Consistent perturbative light-front formulation of quantum electrodynamics

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A new light-front formulation of QED is developed, within the framework of standard perturbation theory, in which x^+ plays the role of the evolution parameter and the gauge choice is $A_+=0$ (light-front "temporal" gauge). It is shown that this formulation leads to the Mandelstam-Leibbrandt causal prescription for the noncovariant singularities in the photon propagator. Furthermore, it is proved that the dimensionally regularized one loop off-shell amplitudes exactly coincide with the correct ones, as computed within the standard approach using ordinary space-time coordinates. [S0556-2821(98)10820-2]

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I. INTRODUCTION

The light-front formulation of gauge quantum field theories has become more and more popular in the past few years. In the Abelian case, i.e., standard QED or the Abelian Higgs model, the renewal of the interest in this subject is mainly because of two reasons. On the one hand, the light-front Hamiltonian approach to QED appears to provide an alternative tool to compute the Lamb shift [1] and deal with bound-state problems [2]. On the other hand, some nonperturbative aspects—such as the role of the zero modes [3]—have first to be clearly understood in Abelian models, before going into the much more challenging non-Abelian case.

The original attempts to set up canonical quantization of QED in the framework of light-front—or null-plane—dynamics date back to the early 1970s [4]. In the original approach, the light-cone coordinate (LCC) $x^+=(x^0+x^3)/\sqrt{2}$ plays the role of the evolution parameter and the standard gauge choice is $A_-=0$, in such a way to stay as close as possible to the axial gauge formulation of QED in standard space-time coordinates (STC).

After a considerable amount of work has been done along this line, it was definitely discovered [5] that perturbation theory, based upon the original light-front quantization scheme for gauge theories, is inconsistent, owing to loop integrations, just because the above scheme necessarily entails the Cauchy principal value (CPV) prescription to understand the spurious noncovariant poles in the gauge particle vector propagator. This means that quite basic features of the standard perturbative approach for gauge theories are lost, such as power counting renormalizability, unitarity, covariance and causality. In other words, the original approach to light-front quantization of gauge theories is certainly not equivalent to the standard covariant formulation, already at the perturbative level; it is *a fortiori* hard to believe that the same approach could provide useful hints beyond perturbation theory, in the absence of deep modifications. *En passant*, it is really curious and rather surprising that a nonnegligeable fraction of the field theorists community seems to have nowadays not yet fully gathered and appreciated this rough breakdown of the conventional old light-front approach to gauge theories. For instance, even the one loop QCD beta function does not result to be, within that context, the correct covariant one [5].

It has been noticed some time ago [6] that, in order to restore at least causality for the free propagator of the gauge fields in the light-cone gauge, a special prescription, thereof called the Mandelstam-Leibbrandt (ML) prescription, has to be employed, in order to regulate the spurious noncovariant singularities. Shortly afterwards, it has been realized that the ML prescription arises from the canonical quantization in standard STC, provided some special unphysical (ghostlike) degrees of freedom are taken into account [7]. Even more, it has been proved that, within that framework, gauge theories in the light-cone gauge are renormalizable, unitary and covariant order-by-order in perturbation theory [8]. It is worthwhile to emphasize how this remarkable result crucially stems from the presence of the above mentioned unphysical degrees of freedom: as soon as they are correctly taken into account, the equivalence between the covariant and light-cone gauges is established, within the standard perturbative approach in STC.

The open issue, which is still there, is to find a light-front formulation for quantum gauge theories, which turns out to be equivalent to the conventional one in ordinary STC, at least in perturbation theory. It is definitely clear, from the above considerations, that such a new formulation, whatever it is, must lead to the ML prescription for the noncovariant singularities of the gauge particle vector propagator, at variance with the original old one, driving instead to the pathological CPV prescription.

A first step towards this direction has been done quite recently by McCartor and Robertson [9]. They have found an algebraic scheme to quantize the theory on the light-front, taking also the above mentioned unphysical degrees of freedom into account. However, as they use the "temporal"

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LCC as the evolution parameter and the ''spatial'' gauge choice $A_-=0$, the above algebraic setting is done after quantization of physical and unphysical degrees of freedom on different characteristic surfaces, i.e., light-front hyperplanes. Beside being somewhat unnatural,[1] this approach does not drive exactly to the standard form of the photon propagator with the ML prescription for the spurious singularity. It is one of the aims of the present paper to show how the latter drawbacks in the McCartor and Robertson approach could be indeed overcome, without spoiling its correct content of an enlarged light-front operator algebra.

In order to achieve this goal, we simply make the transition from the ''spatial'' light-cone gauge $A_-=0$ to the ''temporal'' light-cone gauge $A_+=0$, the ''temporal'' LCC x^+ being kept as the evolution parameter within the light-front formulation. In so doing, on the one hand the free field operator algebra for the whole set of fields is naturally defined on the ''spatial'' hyperplanes $x^+=\text{const}$. On the other hand, the ML prescription is exactly recovered for the propagator of the free radiation field.

These remarkable features allow therefore to correctly develop perturbation theory, once the corresponding interaction Hamiltonian has been single out from constraints analysis of (pseudo-)classical QED in LCC, including unphysical degrees of freedom (i.e., in an enlarged phase space). This leads to obtain the set of light-front QED Feynman's rules, which will be shown to involve an infinite set of special noncovariant vertices. It is then amusing to check, at one loop, that truncated light-front Green's functions—i.e., vacuum expectation values of light-front-time ordered product of field operators—are exactly the same as in the usual STC formulation, provided the gauge invariant dimensional regularization scheme is embodied.

The paper is organized as follows. In Sec. II, we give a critical reading of the McCartor and Robertson approach to light-front quantization of the free radiation field. In so doing, we point out where this approach reveals to be unsatisfactory and how to implement it, in order to reproduce the ML form of the free propagator. In Sec. III, we briefly review the light-front quantization of the free Dirac's field, in order to also establish our notations for the light-front treatment of spinorial matter. In Sec. IV we perform the canonical light-front quantization of QED in the ''temporal'' light-cone gauge $A_+=0$, by means of the standard Dirac's procedure for constrained systems. Section V is devoted to perturbation theory: namely, we derive Feynman's rules and show that, up to the one loop approximation, dimensionally regularized truncated and connected light-front Green's functions are the same, as computed out of the standard canonical framework in usual STC. Section VI contains some further comments and remarks, as well as an outlook on future developments.

[1]Actually, in the presence of interaction, the simultaneous occurrence of ''spatial'' and ''temporal'' light-front hyperplanes, to specify the operator's algebra, makes the treatment somewhat complicated.

II. LIGHT-FRONT QUANTIZATION OF THE FREE RADIATION FIELD

Some time ago [7] it has been shown that the canonical quantization of the free radiation field in the light-cone gauge $n^\mu A_\mu \equiv A_-=0,(n^2=0)$, is suitably formulated using standard space-time coordinates (STC) and leads, eventually, to the ML prescription for the spurious singularities in the propagator. It is worthwhile to stress that, in the derivation of the above result, the unphysical components of the gauge potential play a fundamental role. On the other hand, within the original approach to light-front quantization using light-cone coordinates (LCC) [4], those unphysical degrees of freedom turn out to satisfy constraint equations instead of genuine equations of motion. Thereby, they are eliminated after imposing suitable boundary conditions and, consequently, only the physical degrees of freedom are indeed submitted to canonical quantization. In so doing, unfortunately, the spurious singularity in the vector propagator results to be prescribed as Cauchy principal valued and leads to an inconsistent meaningless perturbation theory.

It is our aim to show in this section how some light-front quantization scheme exists for the radiation field in LCC, which drives eventually to the ML prescription for the spurious singularity of the vector propagator, just like the standard STC formulation does. In order to achieve this goal, we will develop and improve a recent attempt [9], in which the above mentioned unphysical components of the gauge potential are retained and quantized in LCC according to a new procedure. Let us first briefly review the main points of this approach.

