Role of fixed scalars in scattering off a 5D black hole

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We discuss the role of fixed scalars (ν, λ) in scattering off a five-dimensional black hole. The issue is to explain the disagreement of the greybody factor for λ between the semiclassical and effective string calculations. In the effective string approach, this is related to the operators with dimension (3,1) and (1,3). In the semiclassical calculation, this originates from a complicated mixing between λ and other fields. Hence it may depend on the decoupling procedure. It is shown that λ depends on gauge choices such as the harmonic, dilaton gauges, and the Krasnitz-Klebanov setting for $h_{\mu\nu}$. It turns out that ν plays a role of test field well, while the role of λ is obscure. [S0556-2821(98)03920-4]

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I. INTRODUCTION

Recently there has been great progress in a certain class of five-dimensional (5D) black holes with three U(1) charges. This progress was achieved in both the Bekenstein-Hawking entropy (S_{BH}) and absorption cross section (σ_{abs}). The semiclassical calculations of the cross section (greybody factor) in extremal and near extremal black holes are important to compare them with the result of D-branes.

Apart from counting the microstates [1] of a black hole through D-brane physics, a dynamical consideration becomes an important issue [2–6]. This is so because the greybody factor for the black hole arises as a consequence of the gravitational potential barrier surrounding the horizon. That is, this is an effect of spacetime curvature. In the stringy description, their origin comes from the thermal distribution for excitations of the $D1$-$D5$ bound state. Together with the Bekenstein-Hawking entropy, this seems to be a strong hint of a deep and mysterious connection between curvature and statistical mechanics [7]. The cross section calculation for a minimally coupled scalar is straightforward in both semiclassical and effective string models. The s-wave cross section is not sensitive to the moduli and energy (ω). This depends on the area of horizon [3]. However, this is true when the area of the horizon is not zero, e.g., for a 5D black hole with three charges. When a 5D black hole has only two charges, the absorption cross section depends on both moduli and energy [7].

A better test of the agreement between semiclassical and effective string calculations is provided by the fixed scalars. The effective string calculation is well performed in the dilute gas approximation. However, the semiclassical calculations are difficult even for the dilute gas limit, because of a complicated mixing between fixed scalars and other fields (metric and gauge fields). One of fixed scalars (ν) is coupled to an operator of dimension (2,2) in the effective string model. When the $D1$-brane charge (Q_1) is equal to the $D5$-brane charge (Q_5), the string calculation of σ_{abs} yields precise agreement with the semiclassical greybody factor [4]. But the greybody factor for the other (λ) is not in agreement

for $Q_1 = Q_5$. This disagreement is related to the presence of the chiral operators with (3,1) and (1,3) in the effective string approach. On the other hand, this originates from a complicated mixing between λ and other fields in the semiclassical calculation. Thus it may depend on the decoupling procedure. Here we deal mainly with this problem.

In this paper, we shall perform a complete, semiclassical analysis for a 5D black hole with three U(1) charges. This is similar to the 4D $N=4$ black hole with two U(1) charges [8], which provides us a simple model for getting the s-wave cross section of the fixed scalar [9]. Here we consider all perturbing equations around a 5D black hole to find the consistent solution. In the s-wave calculation two fixed scalars are physically propagating modes, whereas other fields become the redundant ones. Hence our main task is to decouple the fixed scalars from all other fields. In order to achieve this, we first consider the general perturbation for the graviton $h_{\mu\nu}$. We choose either the harmonic gauge ($\nabla_\mu \hat{h}^{\mu\nu} = 0$, $\hat{h}^{\mu\nu} = h^{\mu\nu} - 1/2 g^{\mu\nu} h$) or the dilaton gauge ($\nabla_\mu \hat{h}^{\mu\rho} = h^{\mu\nu} \Gamma^\rho_{\mu\nu}$). It turns out that for $Q_1 = Q_5$, ν is independent of the gauge fixing, while λ depends on the gauge choice. This may explain the agreement of greybody factor for ν and the disagreement for λ. For an explicit calculation we choose the Krasnitz-Klebanov (K-K) setting for $h_{\mu\nu}$ as in Ref. [5]. This is not suitable for studying the higher angular momentum modes ($l \geq 1$) [10]. In order to study higher modes, we need to consider the general perturbation as in Refs. [11,12].

The organization of our paper is as follows. In Sec. II, we review the revelant part of a 5D black hole briefly. We set up the perturbation for all fields around 5D black hole solutions in Sec. III. The s-wave absorption cross section is calculated in Sec. IV. Finally, we discuss the role of fixed scalars as the test fields in Sec. V.

