Twist four longitudinal structure function in light-front QCD

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To resolve various outstanding issues associated with the twist four longitudinal structure function $F_L^{\tau=4}(x)$ we perform an analysis based on the BJL expansion for the forward virtual photon-hadron Compton scattering amplitude and equal (light-front) time current algebra. Using the Fock space expansion for states and operators, we evaluate the twist four longitudinal structure function for dressed quark and gluon targets in perturbation theory. With the help of a new sum rule which we have derived recently we show that the quadratic and logarithmic divergences generated in the bare theory are related to the corresponding mass shifts in old-fashioned light-front perturbation theory. We present numerical results for the F_2 and F_L structure functions for the meson in two-dimensional QCD in the one pair approximation. We discuss the relevance of our results for the problem of the partitioning of the hadron mass in QCD. [S0556-2821(98)02721-0]

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I. INTRODUCTION

An important problem in applying QCD to deep inelastic scattering is the existence of power corrections to scaling, more commonly known as higher twist effects. They are essential for making precision tests of QCD. From the early days of the establishment of QCD as the underlying theory of strong interactions, the importance of a proper understanding of power corrections was recognized [1]. Subsequently, leading $1/Q^2$ corrections to the unpolarized leading twist structure function F_2 and the longitudinal structure function F_L were analyzed by the operator product expansion (OPE) [2] and Feynman diagram [3] approaches. Later Qiu [4] gave an alternate method based on special propagators (utilizing unique features of light-cone coordinates) to simplify the analysis.

The power correction to F_L is especially interesting since the leading (twist two) contribution to F_L is perturbative in origin in contrast with the case of F_2. Thus the first, nonperturbative contributions to F_L occur at $1/Q^2$ order. The complexity of the problem of higher twist appears in the OPE analysis which utilizes a collinear basis, since at twist four there appears a proliferation of operator structures. In the Feynman diagram approach it has been shown, using a transverse basis, that contact could be made with light-front current algebra analysis with the result that the twist four part of F_L is given by the Fourier transform of the hadron matrix element of the minus component of the bilocal vector current. Since the minus component of the current involves the constrained fermion field, the relevant operator has explicit dependence on the interaction in contrast with the well-known result for the leading twist contribution to F_2 which involves the plus component of the bilocal current. Even after many years of investigation, an intuitive physical understanding of the interaction dependence in the structure of F_L has been elusive. We provide herein a physically intuitive picture.

Another important problem of current interest is the perturbative aspects of the twist four matrix element. Simple power counting indicates that in the bare theory the twist four matrix element will be afflicted with quadratic divergences [7]. Understanding the origin and the nature of these divergences will be quite helpful in finding procedures to remove them (the process of renormalization).

A third motivation to study the twist four part of F_L comes from the present status of deep inelastic scattering experiments. Measurements [5] of the ratio of the longitudinal to transverse cross section in unpolarized deep inelastic scattering show [6] that power corrections play an important role in nucleon structure experiments in the SLAC kinematic range. It is important to go beyond phenomenological parametrizations for a proper understanding of the nonperturbative nature of these corrections.

Light-front analysis of deep inelastic scattering provides an intuitive physical picture of various structure functions at the twist two level. Recently, the resolution of an ambiguity at the operator level and the parton interpretation of the transverse component of the bilocal current have been achieved in an approach based on light-front field theory [8]. The physical picture of the transverse polarized structure function [9] and a critical examination of the Wandura-Wilczek sum rule in perturbation theory [10] have also been provided in the same approach. Both nonperturbative and perturbative issues can be addressed in the same language in this formalism which uses the Fock space expansion for all the operators and multiparton wave functions for the state [11]. The approach also provides insights into various renor-

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malization issues associated with the different components of currents and the Hamiltonian.

In this work, we show that using the same framework one can resolve outstanding issues associated with the twist four contributions to the longitudinal structure function. A brief summary of some of our results is presented in Ref. [12]. In this work we extend our previous calculations and also present several new results. Our starting point is the Bjorken-Johnson-Low (BJL) expansion for the forward virtual photon-hadron Compton scattering amplitude. This leads us to the commutator of currents which we present in detail for arbitrary flavors in SU(3). Next we consider the specific case of electromagnetic currents and arrive at expressions for the twist two part of F_2 and twist four part of F_L in terms of specific flavor-dependent form factors. In the rest of the paper we consider the flavor singlet part of the structure functions. We identify the integral of $F_L(x)/x$ with the fermionic part of the light-front QCD Hamiltonian density. The consideration of mixing in the flavor singlet channel leads us to the definition of the twist four longitudinal gluon structure function and then we find a sum rule, free from radiative corrections.

The sum rule which the physical structure function has to satisfy involves the physical mass of the hadron which is a finite quantity. A theoretical evaluation of the sum rule which starts with the bare theory, on the other hand, will be afflicted with various divergences (see Sec. IV) depending on the regulator employed. In order to compare with the physical answer resulting from the measurement, we need to renormalize the result by adding counterterms. For the dressed parton target, for example, these counterterms are dictated by mass counterterms in light-front Hamiltonian perturbation theory. For a dressed gluon target, calculations in Sec. IV B show that quadratic divergences are generated and one does not automatically get the result expected for a massless target. The divergence generated is shown to be directly related to the gluon mass shift in old-fashioned perturbation theory. To a given order in perturbation theory, counterterms have to be added to the calculated structure function. The precise selection of counterterms is dictated entirely by the regularization and renormalization of the light-front QCD Hamiltonian. The choice of counterterms in the Hamiltonian, in turn, determines the counterterms to be added to the longitudinal structure function which results in a theoretical prediction of the physical longitudinal structure function. Recall that in Hamiltonian perturbation theory we cannot automatically generate a massless gluon by a clever choice of regulators. The point we emphasize is that the twist four longitudinal structure function is one-to-one related to the Hamiltonian density and that there is no arbitrary freedom in this relationship.

We also note that in the pre-QCD era, there were discussions about a possible $\delta(x)$ function contribution to the longitudinal structure function which may appear to invalidate the sum rule derived ignoring such subtleties. In two-dimensional QCD Burkardt has shown [13] that F_L/x^2 has a delta function contribution and he has discussed implications of this for the sum rule for F_L/x^2. Obviously, F_L/x will not be affected by such a singular contribution and we show explicitly in Sec. V that the sum rule is verified in two-dimensional QCD by virtue of the 't Hooft equation.

To gain an understanding of the nature of quadratic divergences, we evaluate the twist four longitudinal structure functions for quark and gluon targets each dressed through lowest order in perturbation theory. The sum rule allows us to relate these divergences to quark and gluon mass corrections in QCD in time-ordered light-front perturbation theory. We also verify the sum rule in a nonperturbative context in two-dimensional QCD. We also present numerical results for F_2 and F_L structure functions in this model using wave functions calculated in a variational approximation. Finally we discuss the relevance of our results for the problem of the partitioning of hadron masses in QCD.

