–Jona-Lasinio model [14] and also for the Schwinger model [15]. This phase was argued to show parity-flavor breaking, which might explain the pion mass to vanish for $\kappa \rightarrow \kappa_c$. But, the existence of a parity-flavor breaking phase may be a pure strong coupling artifact (see Ref. [16]). The hope that the cusp of this phase could be extended into the weak coupling region is based on an analogy with the Gross-Neveu model with $N_{\text{color}} = \infty$ [5]. However, $N_f = 2$ QCD on a symmetric lattice does not exhibit such a phase for $\beta = 6/g^2 \geq 5.0$ [17] and the temperature dependence of the tip position of the cusp of this phase turns out to be rather weak, such that the cusp seems to remain in the strong coupling region [18]. Whether elongated lattices can finally ‘‘push’’ the cusp into the weak coupling region [19], remains an open question.

From this reasoning it is interesting to examine the mechanism of chiral symmetry breaking also in four-dimensional (4D) compact lattice QED with dynamical Wilson fermions taken into account. It looks as though the mere existence of the strong coupling upper phase is model independent.

In this paper we want to explore the behavior of the theory near to the chiral transition more thoroughly. We are going to study the κ dependence of the pseudoscalar observables, in particular the pion norm Π and the mass m_π of the pseudoscalar particle as given in the following.

The ‘‘pion norm’’

$$\langle \Pi \rangle = \frac{1}{4V} \cdot \langle \text{Tr}(\mathcal{M}^{-1} \gamma_5 \mathcal{M}^{-1} \gamma_5) \rangle_G, \quad (5)$$

is a good indicator for small eigenvalues of the fermionic matrix. The mass of the pseudoscalar particle m_π is extracted from the nonsinglet pseudoscalar zero-momentum correlator

$$\Gamma(\tau) = -\frac{1}{N_s^6} \cdot \sum_{x,y} \langle \bar{\psi} \gamma_5 \psi(\tau, \vec{x}) \cdot \bar{\psi} \gamma_5 \psi(0, \vec{y}) \rangle$$

$$\equiv \frac{1}{N_s^6} \cdot \sum_{x,y} \langle \text{Sp}(\mathcal{M}_{xy}^{-1} \gamma_5 \mathcal{M}_{yx}^{-1} \gamma_5) \rangle_G. \quad (6)$$

In Eqs. (5), (6) $\langle \rangle_G$ indicates averaging over gauge field configurations, and $V = N_\tau \cdot N_s^3$ is the number of sites. Sp means the trace with respect to the Dirac indices. Other observables such as $\langle \rho_{\text{mon}} \rangle$ —the density of DeGrand-Toussaint monopoles [20]—and the photon correlator have also been determined.

Formally we define the bare fermion mass parameter m_q by

$$m_q = \frac{1}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_c(\beta)} \right). \quad (7)$$

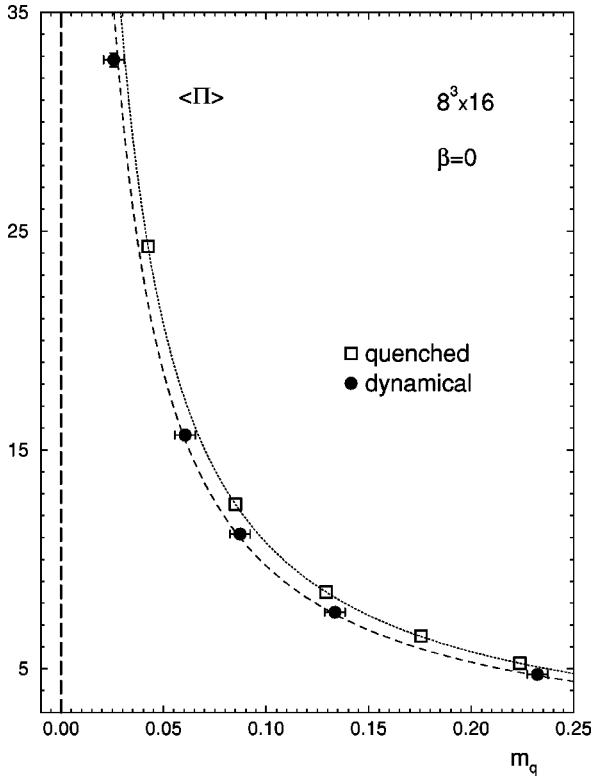


FIG. 2. The pion norm $\langle \Pi \rangle$ vs m_q at $\beta=0$. The curved lines correspond to Eq. (8) with C_0, C_1 given in Table I.

For the determination of the κ_c see below.

