Evaluation of glueball masses from supergravity

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In the framework of the conjectured duality relation between large N gauge theory and supergravity the spectra of masses in large N gauge theory can be determined by solving certain eigenvalue problems in supergravity. In this paper we study the eigenmass problem given by Witten as a possible approximation for masses in QCD without supersymmetry. We place a particular emphasis on the treatment of the horizon and related boundary conditions. We construct exact expressions for the analytic expansions of the wave functions both at the horizon and at infinity and show that requiring smoothness at the horizon and normalizability gives a well defined eigenvalue problem. We show, for example, that there are no smooth solutions with a vanishing derivative at the horizon. The mass eigenvalues up to $m^2=1000$ corresponding to smooth normalizable wave functions are presented. We comment on the relation of our work with the results found in a recent paper by C. Csáki *et al.*, hep-th/9806021, which addresses the same problem. [S0556-2821(98)09620-9]

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I. INTRODUCTION

The problem of solving QCD in the nonperturbative large N limit has been outstanding for several decades [1,2]. It has been suspected that the answer to this question will come from string theory. Recently a very interesting proposal [4] has been introduced and further explored [6–8] which involves a relationship between large N super Yang-Mills theory and anti–de Sitter space (AdS) supergravity [5–25]. This correspondence which was first investigated in studies of 3-branes gives the possibility of studying large N properties of Yang-Mills theories using classical supergravity [3]. The later is expected to give results that should be valid for the strongly coupled gauge theory. At the present time comparison of the two theories has been done for operators and correlators protected by supersymmetry [13–21]. Other quantities such as the entropy or Wilson loops represent predictions of the conjecture [4–25]. For general systems involving p branes a notion of generalized conformal symmetry was found in [23]. One can expect that a similar correspondence holds also in theories without supersymmetry and ultimately in QCD. Witten has presented such an extension where properties of finite temperature Yang-Mills theories are to be computed using AdS black hole backgrounds in gravity [8].

According to Witten's generalization of the conjecture by Maldacena in [4], in order to study $\mathcal{N}=4$ super Yang-Mills theory at large N, high temperature and strong 't Hooft coupling, one should consider the Euclidean Schwarzchild black hole solution in $AdS_5\times S^5$ space-time in the limit where the black hole mass is large [8]. In this limit the metric can be written as

$$ds^2=\left(\frac{r^2}{b^2}-\frac{b^2}{r^2}\right)d\tau^2+\frac{dr^2}{(r^2/b^2-b^2/r^2)}+r^2\sum_{i=1}^{3}dx_i^2+b^2d\Omega_5^2, \tag{1}$$

where $d\Omega_5^2$ is the round metric on S^5, $r=b$ is the horizon radius and the coordinate τ is the Euclideanized periodic time coordinate. This metric is obtained as a solution to the type IIB supergravity equations of motion following from the $\gamma\to 0$ limit of the action

$$S=-\frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{g}\left[R-\frac{1}{2}(\partial\phi)^2+\cdots+\gamma\exp\left(-\frac{3}{2}\phi\right)W+\cdots\right], \tag{2}$$

where the order $\gamma=1/8\zeta(3)\alpha'^3$ terms contain the first string corrections to supergravity and where W is a certain combination of terms quartic in the Weyl tensor [24,25]. The inclusion of stringy α' corrections corresponds to including strong coupling expansion corrections in the gauge theory [4,7,8].

In this paper we would like to study the proposal in [8] for the supergravity calculation of the mass gap in QCD. In the next section we will examine the equations of motion for free scalar field propagation on AdS black hole backgrounds by rewriting them in the form of a Hamiltonian problem. We will then address the problem of the behavior of the wave function at the horizon. In Sec. III we present our exact results for the wave functions and show that there are no normalizable smooth solutions with vanishing derivative at the

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horizon. Using the exact form of the solutions we then exhibit the glueball mass eigenvalues predicted by supergravity. Finally, in Sec. IV we close with some conclusions.

