Rescattering information from $B \rightarrow K\bar{K}$ decays

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Rescattering effects can modify the dependence on the weak phase $\gamma = -\operatorname{Arg}(V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$ of the ratio of rates for $B^{\pm} \rightarrow K \pi^{\pm}$ and $B \rightarrow K^{\pm} \pi^{\mp}$. A test for these effects based on the processes $B^{\pm} \rightarrow K^{\pm}K$ has been suggested. It is pointed out that the rates for the processes $B \rightarrow K^+K^-$, which are expected to be *dominated* by rescattering and for which considerably better experimental bounds exist, are likely to provide a more stringent constraint on these effects. [S0556-2821(98)00223-9]

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I. INTRODUCTION

The decays of *B* mesons have provided useful insight into the pattern of weak charge-changing transitions. *B* decays may serve as a new arena for the study of *CP* violation, and may permit the direct measurement of phases of weak couplings even when *CP*-violating effects have not been seen. Such is the case, for example, when one compares rates for the decays $B^{\pm} \rightarrow K \pi^{\pm}$ and $B \rightarrow K^{\pm} \pi^{\mp}$ [1–3]. (States without superscripts will denote neutral mesons or their charge conjugates.) In the simplest picture, the decays $B^{\pm} \rightarrow K \pi^{\pm}$ are dominated by a "penguin" amplitude with a weak phase π , while the decays $B \rightarrow K^{\pm} \pi^{\mp}$ should contain a small additional contribution from a "tree" amplitude with a weak phase γ [1,2,4]. The ratio

$$R = \frac{\Gamma(B^0 \to K^+ \pi^-) + \Gamma(\bar{B}^0 \to K^- \pi^+)}{\Gamma(B^+ \to K^0 \pi^+) + \Gamma(B^- \to \bar{K}^0 \pi^-)}$$
(1)

was shown to provide useful information on the relative importance of different weak subprocesses and hence on the weak phase $\gamma = -\operatorname{Arg}(V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$, especially when complemented with information on *CP*-violating asymmetries such as parametrized by the ratio

$$A_{0} \equiv \frac{\Gamma(B^{0} \to K^{+} \pi^{-}) - \Gamma(\bar{B}^{0} \to K^{-} \pi^{+})}{\Gamma(B^{+} \to K^{0} \pi^{+}) + \Gamma(B^{-} \to \bar{K}^{0} \pi^{-})}.$$
 (2)

A number of recent works [5] have noted that rescattering effects, if sufficiently large, could obviate the above results. One test for such effects [6,7] relies on an SU(3) relation between their contributions in $B^{\pm} \rightarrow K^{\pm} \pi$ and $B^{\pm} \rightarrow K^{\pm} K$ decays. In the present paper we analyze relations among such effects in *all* $B \rightarrow K\bar{K}$ charge states. We find that the rates for the processes $B \rightarrow K^+ K^-$, which are expected to be *dominated* by rescattering and for which better experimental bounds exist, are likely to provide a more stringent constraint

on these effects. We have previously emphasized the role of processes such as $B \rightarrow K^+ K^-$ in evaluating the importance of rescattering [8].

In Sec. II we recapitulate previous results on the determination of γ through bounds [4] based on the ratio R and through the combination of R with CP-violating asymmetry information as provided, for example, by A_0 [2]. We discuss the criticisms raised in Ref. [5] in Sec. III, where we also explain the relation between rescattering in $B^{\pm} \rightarrow K^{\pm} \pi$ and $B^{\pm} \rightarrow K^{\pm} K$. Examples are given in Sec. IV of rescattering via specific intermediate states, where relations among all charge states in $B \rightarrow K\bar{K}$ occur. We remark briefly on the effect of rescattering in extracting the ratio of tree to penguin contributions in $B \rightarrow K^{\pm} \pi^{\mp}$ in Sec. V, and summarize in Sec. VI.

When studying rescattering effects we concentrate on two-body and quasi-two-body intermediate states. It is likely that multiparticle intermediate states play a dominant role in rescattering [9]. Whereas quantitative studies of rescattering effects via intermediate (quasi-)two-body intermediate states are crude and involve various dynamical assumptions [6,10], our present qualitative discussion of such states will employ simple quark diagrams demonstrating general conservation laws.

II. REVIEW OF PREVIOUS RESULTS

A. Flavor-SU(3) decomposition

The decays of *B* mesons to two flavor-octet light pseudoscalar mesons are characterized by 5 flavor-SU(3) invariant amplitudes [11]. An equivalent graphical description [12] in terms of an overcomplete set of six amplitudes displays the contributions in a manner which shows the flow of flavor and color. We use unprimed amplitudes to denote strangenesspreserving ($\Delta S=0$) *b* decays and primed amplitudes to denote *b* decays leading to one unit of net strangeness ($|\Delta S|=1$).

The amplitudes describing $B \rightarrow P_1 P_2$ decays, where P_i denotes one of the pseudoscalar SU(3)-octet mesons, are as follows:

(1) A *tree* amplitude T(T') involves the subprocess $\overline{b} \rightarrow \overline{u}u\overline{d}(\overline{b} \rightarrow \overline{u}u\overline{s})$ in which the $u\overline{d}(u\overline{s})$ produced by the weak current materializes into a single meson. Such a

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process is *color-favored* in the sense that it is of leading order in an expansion of amplitudes in inverse powers of the number N_c of quark colors.

- (2) A *color-suppressed* amplitude C(C') involves the same subprocess as the corresponding tree amplitude, but the quark and antiquark produced by the weak current end up in different mesons. This amplitude is expected to be suppressed by a factor of $1/N_c$ with respect to the tree amplitude.
- (3) A *penguin* amplitude P(P') has the flavor structure b
 →d(b→s), where the light antiquark d(s) ends up in
 one of the final mesons, the spectator quark in the initial
 B ends up in the other, and a light quark-antiquark pair is
 produced in an SU(3)-flavor-singlet state. Electroweak
 penguin amplitudes violate this last condition and will be
 discussed separately.
- (4) An *annihilation* amplitude A(A') involves the annihilation of the \overline{b} and the *u* in a decaying B^+ into a weak current, which then materializes into a pair of light pseudoscalar mesons.
- (5) An *exchange* amplitude E(E') involves the subprocess $\overline{b}d \rightarrow \overline{u}u(\overline{b}s \rightarrow \overline{u}u)$, where the initial light quark is in the decaying particle, and thus contributes only to B^0 (B_s) decays.
- (6) A penguin annihilation amplitude PA(PA') involves the annihilation of a \overline{b} and d (\overline{b} and s) into a state with vacuum quantum numbers, with subsequent production of a pair of light pseudoscalar mesons.