In this work we discuss data from $8^3 \times 16$ and $16^3 \times 32$ lattices for different values of β within the confinement phase. For the production of dynamical gauge field configurations we employed a Cray-T3D implementation of the hybrid Monte Carlo (HMC) method. A detailed presentation of algorithmic issues, such the tuning of the HMC parameters when approaching κ_c will be published elsewhere.

III. EFFECTS OF DYNAMICAL WILSON FERMIONS

First, we shall discuss the behavior of the bulk observable $\langle \Pi \rangle$ when approaching the line $\kappa_c(\beta)$ at fixed β within the range $0 \leq \beta < \beta_0$ (confinement phase). As in our previous work for representative β values we have chosen $\beta=0.8$ and the strong coupling limit $\beta=0$. In the latter limit the comparison with analytical results is possible (e.g., Refs. [3,21]).

Provided the pseudoscalar mass vanishes for $m_q \rightarrow 0$ it will yield the dominant contribution to the pion norm $\langle \Pi \rangle \sim 1/m_\pi^2$. In case of a PCAC-like relation between m_π and m_q the pion norm can be expressed in the following form:

$$\langle \Pi \rangle = \frac{C_0}{m_q} + C_1, \quad m_q \rightarrow 0, \quad (8)$$

where $C_0 > 0$ —up to a factor—is the subtracted chiral condensate [22] (see also Ref. [23]). $C_1 = \langle \Pi_m \rangle$ comes from the contribution of the massive modes.

In Fig. 2 data for the quenched and dynamical cases at $\beta=0$ are presented to view. The quenched approximation

TABLE I. Compilation of different parameters (see text) for the dynamical and quenched theories at $\beta=0$ and $\beta=0.8$ on an $8^3 \times 16$ lattice.

		κ_c	B	C_0	C_1
$\beta=0.0$	dynamical	0.2450(6)	4.87(5)	0.895(1)	0.88(1)
	quenched	0.2502(1)	4.91(4)	0.996(2)	0.80(1)
$\beta=0.8$	dynamical	0.1832(3)			
	quenched	0.2171(1)	3.42(3)	0.94(2)	0.72(1)

follows the m_q dependence of the full pion norm $\langle \Pi \rangle$ very well, even quantitatively. As $m_q \rightarrow 0+$ the singularity of $\langle \Pi \rangle$ at $\beta=0$ is well described in both cases by Eq. (8) with values of C_0, C_1 listed in Table I. Note that C_0, C_1 differ somewhat for the quenched and dynamical cases and could not be forced to coincide by shifting κ_c . The quenched and dynamical theories nevertheless exhibit the same functional dependence on m_q when $m_q \rightarrow 0$.

For $\kappa \geq \kappa_c$ the averages of fermionic bulk observables in the quenched approximation become poorly defined due to large fluctuations caused by “exceptional” configurations [23,24]. In the theory with dynamical fermions we also observe increasing fluctuations of, e.g., Π when κ is tuned towards κ_c from below. We were able to proceed to $\kappa = 0.242$ ($m_q \sim 0.0253$) at $\beta=0$, before being faced with serious problems with the acceptance rate of the HMC method.

The situation changes drastically when the gauge coupling β is increased. By examining time histories of Π and other observables at $\beta=0.8$ we observe the formation of metastable states in a “critical” region around κ_c in the presence of dynamical fermions. Figure 3(a) illustrates a clearly double peaked distribution of Π close to κ_c for the case of dynamical fermions.

As an example concerning the behavior of other observables, the dependence of $\langle \rho_{\text{mon}} \rangle$ on κ is depicted in Fig. 3(b). The evolution of $\langle \rho_{\text{mon}} \rangle$ resembles the situation of the confinement-deconfinement transition at β_0 for a sufficiently small fixed κ and varying β [13].

Measurements of the effective photon energy extracted from plaquette-plaquette correlators for nonzero momentum confirm that in the case of dynamical fermions the system undergoes a confinement-deconfinement transition at κ_c (see Ref. [13]). With increasing κ the effective energy of the photon rapidly decreases around κ_c and becomes well consistent with the lattice dispersion relation for a zero-mass photon.

We used further “order parameters” to determine κ_c and to make sure that there are no other transition points different from that within the investigated κ range. For example, the variance $\sigma^2(\Pi)$ which is a suitable parameter to locate the line $\kappa_c(\beta)$ within the Coulomb phase [13] peaks at the same κ_c .

