

Neutron and lepton electric dipole moments in the minimal supersymmetric standard model, large CP violating phases, and the cancellation mechanism

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An analysis of the electric dipole moment (EDM) of the neutron and of the lepton in the minimal supersymmetric standard model (MSSM) with the most general allowed set of CP violating phases without generational mixing is given. The analysis includes the contributions from the gluino, the chargino and the neutralino exchanges to the electric dipole operator, the chromoelectric dipole operator, and the CP violating purely gluonic dimension six operator. It is found that the EDMs depend only on certain combinations of the CP phases. The independent set of such phases is classified. The analysis of the EDMs given here provides the framework for the exploration of the effects of large CP violating phases on low-energy phenomena such as the search for supersymmetry at colliders, and in the analyses of dark matter consistent with the experimental limits on EDMs via the mechanism of internal cancellations. [S0556-2821(98)50621-2]

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It is well known that supersymmetric (SUSY) theories contain many new sources of CP violation and can produce large contributions to the electric dipole moments (EDMs) of the neutron and of the electron [1–5]. With normal size CP violating phases, i.e., phases $O(1)$, and with the SUSY spectrum in the TeV range, the neutron and the electron EDMs already lie in excess of the current experimental limit, which for the neutron is [6] $d_n < 1.1 \times 10^{-25} e \text{ cm}$ and for the electron is [7] $d_e < 4.3 \times 10^{-27} e \text{ cm}$. Two approaches have usually been adopted to rectify this situation. The first is to make the phases small, i.e., $O(10^{-2})$ [1], and the other is to use mass suppression by making the SUSY spectrum heavy, i.e., in the several TeV range [4]. The first case, however, represents fine tuning, while the second violates naturalness and also makes the SUSY spectrum so heavy that it may not be accessible even at the CERN Large Hadron Collider (LHC). Recently, a third possibility was proposed [8], i.e., that of internal cancellations in EDMs reducing them below the experimental limits even for CP violating phases $O(1)$.

In recent works the importance of CP violating phases on low-energy phenomena has been recognized [9,10,11]. In Ref. [10] it is shown that large CP violating phases can affect sparticle searches at colliders and in Ref. [11], it is found that large CP violating phases can produce large effects on the neutralino relic density consistent with the experimental constraints on the neutron and on the electron EDM via the cancellation mechanism [8]. However, as one goes beyond the framework of minimal supergravity to include nonuniversalities in the soft SUSY breaking parameters, one finds that new CP violating phases arise which also affect low-energy phenomena. Currently, the effect of CP violating phases beyond the two CP phases allowed by

the minimal supergravity cannot be investigated because the analytic computations of the EDMs in terms of these phases do not exist in the literature and consequently, the EDM constraints arising from experiment cannot be implemented. The purpose of this Rapid Communication is to provide the analytic results for the EDMs beyond the minimal supergravity model by inclusion of all CP violating phases in the framework of MSSM. As is conventional, we ignore generational mixings whose effects are known to be small. We analyze all one loop diagrams with the gluino, the chargino, and the neutralino exchanges for the electric dipole, and the chromoelectric dipole operators allowing for all phases. We also analyze the two loop diagrams which contribute to the purely gluonic dimension six operator allowing again for all CP violating phases.

In MSSM the CP violating phases relevant for the analysis of the EDMs arise from the soft SUSY breaking sector of the theory. We display this sector below [12]:

$$\begin{aligned}
 V_{SB} = & m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - [B\mu \epsilon_{ij} H_1^i H_2^j + \text{H.c.}] \\
 & + M_Q^2 [\tilde{u}_L^* \tilde{u}_L + \tilde{d}_L^* \tilde{d}_L] + M_U^2 \tilde{u}_R^* \tilde{u}_R + M_D^2 \tilde{d}_R^* \tilde{d}_R \\
 & + M_L^2 [\tilde{\nu}_e^* \tilde{\nu}_e + \tilde{e}_L^* \tilde{e}_L] + M_E^2 \tilde{e}_R^* \tilde{e}_R \\
 & + \frac{gm_0}{\sqrt{2}m_W} \epsilon_{ij} \left(\frac{m_e A_e}{\cos \beta} H_1^i \tilde{l}_L^j \tilde{e}_R^* + \frac{m_d A_d}{\cos \beta} H^i \tilde{q}_L^j \tilde{d}_R^* \right. \\
 & \left. - \frac{m_u A_u}{\sin \beta} H_2^i \tilde{q}_L^j \tilde{u}_R^* + \text{H.c.} \right) \\
 & + \frac{1}{2} [\tilde{m}_3 \tilde{g} e^{-i\gamma_5 \xi_3} \tilde{g} + \tilde{m}_2 \tilde{W}^a e^{-i\gamma_5 \xi_2} \tilde{W}^a \\
 & + \tilde{m}_1 \tilde{B} e^{-i\gamma_5 \xi_1} \tilde{B}] + \Delta V_{SB}, \tag{1}
 \end{aligned}$$