The starting point is the Lagrangian density of the free radiation field

$$\mathcal{L}_{\text{rad}}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\Lambda n^\mu A_\mu, \tag{2.1}$$

where $n_\mu=(n_+,n_\perp,n_-)=(1,0,0,0)$, in such a way that $n^\mu A_\mu=A_-$, and Λ is a Lagrange multiplier which enforces the gauge constraint.

The Euler-Lagrange equations lead to

$$\partial_\mu F^{\mu\nu}=n^\nu\Lambda, \tag{2.2a}$$

$$A_-=0. \tag{2.2b}$$

It is convenient to introduce some new field variables as follows: namely,

$$A_\alpha=T_\alpha-\frac{\partial_\alpha}{\partial_\perp^2}\varphi, \tag{2.3a}$$

$$A_+=\frac{\partial_\alpha}{\partial_-}T_\alpha-\frac{\partial_+}{\partial_\perp^2}\varphi-\frac{1}{\partial_\perp^2}\Lambda; \tag{2.3b}$$

then Eqs. (2.2a),(2.2b) become

$$(2\partial_+\partial_--\partial_\perp^2)T_\alpha=0, \tag{2.4a}$$

$$\partial_-\varphi=\partial_-\Lambda=0. \tag{2.4b}$$

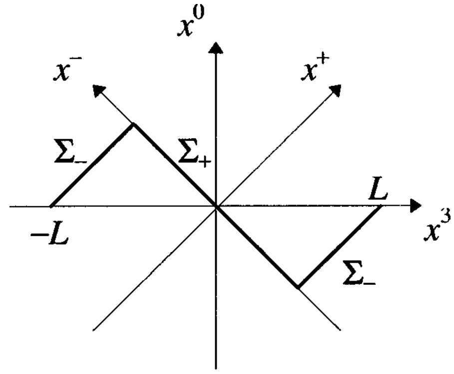

FIG. 1. The null hyperplanes Σ_+ and Σ_-.

We notice that, as the fields $T_\alpha(x)$ fulfill free D'Alembert's equations of motion, then the inverse of the light-front-space derivative in Eq. (2.3b) is understood here to be $(1/\partial_-)\equiv(2\partial_+/\partial_\perp^2)$. Furthermore, from Eq. (2.4b) we can easily see that the fields φ and Λ do not fulfil evolution equations—remember that here it is the LCC x^+ which plays the role of the evolution parameter—but, as previously noticed, they satisfy constraint equations and, therefrom, cannot be canonically quantized on the null hyperplanes at constant x^+.

Now, it has been suggested [9,10] a new light-front quantization procedure, in which the transverse fields T_α are quantized on null hyperplanes at equal x^+, according to the original light-front recipe, while the longitudinal fields φ and Λ at equal x^-. Following this procedure, one can set up the generators of the translations on the null hyperplanes Σ_+ and Σ_-, in the limit $L\to\infty$ (see Fig. 1), and obtain, taking the Heisenberg equations of motion (2.4a),(2.4b) into account, the commutation relations

$$[T_\alpha(x),T_\beta(y)]_{x^+=y^+} = -\frac{i}{4}\,\delta_{\alpha\beta}\delta^{(2)}(x^\perp-y^\perp)\mathrm{sgn}(x^--y^-), \quad (2.5a)$$

$$[\varphi(x),\Lambda(y)]_{x^-=y^-} = i\,\delta(x^+-y^+)\partial_\perp^2\,\delta^{(2)}(x^\perp-y^\perp), \quad (2.5b)$$

$$[T_\alpha(x),\varphi(y)]=[T_\alpha(x),\Lambda(y)] =[\varphi(x),\varphi(y)] =[\Lambda(x),\Lambda(y)]=0, \quad (2.5c)$$

where sgn(x) denotes the usual sign distribution. In so doing, the authors of Ref. [9] suggest that the light-cone-time ordered product of the gauge potential operators defined by

$$D^+_{\mu\nu}(x-y)\equiv\theta(x^+-y^+)\langle 0|A_\mu(x)A_\nu(y)|0\rangle + \theta(y^+-x^+)\langle 0|A_\nu(y)A_\mu(x)|0\rangle, \quad (2.6)$$

might eventually give rise to the ML form of the gauge field propagator. Actually we shall show below that this is not exactly true, owing to the presence of some ill-defined products of tempered distributions.

As a matter of fact, if we consider the transversal physical part of the vector potential, namely,

$$T_\mu=\left(T_+\equiv\frac{\partial_\alpha}{\partial_-}\,T_\alpha,T_\beta,T_-=0\right),\quad \alpha,\beta=1,2, \quad (2.7a)$$

then it has been well known for a long time [4] that the x^+-ordered product of two such physical components leads, in momentum space, to the expression

$$\tilde{T}_{\mu\nu}(k)=\frac{i}{k^2+i\varepsilon}\left\{-g_{\mu\nu}+(n_\mu k_\nu+n_\nu k_\mu)\times\mathrm{CPV}\left(\frac{1}{k_-}\right)\right\}-in_\mu n_\nu\,\frac{1}{k_-^2}. \quad (2.7b)$$

We notice, *en passant*, that the first term in the right-hand side (RHS) of Eq. (2.7b) is a well defined tempered distribution, whereas it is not necessary to specify any prescription to define[2] the very last contact (or ''instantaneous'') term, cause it is also well known since the early 1970s [4] that a complete algebraic cancellation indeed occurs, in perturbation theory for QED, just between those contact terms and the corresponding ones arising from the spinor interaction Hamiltonian.

On the other hand, if we consider the longitudinal components of the gauge potential, namely,

$$\Gamma_\mu=-\frac{1}{\partial_\perp^2}\,(\partial_\mu\varphi+n_\mu\Lambda), \quad (2.8a)$$

then a straightforward calculation yields

$$\langle 0|\Gamma_\mu(x)\Gamma_\nu(y)|0\rangle=\int\frac{d^4k}{(2\pi)^3}\,e^{ik(x-y)}\theta(-k_+)\times\delta(k_-)\,\frac{n_\mu k_\nu+n_\nu k_\mu}{k_\perp^2}. \quad (2.8b)$$

After multiplication, for instance, with $\theta(x^+-y^+)$ and taking the Fourier transform we formally get the convolution

$$\int_{-\infty}^{0}\frac{d\xi}{2\pi i}\,\frac{\delta(k_-)}{(k_+-\xi)-i\epsilon}\times\left[\frac{n_\mu k_\nu+n_\nu k_\mu}{k_\perp^2}\right]_{k_+=\xi}. \quad (2.8c)$$

One can easily convince himself that the above expression does not define a tempered distribution—owing to the logarithmic and linear divergences in the ξ-integration—which means, in turn, that the propagator in Eq. (2.6) is not properly understood from the mathematical point of view, as it is

[2]Nevertheless, one can always define it in the sense of tempered distributions as, for instance, minus the derivative with respect to k_- of CPV $(1/k_-)$.

obtained summing up a well defined distribution (the transversal part) and a meaningless quantity (the longitudinal part).

Nonetheless, it is indeed remarkable that the main idea behind the quantization procedure in Ref. [9], i.e., the enlarged algebra on the characteristic surfaces in order to satisfy causality, is suggestive, albeit troubles arise when dealing with the evolution. It should be apparent that, in fact, the very same reasons preventing us from specifying the algebra of the longitudinal field operators at equal x^+, also prevent us from propagating the unphysical degrees of freedom along x^+. The simplest way to circumvent these difficulties and to build up a consistent light-front dynamics turns out to be a change of the null gauge vector,[3] i.e., we replace $n_\mu \mapsto n^*_\mu \equiv (0,0,0,1)$ in such a way that $n^{*\mu}A_\mu = A_+ = 0$.

