Radiative corrections to W and quark propagators in the resonance region

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We discuss radiative corrections to W and quark propagators in the resonance region $|s-M^2|\lesssim M\Gamma$. We show that conventional mass renormalization, when applied to photonic or gluonic corrections, leads in next to leading order (NLO) to contributions proportional to $[M\Gamma/(s-M^2)]^n$, $(n=1,2,\ldots)$, i.e., to a non-convergent series in the resonance region, a difficulty that affects all unstable particles coupled to massless quanta. A solution of this problem, based on the concepts of pole mass and width, is presented. It elucidates the issue of renormalization of amplitudes involving unstable particles, and automatically circumvents the problem of apparent on-shell singularities. The roles of the Fried-Yennie gauge and the pinch technique prescription are discussed. Because of special properties of the photonic and gluonic contributions, and in contrast with the Z case, the gauge dependence of the conventional on-shell definition of mass is unbounded in NLO. The evaluations of the width in the conventional and pole formulations are compared and shown to agree in NLO but not beyond. [S0556-2821(98)01019-4]

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I. INTRODUCTION

The aim of this paper is to study the radiative corrections to W and unstable fermion propagators in the resonance region. Calling s the invariant momentum transfer, this is the region $|s-M^2|\lesssim M\Gamma$, where M and Γ are the mass and the width of the unstable particles. The W analysis is a natural counterpart of the study of the Z^0 propagator that has played a major role in the interpretation of electroweak physics in the resonance region. For some time it has been known that the conventional on-shell definition of mass,

$$M^2=M_0^2+\mathrm{Re}\,A(M^2), \tag{1.1}$$

where M_0 is the unrenormalized mass and $A(s)$ is the transverse boson self-energy (including tadpole contributions), is gauge dependent in $O(g^4)$ and higher [1–3]. In the Z^0 case, the gauge dependence of M is $\lesssim 2$ MeV in $O(g^4)$ but becomes unbounded in $O(g^6)$ [3]. On the other hand, the complex-pole position

$$\bar{s}=m_2^2-im_2\Gamma_2=M_0^2+A(\bar{s}) \tag{1.2}$$

is gauge-invariant. Thus, a gauge-invariant definition can be achieved by identifying the mass with m_2 or appropriate combinations of m_2 and Γ_2. In particular, it has been shown [1] that the Z mass measured at the CERN e^+e^- collider LEP can be identified with

$$m_1=(m_2^2+\Gamma_2^2)^{1/2}. \tag{1.3}$$

In Eqs. (1.2), (1.3) we have followed the notation introduced in Eqs. (4) and (15) of Ref. [1].

In the W case one expects similar theoretical features. However, as shown in Sec. II, a new problem emerges: in the treatment of the photonic corrections the conventional mass-renormalization procedure generates contributions proportional to $[M\Gamma/(s-M^2)]^l$, $(l=1,2,\ldots)$, in next to leading order (NLO). Thus, one obtains an expansion that does not converge in the resonance region! These theoretical features are generally present whenever the unstable particle is coupled to massless quanta. In Sec. II we present a solution of this problem based on the concepts of pole mass and width. It automatically circumvents the problem of apparent on-shell singularities and, more generally, it elucidates the issue of renormalization of amplitudes involving unstable particles. The roles of the Fried-Yennie gauge and the pinch technique are discussed in Sec. III. In contrast with the Z case, we show that, because of special features of the bosonic and gluonic contributions, the gauge dependence of the conventional on-shell definition of mass is unbounded in NLO. Section IV discusses the overall corrections to the W propagator in NLO. In Sec. V the modified and conventional formulations of the W width are compared and shown to agree in NLO, but not beyond. Potential problems of the conventional definition of width emerging in high orders of perturbation theory are discussed. As a further illustration, in Sec. VI we discuss the QCD corrections to an unstable quark propagator in the resonance region.

II. PHOTONIC CORRECTIONS TO THE W PROPAGATOR IN THE RESONANCE REGION

In order to illustrate the difficulties emerging in the resonance region when the conventional mass renormalization is employed, we consider the contributions of the transverse part of the W propagator in the loop of Fig. 1, with l self-energy insertions. Writing the transverse W self-energy in the form

$$\Pi_{\mu\nu}^{(T)}(q)=t_{\mu\nu}(q)A(s), \tag{2.1}$$

where $s\equiv q^2$ and $t_{\mu\nu}(q)=g_{\mu\nu}-q_\mu q_\nu/q^2$, the contribution $A_{W\gamma}^{(l)}(s)$ from Fig. 1 to $A(s)$ is given by

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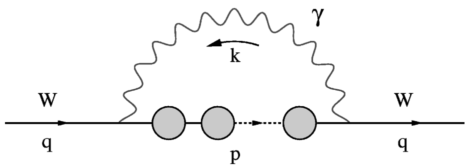

FIG. 1. A class of photonic corrections to the W self-energy. The inner solid and dashed lines and blobs represent transverse W propagators and self-energies.

$$A_{W\gamma}^{(l)}(s)=ie^2(\mu)\,\frac{t_{\mu\nu}(q)}{(n-1)}\,\mu^{4-n}\times\int\frac{d^nk}{(2\pi)^n}\,\mathcal{D}_{\rho\beta}^{(\gamma)}(k)\mathcal{D}_{\lambda\alpha}^{(W,T)}(p)\mathcal{V}^{\rho\lambda\nu}\mathcal{V}^{\beta\alpha\mu}\times\left[\frac{A^{(s)}(p^2)}{p^2-M^2+i\epsilon}\right]^l, \quad (2.2)$$

where $p=q+k$ is the W loop-momentum,

$$\mathcal{D}_{\rho\beta}^{(\gamma)}(k)=-\frac{i}{k^2}\left(g_{\rho\beta}+(\xi_\gamma-1)\,\frac{k_\rho k_\beta}{k^2}\right), \quad (2.3)$$

$$\mathcal{D}_{\lambda\alpha}^{(W,T)}(p)=\frac{-i}{p^2-M^2+i\epsilon}\left(g_{\alpha\lambda}-\frac{p_\alpha p_\lambda}{p^2}\right), \quad (2.4)$$

$$\mathcal{V}^{\beta\alpha\mu}=(2p-k)^\beta g^{\alpha\mu}+(2k-p)^\alpha g^{\beta\mu}-(k+p)^\mu g^{\beta\alpha}, \quad (2.5)$$

ξ_γ is the photon gauge parameter and $A^{(s)}(p^2)$ stands for the W transverse self-energy with the conventional mass renormalization subtraction:

$$A^{(s)}(p^2)=\mathrm{Re}(A(p^2)-A(M^2))+i\ \mathrm{Im}\,A(p^2)=A(p^2)-A(M^2)+i\ \mathrm{Im}\,A(M^2). \quad (2.6)$$

