New glueball-meson mass relations

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Using the ''glueball dominance'' picture of the mixing between $q\bar{q}$ mesons of different hidden flavors, we establish new glueball-meson mass relations which serve as a basis for glueball spectral systematics. For the tensor glueball mass 2.3 ± 0.1 GeV used as an input parameter, these relations predict the following glueball masses: $M(0^{++})\simeq1.65\pm0.05$ GeV, $M(1^{--})\simeq3.2\pm0.2$ GeV, $M(2^{-+})\simeq2.95\pm0.15$ GeV, and $M(3^{--})\simeq2.8\pm0.15$ GeV. We briefly discuss the failure of such relations for the pseudoscalar sector. Our results are consistent with (quasi)linear Regge trajectories for glueballs with a slope $\simeq0.3\pm0.1$ GeV^{-2}.
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I. INTRODUCTION

The existence of a gluon self-coupling in QCD suggests that, in addition to the conventional $q\bar{q}$ states, there may be non-$q\bar{q}$ mesons: bound states including gluons (glueballs and $q\bar{q}g$ hybrids). However, the theoretical quidance on the properties of unusual states is often contradictory, and models that agree in the $q\bar{q}$ sector differ in their predictions about new states. Moreover, the abundance of $q\bar{q}$ meson states in the 1–2 GeV region and glueball-quarkonium mixing makes the identification of the would-be lightest non-$q\bar{q}$ mesons extremely difficult. To date, no glueball state has been firmly established.

Although the current situation with the identification of glueball states is rather complicated, some progress has been made recently in the 0^{++} scalar and 2^{++} tensor glueball sectors, where both experimental and QCD lattice simulation results seem to converge [1]. Recent lattice calculations predict the 0^{++} glueball mass to be 1600 ± 100 MeV [1–4]. Accordingly, there are two experimental candidates [5], $f_0(1500)$ and $f_0(1710)$, in this mass range which cannot both fit into the scalar meson nonet, and this may be considered as strong evidence for one of these states being a scalar glueball (and the other being dominantly $s\bar{s}$ scalar quarkonium).

In the tensor sector, the situation seems cleaner, though less well established. Lattice simulations predict the 2^{++} glueball mass at 2390 ± 120 MeV [4,6], and correspondingly, there are three experimental candidates in this mass region [5]: $f_J(2220)$, $J=2$ or 4, $f_2(2300)$ and $f_2(2340)$. The first candidate is seen in $J/\psi\rightarrow\gamma+X$ transitions but not in $\gamma\gamma$ production [5], while the remaining two are observed in the Okubo-Zweis-Iizuka- (OZI-) rule-forbidden process $\pi p\rightarrow\phi\phi n$ [5], which favors the gluonium interpretation of all three states.

Spin-1 glueballs are more complicated. Lattice studies on the vector glueball are scarce and inconclusive [2], mainly because of the difficulties in constructing the corresponding lattice operators.[1] Various arguments (e.g. [2,7]) suggest that the lowest lying 1^{--} glueball has to consist of at least three constituent gluons. Therefore, it is heavier and more difficult to produce than the scalar and tensor glueballs. On the other hand, once produced it should be easier to identify since it can be expected to mix less with $q\bar{q}$ mesons.[2]

In this paper we wish to undertake an attempt towards glueball spectral systematics using mass relations. Our previous experience with mass relations derived within different approaches to both light and heavy mesons [8–10], and to baryons [11]), shows that these relations can be very successful. They typically hold with an accuracy of a few percent, and often even 1%. In order to relate the glueball masses to the masses of known $q\bar{q}$ mesons, we need to identify processes which are dominated by gluonic intermediate states.

Such processes are, for example, OZI suppressed transitions [12] between different hidden flavor states. For this suppression to hold, it has been shown that contributions from $q\bar{q}$ intermediate states [13] (even though not OZI suppressed, e.g., $\phi\rightarrow K\bar{K}\rightarrow\rho\pi$) must (and do) cancel [14–16]. We assume this cancellation is essentially complete and further assume that of all gluonic intermediate states, the quark $q\bar{q}\leftrightarrow q'\bar{q}'$ transition is dominated by the glueball with the corresponding quantum numbers [17] which is closest in mass to the mixing states. Under these basic assumptions, we relate the mass of the glueball to the masses of the $q\bar{q}$ and $q'\bar{q}'$ mesons.

