TeV scale superstring and extra dimensions

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(Received 2 June 1998; published 22 October 1998)

Utilizing the idea of extra large dimensions, it has been suggested that the gauge and gravity couplings unification can happen at a scale as low as 1 TeV. In this paper, we explore this phenomenological possibility within string theory. In particular, we discuss how the proton decay bound can be satisfied in type I string theory. The string picture also suggests different scenarios of gauge and gravitational couplings unification. The various scenarios are explicitly illustrated with a specific 4-dimensional $\mathcal{N}=1$ supersymmetric chiral type I string model with Pati-Salam-like gauge symmetry and 3 fermion families. We point out certain features that should be generic in other type I strings. [S0556-2821(98)00522-0]

PACS number(s): 11.25.Mj

I. INTRODUCTION

Probably the most important problem in elementary particle theory today is to find out how superstring theory describes our universe. In the standard scenario, the Planck scale M_P (i.e., 10^{19} GeV) defines the string scale to be around 10^{17} GeV, while the unification of the gauge couplings happens at the grand unified scale M_{GUT} (around 10^{16} GeV) [1]. Although the string scale and M_{GUT} are quite close, the discrepancy between them may still be of some concern. However, a more practical problem with this scenario is the difficulty in calculating physical observables. Since the natural string scale of this scenario is between M_{GUT} and M_P, while most of the physical observables are at the electroweak scale M_{EW}, a typical comparison between theory and experiment requires a detailed analysis of a specific string model. Unfortunately, our understanding of the string dynamics is still quite primitive, making such precise calculations essentially impossible. So the connection between string theory and our observable universe is rather tenuous in this scenario at this moment. It is therefore exciting that an alternative scenario has recently emerged.

The idea of extra dimensions have been well studied in Kaluza-Klein theories and string theories. It was suggested by Antoniadis *et al.* [2] that, beyond the usual 4 space-time dimensions that we live in, there are large extra dimensions that may be probed by upcoming experiments. It is by now well known that, in some string models, gravity lives in the bulk, while the gauge and charged matter fields live on the branes (which may be understood as special types of solitons and can have lower dimensions than the bulk [3]); our 4-dimensional universe may actually be inside the branes [4]. In particular, the extra dimensions that gravity feels can be as large as 1 mm, as recently pointed out by Arkani-Hamed, Dimopoulos and Dvali [5], while the extra dimensions that the gauge and other matter couplings feel can be as big as M_{EW}^{-1}, as recently pointed out by Dienes, Dudas and Gherghetta [6]. In this scenario, the Planck scale M_P is traded for the size of the extra dimensions felt by gravity [5,7]. Likewise, gauge coupling unification can be preserved and remain perturbative, but now occurs at scales as low as a TeV [6]. One can therefore now have gravity and gauge coupling unification as low as a few hundred GeV to 1 TeV [5]. In string theory, this means that the string scale m_s can be as low as a TeV [7]. Such a scenario has been suggested previously by Lykken [8]. One advantage of this TeV scale string scenario is obvious. Not only that near future experiments can probe the string scale and the large extra dimensions, it may even help us unravel the string dynamics and pinpoint the string vacuum we live in.

In addition to the advantage of being experimentally testable, this new scenario may offer a simple qualitative explanation to the fermion mass hierarchy problem, as pointed out in Ref. [6]. To be specific, let us suppose the string scale $m_s=1$ TeV. Gravity, but not the standard model gauge and matter fields, lives in n large compactified dimensions, with radii r_i. The radii R_j of the remaining compactified dimensions in which both gravity and gauge fields live are somewhere between m_s^{-1} and M_{EW}^{-1}, so $m_s > R_j^{-1} > M_{EW} \gg r_i^{-1}$. In this scenario, the effective couplings at the m_s scale are all irrelevant operators and so the dimensionless gauge couplings α_i and Yukawa couplings y_f run as powers of the energy scale. If the different Yukawa couplings are comparable at the m_s scale, they can easily differ by orders of magnitude at the electroweak scale due to this power-law behavior. The gauge couplings differ by only one order of magnitude because they are unified at the string scale. Below the R_j^{-1} scales, the dimensionless gravitational coupling runs a function of the energy scale like E^{2+n} to the scales r_i^{-1} and then runs like E^2, yielding a huge M_P [4]. So the presence of the extra dimensions provide a qualitative explanation of the origin of the orders of magnitude differences among the couplings. As pointed out in Ref. [6], $m_s \gg 1$ TeV is perfectly acceptable. However, we have to treat m_s and M_{EW} as two different scales in this situation.

In this paper, we study a number of issues in the 4-dimensional $\mathcal{N}=1$ chiral type I string theory, which is the appropriate framework for the TeV scale string scenario. We put the above ideas in a coherent string picture with an explicit realization. A typical model will have 9-branes, which

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fill the 10-dimensional spacetime, and 5-branes, which fill 6-dimensional spacetime. Both branes have a flat 4-dimensional uncompactified spacetime. The string picture has been discussed previously in Refs. [8,7]. Here, we review and extend the analysis. Among other observations in this paper, we note that:

(1) It is well-known that proton decay can be suppressed by symmetry. Here, we see that proton decay is suppressed by the presence of a custodial U(1) gauge symmetry. The presence of such a U(1) gauge symmetry is generic in type I strings.

(2) As an alternative scenario, the standard model gauge symmetries can come from different types of branes, e.g., QCD SU(3) comes from one type of branes (say, 9-branes) while the weak SU(2) comes from another type (say, 5-branes). Since the 9-brane couplings are in general different from the 5-brane couplings, the standard model gauge couplings do not need to meet at the string scale. Rather, an appropriate choice of the sizes of the compactified dimensions is needed for the couplings to agree with experiment.

(3) Cavendish type experiments have tested Newton's Law to a scale of millimeters [9], providing an upper bound on the large radius. The strong and electroweak scatterings have tested the small extra dimensions to a radius of M_{EW}^{-1}, providing an upper bound on the size of the small extra dimensions. Taking $m_s R_j \sim 1$, the relation between the large radii r_i and M_P is given by

$$M_P^2 \sim 32\pi^2 g^{-4} m_s^2 \prod_{i=1}^{n} (m_s r_i) \tag{1}$$

where n is the number of large compactified dimensions, g is the gauge coupling [7]. The numerical factor follows from string unitarity and duality. Clearly m_s must be bigger than M_{EW}. Assume the gauge coupling $g^2 \sim 1$. For $n=2$, r is about 10^{-4} meter for $m_s = 1$ TeV. As pointed out in Ref. [5], both the 1 mm scale and the 1 TeV scale can be tested by experiments in the near future. We point out that the $n=2$ choice seems natural in a number of string scenarios. For example, in the specific model that we consider in this paper, only the $n=2$ choice gives rise to 3 chiral families.

(4) String theory has no global symmetry. However, some gauge couplings are proportional to r^{-1}. For very large r, they become so weak that the respective gauge symmetries may appear like global symmetries. In some situations, the corresponding matter fields with vanishingly small gauge couplings are suitable candidates for dark matter.

To make the discussion concrete, we construct an explicit model to illustrate the TeV scale string scenario. Our analysis of the model is quite sketchy and cavalier. Our purpose is to draw attention to the model's features that are generic to other type I string models. The model is a $D=4$, $\mathcal{N}=1$ supersymmetric, chiral type I string model, with 9-branes and 5-branes. Their gauge groups G_9 and G_5 (with gauge couplings g_9 and g_5 respectively) are identical: $G_9 = G_5 = \mathrm{U}(4) \otimes \mathrm{U}(2) \otimes \mathrm{U}(2)'$. The massless open string spectrum is given in Table I. The U(1)'s associated with the SU(2)s are anomalous, not unusual in string theories. The U(1) associated with the SU(4) provides the custodial symmetry to suppress proton decay. We shall use this model to discuss the following three scenarios:

(5) One may identify the $\mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)'$ from the 9-brane sector as the Pati-Salam group. Spontaneous symmetry breaking reduces it to $\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)$. At the string scale, both the QCD coupling g_3 and the weak coupling g_2 are equal to the 9-brane coupling g_9, and $\sin^2 \theta_W = 3/8$. There is a $\mathrm{U}(1)_B$ gauge symmetry associated with the baryon number, so the proton decay is suppressed. However, it seems that breaking this $\mathrm{U}(1)_B$ symmetry will also break QCD SU(3). The model has only one chiral family, plus a vector (i.e., a chiral and an anti-chiral) family.