Let us therefore consider the new Lagrangian density

$$\mathcal{L}_{\rm rad} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \Lambda n^{*\mu}A_\mu ; \tag{2.9}$$

as the whole set of fields now satisfies genuine equations of motion, it is convenient to proceed within the framework of Dirac's canonical quantization [11].

The canonical momenta are $[\mathcal{A}_{\rm rad} \equiv \int d^4x \mathcal{L}_{\rm rad}(x)]$

$$\pi^- \equiv \frac{\delta \mathcal{A}_{\rm rad}}{\delta \partial_+ A_-} = F_{+-}, \tag{2.10a}$$

$$\pi^\alpha \equiv \frac{\delta \mathcal{A}_{\rm rad}}{\delta \partial_+ A_\alpha} = F_{-\alpha}, \tag{2.10b}$$

$$\pi^+ \equiv \frac{\delta \mathcal{A}_{\rm rad}}{\delta \partial_+ A_+} = 0, \tag{2.10c}$$

$$\pi^\Lambda \equiv \frac{\delta \mathcal{A}_{\rm rad}}{\delta \partial_+ \Lambda} = 0, \tag{2.10d}$$

whence it follows that there are two primary second class constraints (2.10b) originating from the use of LCC, as well as two primary first class constraints (2.10c),(2.10d).

The canonical Hamiltonian becomes

$$H_{\rm rad} = \int d^3x \left\{ \frac{1}{2}(\pi^-)^2 + \frac{1}{4} F_{\alpha\beta}F_{\alpha\beta} - A_+(\partial_\alpha \pi^\alpha + \partial_- \pi^- - \Lambda) \right\}, \tag{2.11}$$

and, consequently, from the light-front-temporal consistency of the first class constraints (2.10c),(2.10d) we derive the secondary constraints

$$A_+ = 0, \tag{2.12a}$$

$$\partial_\alpha \pi^\alpha + \partial_- \pi^- - \Lambda = 0. \tag{2.12b}$$

The full set of constraints is now second class and thereby we can compute the Dirac's brackets. After choosing as independent fields the following ones,

$$\phi_1 = A_1, \quad \phi_2 = A_2, \quad \phi_3 = A_-, \quad \phi_4 = \pi^-, \tag{2.13}$$

we eventually obtain the Dirac's brackets matrix

$$\Phi_{ab}(\mathbf{x},\mathbf{y}) \equiv \{\phi_a(x), \phi_b(y)\}_D \big|_{x^+=y^+}, \quad a, b = 1,2,3,4,$$

whose matrix elements are integro-differential operators in terms of light-front-space coordinates $\mathbf{x} = (x^1, x^2, x^-)$: namely,

$$\Phi_{ab}(\mathbf{x},\mathbf{y}) \left| \begin{matrix} -\mathbf{1}/2\partial_- & 0 & 0 & \partial_1/2\partial_- \\ 0 & -\mathbf{1}/2\partial_- & 0 & \partial_2/2\partial_- \\ 0 & 0 & 0 & \mathbf{1} \\ -\partial_1/2\partial_- & -\partial_2/2\partial_- & -\mathbf{1} & \partial_\perp^2/2\partial_- \end{matrix} \right| . \tag{2.14}$$

Here the identity **1** means the product $\delta(x^- - y^-)\delta^{(2)}(x^\perp - y^\perp)$, whilst the kernels $(\mathbf{1}/\partial_-)$ and $(\partial_\alpha/\partial_-)$ are shorthands for $\frac{1}{2}\delta^{(2)}(x^\perp - y^\perp)\mathrm{sgn}(x^- - y^-)$ and $\frac{1}{2}(\partial_\alpha \delta^{(2)})(x^\perp - y^\perp)\mathrm{sgn}(x^- - y^-)$, respectively. It should be noticed that the sign distribution is such to enforce standard (anti-)symmetry properties of Dirac's brackets.

After setting the secondary constraints strongly equal to zero in the Hamiltonian (2.11), we obtain the Dirac's form

$$H_D = \int d^3x \left\{ \frac{1}{2}(\pi^-)^2 + \frac{1}{4} F_{\alpha\beta}F_{\alpha\beta} \right\}. \tag{2.15}$$

Now, in order to simplify the equations of motion, it is convenient to make the change of variables similar to the one of Eqs. (2.3a),(2.3b) but tailored to the present light-cone gauge choice $A_+ = 0$: namely,

$$A_\alpha = T_\alpha - \frac{\partial_\alpha}{\partial_\perp^2} \varphi, \tag{2.16a}$$

$$A_- = \frac{2\partial_-}{\partial_\perp^2} \partial_\alpha T_\alpha - \frac{\partial_-}{\partial_\perp^2} \varphi - \frac{1}{\partial_\perp^2} \Lambda; \tag{2.16b}$$

$$\pi^- = \partial_\alpha T_\alpha . \tag{2.16c}$$

The Dirac's brackets among the new independent fields read

$$\Phi'_{ab}(\mathbf{x},\mathbf{y}) = \left| \begin{matrix} -\mathbf{1}/2\partial_- & 0 & 0 & 0 \\ 0 & -\mathbf{1}/2\partial_- & 0 & 0 \\ 0 & 0 & 0 & \partial_\perp^2 \\ 0 & 0 & -\partial_\perp^2 & 0 \end{matrix} \right| , \tag{2.17}$$

where we have set

[3]Actually, an equivalent way to proceed is to keep the previous light-cone gauge choice unaltered and to change the evolution parameter (the light-front-time) from x^+ to x^-, the key point being that the light-front-time and the light-cone gauge vector have to be parallel.

$$\phi_1' = T_1, \quad \phi_2' = T_2, \quad \phi_3' = \varphi, \quad \phi_4' = \Lambda, \tag{2.18}$$

and the canonical Hamiltonian takes its final Dirac's form

$$H_D' \equiv \int d^3x \left\{ \frac{1}{2} \partial_\beta T_\alpha \partial_\beta T_\alpha \right\}, \tag{2.19}$$

whence we obtain the genuine equations of motion

$$\partial_+ T_\alpha = \frac{\partial_\perp^2}{2\partial_-} T_\alpha, \tag{2.20a}$$

$$\partial_+ \varphi = 0, \tag{2.20b}$$

$$\partial_+ \Lambda = 0. \tag{2.20c}$$

The transition to the quantum theory is accomplished under replacement of the Dirac's brackets with canonical equal light-front-time commutation relations, which read

$$[T_\alpha(x), T_\beta(y)]_{x^+=y^+} = -\frac{i}{4} \delta_{\alpha\beta} \delta^{(2)}(x^\perp - y^\perp) \mathrm{sgn}(x^- - y^-), \tag{2.21a}$$

$$[\varphi(x), \Lambda(y)]_{x^+=y^+} = i\delta(x^- - y^-) \partial_\perp^2 \delta^{(2)}(x^\perp - y^\perp), \tag{2.21b}$$

$$[T_\alpha(x), \varphi(y)] = [T_\alpha(x), \Lambda(y)] = [\varphi(x), \varphi(y)] = [\Lambda(x), \Lambda(y)] = 0. \tag{2.21c}$$

It is important to notice that the above canonical commutation relations (CCR) have the very same form as in the McCartor and Robertson quantization scheme, see Eqs. (2.5a)–(2.5c), up to the crucial difference that now the quantization characteristic surface is the same for all the fields.