We recall that, in leading order, $i\ \mathrm{Im}\,A(M^2)=-iM\Gamma$. Equation (2.4) corresponds to the choice $\xi_w=0$ for the W gauge parameter ξ_w (Landau gauge). We note that each insertion of $A^{(s)}(p^2)$ is accompanied by an additional denominator $[p^2-M^2+i\epsilon]$. Thus, Eq. (2.2) may be regarded as the lth term in an expansion in powers of

$$[A(p^2)-A(M^2)+i\ \mathrm{Im}\,A(M^2)](p^2-M^2+i\epsilon)^{-1}.$$

As $A(p^2)-A(M^2)=O[g^2(p^2-M^2)]$ for $p^2\approx M^2$, the contribution $[A(p^2)-A(M^2)](p^2-M^2+i\epsilon)^{-1}$ is of $O(g^2)$ throughout the region of integration. Therefore, each successive insertion leads to corrections of higher order in g^2. However, as $i\ \mathrm{Im}\,A(M^2)\approx-iM\Gamma$ is not subtracted, the combination $i\ \mathrm{Im}\,A(M^2)/(p^2-M^2+i\epsilon)$ may lead to terms of $O(1)$ if the domain of integration $|p^2-M^2|\lesssim M\Gamma$ is important. In fact, the contribution of $[i\ \mathrm{Im}\,A(M^2)/(p^2-M^2+i\epsilon)]^l$ to Eq. (2.2) is, to leading order,

$$A_{W\gamma}^{(l)}(s)=\frac{(-iM\Gamma)^l}{l!}\,\frac{d^l}{d(M^2)^l}A_{W\gamma}^{(0)}(s)+\cdots \quad (2.7)$$

where $A_{W\gamma}^{(0)}(s)$ represents the diagram without self-energy insertions and the dots indicate additional contributions not relevant to our argument.

In the resonance region the inverse zeroth order propagator is proportional to $(s-M^2+iM\Gamma)=O(g^2)$. Therefore, in NLO, contributions of $O[\alpha(s-M^2)]$ are retained but those of $O[\alpha(s-M^2)^2]$ are neglected. Explicit evaluation of $A_{W\gamma}^{(0)}(s)$ in NLO leads to

$$A_{W\gamma}^{(0)}(s)=\frac{\alpha}{2\pi}\left[(\xi_\gamma-3)(s-M^2)\ln\left(\frac{M^2-s}{M^2}\right)+\cdots\right]. \quad (2.8)$$

Inserting Eq. (2.8) into Eq. (2.7) we obtain

$$A_{W\gamma}^{(1)}(s)=\frac{\alpha}{2\pi}(\xi_\gamma-3)(iM\Gamma)\left[\ln\left(\frac{M^2-s}{M^2}\right)+\frac{s}{M^2}\right]+\cdots,$$

$$A_{W\gamma}^{(l)}(s)=\frac{\alpha}{2\pi}(\xi_\gamma-3)\,\frac{(s-M^2)}{l(l-1)}\left(\frac{-iM\Gamma}{s-M^2}\right)^l+\cdots\quad(l\geqslant2). \quad (2.9)$$

We see from Eq. (2.9) that Fig. 1, evaluated with conventional mass renormalization, leads in NLO to a series in powers of $M\Gamma/(s-M^2)$, which does not converge in the resonance region. Thus, rather than generating contributions of higher order in g^2, each successive self-energy insertion gives rise to a factor $-iM\Gamma/(s-M^2)$, which is nominally of $O(1)$ in the resonance region and furthermore diverges at $s=M^2$.

Formally, the series $\Sigma_{l=0}^{\infty}A_{W\gamma}^{(l)}(s)$ with $A_{W\gamma}^{(l)}(s)$ given by Eq. (2.7) can be resummed. In fact, it leads to

$$\sum_{l=0}^{\infty}A_{W\gamma}^{(l)}(s,M^2)=A_{W\gamma}^{(0)}(s,M^2-iM\Gamma)+\cdots. \quad (2.10)$$

Thus,

$$\sum_{l=0}^{\infty}A_{W\gamma}^{(l)}(s)=\frac{\alpha}{2\pi}\left[(\xi_\gamma-3)(s-M^2+iM\Gamma)\times\ln\left(\frac{M^2-iM\Gamma-s}{M^2-iM\Gamma}\right)+\cdots\right]. \quad (2.11)$$

Even if one accepts these ''a posteriori'' formal resummations, the theoretical situation in the framework of conventional mass renormalization is unsatisfactory. In fact, in the conventional formalism, the W propagator is inversely proportional to

$$\mathcal{D}^{-1}(s)=s-M^2+iM\Gamma-(A(s)-A(M^2))-iM\Gamma\ \mathrm{Re}\,A'(M^2) \quad (2.12)$$

where Γ is the radiatively corrected width and we have employed its conventional definition

$$M\Gamma = -\mathrm{Im}\, A(M^2)/[1-\mathrm{Re}\, A'(M^2)]. \tag{2.13}$$

The contribution of Eq. (2.11) to $\mathcal{D}^{-1}(s)$ is

$$-\frac{\alpha}{2\pi}(\xi_\gamma-3)\Bigg[(s-M^2+iM\Gamma)\times\ln\left(\frac{M^2-iM\Gamma-s}{M^2-iM\Gamma}\right)+iM\Gamma\left(1+i\frac{\pi}{2}\right)\Bigg]+\cdots .$$

We note that the last term is a gauge-dependent contribution not proportional to the zeroth order term $s-M^2+iM\Gamma$. As a consequence, in NLO the pole position is $\tilde{M}^2-i\tilde{M}\tilde{\Gamma}$, where

$$\tilde{M}^2 = M^2[1-(\alpha/4)(\xi_\gamma-3)(\Gamma/M)], \tag{2.14}$$

$$\tilde{\Gamma} = \Gamma[1-(\alpha/2\pi)(\xi_\gamma-3)]. \tag{2.15}$$

As the pole position is gauge-invariant, so must be $\tilde{M}$ and $\tilde{\Gamma}$. Furthermore, in terms of $\tilde{M}$ and $\tilde{\Gamma}$, $\mathcal{D}^{-1}(s)$ retains the Breit-Wigner structure. Thus, in a resonance experiment $\tilde{M}$ and $\tilde{\Gamma}$ would be identified with the mass and width of W. The relation $\tilde{\Gamma}=\Gamma[1-(\alpha/2\pi)(\xi_\gamma-3)]$ leads then to a contradiction: the measured, gauge-independent, width $\tilde{\Gamma}$ would differ from the theoretical value Γ by a gauge-dependent quantity in NLO. This contradicts the premise of the conventional formalism that Γ, defined in Eq. (2.13), is the radiatively corrected width and is, furthermore, gauge-independent. We can anticipate that the root of the problem is that Eq. (2.13) is only an approximate expression for the width of the unstable particle. In particular, it is not sufficiently accurate when non-analytic contributions are considered.