II. BLACK HOLE BACKGROUNDS

According to Witten the equations for a free field propagation, $\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu\eta)=0$ for a scalar field, in the five-dimensional space-time described by the first terms of Eq. (1) with τ compactified on S^1 should give glueball masses for three-dimensional QCD (QCD_3). Similarly a computation in an AdS_7 black hole background is expected to be of relevance for glueball masses in QCD_4 [8]. One should look for solutions behaving like (static) plane waves along the x_i directions $\phi\sim\eta(r)e^{ik\cdot x}$ and then demanding normalizability and regularity of the behavior of $\eta(r)$ at $r=b$ and $r=\infty$ will select only certain allowed discrete values for $m^2=-k^2$. These values of m^2 are then interpreted as particle masses in the three-dimensional world parametrized by the x_i. To study the corrections to these masses in the strong coupling expansion one then should work with the $O(\gamma)$ corrections to the background metric (1) and to the dilaton field. To $O(\gamma)$ and for the purpose of computing mass corrections, it is consistent to take a classical solution with a vanishing dilaton field. The $O(\gamma)$ correction to the metric was found in [25] and one uses it to compute the $O(\gamma)$ corrections to the glueball masses.

Consider the general metric for the AdS_{n+1} black hole in the large mass limit [8]

$$ds^2=\left(\frac{r^2}{b^2}-\frac{b^{n-2}}{r^{n-2}}\right)d\tau^2+\frac{dr^2}{(r^2/b^2-b^{n-2}/r^{n-2})}+r^2\sum_{i=1}^{n-1}dx_i^2. \tag{3}$$

The equation of motion for a free scalar field of the form $\phi\sim\eta(r)e^{ik\cdot x}$ is then given by

$$\partial_r\left(r^{n-1}\left(\frac{r^2}{b^2}-\frac{b^{n-2}}{r^{n-2}}\right)\partial_r\eta\right)+r^{n-3}m^2\eta=0, \tag{4}$$

where $m^2=-k^2$ is the $(n-1)$-dimensional mass. Consider the measure coming from the metric (3) above (we set $b=1$ for the remainder of this section)

$$\langle\eta|\eta\rangle=\int_1^\infty dr r^{n-1}\eta(r)\eta^*(r). \tag{5}$$

In order to trivialize the measure we can take a new variable $y=r^{n/2}$ for which with $\Phi(y)=y^{1/2}\eta(y)$ the equation becomes

$$\partial_y((y^2-1)\partial_y\Phi)+\left[-\tfrac{1}{4}(3+y^{-2})+\frac{4m^2}{n^2}y^{-4/n}\right]\Phi=0. \tag{6}$$

Integrating Eq. (6) from the horizon to ∞ against $\Phi^*(y)$ and integrating by parts assuming normalizability and smoothness we obtain a bound on the possible values of m^2. For example, for $n=4$ we find that $m^2>4$. To eliminate the first derivative term in Eq. (4) and to write that equation in terms of a Hamiltonian problem we now take $y=r^{n/2}=\cosh(w)$ and redefine $A(w)=\sinh(2w)^{1/2}\eta(w)$. This gives

$$\tfrac{1}{2}\partial_w^2A(w)-V(w)A(w)=0, \tag{7}$$

where the potential is now given by

$$2V(w)=1-\sinh(2w)^{-2}-\frac{4}{n^2}m^2\cosh(w)^{-4/n}. \tag{8}$$

We are interested in the wave function $A(w)$ for the zero eigenvalue of Eq. (7). If we expand the potential around the horizon $w=0$ we obtain $V(w)=-1/(8w^2)+(2/3-(2m^2)/n^2)+((4m^2)/n^3-2/15)w^2+O(w^4)$. The harmonic oscillator perturbed by a potential of the form $\lambda(1/w^2)$ was examined in [26]. Our potential corresponds precisely to the limiting case $\lambda=-1/8$ in that reference beyond which the Hamiltonian is not bounded below. The indicial equation for Eq. (7) with the potential expanded about the horizon $w=0$, will have a double root $\frac{1}{2}$. Therefore, near the horizon we will have the behaviors $A(w)\sim w^{1/2}$ and $A(w)\sim w^{1/2}\ln(w)$ for the two independent solutions of Eq. (7). Both solutions are normalizable near $w=0$ and we also have a normalizable density of probability currents at the horizon of the form $J(w)\sim A(w)\partial_wA(w)\sim$ const or $\sim\ln^2(w)$. The two solutions give wave functions $\eta(r)$ for Eq. (4) which behave near the horizon like $\eta(w)\sim$ const or $\eta(w)\sim\ln(w)$. The first derivatives then become $d\eta/dr\sim$ const or $d\eta/dr\sim$ const$/w$. We therefore expect that the Neumann boundary condition may never be attained at the horizon for a regular solution. Indeed we note that we have a potential which is singular at the horizon and that it could be expected that it is not possible to demand Dirichlet or Neumann boundary conditions there and as we will see this is what happens in our case. It would be interesting from this general point of view to understand if possible tunneling effects could contribute in a small amount to the values of the eigenmasses. Our solutions of Eq. (4) which we will present in the next section are consistent with the above behavior.