(6) One may identify $\mathrm{SU}(4)_9 \otimes \mathrm{SU}(2)_5 \otimes \mathrm{SU}(2)_5'$ (the subscripts indicate which sectors each comes from) as the Pati-Salam gauge group. The QCD SU(3) comes from the spontaneous symmetry breaking of the $\mathrm{SU}(4)_9 \otimes \mathrm{SU}(2)_5'$ by a bi-fundamental matter field, while the remaining $\mathrm{SU}(2)_5$ is identified with the weak $\mathrm{SU}(2)_L$. The $\mathrm{SU}(4)_5$ may get strong and induce both dynamical supersymmetry breaking and electroweak symmetry breaking. The gauge group $\mathrm{SU}(2)_9 \otimes \mathrm{SU}(2)_9'$ is also broken. At the string scale, $g_3 = g_9$ while $g_2 = g_5$. Now there are two families of quarks and leptons, coming from the 95 sector, while the Higgs fields come from the 55 sector. Again, the perturbative couplings obey the baryon quantum number conservation because of $\mathrm{U}(1)_B$.

(7) The first scenario has one chiral family in the 99 sector while the second scenario has two chiral families in the 59 sector. It turns out that there is another scenario where there are three chiral families. Under diagonal spontaneous symmetry breaking, $\mathrm{SU}(2)_9 \otimes \mathrm{SU}(2)_5$ becomes SU(2). This Higgs mechanism is permitted by the presence of the appropriate bi-fundamental fields, resulting in a model which contains $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ with 3 chiral families: one from the 99 sector and two from the 59 sector. Weak interaction universality automatically follows, independent of the relative values of g_9 and g_5. The standard model couplings are all different. In the case $g = g_9 = g_5$, we have at the string scale,

$$g^2 = g_3^2 = 2g_2^2 = \frac{8}{3} g_Y^2. \tag{2}$$

Hence the electroweak Weinberg mixing angle satisfies $\sin^2 \theta_W = 3/7$ at the string scale. This scenario has one practical advantage. Since $g_3 > g_2 > g_Y$ at the string scale, the running couplings need power-like behavior for only a relatively short range of energies; that is, they do not have to grow much. As a consequence, the string coupling will stay weak, and perturbative type I string theory should be valid for analysis.

It is clear that, among other properties, the presence of U(1)'s (associated with the centers of mass of the branes), the presence of bi-fundamental matter fields, and the identical nature of 9-brane gauge group and 5-brane gauge groups (if present) are generic features of many $D=4$, $\mathcal{N}=1$ supersymmetric, chiral Type I string models. These properties are

TABLE I. The massless open string spectrum of the 4-dimensional type I $\mathbf{Z}_6$ orbifold model with $N=1$ space-time supersymmetry and gauge group $[\mathrm{SU}(2)'\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3]^2$. The U(1)'s come from the traces of U(2)′, U(2) and U(4) respectively. The H-charges in both the -1 picture and the $-1/2$ picture for states in the open string sector are also given. The vector multiplets are not shown. The closed string sectors give rise to the gauge singlets and the gravity supermultiplet. The H-charges are explained in Appendix B.

Sector	Field	$[\mathrm{SU}(2)'\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3]^2$	$(H_1,H_2,H_3)_{-1}$	$(H_1,H_2,H_3)_{-1/2}$
Open 99	S_1	$(\mathbf{3},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(+2,0,0;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	S_2	$(\mathbf{3},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(+2,0,0;0,0,0)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	S_3	$(\mathbf{1},\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,-2,0;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	S_4	$(\mathbf{1},\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,-2,0;0,0,0)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	U_1	$(\mathbf{2},\mathbf{1},\overline{\mathbf{4}};\mathbf{1},\mathbf{1},\mathbf{1})(-1,0,-1;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	U_2	$(\mathbf{2},\mathbf{1},\overline{\mathbf{4}};\mathbf{1},\mathbf{1},\mathbf{1})(-1,0,-1;0,0,0)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	U_3	$(\mathbf{2},\mathbf{1},\mathbf{4};\mathbf{1},\mathbf{1},\mathbf{1})(-1,0,+1;0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	Q_1	$(\mathbf{1},\mathbf{2},\mathbf{4};\mathbf{1},\mathbf{1},\mathbf{1})(0,+1,+1;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	Q_2	$(\mathbf{1},\mathbf{2},\mathbf{4};\mathbf{1},\mathbf{1},\mathbf{1})(0,+1,+1;0,0,0)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	Q_3	$(\mathbf{1},\mathbf{2},\overline{\mathbf{4}};\mathbf{1},\mathbf{1},\mathbf{1})(0,+1,-1;0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	H	$(\mathbf{2},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(+1,-1,0;0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
Open 55	s_1	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})(0,0,0;+2,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	s_2	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})(0,0,0;+2,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	s_3	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3},\mathbf{1})(0,0,0;0,-2,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	s_4	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3},\mathbf{1})(0,0,0;0,-2,0)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	u_1	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{1},\overline{\mathbf{4}})(0,0,0;-1,0,-1)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	u_2	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{1},\overline{\mathbf{4}})(0,0,0;-1,0,-1)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	u_3	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{1},\mathbf{4})(0,0,0;-1,0,+1)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	q_1	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{4})(0,0,0;0,+1,+1)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	q_2	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{4})(0,0,0;0,+1,+1)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	q_3	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\overline{\mathbf{4}})(0,0,0;0,+1,-1)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	h	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2},\mathbf{1})(0,0,0;+1,-1,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
Open 59	ϕ_1', ϕ_2'	$2(\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{1},\mathbf{1})(+1,0,0;+1,0,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	ϕ_1, ϕ_2	$2(\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})(0,-1,0;0,-1,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$\mathcal{Q}_1$, $\mathcal{Q}_2$	$2(\mathbf{1},\mathbf{1},\mathbf{4};\mathbf{1},\mathbf{2},\mathbf{1})(0,0,+1;0,+1,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$\mathcal{U}_1$, $\mathcal{U}_2$	$2(\mathbf{1},\mathbf{1},\overline{\mathbf{4}};\mathbf{2},\mathbf{1},\mathbf{1})(0,0,-1;-1,0,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	χ_1, χ_2	$2(\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4})(0,+1,0;0,0,+1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	χ_3, χ_4	$2(\mathbf{2},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}})(-1,0,0;0,0,-1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$

quite compatible with experiments and the extra large dimension scenario. Further investigation along this direction will certainly be worthwhile.

This paper is organized as follows. In Sec. II, the basic idea of the TeV scale string scenario is reviewed. The type I string picture with extra large dimensions is discussed. As an illustration, the Pati-Salam like type I string model is presented in Sec. III. Among other issues, we see how the proton decay bound can be resolved. Section IV contains some discussions on the various issues in the scenario. Section V contains the comments. The details of the construction of the type I string model is contained in Appendix A. It is relegated to an appendix because of its technical nature. In Appendix B, we review how to calculate amplitudes to determine the terms in the superpotential of the model.

II. BRANE PICTURE

The idea of extra large dimensions is most conveniently realized in terms of type I string theory and D-branes [10]. The graviton (coming from the closed string sector) lives in the bulk, while the gauge and charged matter fields (coming from the open string sector) live on the branes (which is $p+1$-dimensional for a Dp-brane). Since gravity and gauge fields see different numbers of dimensions, it is possible to have extra large dimensions without making the gauge couplings extremely small at low energies. In the worldsheet construction of heterotic string model, both gravity and gauge fields live in the same space, and so the idea of extra dimensions is difficult to implement. It is possible to realize the extra dimensions with the solitonic 5-branes in heterotic string theory. However, the techniques in constructing het-

erotic string model with these solitonic 5-branes are not very well developed. As a result, type I string theory and D-branes provide the most natural setting to understand the generic features of extra dimensions. Here, we review and expand on the earlier discussions [8,7].

A. Supersymmetric type I string

Various $\mathcal{N}=1$, $D=4$ type I string models have been studied in the past two years [11–18]. They are especially suitable for realizing the idea of extra large dimensions. Gravity lives in the bulk while the gauge fields live on the branes. There are 9-branes, which overlap completely with the bulk. If a model has both p-branes and q-branes, then supersymmetry imposes the restriction $p-q=0 \pmod 4$. To keep the Lorentz property of 4-dimensional spacetime, only 5-branes and 9-branes are permissible. So, for some models, there can be 5-branes as well.