These six amplitudes appear in 5 independent linear combinations, e.g., C+T, C-P, P+A, P+PA, and E+PA, corresponding to the 5 SU(3) invariant amplitudes. Since penguin processes involves loop diagrams with at least one additional power of α_s , they are expected to be modestly suppressed in comparison with tree processes involving comparable sizes of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Since the last three processes involve the participation of the spectator quark, they are expected to be suppressed by a factor of f_B/m_B . The last process should be suppressed by both effects.

Electroweak penguin amplitudes [13] involve no new flavor-SU(3) structures, but require care in identifying weak phases. They may be taken into account by redefining each invariant amplitude to include an electroweak penguin (EWP) contribution [14], $t \equiv T + P_{\text{EW}}^C$, $p \equiv P - (1/3)P_{\text{EW}}^C$, $c \equiv C + P_{\text{EW}}$. We shall ignore these contributions [2,7] for the present discussion.

Application of this SU(3) decomposition relies on associating certain weak phases with some of the six amplitudes. T(T'), C(C'), A(A'), E(E') carry the phase γ . Phases of penguin amplitudes are more involved and require special care when rescattering corrections are considered. For instance, P' is dominated by a weak phase π ; however, rescattering corrections may introduce a significant contribution with phase γ . While such corrections do not affect the SU(3) decomposition, the interpretation of invariant amplitudes can differ significantly from the naive one when re-

TABLE I. Decomposition of $\Delta S = 0$ $B \rightarrow PP$ amplitudes in terms of SU(3) invariant amplitudes.

Decay	Т	С	Р	Ε	Α	PA
$B^+ \rightarrow \pi^+ \pi^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	0	0	0
$K^+ ar K^0$	0	0	1	0	1	0
$B^0 { ightarrow} \pi^+ \pi^-$	-1	0	-1	-1	0	-1
$\pi^0\pi^0$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$1/\sqrt{2}$
K^+K^-	0	0	0	-1	0	-1
$K^0 \overline{K}^0$	0	0	1	0	0	1

scattering is important. We shall give several concrete examples of this circumstance.

We shall discuss here only decays of nonstrange *B* mesons into final states consisting of $\pi\pi$, $K\pi$, and $K\overline{K}$. SU(3)-breaking effects, decays of B_s , and decays involving η and η' states have been treated elsewhere [12,15]. We quote in Tables I and II the decomposition of the relevant decay amplitudes. Overall signs are a consequence of a specific phase convention for meson states [12].

B. Status of data

The CLEO Collaboration [16] has presented evidence for several of the decay modes listed in Tables I and II, and upper limits for others. The branching ratios are summarized in Table III. We also quote our own estimates [17] on the basis of the SU(3) decomposition in Tables I and II and an earlier estimate of the magnitude of invariant amplitudes. We note that these estimates, based on measured $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ rates as input, neglect SU(3) breaking effects and ignore interference between different terms. These estimates will be useful when we come to discuss the contributions of various hadronic states to rescattering processes. We have ignored possible *CP*-violating effects, assuming equal rates for processes and their charge-conjugates.

C. Fleischer-Mannel bound

The predictions of Table III for the decays $B^+ \rightarrow K^0 \pi^+$ and $B^0 \rightarrow K^+ \pi^-$ are based on the assumption that the $|P'|^2$ contribution is the only source of $B^+ \rightarrow K^0 \pi^+$ and is dominant in $B^0 \rightarrow K^+ \pi^-$, where a very small T' contribution is also expected. The equality of the two rates is certainly consistent with present data. However, Fleischer and Mannel [4] have pointed out that if the two rates differ significantly, with R < 1 [see Eq. (1)], one can obtain a useful upper bound on |sin γ |.

TABLE II. Decomposition of $B \rightarrow K\pi$ amplitudes in terms of SU(3) invariant amplitudes.

Decay	T'	<i>C'</i>	P'	E'	Α'	PA'
$B^+ \rightarrow K^0 \pi^+$	0	0	1	0	1	0
$K^+ \pi^0$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
$B^0 \rightarrow K^+ \pi^-$	-1	0	-1	0	0	0
$K^0\pi^0$	0	$-1/\sqrt{2}$	$1/\sqrt{2}$	0	0	0

TABLE III. Branching ratios \mathcal{B} for $B \rightarrow PP$ decays, in units of 10^{-5} . Experimental upper limits are 90% C.L. including systematic errors. Theoretical predictions are based on T(T') and P(P') contributions only, and interference between these two is ignored. Predictions are the same for charge-conjugated states.

Decay	\mathcal{B} (Expt)	\mathcal{B} (Theory)	
$\overline{B^+\!\!\rightarrow\!\pi^+\pi^0}$	<1.6	0.4 ± 0.2	
$K^+ ar{K}^0$	< 0.93	$0.08 {\pm} 0.02$	
$K^0\pi^+$	$1.4 \pm 0.5 \pm 0.2$	1.6 ± 0.4	
$K^+ \pi^0$	$1.5 \pm 0.4 \pm 0.3$	$0.8 {\pm} 0.2$	
$B^0 { ightarrow} \pi^+ \pi^-$	< 0.84	$0.9 {\pm} 0.4$	
$\pi^0\pi^0$	< 0.93	0.04 ± 0.01	
$K^0 \overline{K}^0$	<1.7	$0.08 {\pm} 0.02$	
K^+K^-	< 0.24	a	
$K^+\pi^-$	$1.4 \pm 0.3 \pm 0.1$	1.6 ± 0.4	
$K^0\pi^0$	<4.1	0.8 ± 0.2	

^aNo T or P contributions.

If we ignore a small A' contribution, the amplitude for $B^+ \rightarrow K^0 \pi^+$ may be written

$$A(B^{+} \to K^{0}\pi^{+}) = -|P'|, \qquad (3)$$

where we have taken account of the weak phase $\operatorname{Arg}(V_{tb}^*V_{ts}) = \pi$, and have assumed that the phase of the $\overline{b} \rightarrow \overline{s}$ penguin amplitude is dominated by the top quark contribution. Nothing changes in this discussion if one adds contributions from an internal c-quark with weak phase $\operatorname{Arg}(V_{cb}^*V_{cs}) = 0$, as has been suggested recently [18]. An immediate test of the dominance of this process by a single weak phase is the equality of the rates for $B^+ \rightarrow K^0 \pi^+$ and $B^- \rightarrow \overline{K}^0 \pi^-$ [5].