To formulate the eigenvalue problem, one fixes the behavior at ∞ such that the solution is normalizable. Then demanding regularity of the solution at the horizon determines a discrete set of masses. The equations that describe the wavefunctions corresponding to motion in the AdS black hole backgrounds have regular singular points at 0,1,horizon and ∞ and also at other points according to the value of n. In view of the discussion above, it might be tempting to ask for solutions which are regular at the origin (instead of the horizon) and which decay well enough at ∞, and hope that this would define an interesting eigenmass problem. However, a closer look at Eq. (4) shows that the eigenvalues m^2 even if they exist are not guaranteed to be positive in that situation.

III. CALCULATIONS AND RESULTS

To leading order in γ the equation of motion for the quadratic fluctuation η_0 of the dilaton field is [8]

$$\partial_r(r(r^4-b^4)\partial_r\eta_0)+m_0^2b^2r\eta_0=0, \tag{9}$$

where one takes $b<r<\infty$ and where m_0 is the leading contribution to the mass in the strong coupling expansion. The eigenvalues m_0^2 will provide the masses of the scalar glueball O^{++} states. Considering first the behavior of the solution at infinity it is useful to rewrite the equation in the variable $z=b/r$ with $0<z<1$:

$$\frac{d}{dz}\left(z\left(\frac{1}{z^4}-1\right)\frac{d\eta_0}{dz}\right)+\frac{1}{z^3}\frac{m_0^2}{b^2}\eta_0=0. \tag{10}$$

One wants to find normalizable wave function solutions of Eq. (9) and this fixes the behavior at ∞ to be like $\eta_0 \sim 1/r^4$. This $1/r^4$ behavior at ∞ provides us with a Taylor expansion for η_0 around $z=0$ of the form $\eta_0=\Sigma_{n=2}^{\infty}c_nz^{2n}$ where to fix the overall normalization of η_0 we take $c_1\equiv 0$, $c_2=1$ and then obtain the recursion relation for $n\geqslant 2$:

$$c_{n+1}=-\frac{c_n(m_0^2/b^2)-c_{n-1}(2(n-1)(2n-3)+2(n-1))}{(2n+2)(2n+1)-6(n+1)}. \tag{11}$$

We next concentrate on the behavior of the solutions of this equation near the singularity at the horizon. We will first find an expression for the analytic solution at the horizon and use it to reduce the order of the equation and show that the other independent solution is not smooth at the horizon. In order to better describe the vicinity of the horizon let us use the variable $\zeta=b^2/r^2-1=1/z^2-1$ such that the horizon is[1] at $\zeta=0$. The equation becomes

$$\frac{d^2\eta_0}{d\zeta^2}+\left(-\frac{1}{\zeta+1}+\frac{1}{\zeta+2}+\frac{1}{\zeta}\right)\frac{d\eta_0}{d\zeta} -\frac{m_0^2}{8b^2}\left(-\frac{2}{\zeta+1}+\frac{1}{\zeta+2}+\frac{1}{\zeta}\right)\eta_0=0, \tag{12}$$

where we can expand the fractional coefficients in powers of ζ and where we take a power series ansatz $\eta_0(\zeta)=\Sigma_{n=0}^{\infty}b_n\zeta^n$. We obtain that the first coefficient b_0 is free, $b_1=(m_0^2/8b^2)b_0$ and that the other coefficients can be determined in terms of b_0 from the recursion relation

$$(n+2)^2b_{n+2}=\frac{m_0^2}{8b^2}b_{n+1}-\sum_{k=0}^{n}(k+1)(-1)^{n-k} \times\left(\frac{1}{2^{n-k+1}}-1\right)b_{k+1} +\frac{m_0^2}{8}\sum_{k=0}^{n}(-1)^{n-k}b_k\left(\frac{1}{2^{n-k+1}}-2\right). \tag{13}$$