Let us now search for the solutions, in the framework of the tempered distributions, of the equations of motion in the Fourier space. To this aim, it is convenient to introduce again the longitudinal (unphysical) components of the radiation field

$$\Gamma_\mu = -\frac{1}{\partial_\perp^2} (\partial_\mu \varphi + n_\mu^* \Lambda), \tag{2.22}$$

in such a way that

$$T_\mu(x) \equiv A_\mu(x) - \Gamma_\mu(x). \tag{2.23}$$

For the transverse components we easily get

$$T_\mu(x) = \int \frac{d^2k_\perp dk_-}{(2\pi)^{3/2}} \frac{\theta(k_-)}{\sqrt{2k_-}} \varepsilon_\mu^{(\alpha)}(k_\perp, k_-) \times \{ a_\alpha(k_\perp, k_-) e^{-ikx} + \alpha_\alpha^\dagger(k_\perp, k_-) e^{ikx} \}_{k_+ = k_\perp^2/2k_-}, \tag{2.24}$$

where the (real) polarization vectors are given by

$$\varepsilon_\mu^{(1)}(k_\perp, k_-) = \left| \begin{matrix} 0 \\ 1 \\ 0 \\ 2k_1 k_- / k_\perp^2 \end{matrix} \right|, \quad \varepsilon_\mu^{(2)}(k_\perp, k_-) = \left| \begin{matrix} 0 \\ 0 \\ 1 \\ 2k_2 k_- / k_\perp^2 \end{matrix} \right|, \tag{2.25}$$

whilst the longitudinal components read

$$\Gamma_\mu(x) = \int \frac{d^2k_\perp dk_-}{(2\pi)^{3/2}} \frac{\theta(k_-)}{\sqrt{k_\perp}} \times \left\{ \left[-\frac{k_\mu}{k_\perp} f(k_\perp, k_-) + n_\mu^* g(k_\perp, k_-) \right] \times e^{-ikx} + \text{H.c.} \right\}_{k_+=0}, \tag{2.26}$$

where $k_\perp \equiv \sqrt{k_1^2 + k_2^2}$. The canonical commutation relations (2.21a)–(2.21c) entail the following algebra of the creation-annihilation operators: namely,

$$[a_\alpha(k_\perp, k_-), a_\beta^\dagger(p_\perp, p_-)] = \delta_{\alpha\beta} \delta^{(2)}(k_\perp - p_\perp) \delta(k_- - p_-), \tag{2.27a}$$

$$[f(k_\perp, k_-), g^\dagger(p_\perp, p_-)] = \delta^{(2)}(k_\perp - p_\perp) \delta(k_- - p_-), \tag{2.27b}$$

$$[g(k_\perp, k_-), f^\dagger(p_\perp, p_-)] = \delta^{(2)}(k_\perp - p_\perp) \delta(k_- - p_-), \tag{2.27c}$$

all the other commutators vanishing.

The canonical commutation relations (2.27b),(2.27c) show that the theory involves an indefinite metric space of states. The physical subspace $\mathcal{V}_{\text{phys}}$, whose metric turns out to be positive semidefinite, is defined through the condition [7]

$$g(k_\perp, k_-)|\mathbf{v}\rangle = 0, \quad \forall |\mathbf{v}\rangle \in \mathcal{V}_{\text{phys}}. \tag{2.28}$$

It should be noted that, as

$$\langle \mathbf{w}|\Lambda(x)|\mathbf{v}\rangle = 0, \quad \forall |\mathbf{w}\rangle, |\mathbf{v}\rangle \in \mathcal{V}_{\text{phys}}, \tag{2.29}$$

the Gauss law is indeed satisfied in $\mathcal{V}_{\text{phys}}$.

Let us finally compute the free vector propagator

$$D_{\mu\nu}^+(x-y) \equiv \theta(x^+ - y^+) \langle 0|A_\mu(x) A_\nu(y)|0\rangle + \theta(y^+ - x^+) \langle 0|A_\nu(y) A_\mu(x)|0\rangle, \tag{2.30}$$

which, after the gauge fixing condition (2.12a), turns out to be properly defined from the mathematical point of view, i.e., the product of the distributions in Eq. (2.30) does indeed exist. Separating the transverse and longitudinal components, setting $a_{\mu\nu}(k) \equiv n^*_\mu k_\nu + n^*_\nu k_\mu$ and going to the momentum space we eventually get

$$\tilde{D}^T_{\mu\nu}(k) = \frac{i}{k^2 + i\epsilon}\left[-g_{\mu\alpha} g^\alpha_\nu + \frac{2k_-}{k_\perp^2}\, a_{\mu\nu}(0, k_\perp, k_-)\right], \tag{2.31}$$

$$\tilde{D}^\Gamma_{\mu\nu}(k) = -i\, \frac{k_-}{k_+ k_- + i\epsilon}\, \frac{a_{\mu\nu}(0, k_\perp, k_-)}{k_\perp^2}. \tag{2.32}$$

Taking into account that

$$\frac{2k_-}{k_\perp^2}\left(\frac{1}{k^2 + i\epsilon} - \frac{1}{2k_- k_+ + i\epsilon}\right) = \frac{1}{k^2 + i\epsilon}\, \frac{1}{[k_+]}, \tag{2.33}$$

where

$$\frac{1}{[k_+]} \equiv \frac{1}{k_+ + i\epsilon\ \mathrm{sgn}(k_-)} \equiv \frac{k_-}{k_- k_+ + i\epsilon}, \tag{2.34}$$

which is nothing but the Mandelstam-Leibbrandt distribution, we finally get the propagator in the momentum space

$$\tilde{D}^+_{\mu\nu}(k) = \frac{i}{k^2 + i\epsilon}\left[-g_{\mu\nu} + \frac{n^*_\mu k_\nu + n^*_\nu k_\mu}{[n^* k]}\right]. \tag{2.35}$$

It has to be stressed that, more than being mathematically well defined, the present form of the free vector propagator exactly coincides with the one obtained in the framework of ordinary time canonical quantization of Ref. [7]. This means that the light-front operator algebra (2.21a)–(2.21c) together with light-front-time propagation are completely equivalent, at the level of the free field theory, to the ordinary time canonical quantization and standard chronological pairing, at variance with the old light-front formulation of Ref. [4]. This nontrivial result, which arises as the correct implementation of the original ideas of Ref. [9], will survive after the switching on of the interaction with spinor matter, as we shall discuss below.

III. LIGHT-FRONT QUANTIZATION OF THE FREE DIRAC FIELD

Before going to the treatment of QED it is useful to briefly review the canonical light-front quantization of the free Dirac field and, in so doing, establish our conventions and notations. First we recall that, in order to obtain the correct canonical anticommutation relations from Dirac's procedure, it is convenient to consider the system at the (pseudo)classical level. This means that we start from spinor fields in terms of Grassmann-valued fields satisfying the graded version of the canonical Poisson's and Dirac's brackets (see, for instance, Ref. [12]). The same formalism will be generalized in the next section, where Bose fields are also included.