It is therefore important to base the calculations in a formalism that avoids awkward resummations of non-convergent series and the pitfalls we have encountered in the previous argument. To achieve this, we return to the transverse dressed W propagator, inversely proportional to $p^2-M_0^2-A(p^2)$. In the conventional mass renormalization one eliminates M_0^2 by means of the expression $M_0^2=M^2-\mathrm{Re}\,A(M^2)$ [cf. Eq. (1.1)]. An alternative possibility is to eliminate M_0^2 by $M_0^2=\overline{s}-A(\overline{s})$ [cf. Eq. (1.2)]. The dressed propagator in the loop integral is inversely proportional to $p^2-\overline{s}-[A(p^2)-A(\overline{s})]$. Expansion of the dressed propagator leads in Fig. 1 to a series in powers of $[A(p^2)-A(\overline{s})]/(p^2-\overline{s})$. As $A(p^2)-A(\overline{s})=O[g^2(p^2-\overline{s})]$ when the loop momentum is in the resonance region, $[A(p^2)-A(\overline{s})]/(p^2-\overline{s})$ is $O(g^2)$ throughout the domain of integration. Thus, each successive self-energy insertion leads now to terms of higher order in g^2 without awkward non convergent contributions. In this modified strategy, the zeroth order propagator in Eq. (2.4) is replaced by

$$\mathcal{D}^{(W,T)}_{\alpha\lambda}(p)=\frac{-i}{p^2-\overline{s}}\left(g_{\alpha\lambda}-\frac{p_\alpha p_\lambda}{p^2}\right). \tag{2.16}$$

We note that the imaginary part in $(p^2-\overline{s})^{-1}$ has the same sign as Feynman's $i\epsilon$ prescription. Therefore, although the poles of Eq. (2.4) in the k^0 complex plane are displaced by the $im_2\Gamma_2$ insertion, they remain in the same quadrants so that Feynman's contour integration or Wick's rotation can be carried out. $A^{(0)}_{W\gamma}(s)$, Fig. 1 without loop insertions, now leads directly to

$$A^{(0)}_{W\gamma}(s)=\frac{\alpha}{2\pi}\left[(\xi_\gamma-3)(s-\overline{s})\ln\left(\frac{\overline{s}-s}{\overline{s}}\right)+\cdots\right]. \tag{2.17}$$

$A^{(l)}_{W\gamma}(s)$ $(l\geqslant 1)$, the terms with l insertions in Fig. 1, give now contributions of $O(\alpha g^{2l})$, the normal situation in perturbative expansions. The W propagator in the modified formalism is inversely proportional to $s-\overline{s}-[A(s)-A(\overline{s})]$. The contribution of Eq. (2.17) to $[A(s)-A(\overline{s})]$ is proportional to $s-\overline{s}$ so that the pole position is not displaced, the gauge-dependent contributions factorize as desired, and the previously discussed pitfalls are avoided. As $A^{(l)}_{W\gamma}(\overline{s})$ is infrared convergent in the modified approach, $A^{(l)}_{W\gamma}(s)$ leads now to contributions to $[A(s)-A(\overline{s})]$ of order $O[(s-\overline{s})\alpha g^{2l}]=O[\alpha g^{2(l+1)}]$ and can therefore be neglected in NLO when $l\geqslant 1$.

The remaining contributions to $A(s)$ from the photonic diagrams, including those from the longitudinal part of the W propagator in the loop of Fig. 1, and from the diagrams involving the unphysical scalar ϕ and the ghost C_γ, have no singularities at $s=M^2$ and can therefore be studied with conventional methods. In particular, in the evaluation of $A(s)-A(\overline{s})$ in NLO it is sufficient to retain their one-loop contributions. In these diagrams the propagators are proportional to $(p^2-M^2\xi_w)^{-1}$ instead of $(p^2-M^2)^{-1}$. As a consequence, they lead to logarithmic contributions proportional to

$$(s-M^2)\left[\frac{s-M^2\xi_w}{M^2}\right]\ln\left(\frac{M^2\xi_w-s}{M^2\xi_w}\right),$$

rather than Eq. (2.8). They have branch cuts starting at $s=M^2\xi_w$, which indicates the unphysical nature of these singularities. Although they must cancel in physical amplitudes, they are present in partial amplitudes such as conventional self-energies and propagators. We briefly discuss how to treat them in NLO. For $|\xi_w-1|\gtrsim\Gamma/M$, the logarithm can be expanded about $s=M^2$ and one obtains

$$(s-M^2)\left[1-\xi_w+\frac{s-M^2}{M^2}\right]\left[\ln\left(\frac{\xi_w-1}{\xi_w}\right)+O\left(\frac{s-M^2}{M^2(1-\xi_w)}\right)\right].$$

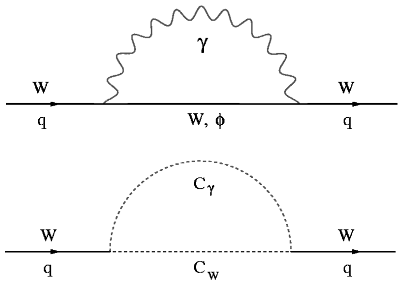

FIG. 2. One-loop photonic diagrams for the W self-energy; ϕ is the unphysical scalar, C_γ and C_W are ghosts.

The contribution from $O[(s-M^2)/M^2(1-\xi_w)]$ is proportional to $(s-M^2)^2/M^2$ and is therefore neglected in NLO. For the same reason, we can neglect $(s-M^2)/M^2$ in the second factor. Therefore, for $|\xi_w-1|\gtrsim\Gamma/M$, in NLO we can approximate this contribution by the simple expression $(s-M^2)(1-\xi_w)\ln[(\xi_w-1)/\xi_w]$. For $|\xi_w-1|\lesssim\Gamma/M$ the expansion of the logarithmic factor is not valid, but we note that the whole contribution is proportional to $(s-M^2)^2$ or $(s-M^2)(1-\xi_w)$ and therefore negligible in NLO. As a consequence, the above mentioned approximation can be used for any value of ξ_w. Calling $A^\gamma(s)$ the overall contribution of the one-loop photonic diagrams to the transverse W self-energy (Fig. 2), in the modified formulation the relevant quantity in the correction to the W propagator is $A^\gamma(s)-A^\gamma(\bar{s})$. In general R_ξ gauge, we find, in NLO,

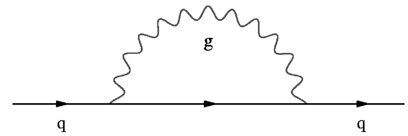

FIG. 3. One loop diagram for the quark self-energy in QCD.

$$A^\gamma(s)-A^\gamma(\bar{s})=\frac{\alpha(m_2)}{2\pi}(s-\bar{s})\left\{\delta\left(\frac{\xi_w}{2}-\frac{23}{6}\right)+\frac{34}{9}-2\ln\left(\frac{\bar{s}-s}{\bar{s}}\right)-(\xi_w-1)\left[\frac{\xi_w}{12}-\left(1-\frac{(\xi_w-1)^2}{12}\right)\ln\left(\frac{\xi_w-1}{\xi_w}\right)\right]-\left(\frac{11}{12}-\frac{\xi_w}{4}\right)\right.$$
$$\left.\times\ln\xi_w+(\xi_\gamma-1)\left[\frac{\delta}{2}+\frac{1}{2}+\ln\left(\frac{\bar{s}-s}{\bar{s}}\right)+\frac{(\xi_w^2-1)}{4}\ln\left(\frac{\xi_w-1}{\xi_w}\right)-\frac{\ln\xi_w}{4}+\frac{\xi_w}{4}\right]\right\}, \tag{2.18}$$

where $\delta=(n-4)^{-1}+(\gamma_E-\ln 4\pi)/2$, we have treated the logarithmic terms according to the previous discussion and set $\mu=m_2$. The corresponding one-loop gluonic contribution to the quark self-energy is depicted in Fig. 3.