At the horizon this solution goes to a constant b_0 and the first derivative $d\eta_0/dr=(-2/b)d\eta_0/d\zeta=(-2/b)b_1=(-m_0^2/4b^3)b_0\neq 0$. We will now use this solution to reduce the order of the equation and find a second linearly independent solution. Let ψ be the analytic solution defined by the recursion relation (13) and set $\eta_0=\psi\cdot g$. Inserting into Eq. (12) we obtain a first order equation for the ζ derivative of g:

$$\frac{d^2g}{d\zeta^2}\left(\frac{dg}{d\zeta}\right)^{-1}+2\frac{d\psi}{d\zeta}\psi^{-1}+\left(-\frac{1}{\zeta+1}+\frac{1}{\zeta+2}+\frac{1}{\zeta}\right)=0, \tag{14}$$

implying that $dg/d\zeta=\text{const}(\zeta+1)/(\zeta(\zeta+2)\psi^2)$. Therefore the second solution to Eq. (12) has a first derivative which blows up at the horizon and is not smooth there. Consequently, there are no smooth solutions with a vanishing first derivative at the horizon. This behavior in the region close to the horizon is consistent with the results of Sec. II where we have seen that the problem reduces to a Schrodinger problem for the harmonic oscillator perturbed by a potential of the form $V(x)=(-1/8)1/x^2$.

We would now like to use the exact form for the series solution at the horizon and try to fix the overall normalization by choosing the coefficient b_0 in such a way that this solution matches with the Taylor expansion (11) obtained by expanding at spatial infinity $z=0$. Of course, such a matching of the two Taylor expansions over an interval will be possible only for certain values of m_0^2 and these are the values for the masses. We have used these exact expressions for the analytic expansions of the wave functions and have evaluated them numerically. We found that in practice this method yields strong conditions on the allowed values of m_0^2 which can be found with an arbitrarily high numerical precision. Indeed we found that for the allowed values of m_0^2 and once we compare the values of the two Taylor expansions at one point to fix the coefficient b_0, the two Taylor expansions actually agree to a very high accuracy over an entire interval thus providing an impressive test of the method. Small changes in the values of m_0^2 away from the correct value are easily detected by the mismatch they produce between the Taylor expansions at the horizon and at ∞. As an example we show the wave function for the 12th eigenvalue $m_0^2=895.8$ in Fig. 1. We plotted the solution in $x=b/r$. The curve starting at the origin is determined from the Taylor

[1]One could take $1/r-1$ as well but it turns out that $1/r^2-1$ provides a much better behavior of the coefficients of the power series for the regular solution and this is important to ensure a proper numerical treatment of the problem.

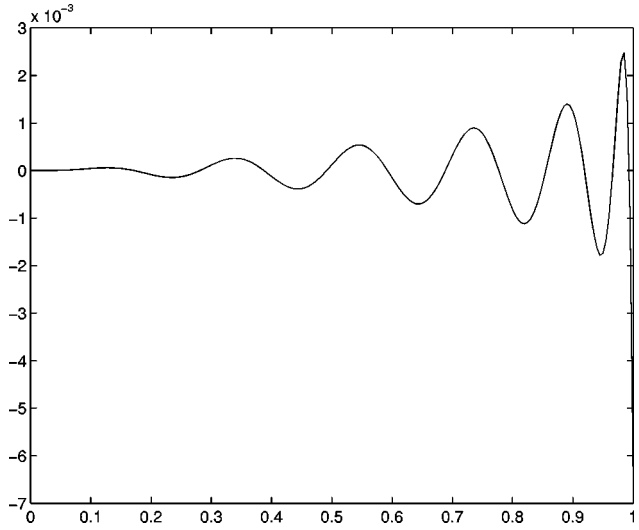

FIG. 1. The exact wave function for $m_0^2=895.5$. The horizontal axis is $x=b/r$. From $x=0$ up to $x=0.9$ we use the Taylor expansion at ∞. From $x=1$ down to $x=0.8$ we use the Taylor expansion about the horizon located at $x=1$.

expansion at infinity which we take up to $x=0.9$. We find that the power series converges extremely well in this region. From $x=1$ we use the Taylor series from the horizon down to $x=0.8$ where again we find a rapid convergence. As is clear from the figure the two expansions match perfectly as expected since we are describing the exact analytic form of the solution. We note that the radial derivative at the horizon is not zero (it is a factor of order one times the x derivative) and is in fact not small. We find similar results for the other mass eigenvalues.

In Table I we reproduce the first twelve values of m_0^2.