Within the framework of the light-front quantization, it is customary to introduce the following representation of the Dirac's matrices: namely,

$$\gamma^+ = \begin{vmatrix} 0 & 0 \\ \sqrt{2}\sigma^1 & 0 \end{vmatrix} \gamma^1 = \begin{vmatrix} -i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{vmatrix}$$

$$\gamma^2 = \begin{vmatrix} i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{vmatrix} \gamma^- = \begin{vmatrix} 0 & \sqrt{2}\sigma^1 \\ 0 & 0 \end{vmatrix}, \tag{3.1}$$

and we write the four-component Dirac's spinor as

$$\Psi \equiv \begin{vmatrix} \psi \\ \chi \end{vmatrix}, \tag{3.2}$$

with ψ, χ two-components complex spinors. Here σ^i, $i = 1,2,3$ are the Pauli's matrices and we also set

$$\tau^1 \equiv \sigma^3, \quad \tau^2 \equiv i\mathbf{1}_2. \tag{3.3}$$

Therefore, the Lagrangian density for the free Dirac's field

$$\mathcal{L}_D = \overline{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi, \tag{3.4}$$

where $\overline{\Psi} \equiv \Psi^\dagger \gamma^0$, $\gamma^0 = 2^{-1/2}(\gamma^+ + \gamma^-)$, may be rewritten as

$$\mathcal{L}_D = \psi^\dagger i\sqrt{2}\partial_+ \psi + \chi^\dagger i\sqrt{2}\partial_- \chi + \psi^\dagger (i\tau^{\alpha\dagger}\partial_\alpha - m\sigma^1)\chi + \chi^\dagger (i\tau^\alpha \partial_\alpha - m\sigma^1)\psi, \tag{3.5}$$

whence the canonical momenta read

$$\pi^\psi = -i\sqrt{2}\psi^\dagger, \tag{3.6a}$$

$$\pi^{\psi^\dagger} = 0, \tag{3.6b}$$

$$\pi^\chi = 0, \tag{3.6c}$$

$$\pi^{\chi^\dagger} = 0. \tag{3.6d}$$

It follows that we have two primary second class constraints (3.6a),(3.6b) and two primary first class constraints (3.6c),(3.6d). The canonical Hamiltonian turns out to be

$$H = \int d^3x \{ -\chi^\dagger i\sqrt{2}\partial_- \chi - \psi^\dagger (i\tau^{\alpha\dagger}\partial_\alpha - m\sigma^1)\chi - \chi^\dagger (i\tau^\alpha\partial_\alpha - m\sigma^1)\psi \} \tag{3.7}$$

and the light-front-temporal consistency of the first class constraints lead to the onset of the secondary constraints

$$i\sqrt{2}\partial_- \chi^\dagger + i\partial_\alpha \psi^\dagger \tau^{\alpha\dagger} + m\psi^\dagger \sigma^1 = 0, \tag{3.8a}$$

$$i\sqrt{2}\partial_- \chi + (i\tau^\alpha \partial_\alpha - m\sigma^1)\psi = 0. \tag{3.8b}$$

Now, the whole set of constraints being second class, the graded Dirac's bracket can be consistently defined and taking ψ and $\psi^\dagger$ as independent fields we readily find

$$\{\psi_r(x),\psi^\dagger_{r'}(y)\}_D|_{x^+=y^+}=\frac{1}{\sqrt{2}}\,\delta_{rr'}\,\delta^{(2)}(x^\perp-y^\perp)\,\delta(x^--y^-),\qquad r,r'=1,2, \tag{3.9}$$

all the other graded Dirac's brackets vanishing.

After solving the secondary constraints (3.8a),(3.8b) in terms of the independent fields ψ, $\psi^\dagger$ the canonical Hamiltonian (3.7) can be cast into Dirac's form: namely,

$$H_D=i\sqrt{2}\int d^3x\left\{\psi^\dagger\,\frac{\partial_\perp^2-m^2}{2\partial_-}\,\psi\right\}, \tag{3.10}$$

from which we obtain the canonical equations of motion

$$\partial_+\psi_r=\frac{\partial_\perp^2-m^2}{2\partial_-}\,\psi_r, \tag{3.11a}$$

$$\partial_+\psi_r^\dagger=\frac{\partial_\perp^2-m^2}{2\partial_-}\,\psi_r^\dagger, \tag{3.11b}$$

showing that the independent fields ψ, $\psi^\dagger$ correctly fulfill the Klein-Gordon equation.

The expansion into normal modes leads to the standard decomposition

$$\psi(x)=\int\frac{d^3k}{(2\pi)^{3/2}}\,\frac{\theta(k_-)}{2^{1/4}}\sum_{s=\pm1/2}\{w^s b_s(k_\perp,k_-)e^{-ikx}+w^{-s}d_s^\dagger(k_\perp,k_-)e^{ikx}\}_{k_+=(k_\perp^2+m^2)/2k_-}, \tag{3.12}$$

and Hermitian conjugate, where the polarization vectors are simply given by

$$w^{s=1/2}\equiv\begin{vmatrix}1\\0\end{vmatrix},\qquad w^{s=-1/2}\equiv\begin{vmatrix}0\\1\end{vmatrix}. \tag{3.13}$$

As it is well known the graded Dirac's brackets (3.9) entail the canonical operator algebra

$$\{b_s(k_\perp,k_-),b^\dagger_{s'}(p_\perp,p_-)\}=\delta_{ss'}\,\delta^{(2)}(k_\perp-p_\perp)\,\delta(k_--p_-), \tag{3.14a}$$

$$\{d_s(k_\perp,k_-),d^\dagger_{s'}(p_\perp,p_-)\}=\delta_{ss'}\,\delta^{(2)}(k_\perp-p_\perp)\,\delta(k_--p_-), \tag{3.14b}$$

all the other anticommutators vanishing.

We are now ready to compute the free light-front fermion propagator which is defined to be

$$iS^+(x-y)\equiv\theta(x^+-y^+)\langle 0|\Psi(x)\overline{\Psi}(y)|0\rangle-\theta(y^+-x^+)\langle 0|\overline{\Psi}(y)\Psi(x)|0\rangle. \tag{3.15}$$

To this aim, it is convenient to introduce the light-front pairing between any two-component spinors $\alpha_r,\beta_{r'}$, $r, r'=1,2$, in such a way that

$$[S^+_{\alpha\beta}(x-y)]_{rr'}\equiv\theta(x^+-y^+)\langle 0|\alpha_r(x)\beta_{r'}(y)|0\rangle-\theta(y^+-x^+)\langle 0|\beta_{r'}(y)\alpha_r(x)|0\rangle; \tag{3.16}$$

then the propagator (3.15) can be cast into a matrix form: namely,

$$iS^+(x-y)=\begin{vmatrix}S^+_{\psi\chi^\dagger}\sigma^1 & S^+_{\psi\psi^\dagger}\sigma^1\\ S^+_{\chi\chi^\dagger}\sigma^1 & S^+_{\chi\psi^\dagger}\sigma^1\end{vmatrix}. \tag{3.17}$$

The only independent light-front pairing turns out to be

$$S^+_{\psi\psi^\dagger}(x-y)=\tau^2\sqrt{2}\partial_-D(x-y;m), \tag{3.18}$$

where

$$D(x-y;m)=\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2+i\epsilon}\,e^{ik(x-y)} \tag{3.19}$$

is the free propagator of the massive real scalar field. Then, from the constraints (3.8a),(3.8b) we eventually obtain

$$S^+_{\psi\chi^\dagger}(x-y)=(i\tau^{\alpha\dagger}\partial_\alpha+m\sigma^1)D(x-y;m), \tag{3.20a}$$

$$S^+_{\chi\psi^\dagger}(x-y)=(i\tau^{\alpha}\partial_\alpha+m\sigma^1)D(x-y;m), \tag{3.20b}$$

$$S^+_{\chi\chi^\dagger}(x-y)=\tau^2\sqrt{2}\,\frac{\partial_\perp^2-m^2}{2\partial_-}\,D(x-y;m)=\tau^2\sqrt{2}\partial_+D(x-y;m)+i\tau^2\,\frac{1}{\sqrt{2}\partial_-}\,\delta^{(4)}(x-y). \tag{3.20c}$$

As a consequence, from Eq. (3.17) and taking Eq. (3.1) into account, the free fermion light-front propagator can be written in the form

$$iS^+(x-y)=(i\gamma^\mu\partial_\mu+m)D(x-y;m)-\frac{\gamma^+}{2\partial_-}\,\delta^{(4)}(x-y), \tag{3.21}$$

where the first term in the RHS is the usual covariant fermion propagator

$$S^{\rm cov}(x-y)=\int\frac{d^4k}{(2\pi)^4}\,\frac{m-\gamma^\mu k_\mu}{k^2-m^2+i\epsilon}\,e^{ik(x-y)}, \tag{3.22}$$

whilst the second one is the so called ''instantaneous'' or ''contact'' term, which is generated by the propagation along the light-cone generating lines. The role of those term will be further elucidated in the next sections; in particular, it will be clear that there is no need to specify any prescription to define the light-front-space antiderivative ∂_-^{-1} which appears in Eq. (3.21).

