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Diquark masses from lattice QCD

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We present the first results for diquark correlation functions calculated in the Landau gauge on the lattice. Masses have been extracted from the long distance behavior of these correlation functions. We find that the ordering of the diquark masses with spin 0 and 1 states in color anti-triplet and sextet channels is in accordance with instanton motivated interaction models. Although we find evidence for an attractive interaction in color anti-triplet states with a splitting between spin 0 and spin 1 diquarks that can account for the mass splitting between the nucleon and the delta, there is no evidence for a deeply bound diquark state.
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I. INTRODUCTION

For a long time it has been speculated that QCD at a finite baryon number density and low temperature may have a much richer phase structure than at high temperature and low or vanishing baryon number density. In general, this is due to the fact that quantum statistics are much more important at low temperature and energetically favors bosonic forms of matter over fermionic matter. At low temperature and low density, the possibility of pion and kaon condensates thus has been discussed. Unlike in the high temperature case, where it is evident that the strongly interacting matter exists in the form of a quark-gluon plasma, it has been speculated that at high baryon density bosonic states—diquarks and even larger quark clusters—may play an important role [1]. Recently, it has been suggested that diquarks may, in fact, form a Bose condensate at high densities and low temperatures [2,3] and also the possibility of a dibaryon phase has again been discussed [4].

The realization of such states of matter crucially depends on details of the interaction among quarks. In particular, the analysis of the fine structure of the hadron spectrum such as the nucleon-delta mass splitting suggests that there exists a strong attractive force between quarks in a color anti-triplet state with anti-parallel spin orientation which leads to the formation of a diquark state. Such a spin dependent interaction naturally arises in the framework of perturbative QCD from one-gluon exchange [5]. Furthermore, it has been realized that a spin and flavor dependent interaction among constituent quarks is needed to account for a satisfactory description of the fine structure of the experimentally observed baryon spectrum [6]. Such an attractive spin and flavor dependent interaction indeed is induced by instantons [7,8]. This raises the possibility for the existence of rather tightly bound spin 0 diquark states. Some indications for such states have been found from the analysis of diquark correlation functions in the instanton liquid model [9]. A quantitative analysis of the interaction between quarks is clearly needed to judge the existence of the interesting diquark phase structure discussed in [2,3]. In particular, the possible coexistence of a chiral symmetry broken phase with a diquark condensate will crucially depend on the value of the diquark masses.

In this paper we will present first results from a calculation of diquark correlation functions on the lattice. In particular, we present results for diquarks in the color anti-triplet representation. Our spectrum calculations have been performed in quenched QCD using perturbatively improved gauge and fermion actions. As quark and diquark correlation functions are gauge-variant observables, we have performed all our calculations in a fixed gauge.[1] We have used the Landau gauge as is commonly done in the analysis of quark [10,11] and gluon masses [12,13] on the lattice.

II. QUARK AND DIQUARK CORRELATION FUNCTIONS

While it is generally expected that the interaction between up and down quarks in a spin 0 color anti-triplet state is attractive, it is not that obvious how quarks in other quantum number channels interact. The analysis of diquark states in the instanton liquid model suggests a quite deeply bound spin 0 diquark state [9]. Such calculations, however, may miss a contribution from the confining part of the quark-quark interaction. In general, the diquark masses will receive contributions from the constituent masses, the confining and the spin-dependent part of the quark-quark interaction. In terms of a simple potential model, these contributions are to leading order additive [6],

$$m_{FSC} = 2\overline{m}_q + V_{conf} + V_S\,, \tag{2.1}$$

where the subscript FSC denotes *flavor, spin and color* quantum numbers, $\overline{m}_q$ denotes the constituent quark mass, V_{conf} and V_S are the contributions from the confining and spin-dependent parts of the quark-quark potential. The latter may include contributions from flavor-spin as well as color-spin dependent interaction terms. Some information on the contribution of the different terms to the diquark masses exists from the analysis of hadron masses within the framework of potential models. In the case of three quark states, the corresponding form of Eq. (2.1) reads, for instance, $m_N = 3\overline{m}_q + \tilde{V}_{conf} + 2V_0 + V_1$ for the nucleon and

[1] Alternatively, one could couple the light diquark states to a heavy quark, which serves to neutralize the color [9]. This, however, only trades *gauge dependence* against *path dependence*.