The 4-dimensional string has the usual 4 spacetime dimensions $(x_0,\ldots,x_3)$, and 6 compactified dimensions. We shall treat this 6 dimensions T^6 as composed of 3 two-tori: T_1 (with coordinates x_8,x_9), T_2 (with coordinates x_6,x_7) and T_3 (with coordinates x_4,x_5), the volumes of which are v_1, v_2 and v_3 respectively. So the volume of the 6 compactified dimensions is $v_1v_2v_3$. Crudely speaking, the volume v_i can be expressed in terms of the compactified radius r_i, $v_i=(2\pi r_i)^2$.[1] So the low energy effective action is given by

$$S=\int d^4x\sqrt{g}\left(\frac{m_s^8 v_1v_2v_3}{(2\pi)^7\lambda^2}R+\frac{1}{4}\frac{m_s^6 v_1v_2v_3}{(2\pi)^7\lambda}F^2 + \frac{1}{4}\sum_{i=1}^{3}\frac{m_s^2 v_i}{(2\pi)^3\lambda}\tilde{F}_i^2+\cdots\right) \tag{3}$$

where m_s is the string scale. F is the field strength of the gauge fields in the 9-branes while the $\tilde{F}_i$ is the gauge field strength on different types of 5-branes (the worldvolumes of which are $\mathcal{M}^4\times T_1$, $\mathcal{M}^4\times T_2$ and $\mathcal{M}^4\times T_3$ respectively; $\mathcal{M}^4$ being the 4 dimensional Minkowski space-time). Here λ is the string coupling, i.e., $\lambda\sim e^{\phi}$, where ϕ is the dilaton field. The relative normalization of the Newton's constant and the gauge coupling (which is related to the D-brane tension) is obtained by factorizing scattering amplitudes into open and closed string channels [10,19]. (In type I string, the N-point open-string one-loop amplitude is equivalent to the closed string scattering to N open strings at the tree level. This relation follows from unitarity.) This should be compared to heterotic string theory where all states are closed string states. The precise numerical factors are determined once we define the string coupling λ to be the ratio of the fundamental string and D-string tensions in type IIB string theory [10].

[1]The radius r_i here does not necessarily have to be the radius R_i of the torus. It is simply a characteristic length scale of the compactified dimension. In the case of a $\mathbf{Z}_N$ orbifold, the volume is given by $\Pi_i v_i=(1/N)\Pi_i(2\pi R_i)^2\equiv\Pi_i(2\pi r_i)^2$.

For simplicity, we will consider only one type of 5-branes in what follows. Let G_9 (G_5) be the gauge group of the 9-brane (5-brane). The 5-branes are compactified on T_3, while the 9-branes are compactified on T^6. The branes and the bulk have a common 4-dimensional uncompactified spacetime. The 4-dimensional Planck mass M_P and the Newton's constant G_N are given by

$$G_N^{-1}=M_P^2=\frac{8m_s^8 v_1v_2v_3}{(2\pi)^6\lambda^2} \tag{4}$$

and the gauge couplings of G_9 and G_5 are

$$g_9^{-2}=\frac{m_s^6 v_1v_2v_3}{(2\pi)^7\lambda},\quad g_5^{-2}=\frac{m_s^2 v_3}{(2\pi)^3\lambda}. \tag{5}$$

These relations are subject to quantum corrections, which we shall ignore for the moment. Recall that the gauge couplings of the standard model are of order 1. In string theory, there is a T-duality symmetry, i.e., physics is invariant under a T-duality transformation. If any of the volume v_i is much smaller than the string scale, i.e., v_i less than m_s^{-2}, the T-dual description is more convenient:

$$\lambda\to(2\pi)^2\frac{\lambda}{v_i m_s^2}$$

$$v_i\to(2\pi)^4\frac{1}{v_i m_s^4}. \tag{6}$$

In this dual picture, the new volume $(2\pi)^4/(v_i m_s^4)$ of the dual T_i torus is large. Under this duality transformation, the Dirichlet and Neumann boundary conditions of the open strings are interchanged, and so the branes are also mapped to other types of branes. For example, for $i=1$, i.e., we T-dual the T_1 torus; the 9-branes become 7-branes $(x_0,\ldots,x_7)$, while the 5-branes become 7-branes $(x_0,\ldots,x_5,x_8,x_9)$. Therefore, they are orthogonal in the compactified space. The effective action becomes

$$S=\int d^4x\sqrt{g}\left(\frac{m_s^8 v_1v_2v_3}{(2\pi)^7\lambda^2}R+\frac{1}{4}\frac{m_s^4 v_2v_3}{(2\pi)^5\lambda}F^2 + \frac{1}{4}\frac{m_s^4 v_1v_3}{(2\pi)^5\lambda}\tilde{F}^2+\cdots\right). \tag{7}$$

If the standard model gauge group is in G_9, then, in this 7-brane picture,

$$g_9^{-2}=\frac{m_s^4 v_2v_3}{(2\pi)^5\lambda}\sim 1. \tag{8}$$

Now, suppose m_s is 1 TeV. To satisfy Eq. (4), i.e., to obtain the $M_P=10^{19}$ GeV, at least one of the 2-volumes must be large. Since the G_9 gauge coupling must be of order 1, the only choice is to take v_1 large. This means the G_5 gauge coupling becomes extremely small, i.e., the gauge fields decouple. The conserved currents that couple to G_5 will appear like those of global symmetries.

We can also keep both G_9 and G_5 gauge couplings of order 1. This can be achieved if we T-dual both the T_1 and T_3 tori to end up with orthogonal (in the compactified space) 5-branes with the following effective action:

$$S=\int d^4x\sqrt{g}\left(\frac{m^8 v_1 v_2 v_3}{(2\pi)^7\lambda^2}R+\frac{1}{4}\frac{m_s^2 v_2}{(2\pi)^3\lambda}F^2 + \frac{1}{4}\frac{m_s^2 v_1}{(2\pi)^3\lambda}\tilde{F}^2+\cdots\right). \tag{9}$$

In this case, we can take v_3 large to satisfy Eq. (4). We shall use this 55′-brane picture later, but we shall still refer to the 5-branes coming from T-dualizing the 9-branes as the 9-branes. To summarize, there are 8 inequivalent scenarios one can entertain: 95-, 77-, 55- and 73-brane configuration, and their T^6-duals (i.e., T-dual along all 6 dimensions): 59-, 77-, 55- and 37-brane configurations. The 95-, 77- and 55-brane effective actions are given above.

(1) We have kept the two radii in each torus to be the same. To build an $\mathcal{N}=1$ supersymmetric model, we need to orbifold the compactified dimensions in the complexified basis. Equal radii in each torus yield discrete symmetries that can be gauged in orbifolds.

(2) If G_9 is identical to G_5, and the matter fields and couplings are symmetric under the interchange of 9- and 5-sectors, then the above 8 cases reduce to 4 inequivalent cases. This seems to be the generic situation in simple type I model-building.

(3) To keep at least one sector of gauge fields visible (i.e., gauge coupling of order 1), we can take at most two T^2's, say T_1 and T_3, with large radii. Equation (9) implies that the product of the two radii is

$$M_P^2=\frac{8m_s^8 v_1 v_2 v_3}{(2\pi)^6\lambda^2}\sim 32\pi^2 m_s^6 r_2^2 r_3^2. \tag{10}$$

If they are equal, then the radius is around 10^{-12} m for $m_s=1$ TeV.

(4) If we want both G_9 and G_5 to be observable, we can take only one T^2 to have large radius. In the effective action (9), we can take v_3 large. This scenario is necessary in any one of the following situations:

(i) the standard model is contained in one sector, say G_9, while a large gauge coupling from G_5 may be needed for a strong interaction to generate dynamical supersymmetry breaking.

(ii) the standard model is contained in both G_9 and G_5 [for example QCD SU(3) in G_9 while weak $SU(2)_L$ in G_5]. In this case, the G_9 and G_5 gauge couplings are in general different even at the string scale:

$$g_3^{-2}=g_9^{-2}=\frac{m_s^2 v_2}{(2\pi)^3\lambda},\quad g_2^{-2}=g_5^{-2}=\frac{m_s^2 v_1}{(2\pi)^3\lambda}. \tag{11}$$

(iii) QCD SU(3) is inside the 9-branes while the weak $SU(2)_L$ comes from the diagonal spontaneous symmetry breaking of a SU(2) inside the 9-branes and a SU(2) inside the 5-branes. In this case, the standard model gauge couplings g_3, g_2 and g_1 are in general different at the string scale, even if $g_9=g_5$.

We shall illustrate each of these possibilities in the next section. From Eq. (4),

$$r_3\sim 10^{-4} g_9 g_5\left(\frac{m_s}{\text{TeV}}\right)^{-2}\ \text{m}. \tag{12}$$

If both g_5 and g_9 are of order unity, and m_s is 1 TeV, then r is 10^{-4} meter. If g_5 becomes small (equivalent to large radius r_1), r_3 just becomes even smaller.

Let us go back to the general case with three types of 5-branes [as in Eq. (3)]. Similar analysis is easy to carry out, so we shall simply restrict ourselves to a few comments. It is easy to see that under T-duality, the rule $p-p'=0 \pmod 4$ is preserved. If v_3 gets large, we see that the gauge couplings of both the 9-brane gauge sector and the third type of 5-brane gauge sector become vanishingly small. As a consequence, the matter fields in this particular 59 sector will essentially decouple from all gauge interactions. They will still couple to other fields via other interactions, including gravity. So they are suitable candidates for dark matter.

