$B \to \gamma\gamma$ decays in a bound state model

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Starting from an effective quark level Lagrangian for $b \to s\gamma$ and $b \to s\gamma\gamma$, we calculate the process $B_s \to \gamma\gamma$ using the bound state model of Holdom and Sutherland. We discuss this model and its limitations. We calculate the contributions from the well-known $b \to s\gamma$ magnetic-moment operator. There is also a non-negligible contribution coming from operators that vanish on the free-quark mass shell. We find a branching ratio of the order of 10^{-7}. [S0556-2821(98)02621-6]

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I. INTRODUCTION

Much attention has been devoted to the decay mode $B \to \gamma K^*$ [1–4]. Recently Holdom and Sutherland [3,5] have invented a model for a heavy-light system of quarks such as the B mesons. In this model matrix elements of quark operators are obtained in terms of loop integrals. The model involves vertices where quarks couple to mesons, essentially as in the chiral quark model [6]. The difference is that, in the present case, the meson-quark vertex includes a form factor such that the effect of high momenta flowing into a light quark is damped. Holdom and Sutherland calculated $B \to K^*\gamma$ within this model with a reasonable result. In this paper we calculate $B_s \to \gamma\gamma$ within the same framework.

In the process $b \to s\gamma$ and its hadronic $B \to K^*\gamma$ counterpart, the light quark is merely a spectator. However, additional effects from the motion of the light quark might be expected in flavor-changing two-photon decays, such as $b \to s\gamma\gamma$ and its hadronic $B_s \to \gamma\gamma$ version, where the light and heavy quarks fuse together.

In a previous paper [7], one of us made an estimate of $B_s \to \gamma\gamma$. In that paper the model was not exploited in detail—just the general idea of the model was used. The underlying quark processes $b \to s\gamma$ and $b \to s\gamma\gamma$ are studied in many papers and are established [7–16]. To calculate the physical process $B_s \to \gamma\gamma$ various model dependent assumptions are used [7,8,10,12,13]. The purpose of the present paper is to study this process using the bound state model of [3,5]. Operators for the electroweak flavor change in $b \to s\gamma\gamma$ transitions contain a genuine off-shell piece, which vanishes on the free-quark mass shell [8], but they are expected to give non-negligible contributions in a bound state calculation [7,17,18].

The parameters of the bound state model [3,5] are determined by certain requirements for the $B_s \to B_s$ amplitude (B_s-meson self-energy) calculated within the model. Unfortunately, these requirements lead to imaginary parts for this amplitude and most amplitudes in general. Because of confinement effects, such imaginary parts should not be there, and we follow the procedure of [3,5] and ignore these imaginary parts. This is of course a drawback of the model. Another drawback is that the obtained value for f_B is of order 1/3 of the measured value.

The outline of the paper is as follows: In Sec. II we present the effective Lagrangian at the quark level. In Sec. III the bound state model is presented, and in Sec. IV the calculation of the $B_s \to 2\gamma$ decay amplitude is performed. Finally, Sec. V contains our conclusions.

II. QUARK EFFECTIVE LAGRANGIAN FOR RADIATIVE $b \to s$ TRANSITIONS

The effective Lagrangian for radiative $b \to s$ transitions are presented in [7–9,17], and we will keep the notation of [7,17]. The one-loop level electroweak transitions $b \to s\gamma$ and $b \to s\gamma\gamma$, related by the Ward identities, can be combined into an effective Lagrangian

$$\mathcal{L}(b \to s)_\gamma = B(M_W)\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}(\overline{s_L}i\overleftrightarrow{D}_\lambda\gamma_\rho b_L), \qquad (1)$$

where the coefficient $B(M_W) \sim eG_F\lambda_{KM}$ contains the result of the relevant loop diagrams, and λ_{KM} is the relevant KM factor for $b \to s$ transitions. The covariant derivative contains both the gluon and photon fields. One can decompose this expression into two pieces $\mathcal{L}(s \to d)_\gamma = \mathcal{L}_F + \mathcal{L}_\sigma$, where $\mathcal{L}_\sigma$ is the well-known off-diagonal magnetic moment term:

$$\mathcal{L}_\sigma = B_\sigma \overline{s}(m_b\sigma_{\mu\nu}F^{\mu\nu}R + m_s\sigma_{\mu\nu}F^{\mu\nu}L)b, \qquad (2)$$