In Fig. 4 as counterpart of Fig. 2 we confront the dependence of $\langle \Pi \rangle$ on m_q in the quenched approximation with the corresponding data when dynamical fermions are taken into account. The general features of the valence approximation at $\beta=0.8$ are the same as at $\beta=0$. $\langle \Pi \rangle$ has a singularity for

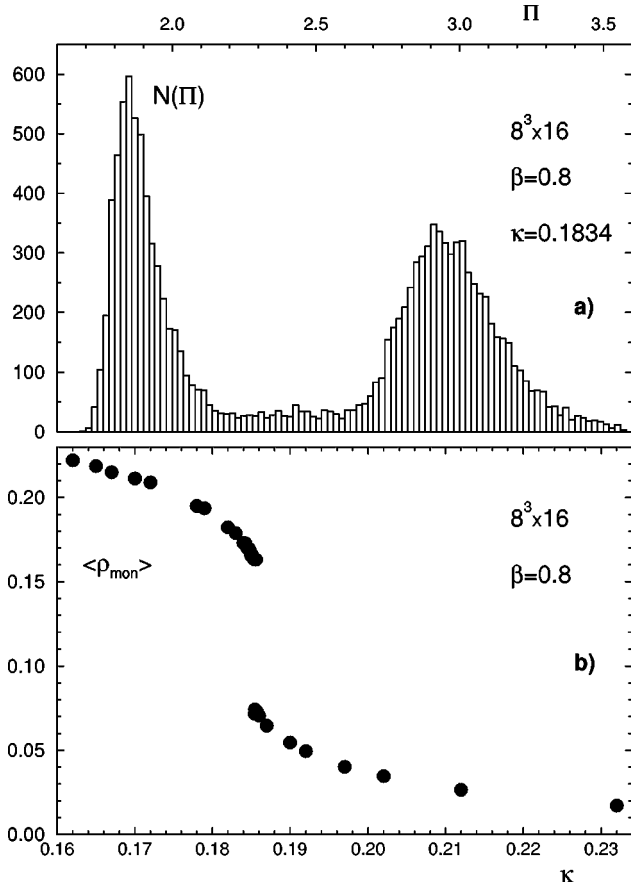


FIG. 3. The unnormalized distribution of Π at $\beta=0.8$ in the vicinity of κ_c (a) and $\langle\rho_{mon}\rangle$ in dependence of κ at the same value of β (b) both for the theory with dynamical fermions.

$m_q \rightarrow 0+$ described very well by Eq. (8) (dashed line in Fig. 4) with parameters C_0, C_1 given in Table I. However, the effect of the fermionic determinant is now seen as a qualitative change in the behavior of the pion norm, which does not behave as $\sim 1/m_q$ but rather exhibits a finite discontinuity accompanied by a metastable behavior [cf. Fig. 3(a)] around κ_c . As shown in the inset of this figure the discontinuity becomes even larger when the lattice size is increased, hardening that it will persist in the infinite volume limit. In the quenched theory averages of Π become statistically not well defined for $\kappa \geq \kappa_c$, as in the case of $\beta=0$, while in the dynamical case at $\beta=0.8$ there is no problem to go beyond κ_c (i.e., into the third phase [13]). The dependence of $\langle\Pi\rangle$ on κ (respectively, m_q) is not symmetric around κ_c .

To substantiate the emerging picture at $\beta=0$ and $\beta=0.8$ we will discuss the evolution of the pseudoscalar mass m_π when $\kappa \rightarrow \kappa_c$. We present in Fig. 5 the dependence of m_π^2 on κ for the full and quenched theories on an $8^3 \times 16$ lattice at $\beta=0$. The quenched data for κ very close to κ_c are obtained by an improved estimator of m_π [23] in order to increase the signal-to-noise ratio. Since the existence of a massless pseudoscalar particle is predicted by strong coupling arguments [3] we extrapolate our dynamical data for m_π^2 linearly to zero in order to determine $\kappa_c(\beta=0)$ as we have done in the quenched case [23]. In Table I we list the