The amplitudes for $B^0 \rightarrow K^+ \pi^-$ and $\overline{B}^0 \rightarrow K^- \pi^+$ are given, under similar assumptions (one uses isospin symmetry to relate the penguin amplitudes in neutral and charged *B* decays to $K\pi$ states), by

$$A(B^{0} \to K^{+} \pi^{-}) = |P'| - |T'| e^{i\delta} e^{i\gamma},$$

$$A(\bar{B}^{0} \to K^{-} \pi^{+}) = |P'| - |T'| e^{i\delta} e^{-i\gamma},$$
 (4)

where δ is a final-state phase difference between penguin and tree amplitudes. The ratio *R* defined in Eq. (1) is then

$$R = 1 - 2r \cos \gamma \cos \delta + r^2, \tag{5}$$

where $r \equiv |T'/P'|$. For fixed R < 1 and any r, δ the minimum of $|\cos \gamma| = |(R-1-r^2)/(2r \cos \delta)|$ occurs when $\cos \delta = 1$ and $r = (1-R)^{1/2}$, leading to the bound

$$\sin^2 \gamma \leqslant R. \tag{6}$$

D. Determination of γ

If one knows *r* in Eq. (5) and measures the *CP*-violating asymmetry in $B \rightarrow K^{\pm} \pi^{\mp}$ decays, one can solve for γ [1–3]. Defining the pseudo-asymmetry

$$A_{0} = \frac{\Gamma(B^{0} \to K^{+} \pi^{-}) - \Gamma(\bar{B}^{0} \to K^{-} \pi^{+})}{\Gamma(B^{+} \to K^{0} \pi^{+}) + \Gamma(B^{-} \to \bar{K}^{0} \pi^{-})},$$
(7)

one has $A_0 = 2r \sin \delta \sin \gamma$; so

$$R = 1 + r^2 \pm \sqrt{4r^2 \cos^2 \gamma - A_0^2 \cot^2 \gamma}.$$
 (8)

This can be formally solved to give

$$4r \sin \gamma = \pm \{ [(1+r)^2 - (R+A_0)] [(R-A_0) - (1-r)^2] \}^{1/2} \\ \pm \{ [(1+r)^2 - (R-A_0)] [(R+A_0) - (1-r)^2] \}^{1/2}.$$
(9)

Estimates of *r* include 0.16 ± 0.06 [2] and 0.20 ± 0.07 [3]. A measurement of γ to an accuracy of $\pm 10^{\circ}$ will require *r* to be known to $\pm 10\%$. This error seems achievable [2].

The simplicity of this method depends on the assumption that the decay $B^+ \rightarrow K^0 \pi^+$ is dominated by the P' amplitude which has a single weak phase. Other contributions from rescattering with a different weak phase would show up as a CP-violating asymmetry in $B^+ \rightarrow K^0 \pi^+$ vs $B^- \rightarrow \overline{K}^0 \pi^-$ decay rates [5]. Fleischer [7] argues that a modified version of the bound (6) can still be written, while rescattering effects might prevent a sufficiently accurate determination of r. In the next two sections we shall relate the rescattering contributions in $B^+ \rightarrow K^0 \pi^+$ to their contributions in $B \rightarrow K\overline{K}$ decays, where of particular interest is $B^0 \rightarrow K^+K^-$ which is dominated by rescattering. The question of rescattering effects on r will be discussed in Sec. V.

III. RESCATTERING EFFECTS

A. Diagrammatic representation

The prediction that $\Gamma(B^+ \to K^0 \pi^+) = \Gamma(B^- \to \overline{K}^0 \pi^-)$ relies on the dominance of a single weak phase (that of the P'amplitude). In the absence of rescattering (we ignore small electroweak penguin effects [2,7]) and if an annihilation contribution A' is as small as expected [12], $A(B^+ \to K^0 \pi^+)$ $= A(B^- \to \overline{K}^0 \pi^-)$. Moreover, rescattering contributions with a *different* weak phase than that of P' are needed in order to violate this relation. Rescattering amplitudes from intermediate charm-anticharm states carrying the same isospin and the same phase (mod π) as P' do not affect the discussion of Secs. II C and II D.

Typical rescattering contributions to $B^+ \rightarrow K^0 \pi^+$ from intermediate states of two charmless pseudoscalar mesons are illustrated in Fig. 1. We consider only processes involving the T' production amplitude for these intermediate states, with the CKM structure $V_{ub}^* V_{us}$. The weak phase of this combination is γ , so rescattering from intermediate states produced via the T' amplitude can contribute to a *CP*-violating asymmetry in $B^{\pm} \rightarrow \pi^{\pm} K$ decays. We omit for now contributions of the color-suppressed C' amplitude, which has the same weak phase as T'. The contributions of Figs. 1(a) and 1(b) should be added coherently with a relative + sign, corresponding to the S-wave nature of the de-



FIG. 1. Typical rescattering contributions to $B^+ \rightarrow K^0 \pi^+$ from intermediate states of two pseudoscalar mesons. Here P^0 denotes π^0, η, η' . (a) Non-strange meson exchange with topology of P'_u or A', depending on how quark lines in P^0 are connected; (b),(c) strange meson exchange with topology of A'. The dashed lines in (b),(c) serve only to guide the eye in determining the topology.

cay. The contribution of Fig. 1(c) may be related to those of Figs. 1(a) and 1(b) in some models (such as Regge pole exchange) but is independent in general.

The topology of quark lines in Fig. 1 illustrates the mixing of invariant flavor-SU(3) amplitudes induced by rescattering. Consider, for example, Fig. 1(a). Viewed as a diagram in which quark lines flow through meson intermediate states from left to right, Fig. 1(a) has the topology of a \overline{b} $\rightarrow \overline{s}$ penguin diagram in which a *u* quark is the intermediate state in the penguin amplitude. We shall denote the corresponding amplitude by P'_u . Similarly, $P'_{c,t}$ will denote penguin amplitudes for $\overline{b} \rightarrow \overline{s}$ with *c*,*t* intermediate states. A corresponding notation $P_{u,c,t}$ will denote penguin amplitudes for $\overline{b} \rightarrow \overline{d}$ transitions.