The plan of this paper is as follows. In Sec. II we derive the expressions for the twist two structure function F_2 and the twist four longitudinal structure function F_L using the BJL expansion and equal-time (x^+) current algebra. The sum rule for F_L is given in Sec. III. In Sec. IV we evaluate F_L for quark and gluon targets dressed through lowest order in perturbation theory and explicitly verify the sum rule. The sum rule is verified explicitly in a nonperturbative context in two-dimensional QCD in Sec. V. In this section, to provide a qualitative picture, we also present numerical results for the F_2 and F_L structure functions in this model. In Sec. VI we discuss the issue of the breakup of hadron mass in QCD in the context of our sum rule. Discussion and conclusions are presented in Sec. VII and our notation and conventions are summarized in an appendix.

II. PRELIMINARIES

In this section we present the expressions for structure functions for arbitrary flavors in SU(3) which follow from the use of the Bjorken-Johnson-Low expansion and light-front current algebra. In terms of the flavor current $J_a^\mu(x) = \overline{\psi}(x)\gamma^\mu(\lambda_a/2)\psi(x)$, the hadron tensor relevant for deep inelastic scattering is given by

$$W_{ab}^{\mu\nu}=\frac{1}{4\pi}\int d^4\xi e^{iq\cdot\xi}\langle P|[J_a^\mu(\xi),J_b^\nu(0)]|P\rangle. \tag{2.1}$$

The forward virtual photon-hadron Compton scattering amplitude is given by

$$T_{ab}^{\mu\nu}=i\int d^4\xi e^{iq\cdot\xi}\langle P|T(J_a^\mu(\xi)J_b^\nu(0))|P\rangle. \tag{2.2}$$

We have

$$T_{ab}^{\mu\nu}(x,Q^2)=2\int_{-\infty}^{\infty}dq'^{+}\frac{W_{ab}^{\mu\nu}(x',Q^2)}{q'^{+}-q^{+}}. \tag{2.3}$$

Using the BJL expansion [14], we have

$$T_{ab}^{\mu\nu}=-\frac{1}{q^-}\int d\xi^- d^2\xi_\perp e^{iq\cdot\xi}\langle P|[J_a^\mu(\xi),J_b^\nu(0)]_{\xi^+=0}|P\rangle + \cdots, \tag{2.4}$$

where the ellipsis represents higher order terms in the expansion which we ignore in the following. In the limit of large q^-, from Eq. (A1), we have

$$W_{ab}^{+-}=\frac{1}{2}F_{L(ab)}+(P^{\perp})^2\frac{F_{2(ab)}}{\nu}+\frac{P^{\perp}\cdot q^{\perp}}{x\nu}F_{2(ab)}, \quad (2.5)$$

with $x=-q^2/2\nu$ and $\nu=P\cdot q$. On the other hand, from Eq. (2.4),

$$\lim_{q^-\to\infty} T_{ab}^{+-}=-\frac{1}{q^-}\int d\xi^- d^2\xi_\perp e^{iq\cdot\xi} \times\langle P|[J_a^+(\xi),J_b^-(0)]_{\xi^+=0}|P\rangle. \quad (2.6)$$

The components of the flavor current $J_a^\mu(x)$ obey the equal-x^+ canonical commutation relation [to be specific, we consider SU(3) of flavors]

$$\begin{aligned}[J_a^+(x),J_b^-(y)]_{x^+=y^+}=&2if_{abc}\overline{\psi}(x)\gamma^-\frac{\lambda_c}{2}\psi(x)\delta^2(x^\perp-y^\perp)\delta(x^--y^-)\\&-\frac{1}{2}\partial_x^+\{\epsilon(x^--y^-)[if_{abc}\mathcal{V}_c^-(x|y)+id_{abc}\overline{\mathcal{V}}_c^-(x|y)]\delta^2(x^\perp-y^\perp)\}\\&+\frac{1}{2}if_{abc}\epsilon(x^--y^-)\partial_x^i\{\delta^2(x^\perp-y^\perp)[\mathcal{V}_c^i(x|y)-\epsilon^{ij}\overline{\mathcal{A}}_c^j(x|y)]\}\\&+\frac{1}{2}id_{abc}\epsilon(x^--y^-)\partial_x^i\{\delta^2(x^\perp-y^\perp)[\overline{\mathcal{V}}_c^i(x|y)+\epsilon^{ij}\mathcal{A}_c^j(x|y)]\}. \end{aligned} \quad (2.7)$$

In deriving the above relations, use has been made of the relation

$$\lambda_a\lambda_b=if_{abc}\lambda_c+d_{abc}\lambda_c. \quad (2.8)$$

We have defined the bilocal currents as follows:

$$\mathcal{V}_c^\mu(x|y)=\frac{1}{2}\left[\overline{\psi}(x)\frac{\lambda_c}{2}\gamma^\mu\psi(y)+\overline{\psi}(y)\frac{\lambda_c}{2}\gamma^\mu\psi(x)\right],\quad \overline{\mathcal{V}}_c^\mu(x|y)=\frac{1}{2i}\left[\overline{\psi}(x)\frac{\lambda_c}{2}\gamma^\mu\psi(y)-\overline{\psi}(y)\frac{\lambda_c}{2}\gamma^\mu\psi(x)\right],$$

$$\mathcal{A}_c^\mu(x|y)=\frac{1}{2}\left[\overline{\psi}(x)\frac{\lambda_c}{2}\gamma^\mu\gamma^5\psi(y)+\overline{\psi}(y)\frac{\lambda_c}{2}\gamma^\mu\gamma^5\psi(x)\right],\quad \overline{\mathcal{A}}_c^\mu(x|y)=\frac{1}{2i}\left[\overline{\psi}(x)\frac{\lambda_c}{2}\gamma^\mu\gamma^5\psi(y)-\overline{\psi}(y)\frac{\lambda_c}{2}\gamma^\mu\gamma^5\psi(x)\right]. \quad (2.9)$$

Further, we introduce the bilocal form factors

$$\langle P|\mathcal{V}_c^\mu(\xi|0)|P\rangle=P^\mu V_c^1(\xi^2,P\cdot\xi)+\xi^\mu V_c^2(\xi^2,P\cdot\xi), \quad (2.10)$$

$$\langle P|\overline{\mathcal{V}}_c^\mu(\xi|0)|P\rangle=P^\mu \overline{V}_c^1(\xi^2,P\cdot\xi)+\xi^\mu \overline{V}_c^2(\xi^2,P\cdot\xi). \quad (2.11)$$

From Eqs. (2.6) and (2.7), we get

$$\begin{aligned}\lim_{q^-\to\infty} q^-T_{ab}^{+-}=&-2if_{abc}P^-\Gamma_c+\frac{q^+}{2}\int d\xi^- e^{(i/2)q^+\xi^-}\epsilon(\xi^-)[f_{abc}\langle P|\mathcal{V}_c^-(\xi|0)|P\rangle+d_{abc}\langle P|\overline{\mathcal{V}}_c^-(\xi|0)|P\rangle]\\&-\frac{q^i}{2}\int d\xi^- e^{(i/2)q^+\xi^-}\epsilon(\xi^-)[f_{abc}\langle P|\mathcal{V}_c^i(\xi|0)|P\rangle+d_{abc}\langle P|\overline{\mathcal{V}}_c^i(\xi|0)|P\rangle]. \end{aligned} \quad (2.12)$$