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where (\tilde{L}, \tilde{q}_L) are the SU(2) (slepton, squark) doublets, $\tan \beta = |\langle H_2 \rangle / \langle H_1 \rangle|$ where H_2 gives mass to the up quark and H_1 gives mass to the down quark and the lepton, ΔV_{SB} is the one loop contribution to the effective potential, and we have suppressed the generation indices. In the above, A_u, A_d, A_e, μ and B are all complex. Additionally, after spontaneous breaking of the electroweak symmetry, the vacuum expectation values of the Higgs fields are in general complex. Some of the phases in Eq. (1) can be eliminated by field redefinitions. However, the choice of which ones to eliminate is arbitrary. Rather, in our analysis we carry all the phases to the end and our final expressions contain only certain specific combinations.

One defines the EDM of a spin- $\frac{1}{2}$ particle by the effective Lagrangian

$$\mathcal{L}_I = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}. \quad (2)$$

In the following we will compute the contributions of the gluino, the chargino and the neutralino exchanges in the minimal supersymmetric standard model (MSSM) keeping all CP violating phases.

The gluino sector contains a phase ξ_3 in the gluino mass term. We make a transformation on the gluino field to move this phase from the mass term to the quark-squark-gluino vertex which is then given by [12]

$$-\mathcal{L}_{q-\tilde{q}-\tilde{g}} = \sqrt{2} g_s T_{jk}^a \sum_{i=u,d} \left(-e^{-i\xi_3/2} \tilde{q}_i^j \frac{1-\gamma_5}{2} \tilde{g}_a \tilde{q}_{iL}^k + e^{i\xi_3/2} \tilde{q}_i^j \frac{1+\gamma_5}{2} \tilde{g}_a \tilde{q}_{iL}^k \right) + \text{H.c.} \quad (3)$$

Here $j, k = 1-3$ are the quark and the squark color indices, $a = 1-8$ are the gluino color indices, and T_a are the SU(3) $_C$ generators. The scalar fields \tilde{q}_L and \tilde{q}_R are in general linear combinations of the mass eigenstates \tilde{q}_i ($i = 1, 2$) so that

$$\tilde{q}_L = D_{q11} \tilde{q}_1 + D_{q12} \tilde{q}_2, \quad \tilde{q}_R = D_{q21} \tilde{q}_1 + D_{q22} \tilde{q}_2, \quad (4)$$

where D_{qij} are the matrices that diagonalize the squark matrix such that $D_q^\dagger M_q^2 D_q = \text{diag}(M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2)$, where

$$M_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{Q}}^2 + m_q^2 + M_z^2 (\frac{1}{2} - Q_q \sin^2 \theta_W) \cos 2\beta & m_q (A_q^* m_0 - \mu R_q) \\ m_q (A_q m_0 - \mu^* R_q) & M_U^2 + m_q^2 + M_z^2 Q_q \sin^2 \theta_W \cos 2\beta \end{pmatrix}. \quad (5)$$

Here $Q_u = 2/3$ ($-1/3$) for $q = u(d)$, $R_q = v_1/v_2^* (v_2/v_1^*)$ for $q = u(d)$, and one parametrizes D_q so that

$$D_q = \begin{pmatrix} \cos \frac{\theta_q}{2} & -\sin \frac{\theta_q}{2} e^{-i\phi_q} \\ \sin \frac{\theta_q}{2} e^{i\phi_q} & \cos \frac{\theta_q}{2} \end{pmatrix}, \quad (6)$$