The paper is organized as follows: Sec. II relates quark mixing amplitudes to the masses of physical mesons. In Sec. III, these amplitudes are expressed in terms of glueball masses, and the new mass relations are derived. We discuss the self-consistency of the calculation and the results. In Sec.

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[1]We thank W. Lee for this remark.

[2]The transitions between glueballs built of three gluons and quarkonia are order $\alpha_S^{3/2}$, compared to order α_S for the glueballs consisting of 2 gluons. If $\alpha_S<1$ at the scale relevant for this transition, vector glueball mixing with quarkonia is suppressed.

IV we show that the glueball masses we find are consistent with the expected gluonic Regge trajectories. The last section contains our summary and conclusions.

II. MESON MASS SQUARED MATRIX

We start with the mass squared matrix in the (constituent) basis $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, modified by the inclusion of the quark mixing amplitudes. (In the following, the symbol for the meson stands for its mass, unless otherwise specified, and we use the notations ρ, K^*, ω and ϕ for the isovector, isodoublet, and two isoscalar states (which will appear later), respectively, of a meson nonet of any spin.) Let

$$\mathcal{M}^2=\begin{pmatrix} \rho^2+A_{uu} & A_{ud} & A_{us} \\ A_{du} & \rho^2+A_{dd} & A_{ds} \\ A_{su} & A_{sd} & 2K^{*2}-\rho^2+A_{ss} \end{pmatrix}. \quad (1)$$

Here ρ^2 and $2K^{*2}-\rho^2$ are the masses squared of the $u\bar{u}$ (or $d\bar{d}$) and $s\bar{s}$ states, respectively, and A's stand for the quark mixing amplitudes: transitions of $q\bar{q}$ into $q'\bar{q}'$; e.g., A_{ud} represents the $u\bar{u}\rightarrow d\bar{d}$ transition, etc. One can expect the mixing amplitudes involving $s\bar{s}$ to differ from the mixing amplitudes involving $u\bar{u}$ and/or $d\bar{d}$ only. This is due to the fact that the mass splitting for the s- and u, d-quarks is much larger than that for the u- and d-quarks themselves, and that the mixing amplitudes (being associated with pair creation and annihilation) should be explicitly dependent on the corresponding quark masses. Therefore, we assume the mixing amplitudes to be isospin- but not SU(3)$_f$-symmetric, i.e., $A_{uu}=A_{ud}=A_{dd}\equiv A$, $A_{us}=A_{ds}\equiv A'$, and $A_{ss}\equiv A''$.

Transforming now the mass squared matrix (1) to the (Gell-Mann) basis $(u\bar{u}-d\bar{d})/\sqrt{2}$, $(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$, $(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$, one obtains

$$\mathcal{M}^2=\begin{pmatrix} \rho^2 & 0 & 0 \\ 0 & \frac{1}{3}(4K^{*2}-\rho^2)+\frac{2}{3}(A-2A'+A'') & -\frac{2\sqrt{2}}{3}(K^{*2}-\rho^2)+\frac{\sqrt{2}}{3}(2A-A'-A'') \\ 0 & -\frac{2\sqrt{2}}{3}(K^{*2}-\rho^2)+\frac{\sqrt{2}}{3}(2A-A'-A'') & \frac{1}{3}(2K^{*2}+\rho^2)+\frac{1}{3}(4A+4A'+A'') \end{pmatrix}, \quad (2)$$

which shows that the mixing amplitudes contribute only to the subspace spanned by the isoscalar (self-conjugate) states (as we have not included the small effect of isospin symmetry breaking). In the following, we restrict ourselves to this subspace alone, namely the lower right 2×2 block (M^2 matrix) of Eq. (2).