Note that matrix elements of $\mathcal{A}_c^\mu(x|y)$ do not contribute to unpolarized scattering. Using the dispersion relation given in Eq. (2.3), together with Eqs. (2.5) and (2.12) and comparing the coefficient of q^i on both sides, we get

$$\frac{F_{2(ab)}(x)}{x}=\frac{i}{4\pi}\int d\eta e^{-i\eta x}[f_{abc}V_c^1(\eta)+d_{abc}\overline{V}_c^1(\eta)]. \quad (2.13)$$

Comparing the coefficients of q^+ on both sides, we get

$$F_{L(ab)}(x)=\frac{1}{Q^2}\frac{i}{\pi}\frac{(q^+)^2}{P^+}\int d\eta e^{-i\eta x}[f_{abc}\langle P|\mathcal{V}_c^-(\xi|0)|P\rangle+d_{abc}\langle P|\overline{\mathcal{V}_c^-}(\xi|0)|P\rangle]$$
$$-\frac{(P^\perp)^2}{Q^2}\frac{i}{\pi P^+}x^2\int d\eta e^{-i\eta x}[f_{abc}\langle P|\mathcal{V}_c^+(\xi|0)|P\rangle+d_{abc}\langle P|\overline{\mathcal{V}_c^+}(\xi|0)|P\rangle]. \tag{2.14}$$

We have introduced $\eta=\frac{1}{2}P^+\xi^-$.

Note that our result for F_L differs from the one given in the literature [15]. The difference can be traced to the expression for F_L that one employs. It is customary [15,3] to ignore the target mass M^2 in the expression for F_L [see Eq. (A2)]. This leads to an incorrect expression for F_L which in turn will lead to an incorrect sum rule (see the following section).

The electromagnetic current

$$J^\mu(x)=J_3^\mu(x)+\frac{1}{\sqrt{3}}J_8^\mu(x). \tag{2.15}$$

From the flavor structure of electromagnetic current, we observe that only d_{abc} contributes to the structure functions in deep inelastic electron-hadron scattering. Explicitly, we have

$$\frac{F_2(x)}{x}=\frac{i}{2\pi P^+}\int d\eta e^{-i\eta x}\langle P|\overline{\mathcal{V}^+}(\xi|0)|P\rangle. \tag{2.16}$$

The longitudinal structure function is given by

$$F_L(x)=\frac{2}{Q^2}\frac{i}{\pi}\frac{(q^+)^2}{P^+}\int d\eta e^{-i\eta x}\langle P|\overline{\mathcal{V}^-}(\xi|0)|P\rangle-2\frac{(P^\perp)^2}{Q^2}\frac{i}{\pi P^+}x^2\int d\eta e^{-i\eta x}\langle P|\overline{\mathcal{V}^+}(\xi|0)|P\rangle. \tag{2.17}$$

We have defined the functions

$$\overline{\mathcal{V}^\pm}(\xi|0)=\left(\frac{2}{3}\right)^{3/2}\overline{\mathcal{V}_0^\pm}(\xi|0)+\frac{1}{3}\overline{\mathcal{V}_3^\pm}(\xi|0)+\frac{1}{3\sqrt{3}}\overline{\mathcal{V}_8^\pm}(\xi|0). \tag{2.18}$$

In arriving at our final results we have used explicit values of the structure constants of SU(3):

$$d_{338}=\frac{1}{\sqrt{3}},\quad d_{888}=-\frac{1}{\sqrt{3}},\quad d_{330}=d_{880}=\sqrt{\frac{2}{3}}. \tag{2.19}$$

$\overline{\mathcal{V}_0^\mu}$ is the flavor singlet component of the fermion bilocal vector current.

III. SUM RULE

Consider the flavor singlet part of the structure functions $F_{2(f)}$ and $F_{L(f)}$ defined by

$$\frac{F_{2(f)}(x)}{x}=\frac{1}{4\pi P^+}\int d\eta e^{-i\eta x}\langle P|[\overline{\psi}(\xi)\gamma^+\psi(0)-\overline{\psi}(0)\gamma^+\psi(\xi)]|P\rangle, \tag{3.1}$$

$$F_{L(f)}(x)=\frac{1}{Q^2}\frac{1}{\pi}\frac{(q^+)^2}{P^+}\int d\eta e^{-i\eta x}\langle P|[\overline{\psi}(\xi)\gamma^-\psi(0)-\overline{\psi}(0)\gamma^-\psi(\xi)]|P\rangle$$
$$-\frac{(P^\perp)^2}{Q^2}\frac{1}{\pi P^+}x^2\int d\eta e^{-i\eta x}\langle P|[\overline{\psi}(\xi)\gamma^+\psi(0)-\overline{\psi}(0)\gamma^+\psi(\xi)]|P\rangle. \tag{3.2}$$

From Eq. (3.1) it follows that $F_{2(f)}(-x)=F_{2(f)}(x)$ and from Eq. (3.2) we explicitly find that $F_{L(f)}^{\tau=4}(-x)=-F_{L(f)}^{\tau=4}(x)$. It can be verified [12] that $F_{L(f)}^{\tau=4}$ satisfies the sum rule

$$\int_0^1 dx\frac{F_{L(f)}^{\tau=4}(x,Q^2)}{x}=\frac{2}{Q^2}\left[\langle P|\theta_q^{+-}(0)|P\rangle-\frac{(P^\perp)^2}{(P^+)^2}\langle P|\theta_q^{++}(0)|P\rangle\right], \tag{3.3}$$

where $\theta_q^{+-}=i\overline{\psi}\gamma^-\partial^+\psi$ is the fermionic part of light-front QCD Hamiltonian density and $\theta_q^{++}=i\overline{\psi}\gamma^+\partial^+\psi$ is the fermionic part of light-front longitudinal momentum density in the light-front gauge $A^+=0$.

Here we have used the fact that the physical structure function vanishes for $x>1$. Neglect of M^2 in the expression [Eq. (A2)] for F_L will lead to $(P^\perp)^2+M^2$ instead of $(P^\perp)^2$ in the above equation which would spoil the correct sum rule given below.

The integral of $F_{L(f)}^{\tau=4}/x$ is therefore related to the hadron matrix element of the (gauge invariant) fermionic part of the light-front *Hamiltonian density*. This result manifests the physical content and the nonperturbative nature of the twist four part of the longitudinal structure function.