The authors of [28] used a ''shooting'' technique and numerically integrated the differential equation using the Taylor expansion at ∞ as an initial condition. To fix the values of m_0^2 one needs to fix the boundary condition at the horizon. If one uses the Neumann boundary condition $\eta_0'=0$ as proposed in [8], one finds numerical values for m_0^2 that are in excellent agreement with the values in Table I even though there are no smooth solutions satisfying that boundary condition. Although the dependence of the eigenmass values on the boundary condition at the horizon is relatively weak, this is of course not true for the wave functions since one is precisely discussing the first derivative at the horizon. The fact that the actual eigenvalues turn out to agree is interesting. It can be explained by the fact that in the ''shooting'' technique $\eta_0'(b)$ is a rapidly varying function of m_0^2 as can be seen from Fig. 2. We can expect that the discrepancy between the exact mass and the one obtained using the Neumann boundary condition will increase with increasing masses since the exact boundary condition has η_0' at the horizon increasing with m_0^2. This point should be taken in consideration in future work on the subject and in particular in future studies of mass spectra from supergravity where one could conceivably demand a high accuracy.

TABLE I. The exact eigenvalue masses m_0^2 for the O^{++} glueball in QCD_3 derived from supergravity. Note that these exact supergravity masses have been rounded to the accuracy shown.

SUGRA m_0^2
11.5877
34.5270
68.9750
114.9104
172.3312
241.2366
321.6265
413.5009
516.8597
631.7028
758.0302
895.8410

We emphasize that our construction is based on matching the analytic forms of the wave functions over an extended interval. Consequently, the size of the error in the determination of the masses in our approach leads us to exclude the possibility of an exact mass formula of the form $m_0^2=6n(n+1)$.

To find the $O(\gamma)$ corrections to the masses of the O^{++} glueballs one needs to study the equations of motion for quadratic fluctuations of the dilaton field in the metric background (1) corrected to leading order in α'. This correction was found in [25] where one can also find the expression for W. One sets $m^2=m_0^2+\gamma m_1^2+O(\gamma^2)$ and $\eta(r)=\eta_0(r)+\gamma\eta_1(r)+O(\gamma^2)$ and perturbs Eq. (2) about the vanishing

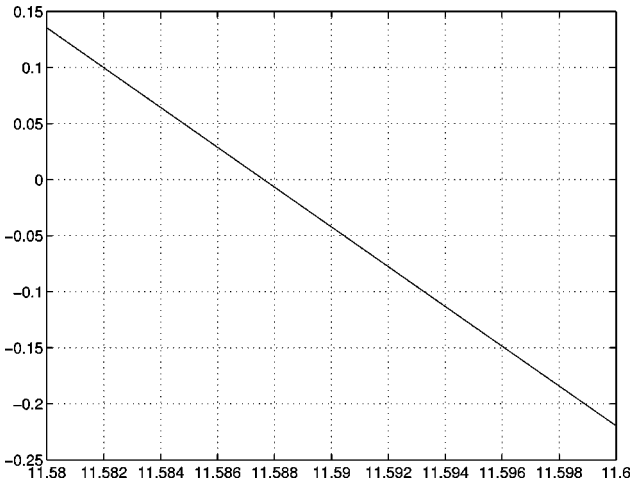

FIG. 2. This plot shows the dependence of the derivative at the horizon $d\eta_0/dr(b)$ as a function of m_0^2 in the ''shooting'' technique. The wave function is normalized so that the first term in the Taylor expansion about ∞ is 1. In this normalization the exact wave function has $d\eta_0/dr=-0.03$ which would put the exact mass at m_0^2 above 11.588 while the Neumann boundary condition would give m_0^2 below 11.588.

dilaton background.[2] This gives the equation of motion (with $b=1$)

$$\frac{d}{dr}\left(r(r^4-1)\frac{d\eta_1}{dr}\right)+rm_0^2\eta_1 = (r^5-r)\left(-\frac{300}{r^5}-\frac{600}{r^9}+\frac{1980}{r^{13}}\right)\frac{d\eta_0}{dr} + \left(-rm_1^2-rm_0^2\left(\frac{75}{r^4}+\frac{75}{r^8}-\frac{165}{r^{12}}\right) + \frac{405}{r^{13}}+m_0^2\frac{120}{r^{11}}\right)\eta_0. \quad (15)$$

Normalizability and the already known behavior of η_0 once again fix the behavior of η_1 at ∞. By an analysis similar to the one we performed above one can show that there are no smooth solutions with a vanishing derivative at the horizon. In this case the expressions for the Taylor expansions become a bit cumbersome. We have calculated the first few mass corrections m_1^2 by the method of matching the two Taylor expansions for the exact solutions. Once again for the same reasons that we explained above, we found values of m_1^2 which coincide with the ones obtained by Csáki *et al.* [28] via the ''shooting'' technique and we will not repeat those values here. Therefore, we also have nothing to add to the physical analysis that was done in that reference and in particular on the comparison with the results from the lattice [27].