The masses of the physical isoscalar states (ω and ϕ) are given by diagonalizing the M^2 matrix,

$$M^2=\begin{pmatrix} \phi^2 & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad (3)$$

and may be obtained from the invariance of the trace and determinant under a unitary transformation, such as rotation in the flavor space, viz.,

$$\mathrm{Tr}\, M^2=2K^{*2}+2A+A''=\omega^2+\phi^2, \quad (4)$$

$$\mathrm{Det}\, M^2=(2K^{*2}+\rho^2+A'')(\rho^2+2A)-2A'^2=\omega^2\phi^2. \quad (5)$$

The transition amplitudes are proportional to unknown matrix elements of the (unspecified) effective Hamiltonian. In order to reduce the number of parameters, we try to relate the amplitudes A, A' and A''. It is plausible to assume that

$$AA''\simeq A'^2. \quad (6)$$

This relation holds, e.g., for the parametrization of the two-gluon-induced transition amplitude in the form

$$A_{ij}=\frac{\Lambda}{M_iM_j}, \quad (7)$$

where Λ is an SU(3)-invariant parameter and M_i, M_j are the constituent quark masses, suggested in Ref. [18]. Here we propose the validity of the relation Eq. (6) for a quark mixing amplitude with an arbitrary number of gluons.

In accordance with Eq. (6), we introduce a parameter r:

$$A'=Ar, \quad A''=Ar^2, \quad (8)$$

Since from Eq. (7) $A>A'>A''$, one can expect $r\leqslant1$.

Equations (4), (5) can be rewritten in terms of r and A as follows:

$$\omega^2+\phi^2=2K^{*2}+A(2+r^2), \quad (9)$$

$$[4K^{*2}-(2+r^2)\omega^2-(2-r^2)\rho^2]\times[(2+r^2)\phi^2+(2-r^2)\rho^2-4K^{*2}]=8r^2(K^{*2}-\rho^2)^2. \quad (10)$$

Equation (10) is a modified version of Schwinger's quartic mass formula [19], in which annihilation effects are taken into account. It is interesting to note that in the case of SU(3)-invariant quark mixing amplitudes (i.e., $r=1$,) it reduces to Schwinger's original relation, *independent of the value of* A.

TABLE I. Meson masses (in GeV). I stands for isospin, and ω, ϕ indicate the isoscalar mostly singlet and octet states, respectively.

J^{PC}	$I=1$	$I=1/2$	$I=0,\ \omega$	$I=0,\ \phi$
0^{-+}	0.1373	0.4957	0.9578	0.5475
0^{++}	1.318	1.429	0.98/1.1	1.5
1^{--}	0.760	0.8961	0.8919	1.0194
2^{++}	1.318	1.429	1.275	1.525
2^{-+}	1.67	1.78	1.65	1.88
3^{--}	1.69	1.78	1.67	1.86

Using Eqs. (4), (5), (8), A and r can be expressed in terms of the meson masses [20]:

$$A=\frac{1}{4}\,\frac{(\omega^2-\rho^2)(\phi^2-\rho^2)}{K^{*2}-\rho^2}, \tag{11}$$

$$r^2=2\,\frac{(\phi^2+\rho^2-2K^{*2})(2K^{*2}-\rho^2-\omega^2)}{(\phi^2-\rho^2)(\omega^2-\rho^2)}. \tag{12}$$

Note that A is small due to the near mass-degeneracy of ρ and ω states which is related to the near-ideal mixing. This smallness is a confirmation of the OZI rule. Since both the denominator and numerator of Eq. (12) contain nearly vanishing factors [the ρ and ω numerator factor of Eq. (11) and a factor which would vanish if the Gell-Mann-Okubo relation were exact, respectively], any small change in the mass values induces a large change in r^2. (For example, a one percent change in the K^* mass makes r imaginary.) Conversely, the masses derived from the relations below are relatively insensitive to the value of r. To determine the glueball masses by those relations, we choose a set of meson masses which give $r\leqslant 1$. Masses we use as input are given in Table I [5,21–23].

III. GLUEBALL-MESON MASS RELATIONS

In this section we derive the meson-glueball mass relations.

The OZI suppression rule may be interpreted in terms of the Feynman box graph connecting the annihilation of $q\bar{q}$ of one flavor and the pair creation of $q'\bar{q}'$ of another flavor, plus all gluonic and quark loop dressings thereof. To obtain a physical interpretation, it is convenient to consider the various time orderings of these graphs in the overall rest frame of the annihilating pair. Here we see that graphs with the form of an overlapping double hairpin reflect multimeson ($q\bar{q}$ meson) intermediate states, while others reflect essentially purely gluonic (ignoring fully closed quark loops such as would be eliminated by quenching in lattice calculations) intermediate states. The results of Ref. [15] suggest that the former strongly cancel, and we extend this to mean their contribution is entirely negligible. For the latter, which describe the usual interpretation of OZI suppression, it is natural to attempt to estimate the strength by saturating with pure glue (glueball) resonances.