Writing

$$1-\frac{s}{\bar{s}}=1-\frac{s}{m_1^2}-i\frac{s}{m_1^2}\frac{\Gamma_2}{m_2}=\rho e^{i\theta}, \tag{2.19}$$

we have

$$\rho=\left[\left(1-\frac{s}{m_1^2}\right)^2+\frac{s^2\Gamma_2^2}{m_1^4m_2^2}\right]^{1/2}, \tag{2.20}$$

$$\rho\sin\theta=-\frac{s\Gamma_2}{m_1^2m_2}, \tag{2.21}$$

where m_1 is defined in Eq. (1.3). Calling $\alpha\equiv\sin^{-1}(\Gamma_2/m_1)$, we have: for $-\infty<s<0$, $\alpha>\theta>0$; for $0<s<m_1^2$, $0>\theta>-\pi/2$; for $m_1^2<s<\infty$, $-\pi/2>\theta>-\pi+\alpha$. In Figs. 4 and 5 the functions $\ln\rho(s)$ and $\theta(s)$ are plotted for $m_1=80.4$ GeV and $\Gamma_1=\Gamma_2 m_1/m_2=2$ GeV over a large range of $\sqrt{s}$ values. Figures 6 and 7 compare these functions with the zero-width approximations over the resonance region. In the limit $\Gamma_2\to 0$, $\theta(s)$ becomes a step function. This corresponds to the well-known expression

$$\mathrm{Im}\left[\ln\left(\frac{M^2-s-i\epsilon}{M^2}\right)\right]=-\pi\theta(s-M^2),$$

where the $i\epsilon$ prescription implies $\theta(0)=1/2$. The zero width approximation, however, is not valid in the resonance region.

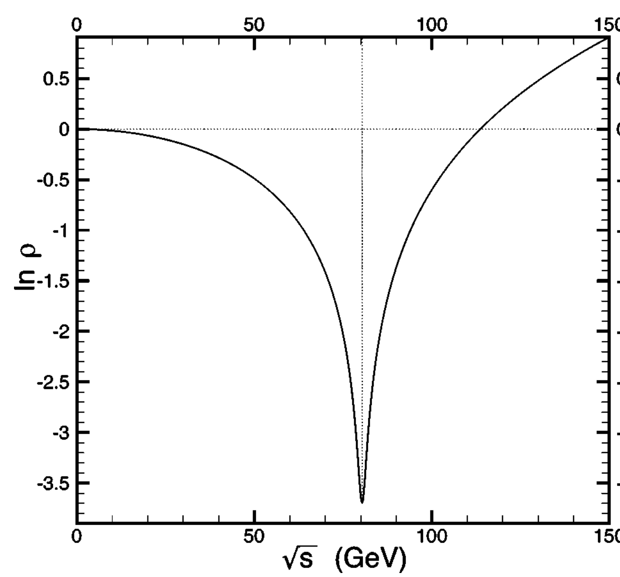

FIG. 4. The function $\ln\rho(s)$ over a large range of $\sqrt{s}$ values, for $m_1=80.4$ GeV and $\Gamma_1=2$ GeV [see Eq. (2.20)]. The minimum occurs at $\sqrt{s}=m_2$.

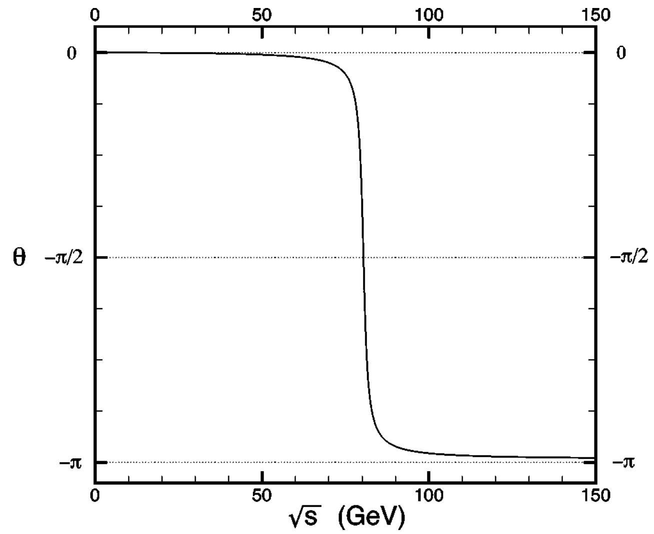

FIG. 5. The function $\theta(s)$ for $m_1=80.4$ GeV and $\Gamma_1=2$ GeV [see Eq. (2.21)]. The value $-\pi/2$ is attained at $\sqrt{s}=m_1$.

Equation (2.18) exhibits a number of interesting theoretical features: (a) the coefficient of $\ln[(\bar{s}-s)/\bar{s}]$ is independent of ξ_w but is proportional to $(\xi_\gamma-3)$. (b) The logarithm $\ln(\xi_w-1)$ contains an imaginary contribution $-i\pi\theta(1-\xi_w)$. This can be understood from the observation that, for $\xi_w<1$, a W boson of mass $s=M^2$ has non-vanishing phase space to ''decay'' into a photon and particles of mass $M^2\xi_w$. As explained before, Eq. (2.18) is only valid in the resonance region.

For completeness, the full one-loop expression for $A^\gamma(s)$ in general R_ξ gauges is given in the Appendix.

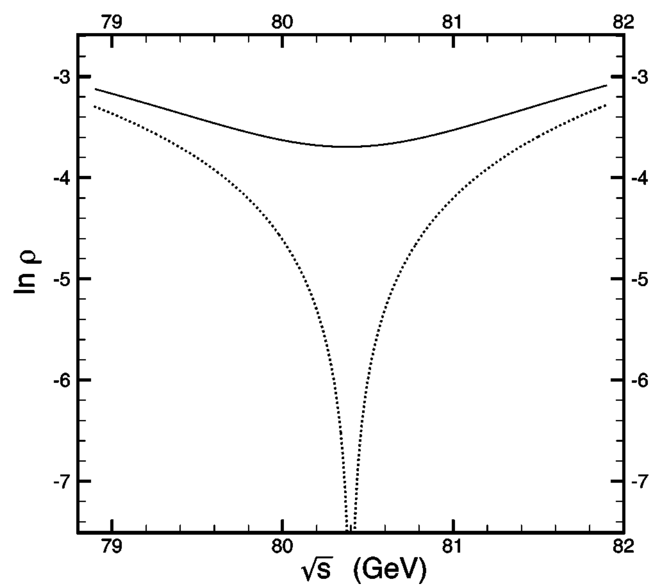

FIG. 6. Comparison of $\ln\rho(s)$ (solid line) with its zero-width approximation $\ln|1-s/m_1^2|$ (dotted line) over the resonance region ($m_1=80.4$ GeV, $\Gamma_1=2$ GeV).