In the limit in which one sums over all meson intermediate states, one may expect a form of quark-hadron duality in which Fig. 1(a) is just equivalent to a short-distance P'_u amplitude, expected to be smaller than $P'_{c,t}$ by a factor $|V^*_{ub}V_{us}/V^*_{cb}V_{cs}|$. This would involve a cancellation of contributions reminiscent of that invoked [19] to suppress D^0 $-\overline{D}^0$ mixing. When certain intermediate states are more important than others this duality could well be violated, leading to large rescattering contributions [5]. Thus, it makes sense to explore the contributions of the lowest-mass intermediate states to gain at least a *qualitative* understanding of relations among rescattering contributions to various processes.

There is another way to connect quark lines entering and leaving the neutral meson P^0 in Fig. 1(a). One could join the

u and \overline{u} on the left with one another and the *u* and \overline{u} on the right with one another, making a pair of "hairpins" on the left and right of P^0 . Such a diagram would have the topology of an "annihilation" diagram, since it is equivalent to the initial \overline{b} and *u* annihilating one another. This "hairpin" diagram is the only one possible in the diagram of Figs. 1(b) and 1(c).

In the limit in which mass differences among π^0 , η , and η' can be neglected, and in which these states are orthogonal combinations of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$, the sum of their contributions to $q_i \bar{q}_i \rightarrow q_j \bar{q}_j$, $i \neq j$, should vanish. This is just the familiar nonet symmetry associated with the Okubo-Zweig-Iikuza (OZI) rule. It probably holds less well for pseudoscalar mesons (which can mix strongly with gluonic intermediate states) than for the vast majority of other mesons. Thus, the graphs of Figs. 1(b) and 1(c) (and hence the topology associated with the A' amplitude) should be important only when intermediate states involving pseudoscalar mesons play a major role in rescattering contributions. If we were to replace the intermediate state K^+P^0 in Fig. 1 by a pair of vector mesons $K^{*+}V^0$, the diagrams of Figs. 1(b) and 1(c) should be highly suppressed, since nonet symmetry is very good for vector mesons. Lipkin has stressed the importance of this feature for *B* decays in other contexts [20].

B. Relation between rescatterings in $B \rightarrow K\pi$ and $B \rightarrow K\overline{K}$

Several authors [6,7] have noted an SU(3) relation between contributions to rescattering in $B^+ \rightarrow K^0 \pi^+$ and B^+ $\rightarrow K^+ \bar{K}^0$. The corresponding $\Delta S = 1$ and $\Delta S = 0$ low energy effective Hamiltonians, describing the subprocesses \bar{b} $\rightarrow \bar{s}\bar{q}q$ and $\bar{b} \rightarrow \bar{d}\bar{q}q$ (q=u,d,s,c), involve each two terms multiplied by CKM factors $V_{cb}^* V_{cs}, V_{ub}^* V_{us}$ and $V_{cb}^* V_{cd}, V_{ub}^* V_{ud}$, respectively. The two pairs of $\Delta S = 1$ and $\Delta S = 0$ effective operators are related to each other by a Uspin reflection $d \leftrightarrow s$. The dominant (direct) amplitudes in $B^+ \rightarrow K^0 \pi^+$ and $B^+ \rightarrow K^+ \bar{K}^0$, which are proportional to $V_{cb}^* V_{cs}$ and $V_{cb}^* V_{cd}$ respectively, obey the hierarchy

$$A_c(B^+ \to K^+ \overline{K}^0) = -\lambda A_c(B^+ \to K^0 \pi^+), \qquad (10)$$

where $\lambda = V_{us}/V_{ud} = 0.22$. On the other hand, the amplitudes in $B^+ \rightarrow K^0 \pi^+$ and $B^+ \rightarrow K^+ \overline{K}^0$, which receive contributions from the subprocesses $\overline{b} \rightarrow \overline{u} u \overline{s}$ and $\overline{b} \rightarrow \overline{u} u \overline{d}$ followed by rescattering, are proportional to $V^*_{ub}V_{us}$ and $V^*_{ub}V_{ud}$, respectively, and obey the opposite hierarchy

$$A_u(B^+ \to K^+ \overline{K}^0) = \frac{1}{\lambda} A_u(B^+ \to K^0 \pi^+).$$
(11)

This relation is expected to hold between the amplitudes $P_u + A$ and $P'_u + A'$ in any description of rescattering which respects flavor SU(3). Examples will be given in the next section.

Thus, the ratio A_u/A_c of amplitudes with different weak phases describing rescattering and direct decays in $B^+ \rightarrow K^+ \overline{K}^0$ should be about $-1/\lambda^2$ times larger than the corresponding ratio in $B^+ \rightarrow K^0 \pi^+$. This makes $B^+ \rightarrow K^+ \overline{K}^0$ particularly sensitive to rescattering effects of this kind. We argued in Ref. [2] that A_{μ}/A_{c} might be as large as unity in $B^+ \rightarrow K^+ \overline{K}^0$, raising the predicted rate by as much as a factor of about 4. This could lead to a prediction $\mathcal{B}(B^+)$ $\rightarrow K^+ \bar{K}^0) \simeq (2-4) \times 10^{-6}$ instead of the value $(8\pm2)\times10^{-7}$ guoted in Table III, and to a sizable CP asymmetry with respect to the charge-conjugate process. The corresponding ratio of amplitudes with different weak phases in $B^+ \rightarrow K^0 \pi^+$ could then be as large as $\lambda^2 \approx 0.05$, sufficient to prevent a very useful determination of γ . Fleischer [7] has used larger rescattering effects (via charmless intermediate states), and argued that conceivable values of the squares of these amplitude ratios could be a factor of 5 above our estimates, leading to possible values of $\mathcal{B}(B^+ \rightarrow K^+ \overline{K}^0)$ as large as 2×10^{-5} . This already exceeds the current experimental bound (see Table III). In the next section we shall show that, at least in a few illustrative examples of intermediate rescattering states, one expects similar or larger values for $\mathcal{B}(B^0)$ $\rightarrow K^+K^-$), for which a much better upper experimental limit $(<2.4\times10^{-6})$ exists.