TABLE II. Predictions for the glueball masses (in GeV) for three input values of the tensor glueball mass: 2.2 GeV, 2.3 GeV, 2.4 GeV.

T	V	T'	V_3	PS	S_1	S_2
2.2	2.934	2.805	2.627	0.877	1.587	1.624
2.3	3.143	2.937	2.740	0.916	1.606	1.646
2.4	3.347	3.069	2.852	0.956	1.623	1.670

Let us therefore assume that the $q\bar{q}\leftrightarrow q'\bar{q}'$ transition proceeds via gluonic intermediate states, viz.,

$$A_{qq'}=\sum_i \frac{\langle q\bar{q}|H|i\rangle\langle i|H|q'\bar{q}'\rangle}{M^2-M^2(i)}, \tag{13}$$

where H is the effective transition Hamiltonian, $|i\rangle$ is a complete set of gluonic states, $M^2(i)$ is mass squared of the intermediate state, and M^2 is mass squared of the initial (and final) state. We now further assume that the sum (13) is saturated by the lowest lying glueball with the corresponding quantum numbers. Therefore, for $q,q'=n(=u,d)$,

$$|A|\equiv|A_{nn}|\simeq\left|\frac{f^2(\omega_n^2)}{\omega_n^2-G^2}\right|,\quad f(\omega_n^2)\equiv\langle G|H|n\bar{n}\rangle|_{q^\mu q_\mu=\omega_n^2}, \tag{14}$$

and for $q,q'=s$,

$$|Ar^2|\equiv|A_{ss}|\simeq\left|\frac{f^2(\omega_s^2)}{\omega_s^2-G^2}\right|,\quad f(\omega_s^2)\equiv\langle G|H|s\bar{s}\rangle|_{q^\mu q_\mu=\omega_s^2}, \tag{15}$$

where $\omega_n^2=\rho^2$ and $\omega_s^2=2K^{*2}-\rho^2$ are the masses squared of pure $q\bar{q}$ counterparts of the physical ω and ϕ states, and G^2 is the corresponding glueball mass squared. Note that we include the lowest lying glueball only. Even though transitions via excited glueballs are suppressed by both the numerator and denominator of Eq. (13), it is of course not *a priori* clear that the sum can be well approximated by the first term only.

To proceed further, it is necessary to have some information regarding the functions $f(\omega^2)$. If one considered an analogous situation in a soluble theory (e.g., nonrelativistic QED for the bound states of lepton-antilepton pairs of various flavors), then one would expect the magnitude of an analog of $f(\omega^2)$ to vary markedly with both orbital and radial quantum numbers. We assume, however, that the prod-

TABLE III. The same as in Table II, according to mass relations of the type given by Eqs. (19), (20).

T	V	T'	V_3	PS	S_1	S_2
2.2	3.016	2.794	2.714	0.919	1.601	1.646
2.3	3.232	2.925	2.836	0.952	1.619	1.672
2.4	3.443	3.056	2.958	0.985	1.638	1.699

uct of $f(\omega_n^2)$ and $f(\omega_s^2)$ in QCD is a constant approximately independent of the quantum numbers of a meson nonet, viz.,

$$f(\omega_n^2)f(\omega_s^2)\approx\text{const.} \tag{16}$$

(This assumption is more general than the one used in Ref. [16] where $f(\omega^2)$ itself is assumed to be independent of ω^2.) We will show that this counterintuitive assumption is justified *ex post facto* by the results obtained, and we will check it for self-consistency below.

Representing now the product $A\cdot Ar^2$ in two ways, viz., from Eqs. (11), (12), and (14), (15), and using (16), one obtains a set of mass relations:

$$\frac{(T^2-a_2^2)(T^2+a_2^2-2K_2^{*2})}{(V^2-\rho^2)(V^2+\rho^2-2K^{*2})}\approx\left(\frac{K_2^{*2}-a_2^2}{K^{*2}-\rho^2}\right)^2\frac{(\phi^2+\rho^2-2K^{*2})(2K^{*2}-\rho^2-\omega^2)(\phi^2-\rho^2)(\omega^2-\rho^2)}{(f_2'^2+a_2^2-2K_2^{*2})(2K_2^{*2}-a_2^2-f_2^2)(f_2'^2-a_2^2)(f_2^2-a_2^2)}, \tag{17}$$