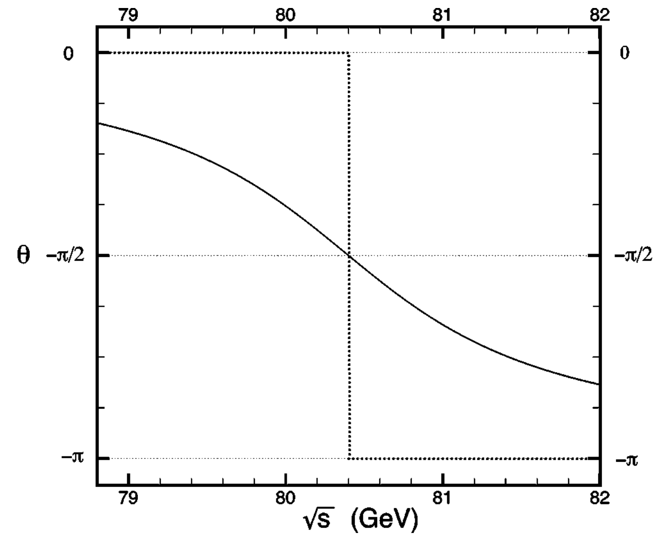

FIG. 7. Comparison of $\theta(s)$ (solid line) with the step function approximation (dotted line) over the resonance region ($m_1=80.4$ GeV, $\Gamma_1=2$ GeV).

III. FRIED-YENNIE GAUGE AND THE PT PRESCRIPTION. GAUGE DEPENDENCE OF THE ON-SHELL MASS

We note that the $\ln[(\bar{s}-s)/\bar{s}]$ terms in Eq. (2.17) and Eq. (2.18) cancel for $\xi_\gamma=3$, the gauge introduced by Fried and Yennie in Lamb-shift calculations [4]. It should be emphasized, however, that a gauge-independent logarithm of this type survives in physical amplitudes involving unstable particles such as W [5]. Thus, the choice $\xi_\gamma=3$ removes this contribution from the propagator's corrections, but not the overall amplitude. In this connection, it is interesting to inquire how the pinch technique (PT) prescription treats these terms. We recall that the PT is a prescription that combines the conventional self-energies with ''pinch parts'' from vertex and box diagrams in such a manner that the modified self-energies are independent of ξ_i $(i=W,\gamma,Z)$ and exhibit desirable theoretical properties. Calling $a(q^2)$ the PT W self-energy, we recall that, in the standard model (SM),

$$a(s)=[A(s)]_{\xi_i=1}-4g^2(\mu)(s-M^2)\times[\cos^2\theta_W I_{WZ}(s)+\sin^2\theta_W I_{\gamma W}(s)], \qquad (3.1)$$

where

$$I_{ij}(s)=i\mu^{4-n}\int\frac{d^nk}{(2\pi)^n}\frac{1}{(k^2-m_i^2)[(k+q)^2-m_j^2]},$$

and tadpole contributions have been included in both $a(s)$ and $A(s)$ [6]. The $I_{\gamma W}(s)$ term leads to a contribution $-(\alpha/\pi)[(s-M^2)^2/s]\ln[(M^2-s)/M^2]$, which is of higher order in $(s-M^2)$. Therefore, in NLO the PT self-energy generates the same $\ln[(\bar{s}-s)/\bar{s}]$ term as the 't Hooft–Feynman gauge ($\xi_\gamma=1$), i.e., $-(\alpha/\pi)(s-\overline{s})\ln[(\overline{s}-s)/\overline{s}]$. The possibility has been suggested in the past to define the on-shell mass in terms of the PT self-energy, namely $M^2=M_0^2+\text{Re } a(M^2)$ [7]. This has the advantage that one is dealing here with a ξ_i-independent amplitude. Repeating the argument after Eq. (2.11), we see however that conventional on-shell renormalization based on $a(s)$ would lead to a contribution $iM\Gamma(\alpha/\pi)[1+i\pi/2]$ which, although ξ_i independent, is not proportional to the zeroth order term $s-M^2+iM\Gamma$. Its removal would require a redefinition of M and Γ, which is inconsistent with the fact that Γ contains all the corrections of $O(\alpha)$. This problem can be circumvented once more by recalling that the PT does not displace the position of the complex pole at least through $O(g^4)$ [8], and expressing the inverse propagator as $s-\overline{s}-[a(s)-a(\overline{s})]$. The contribution of the $(s-\overline{s})\ln[(\overline{s}-s)/\overline{s}]$ terms to $a(s)-a(\overline{s})$ is proportional to $(s-\overline{s})$ and the above mentioned difficulties are avoided.

The difference between m_1, defined in Eq. (1.3), and the conventional on-shell mass M, defined in Eq. (1.1), is

$$M^2-m_1^2=\text{Re } A(M^2)-\text{Re } A(\overline{s})-\Gamma_2^2. \tag{3.2}$$

The contribution of the $(s-\overline{s})\ln[(\overline{s}-s)/\overline{s}]$ term to the right-hand side (RHS) of Eq. (3.2) is

$$\begin{aligned}&\frac{\alpha(m_2)}{2\pi}(\xi_\gamma-3)\left[(M^2-m_2^2)\text{Re }\ln\left(\frac{\overline{s}-M^2}{\overline{s}}\right)\right.\\&\left.\quad-m_2\Gamma_2\text{ Im }\ln\left(\frac{\overline{s}-M^2}{\overline{s}}\right)\right]\\&\quad\approx\frac{\alpha(m_2)}{2\pi}(\xi_\gamma-3)\left[(M^2-m_1^2)\text{Re }\ln\left(\frac{\overline{s}-M^2}{\overline{s}}\right)\right.\\&\left.\qquad+m_2\Gamma_2\frac{\pi}{2}\right].\end{aligned} \tag{3.3}$$

In $\text{Im }\ln[(\overline{s}-M^2)/\overline{s}]$ we have approximated $M^2\approx m_1^2$ and used the fact that $\theta=-\pi/2$ for $s=m_1^2$ [see discussion after Eq. (2.21)]. Thus, we see that

$$M^2-m_1^2=\frac{\alpha(m_2)}{4}(\xi_\gamma-3)m_2\Gamma_2+\cdots, \tag{3.4}$$

where the dots indicate additional contributions. Note that this last equation corresponds to Eq. (2.14) with the identification $\tilde{M}\to m_1$.

As ξ_γ can be arbitrarily large, Eq. (3.4) reveals that in the W case the gauge dependence of the conventional on-shell definition of mass is unbounded in NLO for any value of ξ_w. Similarly, the term proportional to $(s-\overline{s})(\xi_\gamma-1)(\xi_w^2-1)\ln(\xi_w-1)$ in Eq. (2.18) gives an unbounded contribution $(\alpha/8)(\xi_\gamma-1)M\Gamma(\xi_w^2-1)\theta(1-\xi_w)$ to $M^2-m_1^2$ in the restricted range $\xi_w<1$. This situation is to be contrasted with the Z case, where the gauge dependence in NLO is bounded and $\lesssim 2$ MeV [3]. The difference is due to the contribution of the logarithms from the photonic diagrams, which are absent in the Z case. In particular, in the frequently employed 't Hooft–Feynman gauge ($\xi_i=1$), Eq. (3.4) leads to $m_1-M=\alpha(m_2)\Gamma_2/4\approx 4$ MeV. In analogy with the Z case, there are also bounded gauge-dependent contributions to m_1-M arising from non-photonic diagrams in the restricted range $\sqrt{\xi_z}\leqslant\cos\theta_w[1-\sqrt{\xi_w}]$ and from the photonic corrections proportional to $(\xi_w-1)\ln[(\xi_w-1)/\xi_w]$ [cf. Eq. (2.18)].