The fermionic operator matrix elements appearing in Eq. (3.3) change with Q^2 as a result of the mixing of quark and gluon operators in QCD under renormalization. Analyzing the operator mixing we obtain a new sum rule at the twist four level [12]:

$$\int_0^1 \frac{dx}{x} F_L^{\tau=4} = 4\frac{M^2}{Q^2}, \tag{3.4}$$

where M is the invariant mass of the hadron and $F_L^{\tau=4} = F_{L(q)}^{\tau=4}+F_{L(g)}^{\tau=4}$, $F_{L(g)}^{\tau=4}$ is the twist four longitudinal gluon structure function which we define as

$$F_{L(g)}^{\tau=4}(x)=\frac{1}{Q^2}\frac{xP^+}{2\pi}\int dy^- e^{-(i/2)P^+y^-x}\Bigg\{\Bigg[\langle P|(-)F^{+\lambda a}(y^-)F^{-}{}_{\lambda a}(0)+\frac{1}{4}g^{+-}F^{\lambda\sigma a}(y^-)F_{\lambda\sigma a}(0)|P\rangle+(y^-\leftrightarrow 0)\Bigg]$$
$$-\frac{(P^\perp)^2}{(P^+)^2}[\langle P|(-)F^{+\lambda a}(y^-)F^{+}{}_{\lambda a}(0)|P\rangle+(y^-\leftrightarrow 0)]\Bigg\}, \tag{3.5}$$

where $F^{\mu\lambda a}=\partial^\mu A^{\nu a}-\partial^\nu A^{\mu a}+gf^{abc}A_b^\mu A_c^\nu$. Note that in the definition of $F_{L(g)}^{\tau=4}(x)$ the second term where the arguments of $F^{\lambda\sigma a}$ are interchanged is missing in Ref. [12].

To our knowledge, this is the first sum rule at the twist four level for deep inelastic scattering or for QCD in general. The previously known sum rules in deep inelastic scattering are all at the twist two level. The operators involved are kinematical (light-front longitudinal momentum, light-front helicity, etc.) in nature. In contrast, the sum rule we have derived involves a dynamical operator (light-front QCD Hamiltonian), thus revealing a new aspect of the underlying nonperturbative dynamics. Our results show that the measuremnent of the flavor singlet part of the fermionic contributions to the twist four longitudinal structure function in deep inelastic scattering directly reveals the hadron expectation value of the fermionic part of the light-front QCD Hamiltonian density in light-front gauge.

IV. DRESSED PARTON CALCULATIONS

A. Dressed quark with nonzero mass

Next, we investigate the implications of Eq. (3.3) for quadratic divergences in $F_{L(q)}^{\tau=4}$ in perturbation theory. We select the target to be a dressed quark and evaluate the structure functions to order g^2. That is, we take the state $|P\rangle$ to be a dressed quark consisting of bare states of a quark and a quark plus a gluon:

$$|P,\sigma\rangle=\phi_1 b^\dagger(P,\sigma)|0\rangle+\sum_{\sigma_1,\lambda_2}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}$$
$$\times\int\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\sqrt{2(2\pi)^3P^+}\,\delta^3(P-k_1-k_2)$$
$$\times\phi_2(P,\sigma|k_1,\sigma_1;k_2,\lambda_2)b^\dagger(k_1,\sigma_1)a^\dagger(k_2,\lambda_2)|0\rangle. \tag{4.1}$$

In the previous work [12] we have given results for massless quark state. We have shown that the twist four longitudinal structure function has quadratic divergences in perturbation theory. In this section, we show that for a massive quark, in addition to quadratic divergences, logarithmic divergences are generated. We have

$$F_L=\mathcal{M}_1+\mathcal{M}_2, \tag{4.2}$$

where

$$\mathcal{M}_1=\frac{1}{\pi Q^2}\int dy^- e^{(-i/2)P^+y^-x}\langle P|\psi^{+\dagger}(y^-)$$
$$\times[\alpha^\perp\cdot[i\partial^\perp+gA^\perp(y^-)]+\gamma^0 m]$$
$$\times[\alpha^\perp\cdot[i\partial^\perp+gA^\perp(0)]+\gamma^0 m]\psi^+(0)+\text{H.c.}|P\rangle \tag{4.3}$$

and

$$\mathcal{M}_2 = -4\frac{(P^\perp)^2}{Q^2} x F_{2(q)}(x), \tag{4.4}$$

where $|P\rangle$ now has a mass M and m is the bare quark mass.

In the case of quark contributions, the second term in the expression for the bilocal current in Eq. (2.9) vanishes. First we evaluate the contribution $\mathcal{M}_2$ given in Eq. (4.4). We obtain

$$\mathcal{M}_2 = -4\frac{(P^\perp)^2}{Q^2} x^2 \left[\delta(1-x) + \frac{g^2}{8\pi^3} C_f \left(\int d^2k_\perp \frac{\frac{1+x^2}{1-x} k_\perp^2 + (1-x)^3 m^2}{[m^2(1-x)^2 + k_\perp^2]^2} - \delta(1-x) \int dy d^2k_\perp \frac{\frac{1+y^2}{1-y} k_\perp^2 + (1-y)^3 m^2}{[m^2(1-y)^2 + k_\perp^2]^2} \right) \right], \tag{4.5}$$

where $C_f = (N^2-1)/2N$ for SU(N).

Here we have presented the result without working out the transverse integration to maintain a greater degree of transparency.

The contribution from $\mathcal{M}_1$ is split into four parts with additional contributions coming from quark mass terms and can be written as follows:

$$\begin{aligned}
\mathcal{M}_1 = & \frac{1}{\pi Q^2} \int dy^- e^{(-i/2)P^+ y^- x} \langle P | \psi^{+\dagger}(y^-)[-(\partial^\perp)^2 + m^2] \psi^+(0) | P \rangle \\
& + g \frac{1}{\pi Q^2} \int dy^- e^{(-i/2)P^+ y^- x} \langle P | \psi^{+\dagger}(y^-)(i\partial^\perp \cdot \alpha^\perp + \gamma^0 m) \alpha^\perp \cdot A^\perp(0) \psi^+(0) | P \rangle \\
& + g \frac{1}{\pi Q^2} \int dy^- e^{(-i/2)P^+ y^- x} \langle P | \psi^{+\dagger}(y^-) \alpha^\perp \cdot A^\perp(y)(i\partial^\perp \cdot \alpha^\perp + \gamma^0 m) \psi^+(0) | P \rangle \\
& + g^2 \frac{1}{\pi Q^2} \int dy^- e^{(-i/2)P^+ y^- x} \langle P | \psi^{+\dagger}(y^-) A^\perp(y) \cdot A^\perp(0) \psi^+(0) | P \rangle
\end{aligned} \tag{4.6}$$

$$\equiv \mathcal{M}_1^a + \mathcal{M}_1^b + \mathcal{M}_1^c + \mathcal{M}_1^d. \tag{4.7}$$

Since the operators in Eq. (4.3) are taken to be normal ordered, the contribution of $\mathcal{M}_1^d$ vanishes to order g^2.