$$\frac{(T^2-a_2^2)(T^2+a_2^2-2K_2^{*2})}{(V_3^2-\rho_3^2)(V_3^2+\rho_3^2-2K_3^{*2})}\approx\left(\frac{K_2^{*2}-a_2^2}{K_3^{*2}-\rho_3^2}\right)^2\frac{(\phi_3^2+\rho_3^2-2K_3^{*2})(2K_3^{*2}-\rho_3^2-\omega_3^2)(\phi_3^2-\rho_3^2)(\omega_3^2-\rho_3^2)}{(f_2'^2+a_2^2-2K_2^{*2})(2K_2^{*2}-a_2^2-f_2^2)(f_2'^2-a_2^2)(f_2^2-a_2^2)},\quad\text{etc.}, \tag{18}$$

where $V,T,V_3,\ldots$ are the masses of the vector, tensor, $3^{--},\ldots$ glueballs.[3]

It is apparent from these relations that, if one of the glueball masses is chosen as an input parameter, the masses of other glueballs can be predicted, provided the corresponding $(q\bar{q})$ meson masses are known. We choose the mass of the tensor glueball as such an input parameter, $T=2.3\pm0.1$ GeV, and calculate the masses of the 0^{-+}, 0^{++}, 1^{--}, 2^{-+} and 3^{--} glueballs. (We do not calculate the masses of any other glueball states, since the $q\bar{q}$ assignments of the corresponding meson multiplets are not established so far.) Our results are presented in Table II.

In calculating the glueball masses in Table II, we used the meson masses given in Table I. Also, in calculating the value of the scalar glueball mass, mass degeneracy of the corresponding isovector and isodoublet states of the tensor and scalar meson nonets was assumed, in agreement with Ref. [24]: $a_0=a_2$, $K_0^*=K_2^*$.[4] The mass of the isoscalar mostly octet state was taken to be 1.5 GeV [5,27], and two cases were considered: The mass of the isoscalar mostly singlet state was taken to be 0.98 GeV [5] or 1.1 GeV [27] [for which the values of r, as calculated from Eq. (12), are, respectively, 0.82 and 0.90], and the results presented in Table II under S_1 and S_2, respectively.

Approximation $r\approx1$

Let us also consider the approximation of flavor-independent quark annihilation amplitudes. It then follows from Eq. (4) (with $A''=A$) that $A=(\phi^2+\omega^2-2K^{*2})/3$, and Eqs. (17), (18) are replaced by

$$\frac{(T^2-a_2^2)(T^2+a_2^2-2K_2^{*2})}{(V^2-\rho^2)(V^2+\rho^2-2K^{*2})}\approx\left(\frac{\phi^2+\omega^2-2K^{*2}}{f_2^2+f_2'^2-2K_2^{*2}}\right)^2, \tag{19}$$

$$\frac{(T^2-a_2^2)(T^2+a_2^2-2K_2^{*2})}{(V_3^2-\rho_3^2)(V_3^2+\rho_3^2-2K_3^{*2})}\approx\left(\frac{\phi_3^2+\omega_3^2-2K_3^{*2}}{f_2^2+f_2'^2-2K_2^{*2}}\right)^2,\quad\text{etc.} \tag{20}$$

Glueball masses calculated from Eqs. (19), (20) are presented in Table III. Comparison of the results given in Tables II and III shows that they are not very sensitive to the precise values of r, except perhaps for the 1^{--} glueball.

The glueball masses depend only weakly on isovector masses. In order to demonstrate this point, let us rewrite, e.g., Eq. (19) which constitutes a quadratic equation for V^2, and expand it as follows:

$$V^2=K^{*2}+\sqrt{\frac{\Delta_T}{\Delta_V}}\,(T^2-K_2^{*2})-\frac{1}{2}\sqrt{\frac{\Delta_T}{\Delta_V}}\frac{(K_2^{*2}-a_2^2)^2}{(T^2-K_2^{*2})}+\frac{1}{2}\sqrt{\frac{\Delta_V}{\Delta_T}}\frac{(K^{*2}-\rho^2)^2}{(T^2-K_2^{*2})}+\cdots, \tag{21}$$

where $\Delta_V=(\phi^2+\omega^2-2K^{*2})^2$ and $\Delta_T=(f_2^2+f_2'^2-2K_2^{*2})^2$. Note that each of these quantities describes violation of the Gell-Mann–Okubo relation, and so is not large; nonetheless, the values vary from multiplet to multiplet. To the extent that we obtain self-consistency and agreement with experiment, this suggests that we have indeed identified the origin of OZI-violating contributions in terms of glueball intermediate states. [In deriving this relation we used the fact that the SU(3) violating terms ($K^{*2}-\rho^2$, etc.) are small.]