In [28] the authors also studied the spectrum for the O^{--} glueball in three-dimensional QCD and the glueball mass spectra in four-dimensional QCD obtained from the black hole geometry in $AdS_7\times S^4$ [8]. The results of our analysis apply equally well to those cases. We note that in all these cases there is only one smooth solution at the horizon as the indicial equation always has a double root. This is physically interesting since otherwise matching with the behavior at infinity would most probably not put enough restrictions on the wave functions and the eigenmass problem would likely be ill defined.

TABLE II. Values of m_0 for the O^{--} glueball in QCD_3 obtained from matching the exact Taylor expansions at the horizon and ∞ in supergravity. $\tilde{m}_0$ is the same mass normalized such that the lowest O^{++} mass is 4.07.

O^{--} m_0	O^{--} $\tilde{m}_0$
5.1102	6.11
7.8234	9.35
10.3591	12.39
12.8375	15.35
15.2909	18.28
17.7280	21.20
20.1528	24.09
22.5718	26.98
24.9868	29.88
27.3998	32.76
29.8088	35.64

In the case of the O^{--} glueball in QCD_3 one has the eigenvalue problem [28]

$$r(r^4-1)\frac{d^2\eta_0}{dr^2}+(3+r^4)\frac{d\eta_0}{dr}+(m_0^2r-16r^3)\eta_0=0. \quad (16)$$

Normalizability fixes the behavior at ∞ to be $\eta_0(r)\sim r^{-4}$ and the Taylor expansion at ∞ has the form $\eta_0(r)=\Sigma_{n=0}^{\infty}c_nx^{n+2}$ with $x=1/r^2$ (we set $b=1$) and

$$c_n=\frac{(4n(n-1)+12n)c_{n-2}-m_0^2c_{n-1}}{4(n+2)(n+1)-16+4(n+2)}, \quad (17)$$

with $c_0=1$ and $c_1=-m_0^2/20$. At the horizon we use the variable $y=x-1$ and the ansatz $\eta_0(y)=\Sigma_{n=0}^{\infty}b_ny^n$ where we obtain, from Eq. (16),

$$b_n=\frac{(-4(n-1)(5n-2)+m_0^2-16)b_{n-1}+(m_0^2-4(n-2)(4n-3))b_{n-2}-4(n-3)(n-7)b_{n-3}}{8n^2} \quad (18)$$

with b_0 fixed by the Taylor expansion at ∞ and $b_1=(m_0^2-16)b_0/8$ and $b_2=(m_0^2-48)b_1/32+m_0^2b_0/32$. We now find the mass eigenvalues by matching these two expansions. The results for $m_0^2<1000$ are shown in Table II.

We observe that for the O^{--} three-dimensional glueball our exact values for the masses differ slightly from the ones obtained in the shooting technique in [28].[3] As our first eigenvalue essentially agrees with the one in that reference we confirm the agreement between the ratio of the lowest mass O^{--} and O^{++} glueballs in QCD_3 in supergravity and on the lattice reported in [28]. For completeness, in Fig. 3 we plot

[2]Even though the dilaton is corrected to $O(\gamma)$ as was calculated in [25] this does not affect the equation of motion for the fluctuations η to this order.

[3]However, the mass ratios are still in excellent agreement.

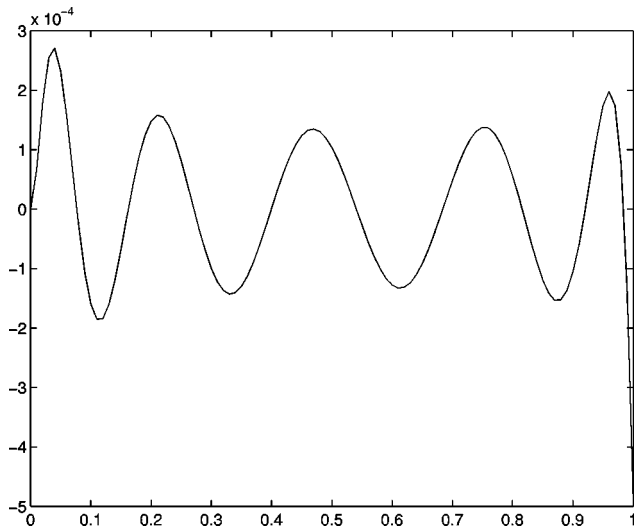

FIG. 3. This plot shows the wave function η_0 for the O^{--} glueball in QCD_3 with $m_0=27.3998$. The plot is obtained from the Taylor expansions at the horizon and ∞ which agree perfectly.