Explicit calculation leads to the diagonal Fock basis contributions

$$\begin{aligned}
(\mathcal{M}_1)_{diag} = \mathcal{M}_1^a = & 4\frac{(P^\perp)^2}{Q^2} x^2 \left[\delta(1-x) + \frac{g^2}{8\pi^3} C_f \left(\int d^2k_\perp \frac{\frac{1+x^2}{1-x} k_\perp^2 + (1-x)^3 m^2}{[m^2(1-x)^2 + k_\perp^2]^2} \right.\right. \\
& \left.\left. - \delta(1-x) \int dy d^2k_\perp \frac{\frac{1+y^2}{1-y} k_\perp^2 + (1-y)^3 m^2}{[m^2(1-y)^2 + k_\perp^2]^2} \right) \right] + \frac{4m^2}{Q^2} \delta(1-x) \left[1 - C_f \frac{g^2}{8\pi^3} \int dy d^2k_\perp \frac{\frac{1+y^2}{1-y} k_\perp^2 + (1-y)^3 m^2}{[m^2(1-y)^2 + k_\perp^2]^2} \right] \\
& + \frac{4C_f}{Q^2} \frac{g^2}{8\pi^3} \int d^2k_\perp (k_\perp^2 + m^2) \frac{\frac{1+x^2}{1-x} k_\perp^2 + (1-x)^3 m^2}{[m^2(1-x)^2 + k_\perp^2]^2}.
\end{aligned} \tag{4.8}$$

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The first term here explicitly cancels the term $\mathcal{M}_2$ given in Eq. (4.5).

Off-diagonal contributions

$$(\mathcal{M}_1)_{nondiag} = \mathcal{M}_1^b + \mathcal{M}_1^c = \frac{C_f}{Q^2} \frac{g^2}{\pi^3} \left[\delta(1-x) \int dy d^2k_\perp \frac{m^2(1-y)}{[m^2(1-y)^2 + k_\perp^2]} - \int d^2k_\perp \frac{k_\perp^2 + m^2(1-x)^2}{(1-x)[m^2(1-x)^2 + k_\perp^2]} \right]. \tag{4.9}$$

Adding all the contributions, we have

$$F_{L(q)}^{\tau=4}(x) = \frac{4m^2}{Q^2}\,\delta(1-x) + \frac{4C_f}{Q^2}\,\frac{g^2}{8\pi^3}\Bigg[\int d^2k_\perp (k_\perp^2+m^2)\frac{\frac{1+x^2}{1-x}k_\perp^2+(1-x)^3m^2}{[m^2(1-x)^2+k_\perp^2]^2}$$

$$-\delta(1-x)m^2\int dy d^2k_\perp \frac{\frac{1+y^2}{1-y}k_\perp^2+(1-y)^3m^2}{[m^2(1-y)^2+k_\perp^2]^2}\Bigg]$$

$$-\frac{C_f}{Q^2}\frac{g^2}{\pi^3}\Bigg[\int d^2k_\perp \frac{k_\perp^2+m^2(1-x)^2}{(1-x)[m^2(1-x)^2+k_\perp^2]} - \delta(1-x)\int dy d^2k_\perp \frac{m^2(1-y)}{[m^2(1-y)^2+k_\perp^2]}\Bigg]. \tag{4.10}$$

Here we have used $M=m$, since the difference that it entails is higher order in the coupling. Note that we are getting back the free quark answer once we switch off the interaction. Also, the dressed massless quark answer can be easily regenerated by putting $M=m=0$. Note that the $k_\perp$ integration now produces logarithmic divergences with the expected quadratic ones, as we remarked earlier.

To check the sum rule explicitly, we evaluate the right-hand side (RHS) of Eq. (3.3) next. A straightforward evaluation leads to

$$\langle P|\theta_q^{+-}(0)|P\rangle_{nondiag} = -C_f\frac{g^2}{2\pi^3}\int dx d^2k_\perp \frac{k_\perp^2+m^2(1-x)^3}{x(1-x)}\frac{1}{[m^2(1-x)^2+k_\perp^2]}, \tag{4.11}$$

$$\langle P|\theta_q^{+-}(0)|P\rangle_{diag} - \frac{(P^\perp)^2}{(P^+)^2}\langle P|\theta_q^{++}(0)|P\rangle_{diag} = 2m^2 + 2C_f\frac{g^2}{8\pi^3}\int dx d^2k_\perp \frac{k_\perp^2+(1-x)m^2}{x}\frac{\frac{1+x^2}{1-x}k_\perp^2+(1-x)^3m^2}{[m^2(1-x)^2+k_\perp^2]^2}. \tag{4.12}$$

Adding the diagonal and off-diagonal contributions from the fermionic part of the Hamiltonian density and multiplying it by $2/Q^2$ one obtains the RHS of the sum rule. Comparing it with the integral of $F_L^{\tau=4}/x$, where F_L is given in Eq. (4.10), one easily sees that the sum rule is verified.

To see the connection of F_L with the fermionic mass shift, we calculate the contribution of the gluonic part of the energy momentum tensor θ^{+-} to the sum rule for the total F_L. Explicit calculation gives

$$\langle P|\theta_g^{+-}(0)|P\rangle_{nondiag} = -C_f\frac{g^2}{2\pi^3}\int dx d^2k_\perp \frac{(1+x)k_\perp^2}{(1-x)^2}\frac{1}{[m^2(1-x)^2+k_\perp^2]}, \tag{4.13}$$

$$\langle P|\theta_g^{+-}(0)|P\rangle_{diag} - \frac{(P^\perp)^2}{(P^+)^2}\langle P|\theta_g^{++}(0)|P\rangle_{diag} = 2C_f\frac{g^2}{8\pi^3}\int dx d^2k_\perp \frac{k_\perp^2}{(1-x)}\frac{\frac{1+x^2}{1-x}k_\perp^2+(1-x)^3m^2}{[m^2(1-x)^2+k_\perp^2]^2}. \tag{4.14}$$

Thus, we get

$$\langle P|\theta_q^{+-}(0)+\theta_g^{+-}(0)|P\rangle_{nondiag} = -C_f\frac{g^2}{2\pi^3}\int dx d^2k_\perp \frac{\frac{(1+x^2)}{1-x}k_\perp^2+(1-x)^3m^2}{x(1-x)}\frac{1}{[m^2(1-x)^2+k_\perp^2]}, \tag{4.15}$$

$$\langle P|\theta_q^{+-}(0)+\theta_g^{+-}(0)|P\rangle_{diag} - \frac{(P^\perp)^2}{(P^+)^2}\langle P|\theta_q^{++}+\theta_g^{++}(0)|P\rangle_{diag}$$

$$=2C_f\frac{g^2}{8\pi^3}\int dx d^2k_\perp \frac{\frac{(1+x^2)}{1-x}k_\perp^2+(1-x)^3m^2}{x(1-x)}\frac{1}{[m^2(1-x)^2+k_\perp^2]}. \tag{4.16}$$

Adding diagonal and off-diagonal contributions, we get

$$\langle P|\theta^{+-}(0)|P\rangle-\frac{(P^{\perp})^2}{(P^+)^2}\langle P|\theta^{++}(0)|P\rangle=-C_f\frac{g^2}{4\pi^3}\int dx d^2k_\perp\frac{\frac{(1+x^2)}{1-x}k_\perp^2+(1-x)^3m^2}{x(1-x)}\frac{1}{[m^2(1-x)^2+k_\perp^2]}. \tag{4.17}$$