where $M_{\tilde{q}_{21}}^2 = |M_{\tilde{q}_{21}}^2| e^{i\phi_q}$ and we choose the range of θ_q so that $-\pi/2 \leq \theta_q \leq \pi/2$ where $\tan \theta_q = 2|M_{\tilde{q}_{21}}^2| / (M_{\tilde{q}_{11}}^2 - M_{\tilde{q}_{22}}^2)$. In terms of the mass eigenstates \tilde{q}_1 and \tilde{q}_2 , the gluino contribution to the EDM of the quark is given by

$$d_{q-\text{gluino}}^E/e = \frac{-2\alpha_s}{3\pi} m_{\tilde{g}} Q_{\tilde{q}} \text{Im}(\Gamma_q^{11}) \left[\frac{1}{M_{\tilde{q}_1}^2} \text{B} \left(\frac{m_g^2}{M_{\tilde{q}_1}^2} \right) - \frac{1}{M_{\tilde{q}_2}^2} \text{B} \left(\frac{m_g^2}{M_{\tilde{q}_2}^2} \right) \right], \quad (7)$$

where $\Gamma_q^{1k} = e^{-i\xi_3} D_{q2k} D_{q1k}^*$, $\alpha_s = g_s^2/4\pi$, $m_{\tilde{g}}$ is the gluino mass, and $\text{B}(r) = [2(r-1)^2]^{-1} [1+r+2r \ln r(1-r)^{-1}]$. An explicit analysis gives $\Gamma_q^{12} = -\Gamma_q^{11}$ where

$$\text{Im}(\Gamma_q^{11}) = \frac{m_q}{M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2} [m_0 |A_q| \sin(\alpha_q - \xi_3) + |\mu| \sin(\theta_\mu + \chi_1 + \chi_2 + \xi_3) |R_q|], \quad (8)$$

which holds for both signs of $M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2$, and the phases χ_i ($i = 1, 2$) are defined so that $v_i = \langle H_i \rangle = |v_i| e^{i\chi_i}$ ($i = 1, 2$). From Eq. (8) we see that the combinations of phases that enter are $(\alpha_q - \xi_3)$ and $\xi_3 + \theta_\mu + \chi_1 + \chi_2$, or alternately one can choose them to be $\alpha_q + \theta_\mu + \chi_1 + \chi_2$ and $\xi_3 + \theta_\mu + \chi_1 + \chi_2$.

To discuss the contribution of the chargino exchanges, we begin by exhibiting the chargino mass matrix

$$M_C = \begin{pmatrix} |\tilde{m}_2| e^{i\xi_2} & \sqrt{2} m_W \sin \beta e^{-i\chi_2} \\ \sqrt{2} m_W \cos \beta e^{-i\chi_1} & |\mu| e^{i\theta_\mu} \end{pmatrix}. \quad (9)$$

It is useful to define the transformation $M_C = B_R M'_C B_L^\dagger$ so that

$$M'_C = \begin{pmatrix} |\tilde{m}_2| & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & |\mu| e^{i(\theta_\mu + \xi_2 + \chi_1 + \chi_2)} \end{pmatrix}, \quad (10)$$

where $B_R = \text{diag}(e^{i\xi_2}, e^{-i\chi_1})$ and $B_L = \text{diag}(1, e^{i(\chi_2 + \xi_2)})$. The matrix M'_C can be diagonalized by the biunitary transformation $U_R^\dagger M'_C U_L = \text{diag}(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+})$. It is clear that the matrix

elements of U_L and U_R are functions only of the combination $\theta = \theta_\mu + \xi_2 + \chi_1 + \chi_2$. We also have $U^* M_C V^{-1} = \text{diag}(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+})$ where $U = (B_R U_R)^T$, and $V = (B_L U_L)^T$. Using the fermion-sfermion-chargino interaction, we find that the chargino contribution to the EDM for the up quark is as follows:

$$d_{u\text{-chargino}}^E/e = \frac{-\alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{k=1}^2 \sum_{i=1}^2 \text{Im}(\Gamma_{uik}) \frac{\tilde{m}_{\chi_i^+}}{M_{dk}^2} \left[Q_{\bar{d}} B \left(\frac{\tilde{m}_{\chi_i^+}^2}{M_{dk}^2} \right) + (Q_u - Q_{\bar{d}}) A \left(\frac{\tilde{m}_{\chi_i^+}^2}{M_{dk}^2} \right) \right], \quad (11)$$

where $A(r) = 2(1-r)^{-2} [3-r+2 \ln r(1-r)^{-1}]$ and

$$\Gamma_{uik} = \kappa_u V_{i2}^* D_{d1k} (U_{i1}^* D_{d1k}^* - \kappa_d U_{i2}^* D_{d2k}^*) \quad (12)$$

and

$$\kappa_u = \frac{m_u e^{-i\chi_2}}{\sqrt{2} m_W \sin \beta}, \quad \kappa_d = \frac{m_d e^{-i\chi_1}}{\sqrt{2} m_W \cos \beta}. \quad (13)$$