which is obtained from $\mathcal{L}(b \to s)_\gamma$ in Eq. (1) when going on shell with the s and b quarks (or using the equations of motion). Then there is a remaining piece

$$\mathcal{L}_F = B_F\overline{s}[(i\gamma\cdot D - m_s)\sigma_{\mu\nu}F^{\mu\nu}L + \sigma_{\mu\nu}F^{\mu\nu}R(i\gamma\cdot D - m_b)]b, \qquad (3)$$

which vanish by applying the (perturbative) equation of motion (or in momentum space, for on-shell free quarks), i.e., for $(i\gamma\cdot D - m_{b,s}) \to 0$. [In Eq. (3) it is understood that the covariant derivative acts on the nearest fermion field to the left or right.]In this case $\mathcal{L}(b \to s)_\gamma$ reduces to its on-shell

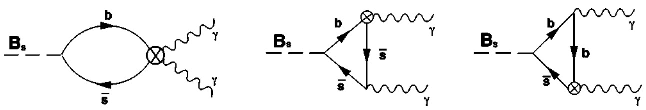

FIG. 1. The two-photon contribution and the one-photon contributions to $B_s \to 2\gamma$. The cross within the circle denotes the effective vertex corresponding to $\tilde{\mathcal{L}}_F$ or $\tilde{\mathcal{L}}_\sigma$ in the last two diagrams.

remnant $\mathcal{L}_\sigma$ in Eq. (2). In [7] it was shown that the genuine off-shell $\mathcal{L}_F$ in Eq. (3) term gives a nonzero contribution for the bound state case.

The expressions (2) and (3) take into account that the coefficients B_F of $\mathcal{L}_F$ and B_σ of $\mathcal{L}_\sigma$, both being equal to $B(M_W)$ at the W scale, evolve differently down to the $\mu = m_b$ scale. This difference between B_F and B_σ is due to different anomalous dimensions of the operators in Eqs. (2) and (3). The B's will have the form [7,9,11]

$$B_{\sigma,F} = \frac{4G_F}{\sqrt{2}} \lambda_{KM} \frac{e}{16\pi^2} C_7^{\sigma,F}, \tag{4}$$

where [11] $C_7^\sigma = -0.28$ and [7]

$$\frac{C_7^F}{C_7^\sigma} \simeq 4/3, \tag{5}$$

at the scale $\mu = m_b$. Bound state models often involve the constituent quark mass instead of the current one. Therefore, we make another decomposition of the total radiative-decay Lagrangian

$$\mathcal{L}(b \to s)_\gamma = \tilde{\mathcal{L}}_F + \tilde{\mathcal{L}}_\sigma, \tag{6}$$

where $\tilde{\mathcal{L}}_F$ is obtained from $\mathcal{L}_F$ simply by replacing the current masses (m_q) by the constituent ones (M_q),

$$\tilde{\mathcal{L}}_F = \mathcal{L}_F(m_q \to M_q), \tag{7}$$

and $\tilde{\mathcal{L}}_\sigma$ is the remainder:

$$\tilde{\mathcal{L}}_\sigma = \overline{s}(\tilde{B}_R \sigma_{\mu\nu} F^{\mu\nu} R + \tilde{B}_L \sigma_{\mu\nu} F^{\mu\nu} L) b, \tag{8}$$

where

$$\tilde{B}_L = B_F(M_s - m_s) + B_\sigma m_s, \quad \tilde{B}_R = B_F(M_b - m_b) + B_\sigma m_b. \tag{9}$$

To calculate the $B_s \to 2\gamma$ amplitude from the Lagrangian $\mathcal{L}(b \to s)_\gamma$ in Eq. (1) one needs some bound state model. Let us in this section be quite general, and just assume that a bound state model involving a meson-quark vertex exists. Moreover, we find it suitable to split the generically off-shell operator $\tilde{\mathcal{L}}_F$ into the one- and two-photon pieces, $\tilde{\mathcal{L}}_F = \tilde{\mathcal{L}}_F^{1\gamma} + \tilde{\mathcal{L}}_F^{2\gamma}$, in an obvious notation.