B. Non-supersymmetric string and the cosmological constant

Supersymmetry was introduced originally to solve the hierarchy problem. Since this hierarchy problem disappears when the string scale is close to the weak scale, we should also consider non-supersymmetric type I models. Generically, besides 9-branes, 7-, 5- and/or 3-branes may be present in a specific model, depending on the details. We also expect a cosmological constant Λ_4 to be present. Again, we can consider the inequivalent scenarios when the various tori become large. Besides the 9753-brane configuration, duality can bring us to the 7975-, 7575- and 7535-brane configurations. Depending on the choice, taking one torus volume large will decouple gauge fields from one or more sectors (if they are present). The analysis is similar to that given for the supersymmetric case and will not be repeated here. There is one important difference between the supersymmetric case and the non-supersymmetric case. For supersymmetry to be unbroken, the 6 dimensional manifold must be a complex manifold. This means that T^6 can always be written as $T^6=T^2\otimes T^2\otimes T^2$, where the two radii in each T^2 are the same, as required by orbifold symmetry. (The exception is the case in which some of the T^2's are twisted only by $\mathbf{Z}_2$ and not by other $\mathbf{Z}_N$ twists. The two radii are unconstrained and one can choose freely only one of them to be large. However, orbifold compactifications of type I string theory involving this type of twists always give rise to non-chiral models [18]). In the non-supersymmetric case, it is possible that only one dimension has large radius (this breaks the complex structure).

Let us comment on the cosmological constant. We have seen how a large Planck mass M_P can be generated from a much smaller string scale m_s. Naively, the same effect happens to the cosmological constant. If there is a 10-dimensional cosmological constant $\Lambda_{10}=m_s^{10}$, then Λ_4

$=\Lambda_{10}v_1v_2v_3$, which is obviously unacceptable. Fortunately, this argument is incorrect. Recall the construction of the string model. We start from a 4-dimensional supersymmetric model toroidally compactified from 10 dimensions; it has no cosmological constant. We reduce the number of supersymmetries by orbifolding or orientifolding. The orbifolding of each of the three tori is needed to break the spacetime supersymmetry and generate chiral fermions, so the mechanism is intimately tied to $D=4$ spacetime. This suggests that $\Lambda_4 = m_s^4$. This is substantially smaller than the previous naive estimate. Unfortunately, this is still unacceptably large, so we need to find some mechanism to suppress it further. Now that we have seen how extra large dimensions can blow up the Planck mass, we are naturally led to ask if the reverse can suppress the cosmological constant.

Suppose we construct a non-supersymmetric string model in 3 spacetime dimensions. (In the construction of non-supersymmetric models, we do not need to complexify the compactified dimensions.) So generically $\Lambda_3 = m_s^3$. Now, let us take the radius r of one of the compactified direction to be large, i.e., decompactify that direction. So the theory essentially describes a 4-dimensional spacetime. The 4-dimensional cosmological constant is given by

$$\Lambda_4 \sim \frac{\Lambda_3}{r} \sim \frac{m_s^3}{r}. \tag{13}$$

For $m_s = 1$ TeV and r the size of the universe, Λ_4 is small enough to be acceptable. This means the supersymmetry breaking mechanism within the string model-building must be intrinsically 3-dimensional. This imposes a strong constraint in non-supersymmetric string model-building. Generically, the theory can decompactify in other directions in the field space, so that Λ_4 ends up of the order of m_s^4. However, Λ_4 measures the vacuum energy density, so it is natural for it to choose the minimum energy path of decompactification. This imposes a strong constraint in model-building. Notice that this mechanism will not work if the string scale is around the grand unified theory (GUT) scale, as is the case in the old scenario.

The above scenario is different from Witten's suggestion [20], which also utilizes the 3 spacetime dimensional picture. In 3 dimensional globally supersymmetric theories, the fermion-boson mass splitting δm is zero, as naively expected, but becomes non-zero in supergravity models. This implies that $\delta m \sim m^2/M$, where m is the typical mass and M is the 3-dimensional Planck mass [21]. So the fermion-boson mass splittings are non-zero while Λ_3 is zero. As we decompactify a direction with radius r, Λ_4 clearly remains zero. However, the 4-dimensional $M_P^2 = M/r$, so, for finite M_P, M goes to infinity as r goes to infinity, and δm goes to zero. This seems to imply that the decompactification of the 3-dimensional supergravity model yields 4-dimensional supergravity. So we believe that non-supersymmetric 4-dimensional models can come from the decompactification of 3-dimensional non-supersymmetric models, but not supersymmetric models.

III. AN EXPLICIT STRING MODEL

In this section, we use an explicit 4-dimensional chiral $\mathcal{N}=1$ supersymmetric type I string model as an illustration of some of the ideas discussed above. Toroidal compactification of type I string theory on a six dimensional torus T^6 gives rise to a four dimensional model with $\mathcal{N}=4$ supersymmetry. One can reduce the number of supersymmetries to $\mathcal{N}=1$ by orbifolding. For example, take $T^6 = T^2 \otimes T^2 \otimes T^2$, where each of the T^2 has a $\mathbf{Z}_3$ and a $\mathbf{Z}_2$ rotational symmetry. The $\mathbf{Z}_3$ generator g and the $\mathbf{Z}_2$ generator R acts on the complex coordinates z_1, z_2, z_3 of the compactified dimensions as follows:

$$gz_1 = \omega z_1, \quad gz_2 = \omega z_2, \quad gz_3 = \omega z_3 \tag{14}$$

$$Rz_1 = -z_1, \quad Rz_2 = -z_2, \quad Rz_3 = z_3 \tag{15}$$

where $\omega = \exp(2\pi i/3)$. The elements g and R generates the group $\mathbf{Z}_6$. If we identify points in T^6 under this discrete rotational symmetry, the resulting orbifold $\mathcal{M} = T^6/\mathbf{Z}_6$ has SU(3) holonomy; only 1 of the 4 gravitinos are kept under the orbifold action. As a result, type I string theory compactified on $\mathcal{M}$ has $\mathcal{N}=1$ supersymmetry in 4 dimensions.

To compute the spectrum, it is convenient to view type I string theory as type IIB orientifold. Type IIB string theory has a worldsheet reversal symmetry. The orientifold projection Ω reverses the parity of the closed string worldsheet (and hence interchanges the role of left- and right-movers in type IIB theory). Gauging this worldsheet parity symmetry results in a theory of unoriented closed strings. Open strings and D-branes are introduced to cancel the divergences (tadpoles) from the Klein bottle amplitude (a one-loop amplitude for *unoriented* closed strings). The orientifold group $\mathcal{O}$ (the discrete symmetries of type IIB theory that we are gauging) contains the elements Ω and ΩR. Tadpole cancellation requires introducing both $D9$- and $D5$-branes. Global Chan-Paton charges associated with the D-branes manifest themselves as gauge symmetry in space-time. As a result, there are gauge fields from both $D9$- and $D5$-branes.

The details of the tadpole cancellation conditions and the construction of $\mathbf{Z}_6$ orientifolds can be found in Appendix A. First, consider the case where the untwisted Neveu-Schwarz–Neveu-Schwarz (NS–NS) sector B-field background is zero; tadpole cancellation implies that $n_9 = n_5 = 32$, where n_9 (n_5) is the number of 9-branes (5-branes). This means that the total rank of the gauge group (which comes from both 9-branes and 5-branes) is 32. This model has gauge group $[\mathrm{SU}(6)\otimes\mathrm{SU}(6)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3]^2$ and was first constructed in Ref. [15]. Although the gauge group contains the standard model gauge group $\mathrm{SU}(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$, the residual gauge symmetry is too large for the model to be phenomenologically interesting.

In the presence of the untwisted NS–NS sector B-field background, it was shown [22,23] that the rank of the gauge group is reduced to $32/2^{b/2}$. Here, b is the rank of the matrix B_{ij} (i,j labels the complex coordinates of T^6). Since we are compactifying type I string theory on a 6 dimensional manifold, $b = 0,2,4,6$. The details of the construction of these models can be found in Appendix A. For $b=2$, the model

has $[SU(2)\otimes SU(2)\otimes SU(4)\otimes U(1)^3]^2$ gauge symmetry, which can be considered as a Pati-Salam like model with some extra global or gauge symmetry depending on the gauge coupling of the 9-brane and 5-brane gauge group. This is the model we are going to study in more details in this paper. For $b=4$, the gauge group is $[SU(2)\otimes SU(2)\otimes U(1)^2]^2$ which is too small to contain the standard model. For $b=6$, the gauge group is $[SU(2)\otimes U(1)]^2$, again does not contain the standard model.