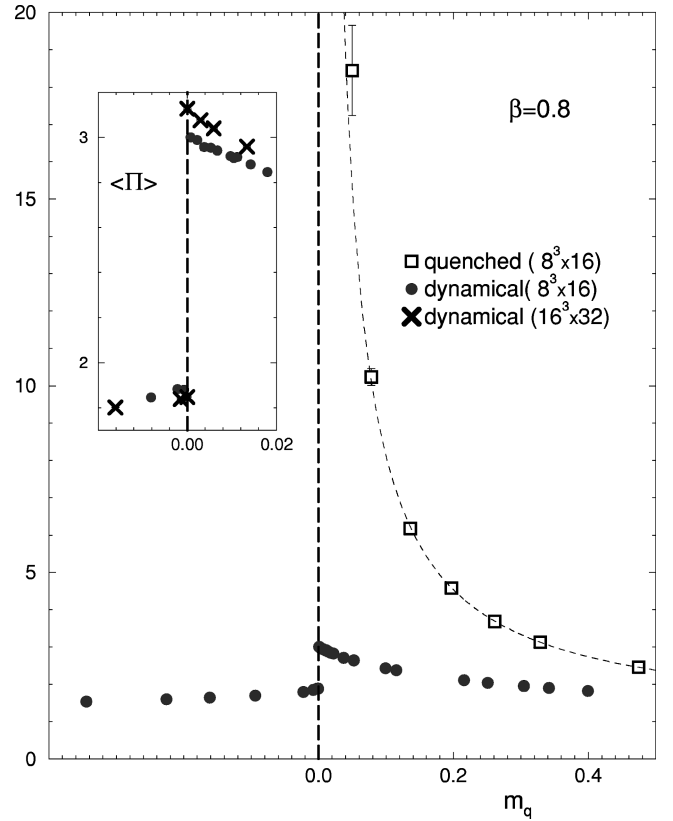


FIG. 4. Counterpart to Fig. 2 at $\beta=0.8$.

values of κ_c obtained for the dynamical and the quenched cases on an $8^3 \times 16$ lattice. The extrapolated values of κ_c for the quenched case are well consistent with the prediction at strong coupling [3]. In both, quenched and dynamical cases we observe the following dependence of m_π^2 on the hopping parameter when approaching κ_c from below

$$m_\pi^2 \sim \left(1 - \frac{\kappa}{\kappa_c}\right), \quad \kappa \leq \kappa_c, \quad (9)$$

which in this limit transforms into a PCAC-like relation between m_π^2 and the bare fermion mass m_q :

$$m_\pi^2 = B \cdot m_q, \quad m_q \rightarrow 0+. \quad (10)$$

The corresponding slopes B for the quenched and full theories coincide within the error bars (see Table I). Thus, Fig. 5 suggests a zero-mass pseudoscalar particle exists. However, this is a necessary but not a sufficient prerequisite for the definition of the chiral limit.

The corresponding behavior of m_π^2 at $\beta=0.8$ for quenched and dynamical fermions is plotted in Fig. 6. From the inset of Fig. 6 it can be seen, that as long as the quenched theory is considered the situation is fully compatible with Eq. (9). Thus Eq. (10) holds as in the case of $\beta=0$. Concerning dynamical fermions, again the situation at $\beta=0.8$ appears to be in sharp contrast to the $\beta=0$ and to the quenched cases, as could be expected from the properties of the pion norm. By approaching the ‘‘critical’’ value $\kappa_c(\beta)$ from below with dynamical fermions, the κ dependence of

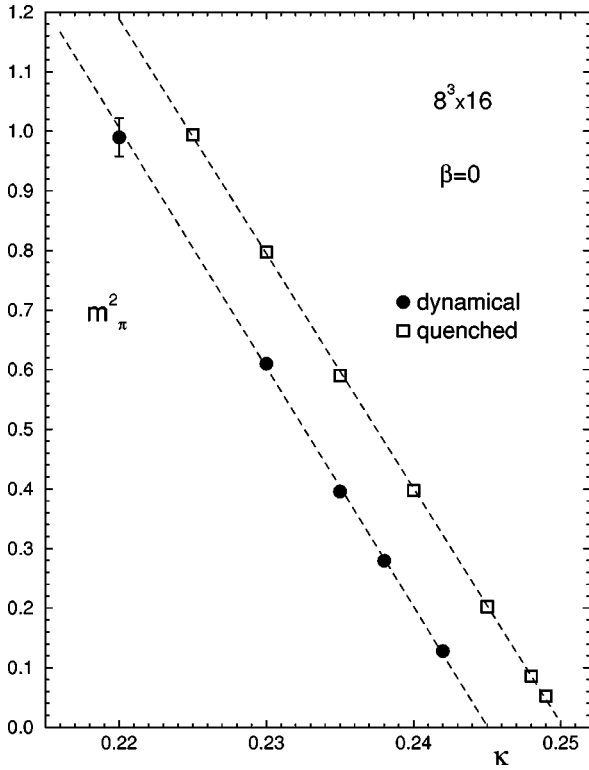


FIG. 5. The κ dependence of m_π^2 for the quenched and dynamical theories at $\beta=0$ on an $8^3 \times 16$ lattice. The broken lines represent linear fits.

m_π^2 is not linear anymore, i.e., is not compatible with Eq. (9). Moreover, close to $\kappa_c(\beta)$ m_π has a comparatively large finite minimal value, which would imply that a zero-mass pseudoscalar particle is not contained in the spectrum of the theory at this particular coupling. Increasing κ beyond κ_c the pseudoscalar mass starts to rise again. Note that in the vicinity of κ_c the dependence of the pseudoscalar mass on κ is different for $\kappa > \kappa_c$ and $\kappa < \kappa_c$, in accordance with the discussion of the pion norm before. As Fig. 6 shows, increasing the lattice size does not qualitatively change the behavior of m_π .