The piece $\tilde{\mathcal{L}}_F^{2\gamma}$ gives rise to the first quark loop diagrams in Fig. 1. Further, the piece $\tilde{\mathcal{L}}_F^{1\gamma}$ gives rise to the second and third diagrams in Fig. 1. (This interaction is represented by the cross within the circle.) In [7] it is shown in detail that even if there is a cancellation between the contributions from $\tilde{\mathcal{L}}_F^{2\gamma}$ and a part of $\tilde{\mathcal{L}}_F^{1\gamma}$, there is a remaining nonzero genuine off-shell contribution which has been neglected in the literature by appealing to the (perturbative) equations of motion. This result is *completely general*, and does *not depend on the details of the bound state dynamics*.

The diagrams for the magnetic moment term $\tilde{\mathcal{L}}_\sigma$ are the same as for $\tilde{\mathcal{L}}_F^{1\gamma}$. It turns out that the magnetic moment term is the most important numerically for $B_s \to 2\gamma$, in contrast with the case $K \to 2\gamma$ where the off-shell contribution was most important [17]. This reflects the fact that bound state effects are more inportant in K mesons compared to B mesons.

III. BOUND STATE MODEL

The flavor-changing radiative vertices at the quark level presented in the previous section have to be supplemented by a quark-meson interaction in order to obtain the amplitude for the physical process $B \to 2\gamma$. Here the heavy B meson cannot be treated as a Goldstone boson like in the chiral quark model adopted earlier for the analogous $K \to \gamma\gamma$ decay [17].

However, the pseudoscalar character of B mesons allows us to parametrize this interaction in a simple way, replacing the term used in chiral quark model calculations [6,17] by

$$\mathcal{L}_{mod} = G_B \overline{s} \gamma_5 b B_s. \tag{10}$$

We use a model of this type proposed by Holdom and Sutherland [3,5]. This is a relativistic model of mesons containing a heavy quark and a light antiquark, and where transition amplitudes are represented by attaching the B_s meson to a loop involving the two constituent quarks. Note that the B_s field is considered as external, such that B_s is not propagating before the quarks are integrated out. The effect of the quarks being confined within the meson is modeled in the meson-quark vertex, and the quarks are technically considered as free in the loop (although they are assigned effective masses which differs from the current masses).

In a meson containing a heavy and a light quark, the heavy quark carries most of the energy and momentum in the system. This means that contributions with large momentum flowing through the light-quark line have to be suppressed [5]. One reproduces this effect by including factors in the vertices which damp the loop integral when the light-quark momentum exceeds some scale Λ_B. The vertex which connects the meson line to the loop is written as

$$G_B = \frac{Z_B^2}{\Lambda_B^2 - q^2}, \tag{11}$$

where q is the light-quark momentum. The light and heavy quarks in the loop are represented by standard fermion propagators with effective masses M_s and M_b, respectively. The effective masses are of order the constituent masses of the quarks, which are expected to differ from the current ones by 2–300 MeV. They are in principle free parameters

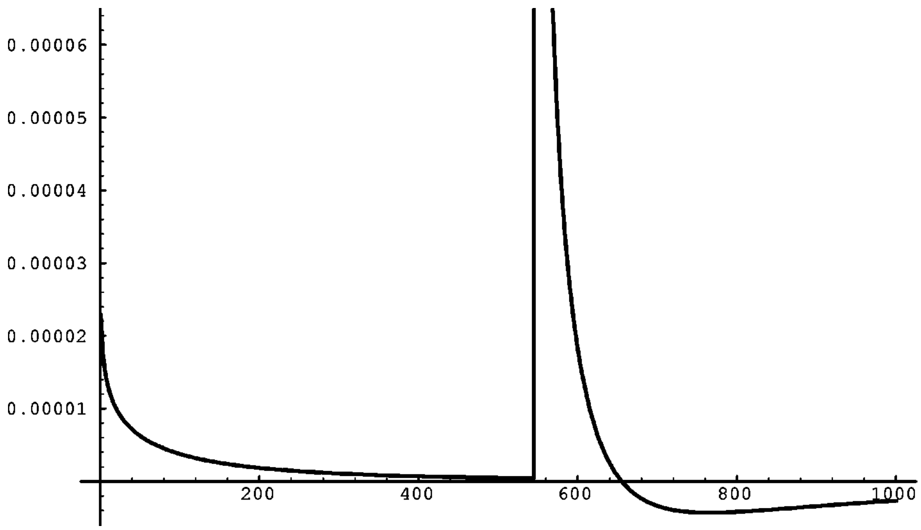

FIG. 2. Re$[\Sigma(\Lambda)]$. We have used $k^2 = M_B^2$, $M_b = 4830$ MeV, and $M_s = 400$ MeV, and we have plotted Re(Σ) as a function of Λ. We find Λ_B to be the value of Λ corresponding to Re$[\Sigma(\Lambda)] = 0$.

of the model. One advantage of this model is that the meson-quark vertex contains a factor of q^{-2} in the denominator, which will make the loop diagrams convergent. These diagrams would be divergent if we had used vertices with no momentum dependence.