[3]In Tables II and III below, T' stands for the mass of the 2^{-+} glueball.

[4]The latter is in agreement with data [5]; the experimental candidate which satisfies the former ($a_0=1322\pm30$ MeV) was seen by LASS [25] and GAMS [26]. As for the candidate $a_0(1450)$ contained in [5], we note that, as we have checked, the results are not very sensitive to the mass of the scalar isovector state: For $a_0\simeq1.4\pm0.25$ GeV, relations of both the type in Eqs. (17) and (18) and in Eqs. (19) and (20) below predict the scalar glueball mass in the range $\simeq1.65\pm0.05$ GeV.

TABLE IV. The values of $f(\omega_n^2)$, $f(\omega_s^2)$, $f(\omega_n^2)f(\omega_s^2)$, for the five meson multiplets. Error estimates were computed from the variations induced by the range of glueball mass values.

J^{PC}	$f(\omega_n^2)$, GeV^2	$f(\omega_s^2)$, GeV^2	$f(\omega_n^2)f(\omega_s^2)$, GeV^4
0^{++}	0.528 ± 0.04	0.232 ± 0.08	0.122 ± 0.05
1^{--}	0.409 ± 0.05	0.303 ± 0.04	0.124 ± 0.03
2^{++}	0.437 ± 0.06	0.273 ± 0.04	0.119 ± 0.03
2^{-+}	0.439 ± 0.07	0.275 ± 0.05	0.121 ± 0.04
3^{--}	0.402 ± 0.07	0.319 ± 0.06	0.128 ± 0.05

It is also important to recognize that in the Gell-Mann–Okubo limit, the ratio of Δ's in Eq. (21) is undefined, so the results of our analysis are highly sensitive to the input meson masses which produce small violations of the Gell-Mann–Okubo relations.

Consistency check

We now return to check the consistency of Eq. (16). Using the glueball masses from Tables II, III, we calculate the values of $f(\omega_n^2)$, $f(\omega_s^2)$, with the help of Eqs. (11),(12),(14) and (15), and the product $f(\omega_n^2)f(\omega_s^2)$ for the five multiplets. (We use $f_0'=980$ MeV for the scalar meson nonet.) The results are shown in Table IV.

Comparison of the results for $f(\omega_n^2)f(\omega_s^2)$ shows that they are consistent with Eq. (16), up to ~7% accuracy, which is in qualitative agreement with the accuracy of the values predicted for the glueball masses (e.g., 3.2 ± 0.2 GeV for the vector glueball is ~6.5% accuracy). It is interesting to note that, in Table IV, both $f(\omega_n^2)$ and $f(\omega_s^2)$ are in even closer agreement for the particular glueball pairs: $(1^{--},3^{--})$ and $(2^{++},2^{-+})$.

Moreover, the assumption (16) seems to be justified for radial excitations as well. We calculated the value of $f(\omega_n^2)f(\omega_s^2)$ for $2\ ^1S_0$ multiplet with $\eta(1295)$, $\pi(1350)$, $K(1430)$ and $\eta(1490)$ as input masses. The result is again in remarkable agreement with the values given in Table IV. Specifically, we obtained $f(\omega_n^2)=0.389$, $f(\omega_s^2)=0.334$ and $f(\omega_n^2)f(\omega_s^2)=0.130$. The origin of the validity of Eq. (16) remains a mystery at present.