Substitution of the form of U and V matrices, gives

$$\Gamma_{ui1(2)} = |\kappa_u| (\cos^2 \theta_d/2) [U_{L2i} U_{R1i}^*] - (+) \frac{1}{2} |\kappa_u \kappa_d| (\sin \theta_d) [U_{L2i} U_{R2i}^*] e^{i(\xi_2 - \phi_d)}. \quad (14)$$

The terms between the brackets [] in Eq. (14) are functions of θ and from the definition of θ_d [as given in the text following Eq. (6)] the terms between the brackets () in Eq. (14) are functions of the combination $\alpha_d + \theta_\mu + \chi_1 + \chi_2$. By taking the imaginary part of Γ and using the definition of ϕ_d [as given in the text following Eq. (6)] one can show that $(\xi_2 - \phi_d)$ depends on the combinations $(\xi_2 - \alpha_d)$, $(\xi_2 + \theta_\mu + \chi_1 + \chi_2)$ and $(\alpha_d + \theta_\mu + \chi_1 + \chi_2)$. So we are left only with the two combinations $\alpha_d + \theta_\mu + \chi_1 + \chi_2$ and $\xi_2 + \theta_\mu + \chi_1 + \chi_2$ with $\xi_2 - \alpha_d$ being just a linear combination of the first two. Similar analyses hold for the chargino contributions to the down quark and one gets only two phase combinations which are identical to the case above with α_d replaced by α_u . For the case of the charged lepton, we find

$$d_{e\text{-chargino}}^E/e = \frac{\alpha_{EM}}{4\pi \sin^2 \theta_W} m_{\nu e}^2 \sum_{i=1}^2 \tilde{m}_{\chi_i^+} \text{Im}(\Gamma_{ei}) A \left(\frac{\tilde{m}_{\chi_i^+}^2}{m_{\nu e}^2} \right), \quad (15)$$

where $\Gamma_{ei} = (\kappa_e U_{i2}^* V_{i1}^*) = |\kappa_e| U_{R2i}^* U_{L1i}$. A direct inspection of Γ_{ei} shows that it depends on only one combination, i.e., $\xi_2 + \theta_\mu + \chi_1 + \chi_2$.

In order to discuss the neutralino exchange contributions, we first exhibit the neutralino mass matrix M_{χ^0} with the most general allowed set of CP violating phases

$$\begin{pmatrix} |\tilde{m}_1| e^{i\xi_1} & 0 & -M_z \sin \theta_W \cos \beta e^{-i\chi_1} & M_z \sin \theta_W \sin \beta e^{-i\chi_2} \\ 0 & |\tilde{m}_2| e^{i\xi_2} & M_z \cos \theta_W \cos \beta e^{-i\chi_1} & -M_z \cos \theta_W \sin \beta e^{-i\chi_2} \\ -M_z \sin \theta_W \cos \beta e^{-i\chi_1} & M_z \cos \theta_W \cos \beta e^{-i\chi_2} & 0 & -|\mu| e^{i\theta_\mu} \\ M_z \sin \theta_W \sin \beta e^{-i\chi_1} & -M_z \cos \theta_W \sin \beta e^{-i\chi_2} & -|\mu| e^{i\theta_\mu} & 0 \end{pmatrix}. \quad (16)$$

Next we make the transformation $M_{\chi^0} = P_{\chi^0}^T M'_{\chi^0} P_{\chi^0}$ where

$$P_{\chi^0} = \text{diag}(e^{i(\xi_1/2)}, e^{i(\xi_2/2)}, e^{-i(\xi_1/2+\chi_1)}, e^{-i(\xi_2/2+\chi_2)}). \quad (17)$$