TABLE I. Diquark states with spin S in flavor (F) and color (C) anti-triplet and sextet representations. The third and fourth column give the relative strength of interaction terms corresponding to a flavor-spin and color-spin coupling, $V_S \sim (\lambda_1^a \lambda_2^a)(s_1 s_2)$, where λ_i^a denote the generators of $SU(3)_{flavor}$ or $SU(3)_{color}$, respectively.

(F,S,C)	State	F-S coupling	C-S coupling
(3*,0,3*)	$\epsilon_{abc}(C\gamma_5)_{\alpha\beta}u^\dagger_{a,\alpha}d^\dagger_{b,\beta}$	-2	-2
(6,1,3*)	$\epsilon_{abc}u^\dagger_{a,\alpha}u^\dagger_{b,\alpha}$	$-1/3$	2/3
(3*,1,6)	$u^\dagger_{c,\alpha}d^\dagger_{c,\alpha}$	2/3	$-1/3$
(6,0,6)	$(C\gamma_5)_{\alpha\beta}u^\dagger_{c,\alpha}u^\dagger_{c,\beta}$	1	1

$m_\Delta = 3\overline{m}_q + \widetilde{V}_{conf} + 3V_1$ for the delta, with $\widetilde{V}_{conf} = 3V_{conf}$. While the masses are expected to receive the largest contributions from the constituent mass term ($\overline{m}_q \simeq 300$ MeV) and the confining part ($V_{conf} \simeq 200$ MeV [6]), the nucleon-delta mass splitting is entirely determined by the difference in the spin dependent part of the potential. The latter also is related to the mass splitting of the $S=0$ and $S=1$ diquarks,

$$m_{613} - m_{303} = \tfrac{1}{2}(m_\Delta - m_N). \tag{2.2}$$

In the following, we will analyze $S=0$ and $S=1$ diquark states in different color and flavor representations. The four different states considered are listed in Table I.

The corresponding diquark correlation functions are built up from the quark correlation function $G^{ab}_{\alpha,\beta}$. For the color anti-triplets we obtain, for instance,

$$\begin{aligned} G^{303}_{cf}(\vec{x},t) &= \epsilon_{abc}\epsilon_{def}(C\gamma_5)_{\alpha\beta}(C\gamma_5)_{\gamma\delta}G^{ad}_{\alpha\gamma}G^{be}_{\beta\delta}, \\ G^{613}_{cf}(\vec{x},t) &= \epsilon_{abc}\epsilon_{def}(G^{ad}_{\alpha\beta}G^{be}_{\alpha\beta} - G^{ae}_{\alpha\beta}G^{bd}_{\alpha\beta}), \end{aligned} \tag{2.3}$$

where Latin (Greek) indices denote color (spinor) degrees of freedom.

We have analyzed these correlation functions in Landau gauge. To be specific, we have calculated the diagonal correlators, $G^{F,S,C}(t) \equiv G^{F,S,C}_{a,a}$ and the scalar part of the quark propagator, $G_q(t) \equiv 4G^{a,a}_{\alpha,\alpha}$ where the sum is taken over a and α and also over the spatial coordinates $\vec{x}$ in order to project onto zero-momentum states.

Our calculations have been performed on lattices of size $16^3 \times 32$. The gauge field configurations have been generated with a tree-level Symanzik improved action at a gauge coupling $6/g^2 = 4.1$. A calculation of the string tension at this value of the coupling leads to $\sqrt{\sigma}a = 0.3773(22)$, i.e. a cutoff $a^{-1} \simeq 1.1$ GeV [14].[2] We have generated 73 gauge field separated by 100 sweeps of 4 overrelaxation and 1 heat bath steps each. These are fixed to Landau gauge using an algorithm based on Fourier accelerated overrelaxation [15]. In the fermion sector, we use the Sheikholeslami-Wohlert action with a tree-level clover coefficient [16]. On each gauge field configuration, the fermion matrix has been inverted four times, i.e., with source vectors at four different lattice sites, for eight different quark mass values. Our analysis thus is based on a sample of 292 quark propagators. Fits have been performed with one and two exponential functions. Leaving out successively data points at small time separations, we look for stable results for the fitted masses. Typical results obtained from single exponential fits of quark, diquark and nucleon correlation functions are shown in Fig. 1. Fits with two exponential functions do reach a plateau earlier and yield consistent results.