We will study only rescattering via charmless intermediate states, although some rescattering could also be due to states involving charm-anticharm. Our purpose is mainly to show that such final state interactions in $B^0 \rightarrow K^+ K^-$ are as important as in $B^+ \rightarrow K^+ \overline{K}^0$, which in turn are enhanced by factor $1/\lambda$ relative to those in $B^+ \rightarrow K^0 \pi^+$ affecting the determination of γ . Final state interactions via charmanticharm intermediate states obey the opposite hierarchy (10) and do not affect the measurement of γ as explained in Sec. II.

IV. RELATIONS AMONG RESCATTERING AMPLITUDES IN $B \rightarrow K\bar{K}$

A. $\pi\pi$ and $\pi\eta$ intermediate states

The dominant direct contributions to $B^0 \rightarrow K^0 \bar{K}^0$ and $B^+ \rightarrow K^+ \bar{K}^0$ are expected to arise from the penguin amplitude P and to lead to a branching ratio for each process of $(8\pm 2)\times 10^{-7}$, as noted in Table III. The direct contributions to the decay $B^0 \rightarrow K^+ K^-$ are only an exchange (*E*) and a penguin annihilation (*PA*) amplitude and thus are expected to be considerably smaller. On the other hand, the (colorfavored) decays $B^0 \rightarrow \pi^+ \pi^-$ and $B^+ \rightarrow \pi^+ \pi^0$ are expected to have branching ratios of about 8×10^{-6} and 4×10^{-6} , respectively. One might expect rescattering from these states into $K\bar{K}$ to be of some importance.

The decays $B \rightarrow \pi\pi$ can only populate two-pion states of isospin I=0 and I=2 by virtue of Bose statistics. The final $K\bar{K}$ states can have only I=0 and I=1. Consequently, rescattering from $\pi\pi$ states must lead uniquely to an I=0 final $K\bar{K}$ state, with the consequence

$$A(B^{0} \rightarrow \pi \pi \rightarrow K^{+} K^{-}) = -A(B^{0} \rightarrow \pi \pi \rightarrow K^{0} \overline{K}^{0}),$$
$$A(B^{+} \rightarrow \pi \pi \rightarrow K^{+} \overline{K}^{0}) = 0, \qquad (12)$$



FIG. 2. Rescattering contributions to $B^0 \rightarrow K\bar{K}$ from $\pi^+\pi^-$ intermediate states. (a) $B^0 \rightarrow K^0\bar{K}^0$ (topology of P_u), (b) $B^0 \rightarrow K^+K^-$ (topology of E).

independent of any detailed mechanisms. In particular, this relation holds in the presence of each separate contribution to $B \rightarrow \pi \pi$, i.e., *C* and *P* as well as the dominant *T*.

To illustrate how graphical contributions satisfy the relations (12), consider Figs. 2 and 3 which illustrate the rescattering into $K\bar{K}$ from the color-favored *T* contribution to $B \rightarrow \pi\pi$. The contributions of Figs. 2(a) and 2(b) are equal and opposite, with the negative relative sign coming from the convention adopted for meson states. In terms of invariant SU(3) amplitudes, however, Fig. 2(a) has the topology of a P_u amplitude, while Fig. 2(b) has the topology of *E*. The



FIG. 3. Rescattering contributions to $B^+ \rightarrow K^+ \overline{K}^0$ from $\pi^+ P^0$ intermediate states. (a) Topology of P_u or A, (b),(c) topology of A. The contributions (a) and (b) must cancel one another exactly for $P^0 = \pi^0$ since $\pi^+ \pi^0$ in an S-wave has isospin I = 2 while $K^+ \overline{K}^0$ in an S-wave has I = 1.

hierarchy of invariant amplitudes noted in [12,14] thus is strongly affected if rescattering is important.

If P^0 in Fig. 3 is taken to denote a π^0 , the contributions from Figs. 3(a) and 3(b) exactly cancel one another as a result of the opposite relative signs of the $u\bar{u}$ and $d\bar{d}$ components of the π^0 , while Fig. 3(c) does not enter into the calculation at all. Note that whereas Fig. 3(a) has the topology of a P_u or A amplitude (depending on how the quark lines entering and leaving the π^0 are connected with one another), Fig. 3(b) has the topology of A.

The U-spin relation mentioned in Sec. III B cannot be applied if one considers only intermediate $\pi\pi$ contributions to $B \rightarrow K\bar{K}$, since $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$ transforms under $d \leftrightarrow s$ into $(s\bar{s} - u\bar{u})/\sqrt{2} = (\sqrt{3}\eta_8 + \pi^0)/2$. Here η_8 denotes the flavor-octet state $\eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$. One should thus consider both $K^+\pi^0$ and $K^+\eta_8$ intermediate-state contributions to $B^+ \rightarrow K^0\pi^+$ (Fig. 1), and hence, for self-consistency, also $\pi^+\eta_8$ contributions to $B^+ \rightarrow K^+\bar{K}^0$ (Fig. 3). The diagram of Fig. 3(c) must then be included for $B^+ \rightarrow K^+\bar{K}^0$. It is equivalent to that of Fig. 1(c) but with the substitution $d \leftrightarrow s$ everywhere. Ignoring the mass difference between the π^0 and η_8 , one confirms Eq. (11):

$$A(B^{+} \to [K^{+} \pi^{0}, K^{+} \eta_{8}] \to K^{0} \pi^{+})$$

= $\lambda A(B^{+} \to [\pi^{+} \pi^{0}, \pi^{+} \eta_{8}] \to K^{+} \overline{K}^{0}).$ (13)

Within a specific model of Regge pole exchange involving just exchange of the leading strange vector and tensor meson trajectories [6,10], the uncrossed graphs of Figs. 3(a), 3(b) and the crossed graph of Fig. 3(c) are related to one another by crossing symmetry [21]. The graphs of Figs. 3(a) and 3(b) give equal amplitudes after S-wave projection. Note that the final particles are interchanged in the two graphs, as in Figs. 1(a) and 1(b).] The amplitude for an uncrossed graph in Fig. 3(a) has a phase $-e^{-i\pi\alpha(t)}$, while the amplitude for an uncrossed graph in Fig. 3(b) has a phase $-e^{-i\pi\alpha(u)}$, before S-wave projection. Here $t \equiv (p_{\bar{K}^0})$ $(-p_{\pi^+})^2$, $u \equiv (p_{K^+} - p_{\pi^+})^2$. The corresponding crossed graph in Fig. 3(c) has a phase -1 relative to the first two before S-wave projection. Here α denotes the exchangedegenerate vector and tensor kaon trajectories, with $\alpha(0)$ $\simeq 0.32$ [22]. One finds

$$A(B^{+} \to [\pi^{+} \pi^{0}, \pi^{+} \eta_{8}] \to K^{+} \overline{K}^{0})$$

= $-\frac{1}{3}(1 + \Delta)A(B^{0} \to \pi^{+} \pi^{-} \to K^{+} K^{-}),$ (14)

where Δ is the ratio of the S-wave projection of a crossed graph to the S-wave projection of an uncrossed graph. Unless $|\Delta|$ is much greater than 1, we expect that the rescattering amplitude for $B^+ \rightarrow K^+ \overline{K}^0$, assuming just $\pi^+ \pi^0$ and $\pi^+ \eta_8$ intermediate states, should be smaller in magnitude than that of the neutral *B* into $K^0 \overline{K}^0$ or $K^+ K^-$.