After the transformation, the matrix M'_{χ^0} takes the form

$$\begin{pmatrix} |\tilde{m}_1| & 0 & -M_z \sin \theta_W \cos \beta & M_z \sin \theta_W \sin \beta e^{-i(\Delta\xi/2)} \\ 0 & |\tilde{m}_2| & M_z \cos \theta_W \cos \beta e^{i(\Delta\xi/2)} & -M_z \cos \theta_W \sin \beta \\ -M_z \sin \theta_W \cos \beta & M_z \cos \theta_W \cos \beta e^{i(\Delta\xi/2)} & 0 & -|\mu| e^{i\theta'} \\ M_z \sin \theta_W \sin \beta e^{-i(\Delta\xi/2)} & -M_z \cos \theta_W \sin \beta & -|\mu| e^{i\theta'} & 0 \end{pmatrix}, \quad (18)$$

where $\theta' = (\xi_1 + \xi_2)/2 + \theta_\mu + \chi_1 + \chi_2$, and $\Delta\xi = (\xi_1 - \xi_2)$. Now the matrix M'_{χ^0} can be diagonalized by the transformation $Y^T M'_{\chi^0} Y = \text{diag}(\tilde{m}_{\chi_1^0}, \tilde{m}_{\chi_2^0}, \tilde{m}_{\chi_3^0}, \tilde{m}_{\chi_4^0})$. It is clear that the transformation matrix Y is a function only of θ' and $\Delta\xi/2$. Combining our results, we find that the complex non-

Hermitian and symmetric matrix M_{χ^0} can be diagonalized using a unitary matrix $X = P_{\chi^0}^\dagger Y$ such that $X^T M_{\chi^0} X = \text{diag}(\tilde{m}_{\chi_1^0}, \tilde{m}_{\chi_2^0}, \tilde{m}_{\chi_3^0}, \tilde{m}_{\chi_4^0})$. We can now write down the neutralino exchange contribution to the fermion EDM as follows:

$$d_{f\text{-neutralino}}^E/e = \frac{\alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{k=1}^2 \sum_{i=1}^4 \text{Im}(\eta_{fik}) \frac{\tilde{m}_{\chi_i^0}}{M_{fk}^2} Q_f B \left(\frac{\tilde{m}_{\chi_i^0}^2}{M_{fk}^2} \right), \quad (19)$$

where

$$\eta_{fik} = (a_0 X_{1i} D_{f1k}^* + b_0 X_{2i} D_{f1k}^* + \kappa_f X_{bi} D_{f2k}^*) \times (c_0 X_{1i} D_{f2k} - \kappa_f X_{bi} D_{f1k}), \quad (20)$$

where $b=3(4)$ for $T_{3q} = -\frac{1}{2}(\frac{1}{2})$, $a_0 = -\sqrt{2} \tan \theta_W (Q_f - T_{3f})$, $b_0 = -\sqrt{2} T_{3f}$, and $c_0 = \sqrt{2} \tan \theta_W Q_f$. We discuss now the phases that appear in the various terms in η_{fik} . The term proportional to $a_0 c_0$ contains the factor $X_{1i}^2 D_{f1k}^* D_{f2k}$. It is easily seen that this term equals $+(-) a_0 c_0 \frac{1}{2} Y_{1i}(\theta, \Delta \xi/2) \sin \theta_f e^{-i(\xi_1 - \phi_f)}$, where the $+(-)$ sign is for $k=1(2)$. By doing the same analysis as for the case of the chargino contribution, we find that the combinations that arise here are θ' , $\Delta \xi/2$ and $\alpha_f - \xi_1$ from which we can construct the three combinations: $\xi_1 + \theta_\mu + \chi_1 + \chi_2$, $\xi_2 + \theta_\mu + \chi_1 + \chi_2$ and $\alpha_f + \theta_\mu + \chi_1 + \chi_2$. A similar analysis for the remaining terms of Eq. (20) gives exactly the same result. The sum of the gluino, the chargino, and the neutralino exchanges discussed above gives the total contribution from the electric dipole operator to the quark EDM.