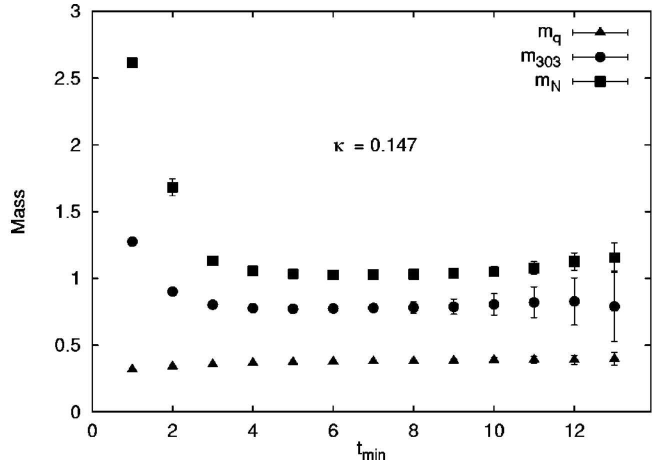

FIG. 1. Masses extracted from single exponential fits to the correlation functions of quark, color anti-triplet spin 0 diquark and nucleon at $\kappa = 0.147$ in the interval $[t_{min}, 32 - t_{min}]$.

In addition to the gauge dependent quark and diquark correlation functions, we also construct the standard hadron correlation functions for the pion, rho, nucleon and delta. The eight different quark masses selected correspond to the interval $0.5 < m_\pi/m_\rho < 0.9$. Our current analysis thus is still restricted to rather heavy quark masses. This is also reflected in the fact that the ratio m_N/m_ρ is still close to that of the additive quark model, i.e., $m_N/m_\rho \simeq 1.5$. Nonetheless, we note that our analysis does yield quite sizeable results for the nucleon-delta mass splitting (Table II) which was determined from a simultaneous fit of the ratio of nucleon and delta correlation functions and the nucleon correlation functions. From an extrapolation to the chiral limit, we obtain $m_\Delta - m_N = 184(63)$ MeV, which is about 40% below the experimental value. It is, however, consistent with earlier calculations with similar parameters [17,18]. For our current analysis, it is reassuring that we can observe a sizeable splitting in baryonic states and are thus sensitive to the spin dependence of hadron masses.

The basic component for the analysis of diquark correlation functions is the quark propagator. Although the quark mass is, in principle, a gauge-variant quantity, it has been found to show only little gauge dependence in a class of covariant gauges, which includes the Landau gauge [10]. The quark masses extracted in Landau gauge are consistent with constituent quark mass values of (350–400) MeV

[2]Here and in the following, we use $\sqrt{\sigma} = 420$ MeV to set the scale for all masses. Although $\sqrt{\sigma}$ is experimentally less well determined than a hadron mass, it is more appropriate to use it to set the scale in a quenched calculation where hadron masses are calculated for various values of the quark mass.

TABLE II. Hadron masses and the nucleon-delta mass splitting for various values of the hopping parameter. The last row gives the results of an extrapolation to the chiral limit obtained from a linear fit in $\kappa^{-1}-\kappa_c^{-1}$. The critical value of the hopping parameter for vanishing pion mass has been determined as $\kappa_c=0.14923(2)$.

κ	m_π	m_ρ	m_N	$m_\Delta - m_N$
0.140	0.910(1)	1.025(2)	1.608(6)	0.065(10)
0.142	0.794(1)	0.932(2)	1.457(6)	0.079(12)
0.144	0.667(1)	0.836(3)	1.298(6)	0.098(16)
0.145	0.596(1)	0.787(4)	1.212(7)	0.114(20)
0.146	0.519(1)	0.739(7)	1.122(10)	0.128(22)
0.147	0.430(2)	0.688(15)	1.029(13)	0.135(41)
0.1475	0.379(2)	0.661(18)	0.982(13)	0.131(66)
0.148	0.316(3)	0.595(66)	0.924(14)	0.147(97)
κ_c		0.579(18)	0.821(13)	0.166(56)