We should remark parenthetically that the use of Regge pole models to estimate S-wave scattering amplitudes for light mesons with c.m. energies of more than 5 GeV is highly dubious. Regge pole exchanges are probably valid mainly for *peripheral* partial waves, i.e., orbital angular momenta *l* corresponding to impact parameters $b \approx l/k \approx 1$ fm, where $k \approx 2.6 \text{ GeV}/c \approx 13 \text{ fm}^{-1}$ is the c.m. 3-momentum. Thus for c.m. energies corresponding to those in *B* decays to a pair of light mesons, peripheral partial waves are of order $l \approx 13$, whereas the central partial waves are likely to be highly subject to absorption (or effects of Regge *cuts*) [23]. Consequently, we are not able to place too much stock in any estimate of Δ , in contrast to other considerations in the present paper which are much less model-dependent.

If one includes also $\pi \eta'$ intermediate states and neglects the mass difference between the π^0 , η , and η' , the diagrams of Figs. 3(b) and 3(c) do not contribute. One then finds

$$A(B^+ \to [\pi^+ \pi^0, \pi^+ \eta, \pi^+ \eta'] \to K^+ \overline{K}^0)$$

= $-A(B^0 \to \pi^+ \pi^- \to K^+ K^-),$ (15)

and hence equal rescattering rates for all three $B \rightarrow K\bar{K}$ processes. So depending on whether we consider just $\pi\pi$, also $\pi\eta$, or all three of $\pi\pi$, $\pi\eta$, and $\pi\eta'$ intermediate states, we obtain a rescattering rate for $B^+ \rightarrow K^+\bar{K}^0$ which is either zero, smaller than, or equal to the rates for the other two $B \rightarrow K\bar{K}$ processes.

B. Vector meson intermediate states

An important class of intermediate states more massive than *PP* which contribute to $B \rightarrow PP$ decays is composed of *VV*, where *V* denotes a vector meson. (Angular momentum and parity conservation forbid rescattering of *VP* states into *PP*). Branching ratios at a level of a few times 10^{-5} were obtained for $B^0 \rightarrow \rho^+ \rho^-$, $B^+ \rightarrow \rho^+ \rho^0$ and $B^+ \rightarrow \rho^+ \omega$ in several model-dependent calculations [24]. The importance (and possibly even dominance) of the corresponding $K^*\rho$ intermediate states in rescattering into $K\pi$ final states has been considered recently [25].

Since $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$ and $\omega = (d\bar{d} + u\bar{u})/\sqrt{2}$ are nearly degenerate, it is sufficient to work in the rotated basis $V_u = (\omega - \rho^0)/\sqrt{2}$ and $V_d = (\omega + \rho^0)/\sqrt{2}$. The diagrams describing rescattering contributions to $B \rightarrow K\bar{K}$ from intermediate vector-meson states produced by the dominant tree (*T*) contributions are shown in Figs. 4 and 5.

As in Fig. 2, the $\rho^+\rho^-$ intermediate state provides equal and opposite contributions to $B^0 \rightarrow K^0 \bar{K}^0$ [Fig. 4(a)] and $B^0 \rightarrow K^+ K^-$ [Fig. 4(b)]. Here the isospin argument of Sec. IV A again applies. Although the I=1 state of $\rho^+\rho^-$ can be produced in the decay, since it can be formed by coupling the spins of $\rho^+\rho^-$ to S=1, their orbital angular momenta to L=1, and $\vec{S} + \vec{L} \equiv \vec{J}$ to J=0, it is forbidden by parity to couple to $K\bar{K}$ in an S-wave. We then find

$$A(B^0 \rightarrow \rho^+ \rho^- \rightarrow K^+ K^-) = -A(B^0 \rightarrow \rho^+ \rho^- \rightarrow K^0 \overline{K}^0).$$
(16)

The graphs of Figs. 4(a) and 5(a) are identical, and the contributions of the graphs of Fig. 5(b) and 5(c) must vanish



FIG. 4. Rescattering contributions to $B^0 \rightarrow K\bar{K}$ from $\rho^+\rho^-$ intermediate states. (a) $B^0 \rightarrow K^0\bar{K}^0$ (topology of P_u), (b) $B^0 \rightarrow K^+K^-$ (topology of *E*).

if the vector mesons respect nonet symmetry and the OZI rule. This implies a simple relation

$$A(B^{+} \rightarrow \rho^{+} V^{0} \rightarrow K^{+} \overline{K}^{0}) = A(B^{0} \rightarrow \rho^{+} \rho^{-} \rightarrow K^{0} \overline{K}^{0})$$
$$= -A(B^{0} \rightarrow \rho^{+} \rho^{-} \rightarrow K^{+} K^{-}).$$
(17)

Thus, the rescattering due to two vector mesons produced via the color-favored *T* amplitude gives equal contributions for all three $B \rightarrow K\bar{K}$ processes.

The U-spin relation of Sec. III B is evident if we perform the interchange $d \leftrightarrow s$ on the graphs of Fig. 5. The results are



FIG. 5. Rescattering contributions to $B^+ \rightarrow K^+ \overline{K}^0$ from $\rho^+ V^0$ intermediate states, where V^0 is a linear combination of ρ^0 and ω . (a) Topology of P_u or A, (b),(c) topology of A. Since V^0 is produced as $V_u = u\overline{u}$ but must rescatter as $V_d = d\overline{d}$ (b) or $V_s = s\overline{s}$ (c), the last two contributions must vanish.

the graphs of Fig. 1, in which K^+P^0 are replaced by $K^{*+}V^0$. Figure 5(a) then describes the decay $B^+ \rightarrow K^0 \pi^+$ via an induced P_u contribution, while Figs. 5(b) and 5(c) continue to give vanishing contributions to this process.