Note that this result is connected to the full fermion mass shift δp_1^- in second order perturbation theory. We have [see Eq. (4.10)], in Ref. [16],

$$\delta p_1^-=-\frac{1}{2P^+}C_f\frac{g^2}{4\pi^3}\int dx d^2k_\perp\frac{\frac{(1+x^2)}{1-x}k_\perp^2+(1-x)^3m^2}{x(1-x)}\frac{1}{[m^2(1-x)^2+k_\perp^2]}. \tag{4.18}$$

B. Dressed gluon

In this section we check the sum rule explicitly for a dressed gluon target. We consider the gluon to be composed of a bare gluon and a quark-antiquark pair:

$$\begin{aligned}|P,\sigma\rangle=&\phi_1 a^\dagger(P,\lambda)|0\rangle+\sum_{\sigma_1,\sigma_2}\int\frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}}\int\frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}}\sqrt{2(2\pi)^3P^+}\,\delta^3(P-k_1-k_2)\\&\times\phi_2(P,\sigma|k_1,\sigma_1;k_2,\sigma_2)b^\dagger(k_1,\sigma_1)d^\dagger(k_2,\sigma_2)|0\rangle.\end{aligned} \tag{4.19}$$

The target gluon and the bare quark and antiquark masses are taken to be zero. Note that, to the order g^2, there will be a contribution from the two-gluon Fock sector due to the non-Abelian nature of the gauge coupling. For simplicity, we exclude that contribution. It is easy to incorporate that contribution by trivially extending our calculation presented here. F_L can be written in terms of $\mathcal{M}_1$ and $\mathcal{M}_2$ given in Eqs. (4.3) and (4.4), where $|P\rangle$ now stands for the dressed gluon represented by Eq. (4.19). Explicit calculation gives

$$\begin{aligned}\mathcal{M}_2&=-4\frac{(P^\perp)^2}{Q^2}xF_{2(q)}^{dressed\ gluon}\\&=-\frac{x^2(P^\perp)^2}{Q^2}\frac{g^2}{\pi^2}N_fT_f[x^2+(1-x)^2]\ln\Lambda^2.\end{aligned} \tag{4.20}$$

Here $T_f=\frac{1}{2}$ and N_f is the number of flavors.

$\mathcal{M}_1$ is again divided into four parts as in Eq. (4.7) and explicit calculation in this case gives the following:

$$\begin{aligned}\mathcal{M}_{1(diag)}=\mathcal{M}_{1(a)}=&\frac{x^2(P^\perp)^2}{Q^2}\frac{g^2}{\pi^2}N_fT_f[x^2+(1-x)^2]\ln\Lambda^2\\&+\frac{\Lambda^2}{Q^2}\frac{g^2}{\pi^2}N_fT_f[x^2+(1-x)^2],\end{aligned} \tag{4.21}$$

$$\begin{aligned}\mathcal{M}_{1(off\text{-}diag)}&=\mathcal{M}_{1(b)}+\mathcal{M}_{1(c)}\\&=-\frac{\Lambda^2}{Q^2}\frac{g^2}{\pi^2}N_fT_f2(1-x).\end{aligned} \tag{4.22}$$

Thus, we get

$$F_L=\frac{\Lambda^2}{Q^2}N_fT_f\frac{g^2}{\pi^2}[x^2+(1-x)^2-2(1-x)]. \tag{4.23}$$

On the other hand, we get

$$\begin{aligned}&\langle P|\theta_q^{+-}(0)|P\rangle_{diag}-\frac{(P^\perp)^2}{(P^+)^2}\langle P|\theta_q^{++}(0)|P\rangle_{diag}\\&=\Lambda^2N_fT_f\frac{g^2}{4\pi^2}\int dx\left[\frac{x^2+(1-x)^2}{x(1-x)}\right]\end{aligned} \tag{4.24}$$

and

$$\langle P|\theta_q^{+-}(0)|P\rangle_{off\text{-}diag}=-\Lambda^2N_fT_f\frac{g^2}{2\pi^2}\int dx\left[\frac{x^2+(1-x)^2}{x(1-x)}\right]. \tag{4.25}$$

Adding diagonal and off-diagonal contributions, we get

$$\begin{aligned}&\langle P|\theta_q^{+-}(0)|P\rangle-\frac{(P^\perp)^2}{(P^+)^2}\langle P|\theta_q^{++}(0)|P\rangle\\&=-\Lambda^2N_fT_f\frac{g^2}{4\pi^2}\int dx\left[\frac{x^2+(1-x)^2}{x(1-x)}\right].\end{aligned} \tag{4.26}$$

Note that this result is connected to the gluonic mass shift δq_2^- due to pair production, since the contribution from the gluonic part of the energy-momentum tensor θ_g^{+-} in this case vanishes. In the massless limit, we have [see Eq. (4.40) in Ref. [16]]

$$\delta q_2^-=-\frac{1}{2P^+}\Lambda^2N_fT_f\frac{g^2}{4\pi^2}\int dx\left[\frac{x^2+(1-x)^2}{x(1-x)}\right]. \tag{4.27}$$

From Eq. (4.23) we compute $\int dx\, F_L/x$. Since x integration is from 0 to 1, it can be written in the following form:

$$\int dx \frac{F_L}{x} = -\frac{\Lambda^2}{Q^2} N_f T_f \frac{g^2}{2\pi^2} \int dx \left[\frac{x^2+(1-x)^2}{x(1-x)} \right]. \tag{4.28}$$

Comparing Eq. (4.26) and Eq. (4.28), one explicitly verifies the sum rule for a dressed gluon target.

As we have emphasized, in the bare theory, the twist four longitudinal structure function is afflicted with divergences. We have to add counterterms to carry out the renormalization procedure so that we have physical answers. The sum rule for the bare theory clearly shows that the quadratic divergences generated are directly related to the gluon mass shift in second order light-front perturbation theory arising from an intermediate quark-antiquark pair. In order to ensure a massless gluon in second order perturbation theory, we have to add the negative of the shift as a counterterm. After adding the counterterm, the gluon mass shift in second order perturbation theory is zero and the twist four longitudinal structure function for a massless gluon becomes zero. Thus, after renormalization, the sum rule is satisfied, with a trivial (i.e., zero) gluon longitudinal structure function.