The situation at $\beta=0.6$ looks similar to that observed at $\beta=0.8$. However, in the full theory the minimum of the ‘‘pion’’ mass comes closer to zero. This comes not unexpected, since we are approaching the endpoint of the first order line at (β_2, κ_2) in Fig. 1.

IV. CONCLUSIONS AND DISCUSSION

We have studied the approach to $\kappa_c(\beta)$ at different β values within the confinement phase of the compact lattice QED with Wilson fermions comparing the full theory with its quenched approximation. We have shown the importance of vacuum polarization effects due to dynamical fermions in the context of the chiral limit.

In the strong coupling limit $\beta=0$ the main effect of dynamical fermions seems to be a renormalization of the ‘‘critical’’ value κ_c , $\kappa_c^{\text{dyn}} \neq \kappa_c^{\text{quen}}$. The functional dependence of the studied observables on κ (respectively, m_q) in the limit

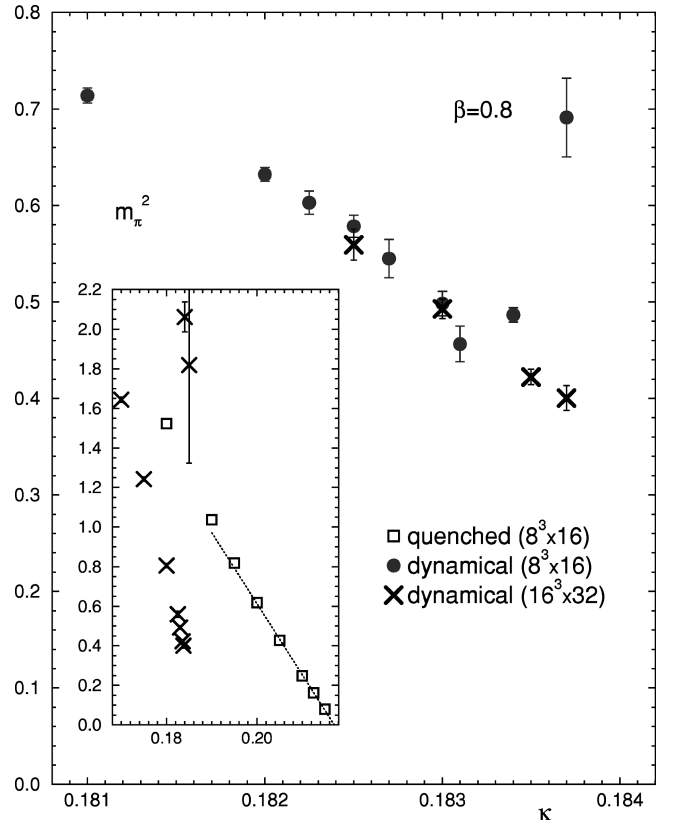


FIG. 6. The behavior of m_π^2 around κ_c at $\beta=0.8$. The κ scale of the smaller plot is condensed in order to permit a direct comparison between the dynamical and quenched data. The straight line corresponds to a linear fit of the quenched data.

$\kappa \rightarrow \kappa_c$ compared to the quenched approximation does not change. Our data suggest, that at $\beta=0$ the pseudoscalar particle becomes massless when $\kappa \rightarrow \kappa_c$.

At $\beta=0.8$ the presence of the dynamical (‘‘sea’’) fermions drastically changes the transition. There we have found a transition which cannot be associated with the zero-mass limit of a pseudoscalar particle anymore, in sharp contrast to the quenched case.

Naively, one would expect that the chiral limit could be established everywhere in the confinement phase when approaching the ‘‘critical’’ line $\kappa_c(\beta)$. This is not the case. At $\beta=0$ we cannot exclude that the upper phase at $\kappa > \kappa_c$ corresponds to the broken parity flavor, which explains the vanishing of the pion mass at κ_c .

It might be interesting to carry out an analogous investigation for staggered fermions. However, this does not mean that we *a priori* expect a universal behavior for the chiral limit of strong coupling lattice QED with different discretizations of the fermion fields.

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