IV. LIGHT-FRONT QED IN THE LIGHT-CONE TEMPORAL GAUGE

We are now ready to discuss the main subject, i.e., the perturbative light-front formulation of spinor QED, in which the LCC x^+ plays the role of evolution parameter, within the light-cone gauge choice $A_+=0$. Owing to this pattern (the controvariant LCC x^+ just corresponds to the covariant component A_+ of the Abelian vector potential), this formulation will be naturally referred to as light-front QED in the light-cone temporal gauge.

The starting point is obviously the Lagrangian density

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\Lambda A_+ +\overline{\Psi}(i\gamma^\mu\partial_\mu-m)\Psi+eA_\mu\overline{\Psi}\gamma^\mu\Psi, \tag{4.1}$$

which can be rewritten, using the notations of the previous section, in the form

$$\begin{aligned}\mathcal{L}=&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\Lambda A_+ +\psi^\dagger i\sqrt{2}\partial_+\psi+\chi^\dagger i\sqrt{2}\partial_-\chi\\&+\psi^\dagger(i\tau^{\alpha\dagger}\partial_\alpha-m\sigma^1)\chi+\chi^\dagger(i\tau^\alpha\partial_\alpha-m\sigma^1)\psi\\&+eA_+\sqrt{2}\psi^\dagger\psi\\&+eA_\alpha(\psi^\dagger\tau^{\alpha\dagger}\chi+\chi^\dagger\tau^\alpha\psi)+eA_-\sqrt{2}\chi^\dagger\chi.\end{aligned}\tag{4.2}$$

As the interaction does not contain derivative couplings, the definitions of the canonical momenta do not change with respect to the free case: then we have

$$\pi^-=F_{+-}, \tag{4.3a}$$

$$\pi^\alpha=F_{-\alpha}, \tag{4.3b}$$

$$\pi^+=0, \tag{4.3c}$$

$$\pi^\Lambda=0, \tag{4.3d}$$

$$\pi^\psi=-i\sqrt{2}\psi^\dagger, \tag{4.3e}$$

$$\pi^{\psi^\dagger}=0, \tag{4.3f}$$

$$\pi^\chi=0, \tag{4.3g}$$

$$\pi^{\chi^\dagger}=0, \tag{4.3h}$$

where, again, Eqs. (4.3b),(4.3e),(4.3f) are primary second class constraints whilst the remaining ones, but Eq. (4.3a), are primary first class. The canonical Hamiltonian reads

$$\begin{aligned}H=\int d^3x\Big\{&\frac{1}{2}(\pi^-)^2+\frac{1}{4}F_{\alpha\beta}F_{\alpha\beta}\\&-A_+(\partial_\alpha\pi^\alpha+\partial_-\pi^--\Lambda)-\chi^\dagger i\sqrt{2}\partial_-\chi\\&-\psi^\dagger(i\tau^{\alpha\dagger}\partial_\alpha-m\sigma^1)\chi-\chi^\dagger(i\tau^\alpha\partial_\alpha-m\sigma^1)\psi\\&-eA_+\sqrt{2}\psi^\dagger\psi-eA_\alpha(\psi^\dagger\tau^{\alpha\dagger}\chi+\chi^\dagger\tau^\alpha\psi)\\&+eA_-\sqrt{2}\chi^\dagger\chi\Big\}\end{aligned}\tag{4.4}$$

and from the light-front temporal consistency of the primary first class constraints the following secondary constraints arise: namely,

$$A_+=0, \tag{4.5a}$$

$$\partial_\alpha\pi^\alpha+\partial_-\pi^--\Lambda-e\sqrt{2}\psi^\dagger\psi=0, \tag{4.5b}$$

$$i\sqrt{2}D_-^*\chi^\dagger+iD_\alpha^*\psi^\dagger\tau^{\alpha\dagger}+m\psi^\dagger\sigma^1=0, \tag{4.5c}$$

$$i\sqrt{2}D_-\chi+(i\tau^\alpha D_\alpha-m\sigma^1)\psi=0, \tag{4.5d}$$

where, as usual, we have set $D_\mu\equiv=\partial_\mu-ieA_\mu$.

The whole set of primary and secondary constraints is now second class and we can proceed to the calculation of graded Dirac's brackets. To this aim, however, it is better to make a preliminary observation. From the constraint equations (4.5c), (4.5d) it is apparent that, if we want to express the two-components spinors χ and $\chi^\dagger$ as functionals of the independent ones ψ and $\psi^\dagger$, we have to invert the differential operator $D_-=\partial_--ieA_-$. In the present context the corresponding Green's function will be understood as a formal series: namely,

$$\frac{1}{D_-}\equiv\frac{1}{\partial_-}\sum_{n=0}^{\infty}\left(ieA_-\frac{1}{\partial_-}\right)^n, \tag{4.6}$$

where each antiderivative acts upon all the factors on its right.

As it will be clear later on, we remark that it is neither necessary nor convenient to specify any kind of prescription, in order to properly define the antiderivative itself. Furthermore, it is unavoidable that the Dirac's Hamiltonian, in which all the constraints are solved in terms of the independent fields, would result into a formal (infinite) power series of the dimensionless electric charge *e*.

Let us turn now to the calculation of the graded Dirac's brackets. As the actual inversion of the constraints matrix is a little bit complicated in the present case, it is convenient to operate iteratively and compute some sequences of preliminary brackets (eventually four sequences). After taking

$$\begin{aligned}&\xi_1\equiv A_1,\quad \xi_2\equiv A_2,\quad \xi_3\equiv A_-,\\&\xi_4\equiv\pi^-,\quad \xi_5\equiv\psi,\quad \xi_6\equiv\psi^\dagger,\end{aligned}\tag{4.7}$$

as independent fields, a straightforward although very tedious calculation leads to the following result: namely,

$$\Xi_{ab}(\mathbf{x},\mathbf{y})=\begin{vmatrix} -\mathbf{1}/2\partial_- & 0 & 0 & \partial_1/2\partial_- & 0 & 0 \\ 0 & -\mathbf{1}/2\partial_- & 0 & \partial_2/2\partial_- & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\ -\partial_1/2\partial_- & -\partial_2/2\partial_- & -\mathbf{1} & \partial_\perp^2/2\partial_- & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i/\sqrt{2} \\ 0 & 0 & 0 & 0 & -i/\sqrt{2} & 0 \end{vmatrix}, \tag{4.8}$$

where, once again, we have denoted the Dirac's brackets matrix as

$$\Xi_{ab}(\mathbf{x},\mathbf{y})\equiv\{\xi_a(x),\xi_b(y)\}_D|_{x^+=y^+},\quad a,b=1,\ldots,6.$$

It is important to realize that the set of the independent interacting fields $\xi_a(x)$, $a=1,\ldots,6$, do obey the very same algebra as the corresponding independent free fields, notwithstanding the fact that the secondary constraints are quite different in the two cases. This feature, as we shall see in the sequel, is of crucial importance in setting up the perturbation theory. Moreover, it has to be gathered that the above property does not hold in general for an arbitrary constrained system, but it depends, in the present case, upon a clever choice of the independent fields.