The chromoelectric dipole moment \tilde{d}^C of the quarks is defined via the effective dimension five operator:

$$\mathcal{L}_I = -\frac{i}{2} \tilde{d}^C \bar{q} \sigma_{\mu\nu} \gamma_5 T^a q G^{\mu\nu a}. \quad (21)$$

Contributions to \tilde{d}^C of the quarks from the gluino, the chargino, and from the neutralino exchange are given by

$$\tilde{d}_{q\text{-gluino}}^C = \frac{g_s \alpha_s}{4\pi} \sum_{k=1}^2 \text{Im}(\Gamma_q^{1k}) \frac{m_{\tilde{g}}}{M_{\tilde{q}k}^2} C \left(\frac{m_{\tilde{g}}^2}{M_{\tilde{q}k}^2} \right), \quad (22)$$

$$\tilde{d}_{q\text{-chargino}}^C = \frac{-g^2 g_s}{16\pi^2} \sum_{k=1}^2 \sum_{i=1}^2 \text{Im}(\Gamma_{qik}) \frac{\tilde{m}_{\chi_i^+}}{M_{\tilde{q}k}^2} B \left(\frac{\tilde{m}_{\chi_i^+}^2}{M_{\tilde{q}k}^2} \right), \quad (23)$$

and

$$\tilde{d}_{q\text{-neutralino}}^C = \frac{g_s g^2}{16\pi^2} \sum_{k=1}^2 \sum_{i=1}^4 \text{Im}(\eta_{qik}) \frac{\tilde{m}_{\chi_i^0}}{M_{\tilde{q}k}^2} B \left(\frac{\tilde{m}_{\chi_i^0}^2}{M_{\tilde{q}k}^2} \right), \quad (24)$$

where $B(r)$ is defined following Eq. (7) and $C(r)$ is given by

$$C(r) = \frac{1}{6(r-1)^2} \left(10r - 26 + \frac{2r \ln r}{1-r} - \frac{18 \ln r}{1-r} \right). \quad (25)$$

We note that all of the CP violating phases are contained in the factors $\text{Im}(\Gamma_q^{1k})$, $\text{Im}(\Gamma_{qik})$, and in $\text{Im}(\eta_{qik})$. But these are

precisely the same factors that appear in the gluino, the chargino, and the neutralino contributions to the electric dipole operator.

Finally, we look at the CP phases that enter in the CP violating purely gluonic dimension six operator. The gluonic dipole moment d^G is defined via the effective dimension six operator

$$\mathcal{L}_I = -\frac{1}{6} d^G f_{\alpha\beta\gamma} G_{\alpha\mu\rho} G_{\beta\nu}^\rho G_{\gamma\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma}, \quad (26)$$

where $f_{\alpha\beta\gamma}$ are the Gell-Mann coefficients, $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor with $\epsilon^{0123} = +1$, and $G_{\alpha\mu\nu}$ is the gluon field strength. Carrying out the analysis including all phases, we get

$$d^G = -3 \alpha_s \left(\frac{g_s}{4\pi m_{\tilde{g}}} \right)^3 [m_t (z_1^t - z_2^t) \text{Im}(\Gamma_t^{12}) H(z_1^t, z_2^t, z_t) + m_b (z_1^b - z_2^b) \text{Im}(\Gamma_b^{12}) H(z_1^b, z_2^b, z_b)], \quad (27)$$

where

$$\Gamma_q^{1k} = e^{-i\xi_3} D_{q2k} D_{q1k}^*, \quad z_\alpha^q = \left(\frac{M_{\tilde{q}\alpha}}{m_{\tilde{g}}} \right)^2, \quad z_q = \left(\frac{m_q}{m_{\tilde{g}}} \right)^2. \quad (28)$$

It is easily seen that the combination of phases involving ξ_3 are similar to the gluino exchange terms discussed earlier. Similar expressions can arise if the quarks in the loop were from the other two generations and one gets the combinations $\alpha_q + \theta_\mu + \chi_1 + \chi_2$, $\xi_3 + \theta_\mu + \chi_1 + \chi_2$.

The contribution to the neutron EDM using the nonrelativistic SU(6) formula is given by $d_n = \frac{1}{3}(4d_d - d_u)$. The above analysis holds at the electroweak scale. To obtain the value at the hadronic scale, one uses renormalization group to evolve it down to that scale. Thus $d_n^E = \eta^E d_n$ where d_n^E is the value at the hadronic scale and η^E is the QCD correction factor. The contributions of the chromoelectric dipole operator, and of the purely gluonic dimension six operator to the quark EDMs are obtained by use of the naive dimensional analysis [13], so that $d_n^C = (e/4\pi) \tilde{d}_n^C \eta^C$ and $d_n^G = (eM/4\pi) d_n^G \eta^G$, where d_n^C and d_n^G are the contributions at the electroweak scale, η^C and η^G are the QCD correction factors and $M = 1.19$ GeV is the chiral symmetry breaking scale.