Let us discuss in more details the spectrum of the model with $[SU(2)\otimes SU(2)\otimes SU(4)\otimes U(1)^3]^2$ gauge symmetry. Open strings start and end on D-branes. Since there are two kinds of D-branes (9-branes and 5-branes), there are three types of open strings that we need to consider: 99, 55 and 59 open strings. The open string spectrum of this model is given in Table I. Here, we consider all D5-branes sitting at the same orbifold fixed point. The fact that the 99 and the 55 sector have the same spectrum follows from T-duality. Since open string has only two end-points, the charged matter fields are either bi-fundamentals or symmetric (or anti-symmetric) representations of the gauge groups. Notice that the first and the second U(1) of both the 99 and the 55 gauge groups are anomalous, with U(1) anomaly equals -16 and $+16$ respectively. We can form a linear combination of these U(1)'s such that only one of them is anomalous [this combination is given by Q_1-Q_2 where $Q_{1,2}$ are the first and the second U(1) charge respectively]. By the generalized Green-Schwarz mechanism [24], some of the fields charged under the anomalous U(1) will acquire vevs to cancel the Fayet-Illiopoulos D-term. In addition to the open string spectrum, there are also closed string states. Since they do not carry Chan-Paton factors, they are singlets under the gauge group.

We see that the model has enough realistic features so that we can use it to study various scenarios discussed earlier. Here we shall consider three different possible ways that the model may be interpreted as an approximate way to describe nature. There is one chiral family in the first scenario, two chiral families in the second scenario, and three chiral families in the third scenario. Our description is sketchy and we shall simply assume the dynamics needed to behave in the way we like. Our purpose is to illustrate some of the features of brane-physics, and draw attention to the model's features that are generic to other type I string models. We shall not worry about which (if any) of the three scenarios is actually realized by the string dynamics.

A. Scenario 1

To describe this scenario, let us go to the T-dual picture where there are two different types of 5-branes [as in Eq. (9)]. For convenience, we will still refer to the 5-branes coming from T-dualizing the 9-branes as the 9-branes. Suppose the standard model $SU(3)\otimes SU(2)\otimes U(1)$ gauge group comes from the 9-brane sector only. In this model, the gauge group is $SU(4)\otimes SU(2)_L\otimes SU(2)_R$, and the 99 sector matter fields are singlet under the 5-brane gauge group. We can make the 5-brane gauge coupling relatively strong, so that $SU(4)_5$ gets strong and may trigger dynamical supersymmetry breaking. It may also cause spontaneous symmetry breaking of $SU(2)_5\otimes SU(2)_5$ so that the 55 and the 59 sector matter fields become heavy. In any case, let us focus our attention on the 99 sector. Here some of the low dimension terms in the superpotential is given by (see Table I for notations)

$$\mathcal{W}=(U_1Q_2+U_2Q_1)H+(U_1S_2+U_2S_1)U_3+(Q_1S_4+Q_2S_3)Q_3+\cdots \tag{16}$$

where we have suppressed the λ dependence and the exact coefficients of the couplings. (The λ dependence of N-point couplings is $g^{N-2}\sim\lambda^{(N-2)/2}$).

To break the gauge group down to $SU(3)\otimes SU(2)\otimes U(1)$, we can move some of the 9-branes away from each other. This mechanism is equivalent to the spontaneous symmetry breaking (SSB) action of the Higgs field in the effective field theory; that is, we can give vacuum expectation value to the Higgs superpartner of one of the U fields. Since the U fields are charged under $U(4)\supset SU(4)\otimes U(1)$, it is more appropriate to consider $SU(4)\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)\supset SU(3)\otimes SU(2)_L\otimes U(1)\otimes U(1)\otimes U(1)$,

$$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})(-1)=(\bar{\mathbf{3}},\mathbf{1})\left(-\frac{1}{3},\frac{1}{2},-1\right)\oplus(\bar{\mathbf{3}},\mathbf{1})\left(-\frac{1}{3},-\frac{1}{2},-1\right)\oplus(\mathbf{1},\mathbf{1})\left(1,\frac{1}{2},-1\right)\oplus(\mathbf{1},\mathbf{1})\left(1,-\frac{1}{2},-1\right). \tag{17}$$

Here, the first U(1) charge is the $B-L$ number, the second U(1) charge is $I_R=SU(2)_R$ isospin, and the third U(1) charge is $3B+L$ which comes from the decomposition $U(4)\supset SU(4)\otimes U(1)$. Notice that the U(1) hypercharge $Y=B-L+2I_R$ and the baryon number $B=(B-L+3B+L)/4$. Therefore, under

$$SU(4)\otimes SU(2)\otimes SU(2)\otimes U(1)\supset SU(3)\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_B\otimes U(1)_{3B+L},$$

$$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})(-1)=(\bar{\mathbf{3}},\mathbf{1})\left(\frac{2}{3},-\frac{1}{3},-1\right)\oplus(\bar{\mathbf{3}},\mathbf{1})\left(-\frac{4}{3},-\frac{1}{3},-1\right)\oplus(\mathbf{1},\mathbf{1})(2,0,-1)\oplus(\mathbf{1},\mathbf{1})(0,0,-1). \tag{18}$$

Here the U(1)s are independent but not orthogonal. If the scalar $(\mathbf{1},\mathbf{1})(0,0,-1)$ acquires a vev, $U(1)_{3B+L}$ is broken, and the fields Q_i and U_i become

$$(\mathbf{4},\mathbf{2},\mathbf{1})(+1)=(\mathbf{3},\mathbf{2})\left(\frac{1}{3},\frac{1}{3}\right)\oplus(\mathbf{1},\mathbf{2})(-1,0)$$

$$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})(-1)=(\bar{\mathbf{3}},\mathbf{1})\left(\frac{2}{3},-\frac{1}{3}\right)\oplus(\bar{\mathbf{3}},\mathbf{1})\left(-\frac{4}{3},-\frac{1}{3}\right)\oplus(\mathbf{1},\mathbf{1})(2,0)\oplus(\mathbf{1},\mathbf{1})(0,0). \tag{19}$$

We see that the Q_i and U_i yield precisely one chiral and one vector (i.e., one chiral plus one anti-chiral) family of the

standard model $SU(3)\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_B$. This also splits the $SU(2)_R$ doublet H into two standard models doublets H_1 and H_2:

$$(\mathbf{1},\mathbf{2},\mathbf{2})(0)=(\mathbf{1},\mathbf{2})(1,0)\oplus(\mathbf{1},\mathbf{2})(-1,0). \tag{20}$$

The μ term $\mu H_1 H_2$ does not appear as lower order terms in the superpotential. In this scenario where there are only 9-branes, $g_3=g_2=g_9$, and $g_Y=\sqrt{3/5}g_9$ at the string scale.

Consider the chiral fermions in the 99 sector before the electroweak symmetry breaking. There is 1 chiral family and 1 vector (chiral plus anti-chiral) family. Generically, a linear combination of U_1 and U_2 will pair up with U_3 to become heavy, while the other linear combination will remain massless. After the SSB to $SU(3)\otimes SU(2)\otimes U(1)$, this $\alpha U_1+\beta U_2$ combination gives the right-handed quarks and leptons. Similarly, a linear combination of Q_1 and Q_2 may pair up with Q_3 to become heavy, while the other linear combination will remain massless. They yield the weak isodoublets of quarks and leptons. So we see that the model has only one chiral family of quarks and leptons.

Now, notice that there are no baryon number violating terms in the superpotential. This is due the third U(1) symmetry. The quarks have $U(1)_3$ charge $+1$, while the anti-quarks have charge -1. The presence of such a U(1) associated with the SU(4) is a generic feature of brane physics [the U(1) factor is the center of mass of the D-branes]. So we should expect the conservation of the baryon number as a generic feature.

Suppose, in Eq. (9), it is v_1, not v_3, that is becoming very large. In this case, the 5-brane sector gauge coupling becomes vanishingly small. So the 5-brane matter fields essentially decouple and can be candidates for dark matter. Notice that these 5-brane gauge fields do not couple to the visible matter fields (except by very weak gravitational interaction), so they are essentially invisible.

B. Scenario 2

Suppose the QCD SU(3) comes from the 9-brane sector while the weak SU(2) comes from the 5-brane sector. To be specific, the gauge group is $SU(4)_9\otimes SU(2)_5\otimes SU(2)_5$. The quarks and leptons come from the 59 sector while the Higgs field comes from the 55 sector. There is a $\mathbf{Z}_2$ symmetry under which all matter fields are odd while the Higgs field is even. The superpotential is given by

$$\begin{aligned}\mathcal{W}=&(U_1Q_2+U_2Q_1)H+(U_1S_2+U_2S_1)U_3\\&+(Q_1S_4+Q_2S_3)Q_3+(u_1q_2+u_2q_1)h\\&+(u_1s_2+u_2s_1)u_3+(q_1s_4+q_2s_3)q_3\\&+\sum_{i=1}^{2}\sum_{j=1}^{2}\mathcal{U}_i\mathcal{Q}_j h+\sum_{i=1}^{2}\sum_{j=3}^{4}\chi_i\chi_j H\\&+\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_i'\mathcal{U}_jU_3+\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_i\mathcal{Q}_jQ_3+\sum_{i=1}^{2}\sum_{j=3}^{4}\phi_i'\chi_ju_3\\&+\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_i\chi_jq_3+\cdots.\end{aligned} \tag{21}$$

Again, we have suppressed the λ dependence and the exact coefficients of the couplings. As before, vev for one of $\mathcal{U}_i$ fields induces SSB: $SU(4)\otimes SU(2)_L\otimes SU(2)_R\supset SU(3)\otimes SU(2)\otimes U(1)_Y$. There are two families of quarks and leptons. As in the previous scenario, conservation of the third U(1) charge prevents any perturbative baryon number violating term. The analysis is quite similar to the above scenario, so we shall not repeat. A crucial difference is that, even at the string scale, the QCD coupling $g_3=g_9$ and the weak coupling $g_2=g_5$; they need not be the same. From Eq. (11), we see that their relative values depend on the compactification volumes. The hypercharge U(1) coupling is a function of g_3 and g_2:

$$g_Y=\frac{\sqrt{3}g_9g_5}{\sqrt{3g_9^2+2g_5^2}}. \tag{22}$$

If $g_9=g_5=g$, then $g_Y=\sqrt{3/5}g$ at the string scale.