IV. OVERALL CORRECTIONS TO *W* PROPAGATORS IN THE RESONANCE REGION

In contrast with the photonic corrections, the non-photonic contributions $A_{np}(s)$ to $A(s)$ are analytic around $s=\overline{s}$. We can therefore write

$$A_{np}(s)-A_{np}(\overline{s})=(s-\overline{s})A'_{np}(m_2^2)+\cdots, \tag{4.1}$$

where the dots indicate higher-order contributions.

In the resonance region, and in NLO, the transverse W propagator is given by

$$\mathcal{D}_{\alpha\beta}^{(W,T)}(q)=\frac{-i(g_{\alpha\beta}-q_\alpha q_\beta/q^2)}{(s-\overline{s})\left[1-A'_{np}(m_2^2)-\dfrac{\alpha(m_2)}{2\pi}F(s,\overline{s},\xi_\gamma,\xi_w)\right]}, \tag{4.2}$$

where $s=q^2$ and $F(s,\overline{s},\xi_\gamma,\xi_w)$ is the expression between curly brackets in Eq. (2.18). An alternative expression, involving an s-dependent width, can be obtained by splitting A'_{np} into real and imaginary parts, and the latter into fermionic $\text{Im } A'_f$ and bosonic $\text{Im } A'_b$ contributions. Neglecting very small scaling violations, we have

$$\text{Im } A'_f(m_2^2)\approx\text{Im } A_f(m_2^2)/m_2^2\approx-\Gamma_2/m_2. \tag{4.3}$$

Equation (4.2) becomes then

$$\mathcal{D}^{(W,T)}_{\alpha\beta}(q)=\frac{-i(g_{\alpha\beta}-q_\alpha q_\beta/q^2)}{\left(s-m_1^2+is\frac{\Gamma_1}{m_1}\right)\left[1-\mathrm{Re}\,A'_{np}(m_1^2)-i\ \mathrm{Im}\,A'_b(m_1^2)-\frac{\alpha(m_1)}{2\pi}F\right]},\tag{4.4}$$

where $\Gamma_1/m_1=\Gamma_2/m_2$. $\mathrm{Im}\,A'_b(m_1^2)$ is nonzero and gauge-dependent in the subclass of gauges that satisfy $\sqrt{\xi_z}\leqslant\cos\theta_w[1-\sqrt{\xi_w}]$. (If this condition is satisfied, a W boson of mass $\sqrt{s}\approx M_W$ has non-vanishing phase space to "decay" into particles of mass $M_W\sqrt{\xi_w}$ and $M_Z\sqrt{\xi_z}$.) Otherwise $\mathrm{Im}\,A'_b(m_1^2)$ vanishes. Although m_1 and Γ_1 are gauge-invariant, $\mathrm{Re}\,A'_{np}(m_1^2)$, $\mathrm{Im}\,A'_{np}(m_1^2)$, and F are gauge-dependent. In physical amplitudes, such gauge-dependent terms cancel against contributions from vertex and box diagrams. The crucial point is that the gauge-dependent contributions in Eq. (4.4) factorize so that such cancellations can take place and the position of the complex pole is not displaced.

V. COMPARISON OF THE W WIDTH IN THE CONVENTIONAL AND MODIFIED FORMULATIONS

In this section we show that the conventional and modified formulations lead to the same result for the W width in NLO. However, the two approaches differ in higher orders. In particular, the conventional formulation is plagued in high orders by severe infrared singularities. Calling $A_0(s,M_0^2)$ the transverse self-energy evaluated in terms of the bare mass M_0, and $A(s,M^2)$ and $\overline{A}(s,\overline{s})$ the expressions obtained by substituting $M_0^2=M^2-\mathrm{Re}\,A(M^2,M^2)$ and $M_0^2=\overline{s}-\overline{A}(\overline{s,s})$, respectively, we have

$$A_0(s,M_0^2)=A(s,M^2)=\overline{A}(s,\overline{s}).\tag{5.1}$$

In the conventional approach the W width is given by Eq. (2.13) or, equivalently,

$$M\Gamma=-\mathrm{Im}\ A(M^2,M^2)+M\Gamma\ \mathrm{Re}\ A'(M^2,M^2),\tag{5.2}$$

where the prime means differentiation with respect to the first argument. Instead, in the modified formulation discussed in the present paper, the width is defined by

$$m_2\Gamma_2=-\mathrm{Im}\ \overline{A}(\overline{s,s}),\tag{5.3}$$

which follows from Eq. (1.2). Combining Eq. (5.3) with Eq. (5.1) we find:

$$\begin{aligned}m_2\Gamma_2&=-\mathrm{Im}\ A(\overline{s},M^2)\\&=-\mathrm{Im}\ A(M^2,M^2)\\&\quad-\mathrm{Im}[(\overline{s}-M^2)A'(M^2,M^2)]+O(g^6).\end{aligned}\tag{5.4}$$

As $\overline{s}-M^2=m_2^2-M^2-im_2\Gamma_2$ and $m_2^2-M^2=O(g^4)$, Eq. (5.4) becomes

$$m_2\Gamma_2=-\mathrm{Im}\ A(M^2,M^2)+m_2\Gamma_2\ \mathrm{Re}\ A'(M^2,M^2)+O(g^6).\tag{5.5}$$

Comparing Eq. (5.2) and Eq. (5.5) we see that indeed

$$\Gamma_2=\Gamma+O(g^6).\tag{5.6}$$

Thus, the two calculations of the width coincide through $O(g^4)$, i.e., in NLO. It is interesting to see how the two formulations treat potential infrared divergences. As is well-known, $\mathrm{Re}\,A'_\gamma(M^2,M^2)$, the photonic contribution to $\mathrm{Re}\,A'(M^2,M^2)$, is logarithmically infrared divergent. Therefore, $M\Gamma\ \mathrm{Re}\,A'(M^2,M^2)$ in the last term of Eq. (5.2) contains a logarithmic infrared divergence in $O(\alpha g^2)$. This is canceled by an infrared divergence in $\mathrm{Im}\,A(M^2,M^2)$ arising from $A^{(1)}_{W\gamma}(M^2,M^2)$, i.e., Fig. 1 with one self-energy insertion. As it is clear from the discussion of Sec. II, the infrared divergence in $A^{(1)}_{W\gamma}(M^2,M^2)$ has its origin in the fact that the self-energy insertion induces a correction factor $i\ \mathrm{Im}\,A(M^2)/(p^2-M^2)$, where p is the W loop momentum.