V. (1+1)-DIMENSIONAL QCD: EXPLICIT CALCULATIONS

In this section, we turn to two-dimensional QCD in order to test the sum rule given in Eq. (3.4) explicitly in a nonperturbative context. In $1+1$ dimensions, in $A^+=0$ gauge, we have

$$\int_0^1 \frac{dx}{x} F_{L(q)}^{\tau=4}(x) = \frac{2}{Q^2} \langle P | [\theta_q^{+-}(0) + \theta_g^{+-}(0)] | P \rangle, \tag{5.1}$$

with $\theta_q^{+-} = 2m^2 \psi^{+\dagger}(1/i\partial^+)\psi^+$ and $\theta_g^{+-} = -4g^2 \psi^{+\dagger} T^a \psi^+ \underline{[1/(\partial^+)^2]} \psi^{+\dagger} T^a \psi^+$. We consider the standard one pair $(q\bar{q})$ approximation to the meson ground state. Explicit evaluations show that

$$\frac{F_{L(q)}^{\tau=4}}{x} = \frac{4}{Q^2} \psi^*(x) \frac{m^2}{x(1-x)} \psi(x) \tag{5.2}$$

and

$$\int_0^1 dx \frac{F_{L(g)}^{\tau=4}}{x} = \frac{4}{Q^2}(-) C_f \frac{g^2}{\pi} \int_0^1 dx \int_0^1 dy\, \psi^*(x) \frac{\psi(y)-\psi(x)}{(x-y)^2}, \tag{5.3}$$

where $\psi(x)$ is the ground state wave function for the meson. Thus

$$\int_0^1 \frac{dx}{x} F_L^{\tau=4}(x) = \frac{4}{Q^2} \int_0^1 dx\, \psi^*(x) \left[\frac{m^2}{x(1-x)} \psi(x) - C_f \frac{g^2}{\pi} \int_0^1 dy \frac{\psi(y)-\psi(x)}{(x-y)^2} \right]. \tag{5.4}$$

By virtue of the bound state equation ('t Hooft equation) obeyed by the ground state wave function $\psi(x)$ for the meson,

$$M^2 \psi(x) = \frac{m^2}{x(1-x)} \psi(x) - C_f \frac{g^2}{\pi} \int dy \frac{\psi(y)-\psi(x)}{(x-y)^2}, \tag{5.5}$$

together with the normalization condition $\int_0^1 dx\, \psi^2(x) = 1$, we easily verify that the twist four longitudinal structure function of the meson obeys the sum rule

$$\int_0^1 \frac{dx}{x} F_L^{\tau=4} = \frac{2}{Q^2} \langle P | \theta^{+-}(0) | P \rangle = 4 \frac{M^2}{Q^2}. \tag{5.6}$$

In the same model, the contribution to the twist two structure function from the fermionic constituents is given by

$$F_{2(q)}(x) = (x+1-x)\psi^*(x)\psi(x) = \psi^*(x)\psi(x). \tag{5.7}$$

Note that, since there are no partonic gluons or sea in this model, the longitudinal momentum of the meson is carried entirely by the valence quark and antiquark. Thus the momentum sum rule is saturated by the fermionic part of the longitudinal momentum density. On the other hand, light-front energy density is shared between fermionic and gauge bosonic parts and as a consequence the fermions carry only a fraction of the hadron mass. This seemingly paradoxical situation further illuminates the difference between the physical content of the F_2 and $F_L^{\tau=4}$ sum rules.

To get a quantitative picture, next, we explicitly calculate the structure functions $F_{2(q)}(x)$ and $F_{L(q)}(x)/x$ for the ground state meson in two-dimensional QCD. We have parametrized the ground state wave function as $\psi(x) = \mathcal{N} x^s (1-x)^s$ and determined the value of s variationally by minimizing M^2 for given values of m^2 and g^2. The factor $\mathcal{N}$ is determined from the normalization condition $\int_0^1 dx\, \psi^*(x)\psi(x) = 1$. The resulting structure functions are presented in Fig. 1 for two different values of m^2.

Since both the quark and antiquark have equal mass in the model, both structure functions are symmetric about $x=\frac{1}{2}$. When the fermions are heavy [Fig. 1(a)], the system is essentially nonrelativistic and the structure functions are significant only near the region $x=\frac{1}{2}$. When the fermions become lighter [Fig. 1(b)], contributions to the structure function from the end-point regions become significant indicating substantial high momentum components in the ground state wave function. Note that $F_{L(q)}/x$ measures the fermion kinetic energy (in light-front coordinates). The exponent s in the wave function is a function of the fermion mass and s decreases as m decreases. In the massless limit, s vanishes [17] so that the wave function for the ground state becomes $\psi(x) = \theta(x)\theta(1-x)$. This results in a flat F_2 structure function. However, because of the presence of m^2, $F_{L(q)}^{\tau=4}$ vanishes. Because of an exact cancellation between the self-energy and gluon exchange contributions, the gluonic part of the $F_L^{\tau=4}$ also vanishes. Thus the sum rule is satisfied exactly since, in the zero-quark-mass limit, the ground state meson is massless in two-dimensional QCD.

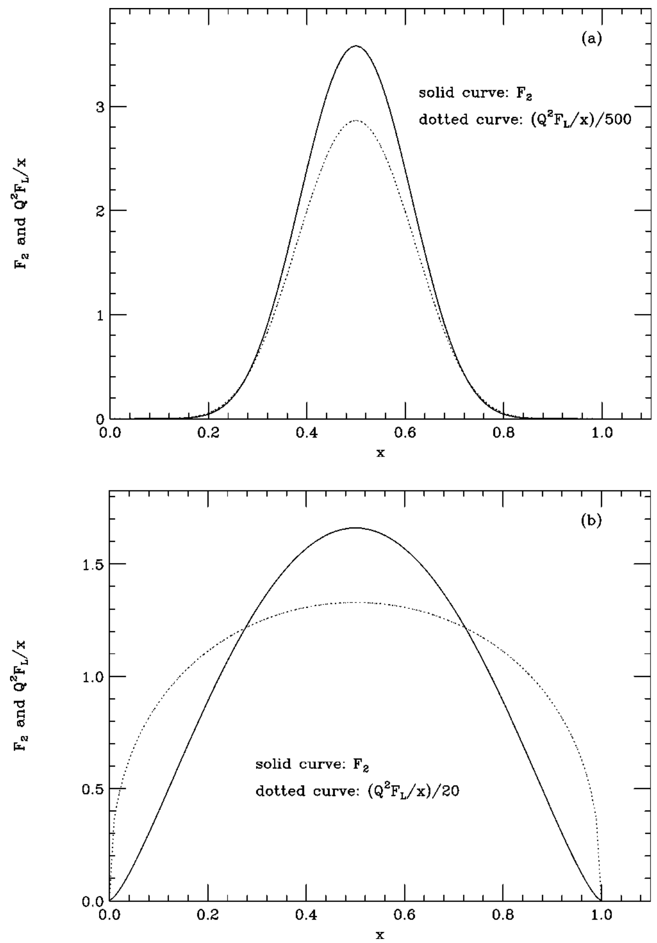

FIG. 1. Fermionic contributions to the structure functions $F_2(x)$ and $F_L^{\tau=4}/x$ for the ground state meson in the 't Hooft model for two different values of m, the quark mass: (a) $m=5$, $s=4.96$. (b) $m=1$, $s=0.70$. The parameter s appearing in the wave function is determined by a variational calculation. We have set $C_f g^2/\pi = 1$.