Finally, after solving the secondary constraints in terms of the independent fields $\xi_a(x)$, $a=1,\ldots,6$, the Hamiltonian (4.4) takes its Dirac's form which becomes

$$H_D=\int d^3x\left\{\frac{1}{2}(\pi^-)^2+\frac{1}{4}F_{\alpha\beta}F_{\alpha\beta} -(i\partial_\alpha\psi^\dagger\tau^{\alpha\dagger}+m\psi^\dagger\sigma^1-eA_\alpha\psi^\dagger\tau^{\alpha\dagger})\frac{1}{i\sqrt{2}D_-} \times(i\tau^\alpha\partial_\alpha\psi-m\sigma^1\psi+eA_\alpha\tau^\alpha\psi)\right\}, \tag{4.9}$$

which is the starting point to develop perturbation theory as we discuss in the next section.

V. PERTURBATION THEORY

In order to separate the interaction Hamiltonian in a constrained system, one has to be very careful in the choice of the independent canonical variables: as a matter of fact, the basic criterion to select the latter ones is eventually dictated by the structure of the Dirac's brackets of the interacting theory.

On the one hand, after choosing $\xi_a(x)$, $a=1,\ldots,6$ as independent fields, we see that the first line of the RHS of Eq. (4.9) does not contain the coupling constant e and, consequently, does not contribute to the interaction Hamiltonian. On the other hand, had we chosen as independent fields the set A_1, A_2, A_-, Λ, ψ, $\psi^\dagger$, which is a perfectly legitimate choice, then, after solving π^- as a functional of the above variables, we find that the first line in the RHS of Eq. (4.9) does indeed contribute to the interaction Hamiltonian through the two terms:

$$-e\sqrt{2}\left\{-\partial_\alpha A_\alpha+\frac{1}{\partial_-}(\partial_\perp^2A_-+\Lambda)\right\}\frac{1}{\partial_-}(\psi^\dagger\psi) +e^2\left\{\frac{1}{\partial_-}(\psi^\dagger\psi)\right\}^2,$$

whence, thereby, a quite different kind of perturbation theory does follow.

In view of the above remark, one could be eventually led to the conclusion that perturbation theory for constrained systems is not univocally determined, owing to the fact that it depends upon the specific choice of the independent fields, in terms of which the constraints are solved. Actually, this apparent ambiguity is not there. As a matter of fact, we recall that perturbation theory stems from the assumption of the existence, at least formally, of the so called evolution operator, which implements the time-dependent unitary transformation relating the interacting to the free fields—see, for instance, [13].

On the other hand, we know that a unitary operator is such to preserve the canonical equal time field algebra. This means that, in the case of constrained systems, the suitable independent interacting fields must satisfy the very same equal time operator algebra as the corresponding free fields do. In terms of those, and only those, independent interacting fields the interaction Hamiltonian has to be expressed and perturbation theory will be safely and consistently developed.

From the constraints (4.3b),(4.5b) and the Dirac's brackets (4.8), it is an easy exercise to show that

$$\{\Lambda(x),\psi(y)\}_D|_{x^+=y^+}=ie\psi(x)\delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{5.1}$$

whereas, in the free field case, the corresponding Dirac's bracket vanishes. As a consequence, the construction of the interaction Hamiltonian as a functional of the fields A_1, A_2, A_-, Λ, ψ, $\psi^\dagger$ does not make sense in order to set up perturbation theory. The interaction Hamiltonian is expressed in terms of the set of independent fields $\xi_a(x)$, $a=1,\ldots,6$, whose Dirac's brackets (4.8) do not depend upon the electric charge e, what makes it now clear why the above algebra (4.8) has been precisely put forward.

We now consider the second line of the Hamiltonian (4.9). As all the field operators in the interaction picture evolve according to free equations of motion, it is convenient to replace with χ and $\chi^\dagger$ those linear combinations of the fields ψ and $\psi^\dagger$, which coincide with the solutions of the free constraint equations (3.8a),(3.8b). After this, we can rewrite the Hamiltonian (4.9) in the form

$$H_D=\int d^3x\left\{\frac{1}{2}(\pi^-)^2+\frac{1}{4}F_{\alpha\beta}F_{\alpha\beta}+(i\sqrt{2}\partial_-\chi^\dagger+eA_\alpha\psi^\dagger\tau^{\alpha\dagger})\frac{1}{i\sqrt{2}D_-}\times(-i\sqrt{2}\partial_-\chi+eA_\alpha\tau^\alpha\psi)\right\}. \tag{5.2}$$

If we now perform, within the second line of the above equation, the following replacements: namely,

$$\chi\mapsto\frac{1}{2}\gamma^+\gamma^-\Psi, \tag{5.3a}$$

$$\tau^\alpha\psi\mapsto\frac{1}{\sqrt{2}}\gamma^+\gamma^\alpha\Psi, \tag{5.3b}$$

$$\chi^\dagger\mapsto\frac{1}{\sqrt{2}}\overline{\Psi}\gamma^-, \tag{5.3c}$$

$$\psi^\dagger\tau^{\alpha\dagger}\mapsto\frac{1}{2}\overline{\Psi}\gamma^\alpha\gamma^+\gamma^-, \tag{5.3d}$$

we eventually obtain

$$H_D=\int d^3x\left\{\frac{1}{2}(\pi^-)^2+\frac{1}{4}F_{\alpha\beta}F_{\alpha\beta}+\overline{\Psi}\gamma^-i\partial_-\Psi-eA_\mu\overline{\Psi}\gamma^\mu\Psi+\frac{e^2}{2}A_\mu\overline{\Psi}\gamma^\mu\frac{1}{iD_-}A_\nu\gamma^\nu\Psi\right\}. \tag{5.4}$$

It is evident, from the above final form of the Dirac's Hamiltonian, that the interaction Hamiltonian density, upon which perturbation theory is set, reads

$$\mathcal{H}_{\rm int}=-eA_\mu\overline{\Psi}\gamma^\mu\Psi+\frac{e^2}{2}A_\mu\overline{\Psi}\gamma^\mu\frac{\gamma^+}{iD_-}A_\nu\gamma^\nu\Psi. \tag{5.5}$$

It is now apparent that, besides the usual covariant vertex of QED, we have to consider, taking the formal definition (4.6) into account, an infinite number of noncovariant vertices. On the other hand, we have seen that also the free Dirac's propagator (3.21) exhibits a noncovariant term besides the usual one. What happens, as we shall here explicitly show up to the one loop order, is that in dimensionally regularized truncated Green's functions all those noncovariant terms cancel, leaving us with the very same renormalizable one loop structures, as found in the standard STC framework [8].

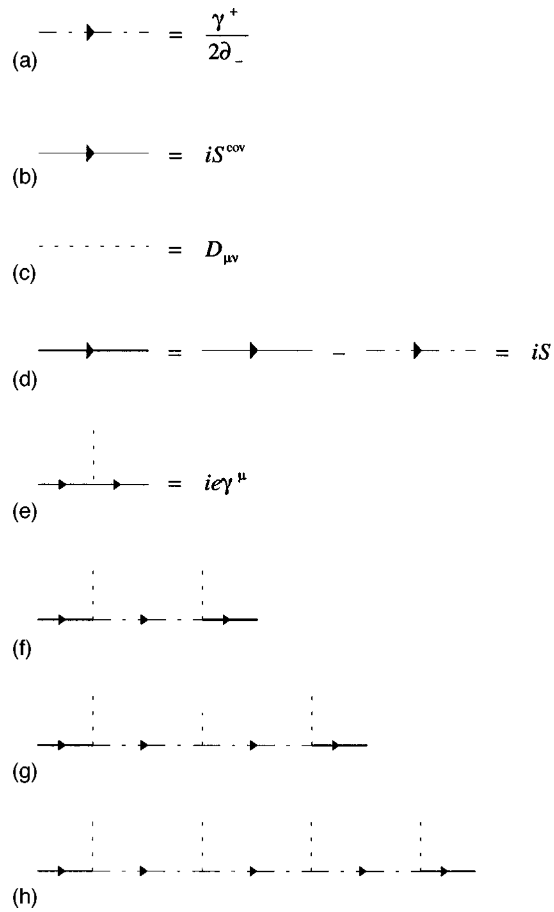

FIG. 2. Feynman's rules.