If one includes color-suppressed contributions to vectormeson pair production, the simple relations (17) no longer seem to hold. However, one expects these contributions to be relatively small.

C. $a_1\pi$ and related intermediate states

The branching ratio of $B^0 \rightarrow a_1^+ \pi^-$ was estimated [26] to be similar to that of $B^0 \rightarrow \rho^+ \pi^-$, a few times 10^{-5} . The $a_1\pi$ intermediate states, produced by dominant tree (*T*) contributions with a weak phase γ , can therefore lead to significant rescattering amplitudes into $K\bar{K}$ states.

In this case, a simple relation among the rescattering amplitudes into the three $K\bar{K}$ states follows from G-parity conservation. Since the G-parity of $a_1\pi$ is +1, and that of $K\bar{K}$ in a state of angular momentum L and isospin I is $(-1)^{L+I}$, an S-wave $K\bar{K}$ state into which $a_1\pi$ states rescatter must be pure I=0. Therefore,

$$A(B^0 \rightarrow a_1 \pi \rightarrow K^+ K^-) = -A(B^0 \rightarrow a_1 \pi \rightarrow K^0 \overline{K}^0),$$
$$A(B^+ \rightarrow a_1 \pi \rightarrow K^+ \overline{K}^0) = 0.$$
(18)

Again, as in the case of intermediate $\pi\pi$ states, this relation can be demonstrated using figures analogous to Figs. 2 and 3.

The I=0 partners of the π are η and η' ; those of the a_1 are $f_1(1285)$ and $f_1(1420)$ or $f_1(1510)$ [27]. These states have even G-parity and probably contribute in color-allowed rescattering processes leading to $B^+ \rightarrow K^+ \overline{K}^0$. As in the case of rescattering from *PP* or *VV* intermediate states, the $K^+ \overline{K}^0$ mode is not likely to be greatly suppressed in a practical calculation. Our purpose was rather to show that the $K^+ K^-$ mode is not likely to be *smaller* than the others when rescattering from a small number of specific intermediate states is dominant.

D. Inclusive intermediate states

We would like to draw a more general conclusion from the previous examples. The generic case of neutral mesons in intermediate states is probably more analogous to the case of Sec. IV B, in which nonet symmetry is valid and transitions $q_i\bar{q}_i \rightarrow q_j\bar{q}_j$ ($i \neq j$) are forbidden. Then Figs. 1(a), 3(a), and 5(a) are interpreted purely as P_u , and contributions of Figs. 1(b),1(c), 3(b),3(c), and 5(b),5(c) should vanish. Hence, one finds no color-favored rescattering contributions to annihilation-type amplitudes. (There will still be colorsuppressed contributions from rescattering to these processes.) Color-favored rescattering processes $B \rightarrow M_1M_2$ $\rightarrow K\bar{K}$ (M_1 and M_2 are light-quark mesons) involving the CKM factor $V_{ub}^*V_{ud}$ will then contribute equal amplitudes in all three $B \rightarrow K\bar{K}$ decays, which we would describe as effective P_u and E contributions.



FIG. 6. Examples of graphs contributing to the short-distance description of rescattering in $B \rightarrow K\bar{K}$ processes. The ovals denote form factors. (a) Processes with topology of a P_u contribution, (b) process with topology of an *E* contribution.

As one sums over more and more intermediate states contributing to the rescattering process and neglects meson mass differences, we would expect the relations among different processes to be more and more accurately described by amplitudes corresponding to quark graphs [12]. This corresponds to a notion of quark-hadron duality akin to that in e $+e^- \rightarrow$ hadrons or $\tau \rightarrow \nu_{\tau}$ + hadrons. When the intermediate hadronic states are broad and overlapping, an effective description in terms of quarks and gluons should become a good approximation. One then needs, of course, to incorporate the free quarks into pairs of light pseudoscalar mesons, which requires the introduction of form factors. The invariant amplitudes introduced in [12] and similar approaches take such form factors into account in a flavor-SU(3)invariant manner. Rescattering contributions then are described in terms of quarks and gluons as well, as illustrated by the examples in Fig. 6. The final quarks, as before, have to be incorporated into hadrons.

Contributions of P_u and P'_u graphs should be evaluable from a short-distance point of view and are expected to be given roughly by [28] $|P_u| \approx |V_{ub}^* V_{ud} / V_{cb}^* V_{cd}| |P|$, $|P'_u| \approx |V_{ub}^* V_{us} / V_{cb}^* V_{cs}| |P'|$. Here one has incorporated unknown form factor information into the amplitude |P'|which we have claimed is the dominant contribution to observed $B \rightarrow K\pi$ decays.

In the absence of significant long-distance effects the contributions of A(A') and E(E') type graphs should contain a factor of f_B/m_B . It is not clear how the form factors [29] in such graphs as Fig. 6(b) compare with those in Fig. 6(a), however. An explicit calculation is needed [30]; we expect it to be a more reliable guide to the magnitude of such rescattering contributions than the popular Regge-pole analyses.

As the hierarchy of amplitudes in terms of a graphical description becomes more and more valid, one should then expect the prediction for the rate for $B^0 \rightarrow K^+K^-$ to drop significantly below that for $B^0 \rightarrow K^0\bar{K}^0$ or $B^+ \rightarrow K^+\bar{K}^0$. A rate for $B^0 \rightarrow K^+K^-$ close to its present upper experimental limit would indicate not only that rescattering contributions are appreciable but that they violate the expected hierarchy of amplitudes. As we have indicated in previous subsections,



FIG. 7. Rescattering contributions to (a) $B^0 \rightarrow K^+ \pi^-$ from $K^{*+}\rho^-$ intermediate states and (b) $B^+ \rightarrow K^0 \pi^+$ from $K^{*+}V^0$ intermediate states.

the decay rate for $B^0 \rightarrow K^+ K^-$ should be comparable to that for the other two $B \rightarrow K\bar{K}$ processes if rescattering is an important contributor to the rates for these processes and is dominated by a few specific intermediate states.