VI. PARTITION OF THE HADRON MASS IN QCD

As is well known, experiments that measure the twist two part of the F_2 structure function yield information on the fraction of longitudinal momenta carried by the charged parton constituents of the hadron (quarks and antiquarks). The sum rule we have derived yields other useful information about the hadron structure. Namely, our sum rule shows that experiments to measure the twist four part of the longitudinal structure function will directly reveal the fraction of the hadron mass carried by charged parton components of the hadron. The light-front Hamiltonian provides theoretical insight into this fraction as follows.

According to our analysis, the twist four part of the longitudinal structure function is directly related to the fermionic part of the light-front QCD Hamiltonian density θ_q^{+-} in the gauge $A_a^+=0$. Explicitly we have

$$\theta_q^{+-} = 2\psi^{+\dagger}[\alpha^\perp\cdot(i\partial^\perp + gA^\perp) + \gamma^0 m] \times \frac{1}{i\partial^+}[\alpha^\perp\cdot(i\partial^\perp + gA^\perp) + \gamma^0 m]\psi^+. \tag{6.1}$$

Thus we have the fermion kinetic energy contribution given by

$$\theta_{q(free)}^{+-} = 2\psi^{+\dagger}[-(\partial^\perp)^2 + m^2]\psi^+ \tag{6.2}$$

and the interaction-dependent part given by

$$\theta_{q(int)}^{+-} = 2g\psi^{+\dagger}\left[\alpha^\perp\cdot A^\perp \frac{1}{i\partial^+}(\alpha^\perp\cdot i\partial^\perp + \gamma^0 m) + (\alpha^\perp\cdot i\partial^\perp + \gamma^0 m)\frac{1}{i\partial^+}\alpha^\perp\cdot A^\perp\right]\psi^+ + 2g^2\psi^{+\dagger}\alpha^\perp\cdot A^\perp \frac{1}{i\partial^+}\alpha^\perp\cdot A^\perp\psi^+. \tag{6.3}$$

Note that the fermion kinetic energy constitutes only a part of the total contribution from fermions. Any theoretical estimate of the fermionic part of the longitudinal structure function necessarily has to involve off-diagonal contributions from Fock states differing in the number of gluons by 1 and 2.

It is important to emphasize the difference between equal-time and light-front Hamiltonians in the context of our calculations. The equal-time Hamiltonian contains the scalar density term ($\overline{\psi}\psi$) accompanying the quark mass m. In contrast, the quark mass appears quadratically in the free part of the light-front Hamiltonian. Recently the question of the partition of hadron masses in QCD has been addressed by Ji [18] in the context of the equal-time Hamiltonian and in terms of twist two and twist three observables. In his analysis, the extraction of the fraction of the hadron mass carried by the fermion constituents is not straightforward because of the presence of the scalar density term. The hadron expectation value of the strange quark scalar density remains unknown (experimentally). Our analysis, however, shows that the twist four longitudinal structure function, once extracted experimentally, directly yields the fraction of the hadron mass carried by fermionic constituents.

VII. DISCUSSION AND CONCLUSIONS

To gain physical intuition on the twist four longitudinal structure function and to understand the occurrence of quadratic divergences and the associated renormalization issues, we have studied the twist four longitudinal structure function in an approach based on Fock space expansion methods in light-front field theory. We have identified the integral of $1/x$ times the twist four part of the fermionic contribution to the longitudinal structure function as the hadron matrix element of the fermionic part of the light-front QCD Hamiltonian density in the light-front gauge apart from an overall constant. We have tested this relation to order g^2 in QCD perturbation theory for both dressed quark and gluon targets. Our result shows that quadratic and logarithmic divergences in the twist four longitudinal structure function are directly related to mass corrections in the light-front theory.

By investigating the mixing of operators in the flavor singlet channel, we have recently derived [12] a new sum rule which involves the invariant mass of the hadron. The validity of the sum rule has been explicitly checked in two-dimensional QCD ('t Hooft model). To get a qualitative picture of the twist four structure function we have computed numerically both F_2 and F_L structure functions in the 't Hooft model using the ground state wave function calculated using a variational ansatz.

We have also discussed the implication of our results for the problem of breakup of hadron masses in QCD in terms of fermionic and bosonic constituents. We have emphasized the differences between equal-time and light-front formulations relevant for this study.

Our results indicate that the experiments to measure the twist four longitudinal structure function reveal the fraction of the hadron mass carried by the charged parton components. Thus these experiments play a complementary role to the longitudinal momentum and helicity distribution information obtained at the twist two level. It is of interest to investigate the feasibility of the direct measurement of the twist four gluon structure function in high energy experiments. Recent work of Qiu and co-workers has shown that semi-inclusive single jet production in deep inelastic scattering [19] and direct photon production in hadron nucleus scattering [20] provide direct measurement of twist four gluon matrix elements.

On the theoretical side we note that some significant progress has been made recently in the bound state problem in light-front QCD [21] based on similarity renormalization group method. In the near future, we plan to undertake a nonperturbative calculation (utilizing Fock space expansion and Hamiltonian renormalization techniques) of the longitudinal structure function for a mesonlike bound state. Such a calculation will undoubtedly help to check the validity of current phenomenological models [6] based on simple assumptions [22] employed in analyzing the data.

Another important problem is the scale evolution of the twist four structure function which is far more complicated than the twist two case. Recently we have provided a physical picture of scale evolution of the F_2 structure function of a composite system in terms of multiparton wave functions in momentum space [23]. We plan to carry out a similar analysis of the twist four longitudinal structure function, elucidating all possible scale dependences and their physical interpretation.

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APPENDIX: SUMMARY OF NOTATION AND CONVENTIONS

The hadron tensor relevant to unpolarized electron-hadron deep inelastic scattering is given by

$$W^{\mu\nu}=\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1(x,Q^2)+\left(P^\mu-\frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu-\frac{P\cdot q}{q^2}q^\nu\right)W_2(x,Q^2). \tag{A1}$$

The dimensionless functions

$$F_L(x,Q^2)=2\left[-W_1+\left(M^2-\frac{(P\cdot q)^2}{q^2}\right)W_2\right], \tag{A2}$$

and

$$F_2(x,Q^2)=\nu W_2(x,Q^2) \tag{A3}$$

are the unpolarized structure functions.

We have defined $-Q^2=q^2=q^+q^- -(q^\perp)^2$.

The light-front coordinates are defined by $x^\pm = x^0 \pm x^3$.

The constraint equation for the fermion field, which follows from the Dirac equation, in $A^+=0$ gauge is given by

$$\psi^-(z)=\frac{1}{4i}\int dy^-\epsilon(z^- - y^-)\times[\alpha^\perp\cdot(i\partial^\perp+gA^\perp)+\gamma^0 m]\psi^+(y^-), \tag{A4}$$

where the antisymmetric step function

$$\epsilon(x^-)=-\frac{i}{\pi}P\int\frac{d\omega}{\omega}e^{(i/2)\omega x^-}. \tag{A5}$$

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