To this aim, let us first obtain the Feynman's rules. From the definition (4.6) together with the identity

$$\frac{\gamma^+}{2}A_-=\frac{\gamma^+}{2}A_\nu\gamma^\nu\frac{\gamma^+}{2}, \tag{5.6}$$

we can formally expand the interaction Hamiltonian density as

$$i\mathcal{H}_{\rm int}=ieA_\mu\overline{\Psi}\gamma^\mu\Psi-ieA_\mu\overline{\Psi}\gamma^\mu\frac{\gamma^+}{2\partial_-}ieA_\nu\gamma^\nu\Psi-ieA_\mu\overline{\Psi}\gamma^\mu\frac{\gamma^+}{2\partial_-}ieA_\rho\gamma^\rho\frac{\gamma^+}{2\partial_-}\times ieA_\nu\gamma^\nu\Psi-ieA_\mu\overline{\Psi}\gamma^\mu\frac{\gamma^+}{2\partial_-}ieA_\rho\gamma^\rho\frac{\gamma^+}{2\partial_-}\times ieA_\sigma\gamma^\sigma\frac{\gamma^+}{2\partial_-}ieA_\nu\gamma^\nu\Psi+\cdots, \tag{5.7}$$

where the antiderivatives (integral operators) act upon all the factors on their right.

From Eqs. (2.35), (3.21) and (5.7), we get the Feynman's rules listed in Fig. 2.

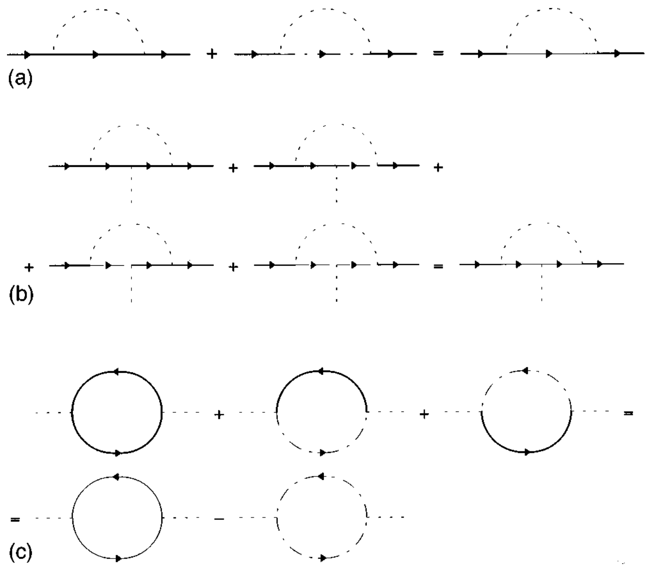

FIG. 3. (a) One loop electron self-energy. (b) One loop electron-positron-photon vertex. (c) One loop photon self-energy.

Using these rules, it is not difficult to check graphically that in the one loop truncated Green's functions, but photon self-energy diagram, all the noncovariant terms cancel algebraically.

For instance, taking two covariant vertices [Fig. 2(e)] and a second order noncovariant one [Fig. 2(f)], we reconstruct the full one loop electron self-energy [see Fig. 3(a)], which, after the removal of the external legs, turns out to be the correct renormalizable one of the standard STC approach.

Moreover, the one loop renormalizable electron-positron-photon proper vertex can be reconstructed [see Fig. 3(b)] taking the covariant vertices of Fig. 2(e) as well as first and second order noncovariant vertices of Figs. 2(f) and 2(g) into account.

Let us come now to the photon one loop self-energy of Fig. 3(c).

After summation of the relevant vertices, we see that, beside the correct standard diagram, a further noncovariant graph is there, whose corresponding integral (in 2ω space-time dimensions) is provided by

$$I^{\rho\sigma}(p_-)=(ie)^2\int \frac{d^{2\omega}l}{(2\pi)^{2\omega}}\mathrm{Tr}\left\{\gamma^\rho\,\frac{\gamma^+}{2i(l_-+p_-)}\,\gamma^\sigma\,\frac{\gamma^+}{2il_-}\right\}. \tag{5.8}$$

However, since

$$\int d^{2\omega}l=\int dl_+\int dl_-\int d^{2\omega-2}l_\perp\,,$$

we immediately see that integration over transverse momenta in Eq. (5.8) gives a vanishing result. This is the only point, up to the one loop approximation, in which the cancellation of noncovariant vertices does not take place algebraically but involves a further analytic tool. Owing to the above cancellation mechanisms, either algebraic or due to dimensional regularization of integrals over transverse momenta, it becomes clear why it is immaterial to specify any prescription to understand noncovariant denominators in fermions propagators as well as in the interaction vertices, at least in perturbation theory.

To sum up, we have shown that, concerning one loop dimensionally regularized truncated Green's functions, the light-front formulation of QED in the light-cone temporal gauge actually reproduces the very same result as in the standard STC renormalizable and (perturbatively) unitary approach [8], in which noncovariant singularities are regulated by means of the ML prescription.

VI. CONCLUSION

A consistent light-front formulation of perturbative QED has been worked out in the light-cone gauge $A_+=0$, in which the LCC x^+ plays the role of the evolution parameter. Owing to this, it is natural, by analogy with the ordinary STC formulation, to refer our choice as to ''temporal'' light-cone gauge, alternative to the original ''axial'' choice $A_-=0$. By consistent, we understand that the quantization scheme here developed reproduces, at least up to the one loop order, the same off-shell amplitudes as computed from the conventional correct approach in usual STC [7,8], which embodies the ML prescription to define the spurious noncovariant singularities of the free photon propagator.

This result is nontrivial and, in turn, also rather surprising. As a matter of fact, it has been thoroughly unravelled [14] that in the quantization of gauge theories in ordinary STC, the use of the temporal (or Weyl) subsidiary condition $A_0=0$ is undoubtedly much more troublesome than the axial one $A_3=0$, which is in turn also affected by subtle mathematical pathologies [15]. Eventually, in spite of the huge number of attempts and efforts, the problem of setting up a fully consistent perturbation theory in the temporal gauge is still to be solved.

On the contrary, within the light-front perturbative formulation of QED, the ''temporal'' gauge choice $A_+=0$ appears to be the safe one, which naturally leads to the ML prescription and thereby to the equivalence with the convention approach in STC, whilst the ''spatial'' choice $A_-=0$ drives to inconsistency [5].

A further comment is deserved to the gauge invariace of the regularization methods in perturbation theory. It clearly appears that, in the present context, the use of dimensional regularization is crucial, in order to provide an infinite set of diagrams cancellation, in the absence of which gauge invariance of QED would be lost. Things are not so lucky for cut-off or Pauli-Villars regularizations, which, thereof, turn out to be quite inconvenient within the perturbative light-front approach.

It should be noticed that the presence of an infinite number of noncovariant vertices, switching on order-by-order in light-front perturbation theory, closely figures the structure of counterterms for the 1PI vertices in the standard STC approach to the light-cone gauges [8]. This feature is probably connected to the specific properties of the ML propagator, i.e., to the kind of structures it generates after loop integrations.

Although graphically transparent, a formal general proof—which is basically by induction—that the cancellation mechanism for noncovariant terms persists, to all order in perturbation theory, will be presented in a forthcoming paper, together with the generalization of the present treatment to the non-Abelian case.

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