IV. FURTHER IMPLICATIONS

Let us examine our results from another point of view. We have found that the vector and spin-3 glueballs have masses around 3 GeV. Can this fact find its simple explanation in, e.g., QCD phenomenology? The answer is positive. Both states are composed of three constituent gluons, and a naive scaling from the two-gluon 2^{++} glueball to the 3-gluon case gives $M(3g)\simeq1.5M(2g)\simeq3.3$ GeV, with $M(2g)\simeq2.2$ GeV. Also, the original constituent gluon model predicts $M(1^{--})/M(2^{++})\simeq M(3^{--})/M(2^{++})\simeq1.5$ [7]. Note that the value of the vector glueball mass obtained in this paper is consistent with the Brodsky-Lepage-Tuan domain [28]

$$|M(1^{--})-M(J/\psi)|<80 \text{ MeV}, \tag{22}$$

obtained from the ratio of the measured widths: $\Gamma(\psi'\to V\to\rho\pi)/\Gamma(J/\psi\to V\to\rho\pi)$.

The value of the pseudoscalar glueball mass obtained here, $\sim0.93\pm0.05$ GeV, however, is inconsistent with the lattice result [4,6] 2490 ± 140 MeV. A possible explanation for this is that we have not included instanton effects. Instanton effects are irrelevant for all other channels, but they may be important for pseudoscalars (and possibly scalars). Indeed, as has been shown separately [29], noninstanton annihilation effects alone cannot provide the mass splitting of ~500 MeV required to reproduce the physical pseudoscalar meson spectrum. The use of the pseudoscalar glueball mass $M(0^{-+})\simeq M(2^{++})=2.2$–$2.4$ GeV in relations of the type (19), (20) leads to $\eta'^2+\eta^2-2K^2=0.08-0.09$ GeV^2, in contrast to the 0.73 GeV^2 required by pseudoscalar meson spectroscopy. The remaining ~0.65 GeV^2 would then be expected to reflect the contribution of instantons [29], so the introduction of instanton effects which are known to be strong in this sector, in addition to those of gluons, changes the situation drastically.

On the other hand, the gluon annihilation effects seem to be sufficient to reproduce the scalar meson spectrum. The value obtained for the scalar glueball mass, ~1650 ±50 MeV, is in agreement with lattice results 1600 ± 100 MeV [2–4]. We note that the mass predicted for the 2^{-+} glueball, $\simeq2.95\pm0.15$ GeV, is also in agreement with lattice QCD results: 3.07 ± 0.15 GeV [6] and $\simeq3$ GeV [2]. Finally, we note that QCD sum rules predict $M(0^{++})=1.5\pm0.2$ GeV, $M(2^{++})\simeq2.0\pm0.1$ GeV, $M(0^{-+})\simeq2.05\pm0.2$ GeV, and find the $3g$-glueball mass to be $\simeq3.1$ GeV [30].

All the glueball masses calculated above, except for the scalar one, are much higher than those of the corresponding mesons, and therefore cannot appreciably mix with the latter.[5] Thus, in addition to the pseudoscalar glueball, only the scalar glueball may be expected to mix considerably with the scalar isoscalar states, since it lies in a mass range spanned by the latter; for any other nonet the use of the 2×2 mass matrix (3) is justified. However, even for the scalar glueball, the mass shift produced by the mixing with quarkonia need not necessarily be large. In a model considered in Ref. [31], for example, the bare glueball mass of 1635 MeV is shifted up to 1710 MeV, and the bare $s\bar{s}$ mass of 1516 MeV is shifted down to 1500 MeV, both modest effects. We also note that relations of the type in Eqs. (17), (18) can be also used to predict masses of problematic quarkonia; e.g., the mass of the $2\ ^3S_1$ isodoublet state.

Glueball Regge trajectories

Finally, we briefly consider the question of the glueball Regge trajectories. A knowledge of these trajectories may be

[5] A mixing of the quarkonia with the four-quark states of the same quantum numbers which may have comparable masses, although not precluded, is not treated here. Although we can no more justify this than any others do, we suspect that justification may be related to the apparently consistent neglect here of multi-($q\bar{q}$-)meson intermediate states in the OZI violating mixing amplitudes.

useful for determining masses of glueballs with exotic quantum numbers (e.g., 0^{+-}, 0^{--}, 1^{-+}, etc.), for which no $q\bar{q}$ counterparts exist so that relations of the type given by Eqs. (17) and (18) or (19) and (20) therefore cannot be applied.