the wave function η_0 corresponding to the mass eigenvalue $m_0=27.3998$.

Finally, we will examine the O^{++} glueball in four-dimensional QCD. The appropriate wave equation is in this case [28]

$$(s^7-s)\frac{d^2\eta_0}{ds^2}+(8s^6-2)\frac{d\eta_0}{ds}+s^3m_0^2\eta_0=0, \qquad (19)$$

TABLE III. Values of m_0^2 and m_0 for the O^{++} glueball in QCD_4 obtained from matching the exact Taylor expansions at the horizon and ∞ in supergravity.

O^{++} m_0^2	O^{++} m_0
26.9498	5.1913
63.8820	7.9926
114.1326	10.6833
177.7429	13.3320
254.7283	15.9602
345.0944	18.5767
448.8437	21.1859
565.9776	23.7903
696.4967	26.3912
840.4013	28.9897
997.6925	31.5863

with $r=s^2$. At ∞ the Taylor expansion is of the form, with $x=1/r$, $\eta_0(x)=\Sigma_{n=0}^{\infty}c_n x^{n+7/2}$ where $c_0=1$, $c_1=-m_0^2/18$ and $c_2=m^4/792$. The recursion relation is

$$c_{n-1}=\frac{(4(n-1/2)(n-3/2)+2(n-1/2))c_{n-4}-m_0^2c_{n-2}}{4(n+5/2)(n+3/2)-10(n+5/2)}. \qquad (20)$$

At the horizon we use the variable $y=x-1$ and the ansatz $\eta_0(y)=\Sigma_{n=0}^{\infty}b_n y^n$ which gives

$$b_{n+1}=\frac{-2(n-2)(2n-5)b_{n-2}-2(n-1)(3+8(n-2))b_{n-1}-(24n(n-1)-m^2+6n)b_n}{12n(n+1)+12(n+1)}, \qquad (21)$$

where b_0 is fixed by matching with the Taylor expansion from ∞ and $b_1=-m_0^2b_0/12$ and $b_2=m_0^2(m_0^2-6)b_0/576$. The masses we obtained are exhibited in Table III.

In this case our exact mass eigenvalues are also in close agreement with the ones presented in [28].

IV. CONCLUSIONS

We have examined the eigenvalue problems that feature in Witten's generalization of the conjecture by Maldacena regarding large N supersymmetric gauge theory at high temperature and strong coupling. We have studied the eigenvalue problem through exact analytical expansions (at both the horizon and infinity) and evaluated these exact expressions numerically. We have analyzed carefully the behavior of eigenfunctions at the horizon and discussed the boundary conditions. We have emphasized the fact that the correct criteria for selecting the wave eigenfunctions are normalizability and smoothness at the horizon, have shown that no smooth solutions exist with vanishing derivative (Neumann boundary condition) at the horizon, and have given a construction of such smooth solution. Given that we are using exact analytic expressions for the various wave functions, our mass eigenvalues can be determined to any desired precison.

Our values for the glueball masses are in agreement with the ones found in [28] and we have explained why the two techniques give identical results to this level of accuracy. Since we used exact formulas for the analytic expansions of the wave function solutions we believe that our work reinforces the good agreement between various glueball spectra obtained in supergravity and on the lattice as was already described in [28]. We hope that the results of this analysis will be of use for further studies of the conjecture. We expect that indeed they will be necessary as soon as higher precision in the mass values becomes important.

Note added. While the present work was in preparation, we received the paper [28] which examines the same prob-

lem. In the body of the text we have therefore described the relation between these two studies. After completion of this work the paper hep-th/9806128 by M. Zyskin which has some overlap with our paper also appeared. We also received comments on the correct treatment of the boundary condition at the horizon by E. Witten and A. Hashimoto and I. Klebanov (private communications).

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