C. Scenario 3

We see that the model has 1 chiral family in the 99 sector and 2 chiral families in the 59 sector. Furthermore, there is a $\mathbf{Z}_2$ symmetry between the 9-brane and the 5-brane. We can construct a new model by gauging this $\mathbf{Z}_2$ symmetry, or part of it, i.e., a $\mathbf{Z}_2$ orbifold of the original model. The $\mathbf{Z}_2$ symmetry we want to orbifold is an outer-automorphism. In terms of current algebra in conformal field theory, such an orbifold converts level-1 current algebra to level-2 current algebra.

Similar procedures can be carried out in the effective field theory without having to impose the condition that $g_9=g_5$ [25]. The basic idea is as follows. We start from a product gauge group $SU(N)\otimes SU(N)$, with gauge couplings g' and g'' respectively. By giving vev to the bi-fundamental field $\phi=(\mathbf{N},\mathbf{N})$ along the flat direction $\langle\phi\rangle=vI_N$ (where I_N is an $N\times N$ identity matrix), the gauge group is broken to $SU(N)$.

In the specific model that we consider in this paper, the fields ϕ_1,ϕ_2 are bi-fundamentals under the $U(2)_9\otimes U(2)_5$ gauge group. Similarly, ϕ_1',ϕ_2' are bi-fundamentals under $U(2)_9'\otimes U(2)_5'$. By giving vevs to ϕ_i's and ϕ_i''s of the above form (with $N=2$):

$$\begin{aligned}&U(2)_9\otimes U(2)_5\rightarrow SU(2)_L\otimes U(1)\\&U(2)_9'\otimes U(2)_5'\rightarrow SU(2)_R\otimes U(1).\end{aligned} \tag{23}$$

The gauge couplings of $SU(2)_L$, $SU(2)_R$ and the accompanying U(1)'s are given by $g=g_9g_5/\sqrt{g_9^2+g_5^2}$. The U(1)s are broken by the Green-Schwarz mechanism, so the resulting model has Pati-Salam gauge group $SU(4)_9\otimes SU(2)_L\otimes SU(2)_R$ with additional custodial $SU(4)_5\otimes U(1)^2$ symmetry. There are three families of chiral fermions under the Pati-Salam gauge group. Two of them come from the 59 sector: $\mathcal{U}_i$ give rise to two families of right-handed quarks and lep-

tons, while $\mathcal{Q}_i$ give rise to two families of left-handed quarks and leptons. The remaining family comes from the 99 sector: the right-handed quarks and leptons come from a linear combination of U_1 and U_2, and the left-handed quarks and leptons come from a linear combination of Q_1 and Q_2. It is interesting to note that one of the three families has a different origin. Whether this will offer an explanation to the fact that there is one heavy family deserves further investigation. Note that weak interaction universality is automatic.

The SSB of $SU(4)\otimes SU(2)_L\otimes SU(2)_R\supset SU(3)\otimes SU(2)_L\otimes U(1)_Y$ is essentially the same as in the first scenario, that is, giving a vev to the one of the U fields. The gauge couplings of the standard model gauge groups do not need to meet at the string scale and are given by

$$g_3=g_9$$

$$g_2=\frac{g_9 g_5}{\sqrt{g_9^2+g_5^2}}$$

$$g_Y=\frac{\sqrt{3}g_3 g_2}{\sqrt{3g_3^2+2g_2^2}} \tag{24}$$

so that $\sin^2\theta_W$ is given by

$$\sin^2\theta_W=\frac{3g_3^2}{6g_3^2+2g_2^2}. \tag{25}$$

If $g_9=g_5=g$, we see that $g_3=g$, $g_2=g/\sqrt{2}$, $g_Y=\sqrt{3/8}g$ and $\sin^2\theta_W=3/7$ at the string scale.

What about the $U(1)_B$ gauge boson associated with the baryon number conservation? Even if its coupling is very weak, it certainly must pick up a mass for the model to be phenomenologically viable. Suppose spontaneous symmetry breaking takes place when one of the scalar field charged under this $U(1)_B$ develops a vev. This vev v must be small so that the baryon number violating terms are suppressed by powers of v/m_s. The exact amount of suppression depends on the details. Consider the scenario in which this spontaneous symmetry breaking happens simultaneously when QCD SU(3) is broken as well, which implies that free quarks and gluons can exist. Suppose the $U(1)_B$ boson picks up a mass μ, then, following Ref. [26], there are free quarks and gluons with mass about $(1\ \mathrm{GeV})^2/\mu$. For $\mu=10$ keV, we see that a free quark or a free gluon will have a mass around 100 TeV.

D. Another string model

Let us consider another $\mathcal{N}=1$, $D=4$ chiral type I model, namely the $\mathbf{Z}_3\otimes\mathbf{Z}_2\otimes\mathbf{Z}_2$ model recently constructed by Kakushadze [27]. This model has 9-branes and three types of 5-branes as given in Eq. (3), all of them have identical gauge groups, so the resulting gauge group is $[U(6)\otimes SO(5)]^4$. Let us assume that QCD SU(3) comes from one of the U(6), while the weak SU(2) comes from one of the SO(5). It seems there are enough Higgs fields to break one of the SU(6) down to SU(4) and then to SU(3), and one of the SO(5) to SU(2). Again, we see that the U(1) carrying baryon numbers is present. However, this U(1) is anomalous, so it will pick up a mass via the Green-Schwarz mechanism automatically. Consider the situation where the torus T_3 is very large. Following Eq. (3), we see that both the gauge couplings of the 9-brane and the third 5-brane sectors become vanishingly small. In particular, the 9-brane matter fields essentially decouple and can be candidates for dark matter.

IV. DISCUSSION

It is clear that, among other properties, perturbative $D=4$, $\mathcal{N}=1$ supersymmetric, chiral type I string models have some very attractive features for the study of the TeV scale string scenario:

(i) Gravitons live in the bulk while gauge and charged matter fields live on the branes.

(ii) The presence of U(1)s (associated with the centers of mass of the branes) which help to stabilize the proton.

(iii) The identical nature of 9-brane gauge group and 5-brane gauge groups (if present) allows different standard model gauge couplings at the string scale.

(iv) the presence of bi-fundamental matter fields allows diagonal spontaneous symmetry breaking; again this mechanism allows different standard model gauge couplings at the string scale. This feature may validate the weak string coupling description of type I string.

These properties are quite compatible with present experiments and allow the future tests of the extra large dimension scenario.

There are a number of reasons why this TeV scale superstring scenario was not seriously considered earlier. In the old string phenomenology framework (i.e., pre-string-duality days), gravity and gauge interactions live in the same space. Since gauge interactions clearly live in an effective 4 spacetime dimensions, at least up to the electroweak scale, the largest the extra dimensions can be is M_{EW}^{-1}, as considered in [2]. However, generically, the string scale is above M_{GUT} to satisfy the proton decay bound. The reason is following. Before our understanding of string duality, all phenomenologically interesting string models are within the heterotic string theory in the conformal field theory framework, where the original rank of the gauge group is 22. Although the rank of the massless gauge symmetry can be substantially reduced, the massive sector retains (at least some of) the original large group feature. A typical heterotic string model that contains the standard model of strong and electroweak interactions in its low energy sector will contain massive bosons that can mediate proton decay. Since these massive bosons have masses of the string scale, we must keep the string scale high enough, say around M_{GUT}, to satisfy the proton decay bound. Generically, the proton decay bound requires the absence of dimension-4 and -5 baryon-number violating operators. If the string scale is around 1 TeV, the higher-dimensional (up to dimension-18) baryon-number violating operator terms can be dangerous. To prevent their appearance, some discrete symmetry or custodial gauge symmetry is necessary. However, the presence of such symmetry is not generic in the old heterotic string theory. In comparison, the U(1)'s in type I strings are very generic; they correspond to

the center of mass of the D-branes. As we have seen in some cases, the difficulty is how to make them massive.