The main results of the paper are given by Eqs. (7), (11), (15), and (19) for the contribution to the EDMs by the electric dipole operator, by Eqs. (22)–(24) for the contribution to the EDMs by the chromoelectric dipole operator and by Eq. (27) for the contribution to the EDM by the purely gluonic dim 6 operator. These formulas give the contributions to the EDMs with the most general set of CP violating phases with no generational mixings. The phases that enter in the quark and in the lepton EDM's are summarized in Table I. As seen from Table I, the electric dipole and the chromoelectric dipole contributions to the neutron EDMs depend on five phases which can be chosen to be $\xi_i + \theta_1$ ($i=1,2,3$), and $\alpha_k + \theta_1$ ($k=u,d$) where $\theta_1 = \theta_\mu + \chi_1 + \chi_2$. The purely glu-

TABLE I. CP violating phases in d_q and d_l in MSSM.

Exchange	u quark	d quark	Charged leptons
\tilde{g}	$\alpha_u + \theta_1$	$\alpha_d + \theta_1$	
	$\xi_3 + \theta_1$	$\xi_3 + \theta_1$	
χ^+	$\alpha_d + \theta_1$	$\alpha_u + \theta_1$	
	$\xi_2 + \theta_1$	$\xi_2 + \theta_1$	$\xi_2 + \theta_1$
χ^0	$\alpha_u + \theta_1$	$\alpha_d + \theta_1$	$\alpha_l + \theta_1$
	$\xi_1 + \theta_1$	$\xi_1 + \theta_1$	$\xi_1 + \theta_1$
	$\xi_2 + \theta_1$	$\xi_2 + \theta_1$	$\xi_2 + \theta_1$
dim 6	$\alpha_k + \theta_1$	$\alpha_k + \theta_1$	
	$k = u, d, c, s, t, b$	$k = u, d, c, s, t, b$	

onic dimension six operator contribution to the quarks depends on four additional phases: $\alpha_k + \theta_1$ ($k = t, b, c, s$), and thus the neutron EDM depends on nine independent phases. The electron EDM depends on just three independent phases: $\xi_i + \theta_1$ ($i = 1, 2$), and $\alpha_e + \theta_1$. Thus the neutron and the electron EDM together depend on ten independent phases. If we include the muon and the tau EDMs, then the neutron and the lepton EDMs altogether depend on twelve phases, i.e., $\xi_i + \theta_1$ ($i = 1, 2, 3$), $\alpha_k + \theta_1$ ($k = u, d, t, b, c, s; e, \mu, \tau$). If we retain only the dominant top-stop contribution to the purely gluonic dimension six operator, then the total number of phases reduces from 12 to 9.

For the case of minimal supergravity, all α_k evolve from the phase α_0 of A_0 where A_0 is the common value of A_i at the grand unified theory (GUT) scale. Similarly, all ξ_i evolve from the phase $\xi_{1/2}$ of the universal gaugino mass $m_{1/2}$ at the GUT scale. In this case the ten phase combinations that appear in d_n and d_e collapse to just two independent ones: $\alpha_0 + \theta_1$, and $\xi_{1/2} + \theta_1$. Often one sets $\xi_{1/2} = 0$, $\chi_1 = 0 = \chi_2$ and chooses the independent phases in minimal supergravity to be α_0 and θ_μ . In that limit the results of this Rapid Communication limit to the results of Ref. [8].

The analysis presented here is the first complete analysis of the neutron and of the lepton EDMs with all allowed CP phases in MSSM under the restriction of no generational mixings. In Ref. [8] consistency with large CP violating phases was achieved by internal cancellations to satisfy the experimental EDM constraints with just two CP phases. In the present analysis, since the electron EDM depends on three independent phases and the neutron EDM depends on nine independent phases, the satisfaction of the experimental EDM constraints with large CP violating phases can be achieved over a much large region. The analysis provides the framework for the investigation of the effects of large CP violating phases on low-energy physics.

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