The parameters Z_B and Λ_B are different for different meson flavor and spin. Our first task is to find the parameters of this model for the B_s meson.

A. Determining the parameters of the model

As mentioned in the previous section, the quark masses, in our case M_b and M_s, are in principle free parameters, and should strictly speaking be determined from some physical requirements. We know, however, that in order to have a realistic model, the quark masses cannot be completely arbitrary, and we know approximately within what range they should be. In [3], where the process $B_d \to K^* \gamma$ is studied, the effective masses M_b and M_s are found to be [3]

$$M_b = 4830 \text{ MeV}, \quad M_s = 400 \text{ MeV}. \tag{12}$$

In our paper the relevant meson is B_s; so we must in addition find the parameters Z_B, Λ_B for this meson, having the physical mass $M_B = 5375$ MeV [4].

To find Λ_B, we first calculate the Feynman diagram representing the $B_s \to B_s$ amplitude, i.e., the meson self-energy $\Sigma(k^2)$, given by

$$-i\Sigma(k) = (-1) N_c \int \frac{d^4 q}{(2\pi)^4} \times \mathrm{Tr}\{ i\gamma_5 G_B i S_{F_s}(q) i\gamma_5 G_B i S_{F_b}(q+k)\}, \tag{13}$$

where G_B corresponds to Eq. (11) for the B_s meson, N_c is the number of color, and q is the s-quark momentum. This self-energy is now required to vanish at the B_s-meson mass shell:

$$\Sigma(k^2 = M_B^2) = 0, \tag{14}$$

where k is the B_s-meson momentum. Now we insert $k^2 = M_B^2$ (for $M_B = 5375$ MeV) and the values for the quark masses (12) in the expression for $\Sigma(k)$. There is now only one variable in Σ, namely, Λ_B. (Z_B only enters as an overall factor.) When we now plot $\Sigma(\Lambda_B)$ with respect to Λ_B, we want to determine the value of Λ_B which gives $\Sigma(\Lambda_B) = 0$ according to the requirement in Eq. (14). When plotting $\Sigma(\Lambda_B)$, however, we observe that there arises an imaginary part because $M_s + M_b < M_B$. This imaginary part will be ignored [3,5]. The value of Λ_B is then determined by the requirement that Re$[\Sigma(\Lambda_B)] = 0$.

Figure 2 shows the graph for Re$[\Sigma(\Lambda)]$. In this plot, we have used $k^2 = M_B^2$, and the quarks masses of Eq. (12). We observe that Re$[\Sigma(\Lambda_B)] = 0$ for $\Lambda_B \approx 655$ MeV.

We now want to determine the last unknown parameter of the model, namely, Z_B. To do this, we write the meson self-energy $\Sigma(k^2)$ in Eq. (13) as

$$\Sigma(k^2) = \Sigma(k^2 = M_B^2) + (k^2 - M_B^2)\hat{\Sigma}(k^2), \tag{15}$$

and we use the requirement

$$\hat{\Sigma}(k^2 = M_B^2) = 1, \tag{16}$$

which will determine Z_B. The model assumes that the B_s-meson field is only present in the Lagrangian density term $\mathcal{L}_{mod}$ of Eq. (10), and that there are no terms for propagating mesons. The requirement (16) garantees that the one-loop diagram for $B_s \to B_s$, obtained from using Eq. (10) twice, gives the Klein-Gordon equation for B_s with no extra wave function renormalization.

Our numerical analysis shows that Eq. (16) is satisfied for the value of $Z_B = 904$ MeV. The values of the parameters are summarized in Table I.

B. Decay constant f_B within the model

Because we consider a system of a quark and an antiquark, we want to check if the obtained values of the parameters give us a physically reasonable value for the B_s decay constant f_B. We calculate the Feynman diagram for the ma-

TABLE I. The parameters of the model.