Suppose we consider the heterotic string beyond the world-sheet construction. For example, solitonic 5-branes can contribute to the massless spectrum in non-perturbative heterotic string, which may have properties that are suitable for phenomenology. However, the analysis of non-perturbative heterotic string is difficult. Hopefully, duality between the type I and the heterotic string [28] allows us to treat more fully the non-perturbative effects.

The string model that we have presented here is constructed from perturbative type I string theory. If the gauge coupling is of order 1, and we expect $m_s R > 1$, Eq. (9) implies that the string coupling λ is small. One would still like to know the energy regime where the perturbative type I picture may become invalid [29]. Naively, one may expect the 4-dimensional low energy effective field theory to be valid at momentum scales below r^{-1}. This is because the low energy effective couplings are small (except for the strong QCD coupling). Quantum corrections coming from the massive string modes are negligible at low energies. Above this scale, one expects the $(4+n)$-dimensional effective field theory to be valid. At scales above R^{-1} but below m_s, we should move from effective field theory to string theory, where perturbative type I string theory is likely to be valid. When the energy-momentum scale is around the string scale m_s, the type I string perturbative description may or may not remain valid. This may depend on the particular scenario and the particular process one is interested in. In view of type I–heterotic duality, one would ask if the weakly-coupled heterotic string description should take over in this regime. However, the techniques in constructing heterotic string vacua with NS 5-branes (the NS 5-branes are dual to the $D5$-branes in the type I theory) are not well developed. Since type I string theory provides a natural setting to realize the idea of extra large dimensions, it is likely that the scenarios that we presented here capture the important features which persist in the large λ regime.

V. COMMENTS

It is interesting to compare the merits of the two scenarios of string phenomenology: the old scenario with the string scale around the GUT scale, and the new scenario [8,5,6] with the string scale around the electroweak scale. Experimentally, the new scenario is clearly superior. High energy scatterings can probe the extra small dimensions while gravity can probe the extra large dimensions. These experiments are coming in the near future. If this scenario is correct, we can expect a lot of experimental information on the detailed structure, which can provide valuable guidance on the precise way nature is realized within string theory. At this moment, before the availability of the experimental data, we can still ask which scenario is more appealing from the theoretical perspective. Without detailed realistic models, any comparison is quite subjective. Nevertheless, we believe the exercise can be illuminating. A scenario may be deemed more natural than another if it has fewer number of disparate scales. Let us give a naive counting of the number of scales in each scenario.

In the old scenario where the string scale is around the Planck scale M_P, we also have the electroweak scale. The Planck scale M_P is about three orders of magnitude above the GUT scale; this discrepancy is different enough to require some new physics ingredients to explain. Let us count this situation as 3 scales. The quark and lepton masses are very different. For example, the mass of the top quark is more than 10^5 that of the electron. Let us assume that the fermion mass splitting introduces another scale that needs understanding. Including the cosmological constant, we have 5 different scales. Let us take one of them, say the electroweak scale, to set the overall normalization. Unification of the gauge couplings provides a nice explanation of the GUT scale, so there remains 3 scales that remain to be understood. If one wants to treat the GUT scale and the Planck scale as close enough to be considered as one, we still have two scales that beg for an explanation.

In the new scenario, we have the string scale around 1 TeV, which is close enough to the electroweak scale to be considered as a single scale. Similarly, the small compactification radii between the electroweak and the string scale should not be treated as new scales. Suppose the standard model gauge couplings are unified at the string scale. Since the gauge and matter fields are living in extra dimensions, say 8 total spacetime dimensions, the gauge couplings are irrelevant operators. So these couplings run as powers and diverge rapidly as we move to lower energies. Once the energies involved go below the scale of the small radii, they become marginal operators and vary only logarithmically. Suppose the Yukawa couplings at the string scale are different but comparable. Again, as irrelevant operators, they diverge rapidly, so they can easily differ by orders of magnitude at scales below the electroweak scale [6]. This provides a qualitative explanation for the fermion mass hierarchy. So we shall not count the fermion mass splittings as an extra scale. Now we can count the number of scales in this scenario: using the string/electroweak scale to set the overall normalization, we have only two scales that beg for an explanation: the cosmological constant and the large radius of $r = 1\ \text{mm} = 10^{16}/\text{TeV}$. (In fact, a cosmological constant of the order r^{-4} is quite compatible with observations.)

Theoretically, it seems that the new scenario looks slightly better than, or at least comparable to, the old scenario. Experimentally, the new scenario is much more testable or reachable and hence superior. So overall, the new scenario certainly deserves further investigation.

ACKNOWLEDGMENTS

We would like to thank Philip Argyres, Chong-Sun Chu, Keith Dienes, Alon Faraggi, Piljin Yi, and especially Zurab Kakushadze for valuable discussions. The research of G.S. and S.-H.H.T. was partially supported by the National Science Foundation. G.S. would like to thank the kind hospitality of the Institute for Theoretical Physics at Stony Brook during his stay. G.S. would also like to thank Joyce M. Kuok Foundation for financial support.

APPENDIX A: CONSTRUCTION OF THE MODEL

In this appendix, we give the details of how to construct from D-branes and orientifolds the $\mathcal{N}=1$, $D=4$ chiral string model with $[\mathrm{SU}(4)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)^3]^2$ gauge symmetry presented in Sec. III. The model also exhibits some novel features [22] of the untwisted NS-NS sector B-field background recently discussed in [23].

We start from type IIB string theory compactified on $T^6=T^2\otimes T^2\otimes T^2$, where each of the two-tori has a $\mathbf{Z}_3$ and a $\mathbf{Z}_2$ rotational symmetry. The $\mathbf{Z}_3$ generator g and the $\mathbf{Z}_2$ generator R acts on the complex coordinates z_1,z_2,z_3 of T^6 as follows:

$$gz_1=\omega z_1,\quad gz_2=\omega z_2,\quad gz_3=\omega z_3 \tag{A1}$$

$$Rz_1=-z_1,\quad Rz_2=-z_2,\quad Rz_3=z_3 \tag{A2}$$

where $\omega=\exp(2\pi i/3)$. The elements g and R together generate the Abelian group $\mathbf{Z}_6$.

Let us consider type I string theory compactified on the toroidal orbifold $\mathcal{M}=T^6/\mathbf{Z}_6$. It is convenient to view type I compactification as type IIB orientifold. The orientifold projection Ω reverses the parity of the closed string worldsheet. This results in a theory of unoriented closed strings. One-loop finiteness generically requires introducing open strings starting and ending on D-branes, so that the divergences (tadpoles) coming from the cylinder, Mobius strip and Klein bottle amplitudes cancel. The orientifold group $\mathcal{O}=\{\Omega^a R^b g^c | a=0,1;\ b=0,1;\ c=0,1,2\}$ contains both the elements Ω and ΩR. Therefore, one has to introduce both $D9$- and $D5$-branes to cancel the tadpoles. The global Chan-Paton charges associated with the D-branes manifest themselves as gauge symmetries in space-time. Hence, there are gauge bosons from both 99 and 55 open strings.

The orbifold action on the Chan-Paton factors is described by unitary matrices $\gamma_{k,p}$ that acts on the string end-points (k labels the orbifold group element, p labels the type of branes). Let $|\psi,ij\rangle$ be an open string state, where ψ is the state of the worldsheet fields and i,j are the Chan-Paton factors of the string end-points (the open string starts on a p-brane and ends on a q-brane). The action of the orbifold element k is given by

$$k:\quad |\psi,ij\rangle\rightarrow(\gamma_{k,p})_{ii'}|k\cdot\psi,i'j'\rangle(\gamma_{k,q}^{-1})_{j'j}. \tag{A3}$$

Tadpole cancellation determines the form of the $\gamma_{k,p}$ matrices. There are two types of constraints that we need to consider. The first one comes from the cancellation of the untwisted tadpoles for the $D9$-branes and the $D5$-branes respectively. This type of constraint determines the number of $D9$- and $D5$-branes. In the general case where the untwisted NS-NS sector B-field can be non-vanishing (with b equals the rank of the matrix B_{ij}, which is always even), tadpole cancellation for the untwisted R-R 10-form potential gives [22,23]

$$2^b(\mathrm{Tr}(\gamma_{1,9}))^2-2^{b/2}64\ \mathrm{Tr}(\gamma_{1,9})+(32)^2=0. \tag{A4}$$

Therefore, the number of D9-branes is given by $n_9=32/2^{b/2}$. Similarly, tadpole cancellation condition for the untwisted Ramond-Ramond (R-R) 6-form potential gives [23]

$$(\mathrm{Tr}(\gamma_{1,5}))^2-\frac{64}{2^{b/2}}\mathrm{Tr}(\gamma_{1,5})+\frac{1}{2^b}(32)^2=0. \tag{A5}$$

Therefore, the number of D5-branes is given by $n_5=32/2^{b/2}$. This was also expected from T-duality between D9- and D5-branes.