[10,11]. Our results for the quark correlation function are consistent with these earlier findings. We generally observe that local masses extracted from $G_q(t)$ rise with increasing temporal distance t and develop a plateau for $t \gtrsim 6$. In this region we have performed exponential fits to extract the quark mass. The results are summarized in the second column of Table III. Using the five largest κ-values to extrapolate linearly in $\kappa^{-1}-\kappa_c^{-1}$ to the chiral limit, we find $\overline{m}_q/\sqrt{\sigma}=0.813(30)$ or $\overline{m}_q \simeq 342(13)$ MeV.

The diquark correlation functions corresponding to the color anti-triplet representation show a remarkably clean exponential decay. This is reflected in the small t-dependence of local masses shown in Fig. 1 and also suggests the existence of an attractive interaction in this channel. Although the diquark correlation functions are also gauge variant, the local masses show a behavior very similar to that of ordinary hadron masses. They approach a plateau from above. In the case of the spin 0 diquark state, this is typically reached already for $t \gtrsim 4$. For the spin 1 correlation functions, we observe larger contributions from excited states at short distances and consequently the plateau is reached only for $t \gtrsim 6$. The situation is improved when we perform fits with two exponentials. These yield stable results for distances $t \gtrsim 2$. The mass values obtained from such fits are given in Tables II and III. In Fig. 2 we show results for half the spin 0 diquark masses and a combination of nucleon and delta masses, $(m_\Delta + m_N)/6$, which generally is used as a definition of the constituent quark mass. As expected from potential models, $m_N/3$ is significantly lighter than the quark masses calculated by us in Landau gauge. We also note that in the chiral limit ($\kappa \equiv \kappa_c$), the quark mass agrees quite well with $(m_\Delta + m_N)/6$. This is quite astonishing, as this phenomenologically defined constituent quark mass receives, at least in the context of potential models, additional contributions from the confining part of the potentials as well as the spin-dependent parts. This difference also is reflected in the different hopping parameter (bare quark mass) dependence seen in Fig. 2.

TABLE III. Quark and diquark masses for various values of the hopping parameter. The last row gives the results of an extrapolation to the chiral limit obtained from a linear fit in $\kappa^{-1}-\kappa_c^{-1}$.

κ	$\overline{m}_q$	m_{303}	m_{613}
0.140	0.586(5)	1.190(9)	1.207(11)
0.142	0.531(5)	1.079(12)	1.102(13)
0.144	0.472(6)	0.962(14)	0.993(14)
0.145	0.442(6)	0.901(15)	0.936(15)
0.146	0.410(8)	0.839(15)	0.880(26)
0.147	0.378(11)	0.774(15)	0.827(33)
0.1475	0.361(12)	0.737(18)	0.806(35)
0.148	0.344(11)	0.696(18)	0.806(45)
κ_c	0.307(11)	0.623(19)	0.727(37)

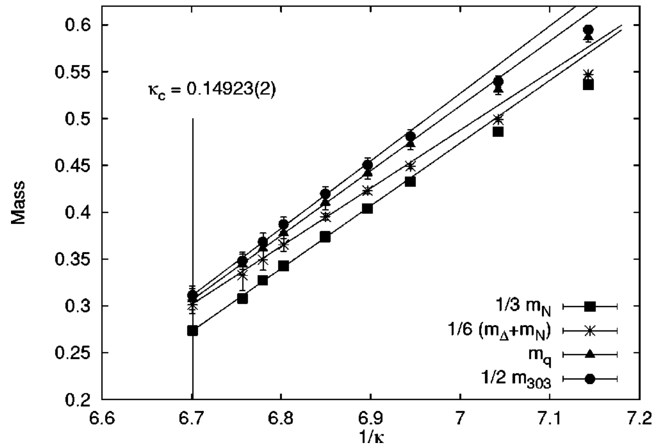

FIG. 2. 1/2 diquark(303), 1/3 nucleon and 1/6 (nucleon+delta) in comparison with the quark mass calculated in Landau gauge.