In higher orders the problem of infrared divergences in the conventional approach becomes severe. It follows from Eq. (2.9) that the diagrams in Fig. 1 generate infrared divergences of $O[\alpha(-i)^l M\Gamma(\Gamma/\lambda_{min})^{l-1}]$ in $A(M^2,M^2)$, where λ_{min} is the infrared cut-off. As a consequence, Eq. (5.2), the width evaluated in the conventional formulation, contains a power-like infrared divergence of $O[\alpha(\xi_\gamma-3)M\Gamma(\Gamma/\lambda_{min})^2]$ which appears in $O(\alpha g^6)$. Similarly, the conventional mass renormalization counterterm $\delta M^2=\mathrm{Re}\,A(M^2,M^2)$ contains an infrared divergence of $O[\alpha(\xi_\gamma-3)M\Gamma^2/\lambda_{min}]$ that appears in $O(\alpha g^4)$. One can avoid these leading infrared divergences by resumming the contributions of the $\mathrm{Im}\,A(M^2,M^2)\approx-M\Gamma$ insertions in Fig. 1. As explained in Sec. II, this would lead to the replacement

$$\begin{aligned}&\frac{\alpha}{2\pi}(\xi_\gamma-3)(s-M^2)\ln\left(\frac{M^2-s}{M^2}\right)\\&\quad\to\frac{\alpha}{2\pi}(\xi_\gamma-3)(s-M^2+iM\Gamma)\ln\left(\frac{M^2-s-iM\Gamma}{M^2-iM\Gamma}\right).\end{aligned}$$

Unfortunately, the contribution of this resummed expression to the right-hand side of Eq. (5.2) is $(\alpha/2\pi)(\xi_\gamma-3)M\Gamma$, a gauge-dependent contribution of $O(\alpha g^2)$ to the width. In contrast, in the modified formulation the corresponding expression is $(\alpha/2\pi)(\xi_\gamma-3)(s-\overline{s})\ln[(\overline{s}-s)/\overline{s}]$ and causes no problem since it gives no contribution to Eq. (5.3). It is also important to note that $\overline{A}_{W\gamma}(\overline{s,s})$ is infrared convergent in all orders, since the self-energy insertions induce a correction factor $[\overline{A}(p^2,\overline{s})-\overline{A}(\overline{s},\overline{s})]^l/(p^2-\overline{s})^l$ in the integrand of Fig. 1, and this converges, modulo logarithms, as $p^2\rightarrow\overline{s}$. In particular, it is easy to check that the contributions of $O[\alpha(\xi_\gamma-3)M\Gamma(\Gamma/\lambda_{min})^2]$ to the width mentioned above are canceled by terms of $O(\alpha g^6)$ in the expansion of Eq. (5.4). In the conventional formulation such terms are not included [cf. Eq. (5.2)] and this leads to the problem of uncompensated infrared singularities in high orders of perturbation theory. Other theoretical difficulties of the conventional definition of width and the need to replace it by Eq. (5.3) have been emphasized in Ref. [9] and Ref. [10].

VI. QCD CORRECTIONS TO QUARK PROPAGATORS IN THE RESONANCE REGION

In pure QCD quarks are stable particles. However, they become unstable when weak interactions are switched on. An example of a reaction that may probe the top quark propagator in the resonance region is $W^++b\rightarrow t\rightarrow W^++b$. In this section we discuss in NLO the QCD part of the corrections to the quark propagator in the resonance region. The relevant diagram is depicted in Fig. 3. Because the gluons are massless, we anticipate problems analogous to those discussed in Sec. II. Therefore, we work from the outset in the complex pole formulation. Denoting the position of the complex pole by $\overline{m}=m-i\Gamma/2$, we observe that Γ arises from the weak interactions. For example, in the top case Γ emerges in lowest order from the imaginary part of the Wb and ϕb contributions to the top self-energy. If we treat Γ in lowest order in the weak interactions, but otherwise neglect the remaining weak interaction contributions to the self-energy, the dressed quark propagator can be written as

$$S_F'(\not{q})=\frac{i}{\not{q}-\overline{m}-(\Sigma(\not{q})-\Sigma(\overline{m}))}, \tag{6.1}$$

where $\Sigma(\not{q})$ is the pure QCD contribution.

Decomposing

$$\Sigma(\not{q})=\overline{m}A(q^2)+\not{q}B(q^2), \tag{6.2}$$

and using $i/(\not{p}-\overline{m})$ as loop propagator, we find from Fig. 3:

$$A(q^2)=\frac{\alpha_s(m)}{3\pi}\left\{-2+(\xi_g+3)\left[-2\delta+2+\left(\frac{\overline{m}^2-q^2}{q^2}\right)\ln\left(\frac{\overline{m}^2-q^2}{\overline{m}^2}\right)\right]\right\}, \tag{6.3}$$

$$B(q^2)=\xi_g\frac{\alpha_s(m)}{3\pi}\left\{2\delta-1-\frac{\overline{m}^2}{q^2}-\left(\frac{\overline{m}^4-q^4}{q^4}\right)\ln\left(\frac{\overline{m}^2-q^2}{\overline{m}^2}\right)\right\}, \tag{6.4}$$

where ξ_g is the gluon gauge parameter and we have set $\mu=m$. In NLO in the resonance region this simplifies to

$$\Sigma(\not{q})=\frac{\alpha_s(m)}{3\pi}\left\{(\not{q}-\overline{m})\left[2(\xi_g-3)\ln\left(\frac{\overline{m}^2-q^2}{\overline{m}^2}\right)+2\delta\xi_g\right]+\overline{m}[4-6\delta]\right\}+\cdots \tag{6.5}$$

and

$$S_F'(\not{q})=\frac{i}{(\not{q}-\overline{m})}\left\{1-\frac{\alpha_s(m)}{3\pi}\left[2(\xi_g-3)\ln\left(\frac{\overline{m}^2-q^2}{\overline{m}^2}\right)+2\delta\xi_g\right]+\cdots\right\}^{-1}. \tag{6.6}$$

As in the W-propagator case, we see that the logarithm vanishes in the Fried-Yennie gauge $\xi_g=3$. In fact, its coefficient can be obtained from the analogous term in Eq. (2.17) by substituting $\alpha\rightarrow(4/3)\alpha_s(m)$, where 4/3 arises from the color factor. Writing once more $s=q^2$, $1-s/\overline{m}^2=\rho(s)e^{i\theta(s)}$, the functions $\rho(s)$ and $\theta(s)$ are given by Eq. (2.20) and Eq. (2.21) with the identification $m_2=(m^2-\Gamma^2/4)^{1/2}$, $\Gamma_2=m\Gamma/m_2$, and m_1 defined in Eq. (1.3). The difference between m and the on-shell mass $M=m_0+\mathrm{Re}\,\Sigma(M)$ in leading order is

$$M-m=-\frac{\alpha_s(m)}{3\pi}\Gamma(\xi_g-3)\mathrm{Im}\ \ln\left(\frac{\overline{m}^2-M^2}{\overline{m}^2}\right)=\frac{\alpha_s(m)}{6}\Gamma(\xi_g-3), \tag{6.7}$$

which can also be obtained from Eq. (3.4) by substituting once more $\alpha(m_2)\rightarrow(4/3)\alpha_s(m)$. Thus, in analogy with the W case, $M-m$ is unbounded in NLO. In the Feynman gauge $(\xi_g=1)$ Eq. (6.7) leads to $m-M=\alpha_s(m)\Gamma/3\approx 56$ MeV, while in the Landau gauge $(\xi_g=0)$ we have $m-M\approx 84$ MeV.