$M_s = 400$ MeV
$M_b = 4830$ MeV $\Lambda_B = 655$ MeV $Z_B = 904$ MeV

trix element of the axial vector current between the meson and hadronic vacuum, and compare this with the physically measurable decay constant f_B, defined by

$$\langle 0|\overline{s}\gamma^{\mu}\gamma_5 b|B_s\rangle = i f_B p^{\mu}. \tag{17}$$

Within our model f_B is given by the following expression:

$$i f_B p^{\mu} = -N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\{i\gamma_5 G_B i S_{F_s}(q)\gamma_{\mu}\gamma_5 i S_{F_b}(q+p)\}, \tag{18}$$

giving the numerical value $f_B = 67$ MeV, which is of order 1/3 of the physical value. Thus the model only reproduces f_B within an order of magnitude.

IV. QUARK-LOOP $B_s \to \gamma\gamma$ AMPLITUDE

Having determined the parameters of the model, we are in position to calculate the $B_s \to 2\gamma$ amplitude. Using the vertices obtained from the effective Lagrangians $\tilde{\mathcal{L}}_F$ and $\tilde{\mathcal{L}}_\sigma$, containing the relevant KM factors for the $b \to s$ transition, we obtain an amplitude of the form

$$\mathcal{M}(B_s \to \gamma\gamma) = \mathcal{M}_{\mu\nu}\epsilon_{\mu}(k_1)\epsilon(k_2)_{\nu}, \tag{19}$$

where the ϵ's are the photon polarization vectors and

$$\mathcal{M}^{\mu\nu} = \frac{N_c Z_B^2 e}{8\pi^2}\{A_+ i(g^{\mu\nu}k_1\cdot k_2 - k_1^{\nu}k_2^{\mu}) + A_- \varepsilon^{\mu\alpha\nu\beta}(k_1)_{\alpha}(k_2)_{\beta}\}, \tag{20}$$

where the quantities $A_{\pm}$ are obtained from the diagram in Fig. 1 and given in Table II.

The numbers in the coloums named ''numerically'' are the result of the numerical integration. These numbers are very sensitive to the explicit choice of the quark masses and Λ_B. In the coloumn named ''analytically'' we have written down the result of the rough approximation $M_s \simeq \Lambda \ll M_B \simeq M_b$. This approximation gives amplitudes $A_{\pm}$ almost one order of magnitude below the exact numerical evaluation, showing the sensitivity of the involved mass parameters.

The decay rate is given by

$$\Gamma(B_s \to 2\gamma) = \frac{N_c^2 Z_B^4 e^2 M_B^3}{2048\ \pi^5}(|A_+|^2 + |A_-|^2). \tag{21}$$

Using the numbers in Table II (from the coloumn named ''numerically'') combined with the values of $B_{\sigma,F}$ we find the branching rate

$$\mathcal{B}(B_s \to 2\gamma) = (1.4\text{–}2.3)\times 10^{-7}. \tag{22}$$

V. CONCLUSIONS

Within the model invented in Refs. [3,5], we have calculated the branching ratio for the process $B_s \to 2\gamma$ which we find to have a branching ratio of order $\sim 10^{-7}$. This is the same order of magnitude as obtained by other groups, using different assumptions [7,10,12–14].

The model, which works well for $B_d \to \gamma K^*$ [3], has some drawbacks and does not work so well for $B_s \to 2\gamma$, when a heavy and a light quark annihilate: (i) The obtained value for f_B is only 1/3 of the physical one; (ii) the results are sensitive to variations of the involved parameters; (iii) the requirements (14),(16) lead to unphysical imaginary parts. These might be simply dropped, but mathematically, the real parts are also influenced by the unphysical imaginary parts (especially near the threshholds).

Nevertheless, the value in Eq. (22) is what we find if we calculate the branching ratio for $B_s \to 2\gamma$ within the framework of this model. Moreover, in spite of the drawbacks of

TABLE II. A_+ and A_-.