The mass of the spin 0, color anti-triplet diquark is slightly larger than twice the constituent quark mass. Interpreted again in terms of a potential model, this suggests that the positive energy contribution resulting from confinement is just balanced by a contribution from an attractive spin interaction. From Fig. 2 as well as Tables II and III it is, however, obvious that the diquark masses show no indication for a deeply bound diquark state. In the tables we also give the result of an extrapolation to the chiral limit which is based on the five lightest quark masses and is obtained from a fit linear in $\kappa^{-1}-\kappa_c^{-1}$. Using again the string tension to set the scale, we find $m_{303}=694(22)$ MeV.

Similar to what has been observed in the case of the delta-nucleon mass splitting, we find that the difference between the masses of the spin 0 and spin 1 diquarks increases with decreasing quark mass. This is shown in Fig. 3. From an extrapolation to the chiral limit, we find $m_{613}-m_{303}=0.104(42)$, which in fact is about 60% of the splitting found in the nucleon channel. This agrees well with the behavior expected from potential models (Eq. 2.2).

Let us finally comment on the correlation functions for diquarks in a color sextet representation. We find that these have a much smaller amplitude and are therefore much more noisy. Moreover, they receive much larger contributions from excited states at short distances and reach a plateau only very late. The general tendency is that the color-sextet

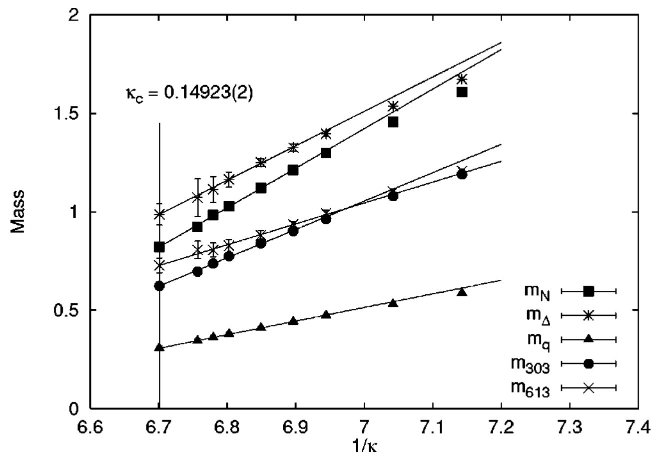

FIG. 3. Mass splitting between color anti-triplet diquarks and nucleon/delta: The lines show fits to the results obtained with the five lightest quark masses.

states have larger masses than the triplet states. This is in accordance with expectations based on a flavor-spin dependent coupling (Table I). We will report on a more detailed analysis of these correlation functions elsewhere.

III. CONCLUSION

We have analyzed color anti-triplet diquark states with spin 0 and 1 as well as quark masses in Landau gauge. We find that the mass splitting between spin 0 and spin 1 diquark states can account for the observed mass splitting between the nucleon and delta.

Although the current analysis has been performed in the quenched approximation of QCD on quite coarse lattices and with fairly large quark masses ($m_\pi/m_\rho \gtrsim 0.5$), the current analysis does seem to rule out a deeply bound diquark state. For the lightest spin 0 state, we find $m_{303} \simeq 700$ MeV which is slightly larger than twice the constituent quark mass.

This makes the existence of a diquark phase coexisting with a chiral symmetry broken phase as suggested in [2] unlikely. However, our finding of a sizeable splitting between the spin 0 and 1 anti-triplet diquark states and the clear exponential decay of the corresponding correlation functions gives some support for an attractive q-q interaction in the spin 0 channel and for the existence of a color superconducting diquark phase at high density. We should stress that our current analysis has been performed in the $T=0$ confining phase of QCD where the attractive interaction due to instantons (flavor-spin coupling) is expected to give the dominant contribution. In the high density regime, it is expected that the instanton induced interactions become suppressed and one-gluon exchange (color-spin coupling) becomes increasingly important for the attractive interaction. Further studies of the temperature and density dependence of diquark masses as well as the nucleon-delta splitting are thus needed. The latter will be difficult to analyze for 3-color QCD due to the well-known algorithmic problems in QCD with nonvanishing chemical potential. The density dependence of the q-q interactions may, however, first be analyzed in 2-color QCD [2].

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