It is widely believed that the tensor glueball is the first particle lying on the (quasi)-linear pomeron Regge trajectory [32], $\alpha(t)=\alpha_{\rm P}(0)+\alpha'_{\rm P}\cdot t$. The parameters have been extracted from experimental data on diffractive deep-inelastic scattering: $\alpha_{\rm P}(0)=1.07\pm0.03$, $\alpha'_{\rm P}=0.25\pm0.01$ GeV^{-2} [33], or $\alpha_{\rm P}(0)=1.086$, $\alpha'_{\rm P}=0.25$ GeV^{-2} [34]. Higher values of $\alpha'_{\rm P}$ have been considered in the literature (in GeV^{-2}): $\alpha'_{\rm P}=0.3$ [35], 0.3–0.36 [36], 0.311 [37], 0.32–0.46 [38]. These differences may be reconciled if one assumes nonlinearity of trajectories as suggested theoretically in Refs. [39] and [40] (taking into account that the slopes have been extracted from data in differing momentum transfer regions), which may have been experimentally observed at CERN [41].

If we take the glueball masses from Table III and ignore the difference in intercepts of parity partner trajectories, then under the assumption of a common linear trajectory for the 2^{++} and 3^{--} glueballs, we obtain

$$\alpha'_G=\frac{1}{M^2(3^{--})-M^2(2^{++})}=0.37\pm0.04, \qquad (23)$$

where the error is taken from the variation induced by the range of input masses for the tensor glueball. (The result from Table II is 25% larger.) Similar results may be obtained from other combinations.

Our results, therefore, are consistent with the standard expectation that the glueballs populate (quasi)-linear Regge trajectories with slope $\simeq 0.3\pm0.1$ GeV^{-2}, in agreement with Refs. [33, 34, 36–38]. It should be kept in mind, however, that the power of this demonstration is limited by the difficulty of examining the entire space of meson masses consistent with $r=1$. So far, we have only studied the single point in that space defined by Table I.

V. SUMMARY AND CONCLUSIONS

In this paper we used OZI suppressed processes in the isoscalar sectors to find new glueball-meson mass relations. First, motivated by a parametrization of the two-gluon amplitude in terms of quark masses and a hadronic scale Λ, we assumed that the transition amplitudes for quark mixing can be related via a parameter r, which can be expected to satisfy $r\leqslant 1$. Since the masses turn out to be insensitive to the exact value of r, r cannot be determined from data, and we fixed the relevant meson masses to give $r\leqslant 1$. We then assumed cancellation of two step hadronic contributions and glueball dominance. Finally, we assumed that matrix elements of the (in practice unknown) interaction Hamiltonian between $q\bar{q}$ and glueball with the corresponding quantum numbers are independent of the quantum numbers of the meson nonet. Under these assumptions, we established new glueball-meson mass relations, and used them to predict the glueball masses. With the tensor glueball mass 2.2 ± 0.1 GeV chosen as an input parameter in these relations, we obtained the following glueball masses:

$$M(0^{++})\simeq 1.65\pm0.05 \text{ GeV},$$

$$M(1^{--})\simeq 3.2\pm0.2 \text{ GeV},$$

$$M(2^{-+})\simeq 2.95\pm0.15 \text{ GeV},$$

$$M(3^{--})\simeq 2.8\pm0.15 \text{ GeV}.$$

We have shown that these glueball masses are consistent with (quasi)-linear Regge trajectories, with slope $\simeq 0.3\pm0.1$ GeV^{-2}. Our calculation is self-consistent, in the sense that the results have not led to any contradictions with our assumptions.

Finally, we should comment on the unnatural, but apparently self-consistent, assumption regarding the behavior of the annihilation transition amplitudes (f), namely the state independence of their light-quark-strange-quark product, as given in Eq. (16), which does not seem to depend upon orbital or even radial quantum numbers of the quarks involved. If the glueball mass spectrum we derive is experimentally confirmed, supporting our plethora of assumptions, it will be necessary to find a physical interpretation of this regularity. Here we merely note that while perturbative analyses would suggest a strong variation, they depend on a specific behavior of the t-channel segment of the quark propagator involved in the annihilation. However, if this exchange is also (quark-) Reggeized [42], then the amplitude could be expected to be dominated by the trajectory intercept, i.e., at the minimum momentum transfer possible. This quantity is indeed multiplet independent, so long as the trajectories involved are degenerate (rather than split by parity, or daughters, for example). It would be intriguing to confirm such a consistency between Regge trajectories for quarks and color singlet hadrons.

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