	Numerically	Analytically
A_+	$-\frac{3}{\Lambda_B^2}[0.2966](\tilde{B}_R - \tilde{B}_L)$ $+\frac{B_F}{\Lambda_B^2}(0.9137M_s - 0.03004M_b)$	$\frac{B_\sigma}{M}\left[-\frac{3}{2} - \frac{M}{m}\frac{\pi}{2} + 2\ln\left(\frac{M}{m}\right) + \frac{3\pi}{4}\frac{m}{M}\right]$ $+\frac{B_F}{2}\left[\frac{1}{m} - \frac{\pi^2}{3M}\right]$
A_-	$-\frac{3}{\Lambda_B^2}[0.2966](\tilde{B}_R + \tilde{B}_L)$ $+\frac{B_F}{\Lambda_B^2}(0.9137M_s + 0.03004M_b)$	$\frac{B_\sigma}{M}\left[-\frac{3}{2} - \frac{M}{m}\frac{\pi}{2} + 2\ln\left(\frac{M}{m}\right) + \frac{3\pi}{4}\frac{m}{M}\right]$ $+\frac{B_F}{2}\left[\frac{1}{m} + \frac{\pi^2}{3M}\right]$

the model, the amplitude is of the right order of magnitude. There is also an irredicible contribution [8,10,13–16] to $b \to s\gamma\gamma$ of fourth order in the photon momenta which has to be taken into account, in order to obtain a precise value of the amplitude. This contribution is directly proportional to matrix element of the axial current, and thus proportional to f_B. Still, the contributions we have considered give the correct order of magnitude.

The bound state model we have considered may of course be modified by dropping or relaxing the requirements (14) and (16). One possibillity is to raise the quark masses in order to avoid unphysical imaginary parts. However, then the requirement (14) cannot be met by any value of Λ_B. Still, for some chosen value of Λ_B, we could determine Z_B to fit the physical value of f_B at the expence of a renormalizing constant $\sqrt{\hat{\Sigma}(k^2=M_B^2)}$ to multiply each physical B_s-meson field. But then there is some arbitrariness present, and the idea of the model is to a large extent lost.

In some recent papers, the process $b \to s\gamma\gamma$ has been considered [12–16]. There is now wide agreement [7,8,10,13–15] that the rate for $B_s \to \gamma\gamma$ is of order 10^{-7} in the standard model, although the estimate in [12] is slightly higher. The decay mode $B_s \to \gamma\gamma$ is also a natural place to look for new physics [15,16]. In [10,12,13] one calculates the free-quark amplitude and uses Eq. (17) and the corresponding relation for the matrix element of the axial density $\overline{s}\gamma_5 b$. Still the amplitude depends on the quark momentum of the light s quark. Appealing to the heavy-quark limit, this momentum is neglected with respect to the heavy-b-quark mass. If the s quark is not neglected, it has to be integrated out, as we have done in this paper. It should be noted that the off-shell Lagrangian $\tilde{\mathcal{L}}_F$ gives a zero contribution in the heavy-quark limit (and $M_B = M_b$). However, it can be shown that the off-shell piece still gives a nonzero contribution by using [10,12,13] relations such as Eq. (17), and some averaging over the s-quark momentum.

The model we have used [3,5] does not use the formalism of heavy-quark effective field theory (HQEFT), where results are obtained in terms of expansions in inverse powers of heavy (c and b) quark masses and, in addition, some bound state parameters [19]. The inclusive process $B_s \to \gamma X_s$, which is also to a large extent determined by the effective Lagrangian in Eq. (2) and thereby related to the process $B_s \to \gamma\gamma$, has been extensively studied in terms of HQEFT [20]. It is therefore natural to try to study $B_s \to \gamma\gamma$ within the same framework. And in [16] some ideas from HQEFT are used to estimate the relevant hadronic matrix elements. A fruitful way to handle the hadronized version of quark processes such as $b \to s\gamma$ and $b \to s\gamma\gamma$ might be to use some heavy-light model where HQEFT is combined with Nambu-type models and/or the chiral quark model [6]. The latter is known to work well for the $K \to \gamma\gamma$ decay [17,18] and for decays of K mesons in general [21]. Such a heavy-light model, where the light-quark momenta are integrated out in a consistent way, is proposed by various groups [22]. However, a systematic study of $B_s \to \gamma\gamma$ will meet some difficulties which must be overcome: Usually, when HQEFT is used, there is a heavy quark going through the process, as in $B \to D$ transitions or in the calculations of the *inclusive rate* for $B_s \to \gamma X_s$. In the case of the exclusive process $B_s \to \gamma\gamma$, there are no heavy particles in the final state, and the energy release is big compared to the hadronic scale of strong interactions ($\simeq 1$ GeV). Still, models of the type of [22] should be able to tell us something about the decay mode $B_s \to \gamma\gamma$.

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