V. RESCATTERING AND TREE-PENGUIN AMPLITUDE RATIO

In the first paper of Ref. [7] it was noted that rescattering could affect the determination of the ratio r = |T'/P'| which was needed to extract the weak phase γ from the ratio of $B^{\pm} \rightarrow K \pi^{\pm}$ and $B \rightarrow K^{\pm} \pi^{\mp}$ rates. This is true to some extent for the determination $r = 0.16 \pm 0.06$ [2], which relied upon information from the decays $B \rightarrow \pi^{+} \pi^{-}$ and $B^{\pm} \rightarrow \pi^{\pm} \pi^{0}$. In that determination it was assumed that these processes were dominated by the color-favored amplitude *T*, and that factorization could be used to relate *T* to the corresponding strangeness-changing amplitude *T'*.

As noted in Ref. [2], a cleaner way to determine the *T* amplitude in the long run will be to use the semileptonic process $(B^0 \rightarrow \pi^- l^+ \nu_l)$, currently measured to have branching ratio [31]

$$\mathcal{B}(B^0 \to \pi^- l^+ \nu_l) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}.$$
 (19)

When the spectrum for this process is well enough measured, one will use the relation

$$\Gamma(B^{0} \to K^{+} \pi^{-})|_{\text{tree}} = 6 \pi^{2} f_{K}^{2} |V_{us}|^{2} a_{1}^{2} \left. \frac{d\Gamma(B^{0} \to \pi^{-} l^{+} \nu_{l})}{dq^{2}} \right|_{q^{2} = m_{K}^{2}}$$

$$(20)$$

to evaluate T'.

The key element in assuming that this factorization approach yields T' arises in the assumption that rescattering effects do not by themselves contribute a significant T' piece in $B \rightarrow K\pi$ decays. Note that T' is defined as an amplitude with weak phase γ . A typical rescattering contribution to $B^0 \rightarrow K^+\pi^-$ carrying this phase is shown in Fig. 7(a). A

corresponding contribution to $B^+ \rightarrow K^0 \pi^+$ is shown in Fig. 7(b). An additional contribution to $B^0 \rightarrow K^+ \pi^-$ of course comes from the elastic intermediate state, whereas no such contribution occurs in $B^+ \rightarrow K^0 \pi^+$.

Using arguments as in Sec. IV, it can be seen that inelastic rescattering is likely to be of comparable importance in $B^0 \rightarrow K^+ \pi^-$ and $B^+ \rightarrow K^0 \pi^+$. For any inelastic channel leading to a $K^+\pi^-$ final state by a diagram of type 7(a) there will be an isospin-related diagram of type 7(b), in which a corresponding intermediate state rescatters to $B^+ \rightarrow K^0 \pi^+$. Using this picture, the only difference between rescattering in the two processes comes from the less important *elastic* channel which only contributes to $B^0 \rightarrow K^+ \pi^-$. Similar elastic rescattering contributions should affect $B^0 \rightarrow \pi^+ \pi^-$ or $B^+ \rightarrow \pi^+ \pi^0$. Their presence would be manifested in the failure of factorization in the comparison of $B \rightarrow \pi l \nu_l$ and colorfavored $B \rightarrow \pi \pi$ decays. There are two ways to gauge the importance of the major (inelastic) rescattering in B^+ $\rightarrow K^0 \pi^+$. One way is to look for rate enhancements in B $\rightarrow K\bar{K}$ as discussed in Sec. IV. The other method [2,7] is by looking for a *CP*-violating rate difference between B^+ $\rightarrow K^0 \pi^+$ and its charge conjugate. Thus, it appears that one will have satisfactory cross-checks of the methods used to extract r from B decays. The method becomes particularly simple if $B \rightarrow K\bar{K}$ rates show no enhancement relative to naive expectations, if no asymmetry is measured between B^+ $\rightarrow K^0 \pi^+$ and its charge-conjugate, and if comparison of B $\rightarrow \pi l \nu_l$ with color-favored $B \rightarrow \pi \pi$ decays supports factorization.

VI. SUMMARY

We have discussed possible ambiguities in the determination of the weak phase γ through a comparison of $B^{\pm} \rightarrow K\pi^{\pm}$ and $B \rightarrow K^{\pm}\pi^{\mp}$ decays. We have shown that satisfactory means exist for measuring the effects of rescattering on these processes by studying the effects in $B \rightarrow K\bar{K}$ decays. Rescattering effects in these processes are enhanced by $1/\lambda^2$ relative to those in $B \rightarrow K\pi$. In particular, the decay $B^0 \rightarrow K^+K^-$ is of great interest since it is dominated by rescattering effects. We demonstrated a few cases in which the rescattering amplitude in this process is expected to be as pronounced as in $B^+ \rightarrow K^+ \overline{K}^0$ and $B^0 \rightarrow K^0 \overline{K}^0$. In the illustrative cases of $\pi\pi$ and $a_1\pi$ intermediate states, rescattering into $K^+ \overline{K}^0$ is allowed while rescattering into $K^+ \overline{K}^0$ is forbidden by isospin and G-parity, respectively. Upper limits on the rates of $B \rightarrow K^+ \overline{K}^-$ can be used to set bounds on rescattering effects in $B^{\pm} \rightarrow K \pi^{\pm}$,

$$|P'_{u}/P'| \simeq \lambda \sqrt{\frac{\Gamma(B^{0} \rightarrow K^{+}K^{-}) + \Gamma(\overline{B}^{0} \rightarrow K^{+}K^{-})}{\Gamma(B^{+} \rightarrow K^{0}\pi^{+}) + \Gamma(B^{-} \rightarrow \overline{K}^{0}\pi^{-})}}.$$
(21)

Whereas estimates of rescattering effects are rather crude and depend on rescattering models (such as Regge-exchange [6,10]), our present considerations were model-independent once one assumed a dominant set of intermediate states contributing to the rescattering. Our results were shown to depend somewhat on the intermediate states through which rescattering occurs.

In the absence of rescattering contributions, or when rescattering contributions respect a hierarchy of amplitudes which predicts a suppression of processes involving the spectator quark, the decays $B \rightarrow K^+ K^-$ are expected to be highly suppressed. A very useful upper limit on the average branching ratio of these processes would be 4×10^{-8} , a factor of 60 below the present limit, which seems achievable in future experiments [32]. In this case the method we have proposed previously should be sufficient for measuring γ to a level of 10° [2]. A more modest limit, 4×10^{-7} , would leave an uncertainty in γ of the order of a few tens of degrees. Conversely, an observation of these decay modes may provide an early warning of the importance of rescattering effects, since present experimental bounds on them are considerably more stringent than on other modes expected to be enhanced by rescattering effects.

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