The other constraint comes from tadpole cancellation of the twisted R-R 6-form potential. Since the twisted closed string states propagating in the tree-channel do not have momentum or winding, the twisted tadpoles remain the same in the presence of the untwisted NS-NS sector B-field background (the effect of which is to shift the left- plus right-moving momentum lattice). The twisted tadpoles for $\mathbf{Z}_N$ orientifolds in 6 dimensions have been computed in [30–32] and generalized to 4 dimensions in Refs. [12,14,15]. Here, we state the results for the $\mathbf{Z}_6$ case:

$$\mathrm{Tr}(\gamma_{g,p})=-(-1)^{b/2}32[\cos(\pi/3)]^3=-(-1)^{b/2}4 \tag{A6}$$

$$\mathrm{Tr}(\gamma_{R,p})=\mathrm{Tr}(\gamma_{Rg,p})=0. \tag{A7}$$

Let us consider the solutions to the above tadpole cancellation conditions for all possible values of b:

(i) For $b=0$, $n_9=n_5=32$

$$\gamma_{R,p}=\mathrm{diag}(iI_{16},-iI_{16}), \tag{A8}$$

$$\gamma_{g,p}=\mathrm{diag}(\omega I_6,\omega^2 I_6,I_4,\omega I_6,\omega^2 I_6,I_4), \tag{A9}$$

where I_M is an $M\times M$ identity matrix. The gauge group from the 99 open strings is $\mathrm{SU}(6)\otimes\mathrm{SU}(6)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3$. The 55 open strings also give rise to the gauge group $\mathrm{SU}(6)\otimes\mathrm{SU}(6)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3$ if the $D5$-branes are located at the same fixed point. The total rank of the gauge group is 32. This model was first constructed in Ref. [15].

(ii) For $b=2$, $n_9=n_5=16$.

$$\gamma_{R,p}=\mathrm{diag}(iI_8,-iI_8), \tag{A10}$$

$$\gamma_{g,p}=\mathrm{diag}(\omega I_2,\omega^2 I_2,I_4,\omega I_2,\omega^2 I_2,I_4). \tag{A11}$$

The gauge group (from both 99 and 55 open strings) is $[\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3]^2$. The total rank of the gauge group is 16. This is the model that we study in this paper.

(iii) For $b=4$, $n_9=n_5=8$.

$$\gamma_{R,p}=\mathrm{diag}(iI_4,-iI_4), \tag{A12}$$

$$\gamma_{g,p}=\mathrm{diag}(\omega I_2,\omega^2 I_2,\omega I_2,\omega^2 I_2). \tag{A13}$$

The gauge group (from both 99 and 55 open strings) is $[\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)^2]^2$. The total rank of the gauge group is 8.

(iv) For $b=6$, $n_9=n_5=4$.

$$\gamma_{R,p}=\mathrm{diag}(iI_2,-iI_2), \tag{A14}$$

$$\gamma_{g,p}=I_4. \tag{A15}$$

The gauge group (from both 99 and 55 open strings) is $[\mathrm{SU}(2)\otimes\mathrm{U}(1)]^2$. The total rank of the gauge group is 4.

The gauge groups of the models for $b=4,6$ are too small to accommodate the standard model, which make them phenomenologically uninteresting. We will focus on the $b=2$ model with $[\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3]^2$ gauge symmetry.

To construct the open string spectrum, we keep all physical states that are invariant under the orbifold action. There are contributions from 99, 55 and 59 open strings. As pointed out in Ref. [23], the 59 open string sector states come with a multiplicity $\xi=2^{b/2}$. (Recall that without B-field, the multiplicity of states in the 59 sector was one per configuration of Chan-Paton charges [31,32].) The open string spectrum of the $[\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(4)\otimes\mathrm{U}(1)^3]^2$ model is given in Table I.

APPENDIX B: *H*-CHARGES, SCATTERINGS AND COUPLINGS

In this appendix, we review the conformal field theory techniques in calculating scattering amplitudes (and hence couplings) in orbifold models. In type I string theory, closed string sector only gives rise to gauge singlets. We will therefore focus on the couplings between open string states.

In the standard orbifold formalism, the internal part of the worldsheet supercurrent can be written as

$$T_F=\frac{i}{2}\sum_{a=1}^{3}\psi^a\partial X^a+\mathrm{H.c.}=\frac{i}{2}\sum_{a=1}^{3}e^{i\rho^a}\partial X^a+\mathrm{H.c.}, \tag{B1}$$

where ψ^a are complex world-sheet fermions, which can be bosonized:

$$\psi^a=\exp(i\rho^a)=\exp(iH\cdot\rho),$$

$$\psi^{a\dagger}=\exp(-i\rho^a)=\exp(-iH\cdot\rho). \tag{B2}$$

Here, H (known as the H-charge) equals (1,0,0), (0,1,0) or (0,0,1) for $a=1,2,3$. The supercurrent is therefore a linear combination of terms with well defined H-charges.

In the covariant gauge, we have the reparametrization ghosts b and c, and superconformal ghosts β and γ [33]. It is most convenient to bosonize the β,γ ghosts: $\beta=\partial\xi e^{-\phi}$, $\gamma=\eta e^{\phi}$, where ξ and η are auxiliary fermions and ϕ is a bosonic ghost field obeying the OPE $\phi(z)\phi(w)\sim\log(z-w)$. The conformal dimension of $e^{q\phi}$ is $-(\frac{1}{2})q(q+2)$. In covariant gauge, vertex operators are of the form $V(z)\lambda_{ij}$, where $V(z)$ is a dimension 1 operator constructed from the conformal fields (which include the longitudinal components as well as the ghosts), and λ_{ij} is the Chan-Paton wave function. The vertex operators for space-time bosons carry integral ghost charges ($q\in\mathbf{Z}$) whereas for space-time fermions the ghost charges are half-integral ($q\in\mathbf{Z}+\frac{1}{2}$). Here, q specifies the picture. The canonical choice is $q=-1$ for space-time bosons and $q=-\frac{1}{2}$ for space-time fermions. We will denote the corresponding vertex operators by $V_{-1}(z)$ and $V_{-1/2}(z)$, respectively. Vertex operators in the $q=0$ picture (with zero ghost charge) is given by *picture changing*:

$$V_0(z)=\lim_{w\to z}e^{\phi}T_F(z)V_{-1}(w). \tag{B3}$$

One can see that besides the supercurrent, open string states also carry H-charges. The vertex operator for gauge bosons in the -1 picture is given by $\psi^\mu\lambda_{ij}$ where μ is the spacetime index. Therefore, they do not carry H-charges. On the other hand, the vertex operator for matter fields in 99 and 55 sector is given by $\psi^a\lambda_{ij}$. Hence, in the -1 picture, $H=(1,0,0)$, (0,1,0) or (0,0,1) depending on which worldsheet fermion is excited. The moding of the worldsheet fermions in the 59 sector is different from that in the 99 sector. Therefore, in the -1 picture, matter fields in the 59 sector carry half-integral H-charges instead of integral H-charges. The H-charges of the massless fields of the $[\mathrm{SU}(4)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)^3]^2$ model is given in Table I.

Having constructed the vertex operators for the massless states, one can in principle compute the scattering amplitudes, or the corresponding couplings in the superpotential. The coupling of M chiral superfields in the superpotential is given by the scattering amplitude of the component fields in the limit when all the external momenta are zero. Due to holomorphicity, one needs to consider only the scatterings of left-handed space-time fermions, with vertices $V_{-1/2}(z)$, and their space-time superpartners. Since the total ϕ ghost charge in any tree-level correlation function is -2, it is convenient to choose two of the vertex operators in the $-1/2$-picture, one in the -1-picture, and the rest in the 0-picture. Using the $\mathrm{SL}(2,\mathbf{C})$ invariance, the scattering amplitude is therefore

$$\begin{aligned}\mathcal{A}_M=&\,g_{\mathrm{st}}^{M-2}\,\mathrm{Tr}(\lambda^1\lambda^2\cdots\lambda^M)\\&\times\int dz_4\cdots dz_M\langle V_{-1/2}(0)V_{-1/2}(1)\\&\times V_{-1}(\infty)V_0(z_4)\cdots V_0(z_M)\rangle,\end{aligned} \tag{B4}$$

where we have normalized the c ghost part of the correlation function $\langle c(0)c(1)c(\infty)\rangle$ to 1. To obtain the open string scattering amplitudes, we have to take the integration variables z_i to the real axis, with $z_i>z_{i+1}$. Now the terms in the superpotential can be read off directly from the resulting scattering amplitudes. For a non-zero coupling, the sum of the H-charges must be zero in the corresponding scattering amplitude. Note that the supercurrent carries terms with different H-charges. Because of picture changing, H-charges are not global charges even though they must be conserved exactly. In additional to the H-charge conservation, there is also a discrete $\mathbf{Z}_2$ symmetry coming from the orbifold twist. For the couplings to be non-zero, the total twist in the scattering amplitude (B4) must be an integer.

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