VII. CONCLUSIONS

We have shown in Sec. II that conventional mass renormalization [Eq. (1.1)], when applied to the photonic and gluonic diagrams, leads to a series in powers of $M\Gamma/(s-M^2)$, which does not converge in the resonance region [Eq. (2.9)]. In Sec. V we have pointed out that this behavior induces in high orders power-like infrared divergences in both M and Γ. In principle, these severe problems can be circumvented by a resummation procedure, explained in Sec. II. Unfortunately, the resummed expressions are incompatible with the conventional definition of width [Eq. (2.13) and Eq. (5.2)]. In fact, combining the resummed expression with these equations, we have encountered gauge-dependent corrections of $O(\alpha\Gamma)$ to the width and resonant propagator, in contradiction with basic theoretical properties of these amplitudes. This clash between the resummed expressions and the conventional definition of width is not difficult to understand. Indeed, the usual derivation of the latter treats the unstable particle as an asymptotic state, which is clearly an approximation. In Sec. II and V we have discussed an alternative treatment of the resonant propagator and the width based on the complex pole position $\bar{s}=M_0^2+A(\bar{s})$. The non-convergent terms in the resonant region and the potential infrared divergences in Γ and M are avoided by employing $(p^2-\bar{s})^{-1}$ rather than $(p^2-M^2)^{-1}$ in the Feynman integrals, where p is the W or quark loop momentum. The one-loop diagrams lead now directly to the resummed expression of the conventional approach, while the multi-loop expansion generates terms which are genuinely of higher order. The non-analytic terms and gauge-dependent corrections in the resonant region cause no problem because they are proportional to $s-\bar{s}$ and exactly factorize. We emphasize that this is a crucial property, since it implies that the pole position is not displaced and the gauge-dependent corrections can be canceled by vertex and box contributions. Furthermore, they do not lead to difficulties in the evaluation of the width because the latter is now defined by Eq. (5.3). In particular, the non-analytic contributions cancel exactly in its evaluation and the answer is infrared convergent to all orders in the perturbative expansion. Comparing the masses defined in the two approaches, in Sec. III we have reached the conclusion that, unlike the Z case, the gauge dependence of the conventional definition of mass [Eq. (1.1)] is unbounded in NLO for any value of ξ_w. In Sec. V it is further shown that the conventional and alternative formulations of the width coincide in NLO, but not beyond. The analysis reveals also a curious and perhaps universal property: in NLO the non-analytic terms in both the W and quark propagators vanish in the Fried-Yennie gauge $\xi_\gamma=3$.

In the past, a number of authors have employed heuristically the replacement $\ln[(M^2-s)/M^2]\to\ln[(M^2-iM\Gamma-s)/(M^2-iM\Gamma)]$ in order to avoid apparent on-shell singularities (see, for example, Refs. [5, 11] and the first article of Ref. [12]). In this paper we have attempted to clarify the theoretical basis for this heuristic procedure and shown how it emerges from the formalism. In fact, the analysis leads to the conclusion that the replacement $M^2\to\bar{s}$ must be made in the complete expression of the non-analytic terms and that, at the same time, the definition of width must be changed from Eq. (5.2) to Eq. (5.3).

The idea of employing $\bar{s}$, rather than the conventional approach, as a basis to define the mass and width of unstable particles and analyze the propagator in the resonance region has been recently advocated, for different theoretical reasons, by a number of theorists [1–3,9,10,12]. The arguments in this paper provide an additional and powerful argument for such an approach.

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APPENDIX: PHOTONIC CORRECTIONS TO THE TRANSVERSE W SELF-ENERGY IN GENERAL R_ξ GAUGES

The conventional evaluation of the contribution of Fig. 2 to the transverse W self-energy is given by

$$A^\gamma(s)=A^\gamma(s)|_{\xi_i=1}+\Delta A^\gamma(s)|_{\xi_i\neq 1}, \tag{A1}$$

where $A^\gamma(s)|_{\xi_i=1}$, the contribution in the 't Hooft–Feynman gauge, is

$$A^\gamma(s)|_{\xi_i=1}=\frac{\alpha}{2\pi}\left\{-\left(\delta+\ln\frac{M}{\mu}\right)\left(\frac{10}{3}s+3M^2\right)+\frac{11}{6}M^2+\frac{31}{9}s-\frac{M^4}{3s}+\frac{1}{3}(M^2-s)L(s,M^2)\left[5+\frac{2M^2}{s}-\frac{M^4}{s^2}\right]\right\}. \tag{A2}$$

The remainder is

$$\begin{aligned}\Delta A^\gamma(s)|_{\xi_i\neq 1}=\frac{\alpha}{4\pi}\Bigg\{&L(s,M^2)\,\frac{(M^2-s)^2}{M^2}\left[\frac{M^2+s}{s}-\frac{(M^2-s)^2}{6s^2}\right]-L(s,M^2\xi_w)(M^2-s)\,\frac{(M^2\xi_w-s)}{M^2}\left[\frac{M^2+s}{s}-\frac{(M^2\xi_w-s)^2}{6s^2}\right]\\&+(\xi_\gamma-1)\left(\frac{M^2-s}{4}\right)\Bigg[-4\left(\delta+\ln\frac{M}{\mu}\right)-\frac{M^2}{s}(\xi_w+1)+\left(1+\frac{s}{M^2}\right)\ln\xi_w-\xi_w-3-\left(\frac{M^2+s}{s^2M^2}\right)\Big((M^2+s)^2L(s,M^2)\\&+(M^4\xi_w^2-s^2)L(s,M^2\xi_w)\Big)\Bigg]+(\xi_w-1)\Bigg[\left(\delta+\ln\frac{M}{\mu}\right)\left(s-\frac{3}{2}M^2\xi_w-\frac{M^2}{2}\right)+\xi_w\left(\frac{11}{24}M^2+\frac{M^4}{6s}\right)+\frac{s}{6}-\frac{17}{24}M^2\\&+\frac{M^4}{6s}\Bigg]+\ln\xi_w\left[M^2-\frac{s}{6}-\frac{5s^2}{6M^2}+\frac{\xi_w}{2}(M^2+s)-\frac{3}{4}M^2\xi_w^2\right]\Bigg\},\end{aligned} \tag{A3}$$

where

$$L(x,y)=\ln\left(\frac{y-x}{y}\right),$$

and δ is defined after Eq. (2.18).

The ξ_i dependence of the complete transverse self-energy $A(s)$ must vanish on-shell (provided the tadpole contributions are included). The photonic diagrams give rise to all the ξ_γ dependent and $L(s,M^2\xi_w)$ contributions in $A(s)$, and we see that these terms indeed cancel when $s=M^2$. The non-vanishing terms in Eq. (A3) are canceled on-shell by non-photonic contributions.

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