II. 5D BLACK HOLES

Here we consider a class of 5D black holes representing the bound state of $n_1 (= VQ_1/g)D1$ strings and $n_5 (= Q_5/g)D5$-branes compactified on $T^5 (= T^4 \times S^1)$. This

black hole can also be obtained as a solution to the semiclassical action of a type-IIB superstring compactified on T^5. The effective action for a 5D black hole with three charges is given by [4,5]

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left\{ R - \frac{4}{3} (\nabla\lambda)^2 - 4(\nabla\nu)^2 - \frac{1}{4} e^{8/3\lambda} F^{(K)2} \right.$$

$$\left. - \frac{1}{4} e^{-4/3\lambda + 4\nu} F^2 - \frac{1}{4} e^{-4/3\lambda - 4\nu} H^2 \right\}, \tag{1}$$

where $F^{(K)}_{\mu\nu}$ is the Kaluza-Klein (KK) field strength along the string direction (S^1), $F_{\mu\nu}$ is the electric components of the Ramond-Ramond (RR) two-form, and $H_{\mu\nu}$ is dual to the magnetic components of the RR two-form. Here we omit the analysis of the 6D dilaton ϕ_6, since it is just a minimally decoupled scalar. On the other hand, the scalars ν and λ interact with the gauge fields and are examples of the fixed scalar. ν is related to the scale of the internal torus (T^4), while λ is related to the scale of the KK circle (S^1). κ_5^2 is the 5D gravitational coupling constant ($\kappa_5^2 = 8\pi G_N^5$, G_N^5 = 5D Newtonian constant). This can be determined by $G_N^5 = G_N^{10}/V_5 = 8\pi^6 g^2/(2\pi)^5 VR = \pi g^2/4VR$ with $V = R_5 R_6 R_7 R_8$ (volume of T^4), $R = R_9$ (radius of S^1), $\alpha' = 1$, and g (= 10D string coupling constant). We wish to follow the Misner-Thorne-Wheeler (MTW) conventions [13].

The equations for action (1) is given by

$$R_{\mu\nu} - \frac{4}{3} \partial_\mu\lambda\partial_\nu\lambda - 4\partial_\mu\partial_\nu\nu - e^{8/3\lambda} \left(\frac{1}{2} F^{(K)}_{\mu\rho} F^{(K)}{}_\nu{}^\rho \right.$$

$$\left. - \frac{1}{12} F^{(K)2} g_{\mu\nu} \right) e^{-4/3\lambda + 4\nu} \left(\frac{1}{2} F_{\mu\rho} F_\nu{}^\rho - \frac{1}{12} F^2 g_{\mu\nu} \right)$$

$$- e^{-4/3\lambda - 4\nu} \left(\frac{1}{2} H_{\mu\rho} H_\nu{}^\rho - \frac{1}{12} H^2 g_{\mu\nu} \right) = 0, \tag{2}$$

$$8\nabla^2\nu - e^{-4/3\lambda + 4\nu} F^2 + e^{-4/3\lambda - 4\nu} H^2 = 0, \tag{3}$$

$$8\nabla^2\lambda - 2e^{8/3\lambda} F^{(K)2} + e^{-4/3\lambda + 4\nu} F^2 + e^{-4/3\lambda - 4\nu} H^2 = 0, \tag{4}$$

$$\nabla_\mu (e^{8/3\lambda} F^{(K)\mu\nu}) = 0, \tag{5}$$

$$\nabla_\mu (e^{-4/3\lambda + 4\nu} F^{\mu\nu}) = 0, \tag{6}$$

$$\nabla_\mu (e^{-4/3\lambda - 4\nu} H^{\mu\nu}) = 0. \tag{7}$$

In addition, we need the remaining Maxwell equations as three Bianchi identities [12,14]

$$\partial_{[\mu} F^{(K)}{}_{\rho\sigma]} = \partial_{[\mu} F_{\rho\sigma]} = \partial_{[\mu} H_{\rho\sigma]} = 0. \tag{8}$$

The black hole solution is given by the background metric

$$ds^2 = -df^{-2/3} dt^2 + d^{-1} f^{1/3} dr^2 + r^2 f^{1/3} d\Omega_3^2 \tag{9}$$

and

$$e^{2\bar{\lambda}} = \frac{f_K}{\sqrt{f_1 f_5}}, \quad e^{4\bar{\nu}} = \frac{f_1}{f_5}, \quad f = f_1 f_5 f_K, \tag{10}$$

$$\bar{F}^{(K)}_{tr} = \frac{2Q_K}{r^3 f_K^2}, \quad \bar{F}_{tr} = \frac{2Q_1}{r^3 f_1^2}, \quad \bar{H}_{tr} = \frac{2Q_5}{r^3 f_5^2}. \tag{11}$$

Here four harmonic functions are defined by

$$f_1 = 1 + \frac{r_1^2}{r^2}, \quad f_5 = 1 + \frac{r_5^2}{r^2}, \quad f_K = 1 + \frac{r_K^2}{r^2}, \quad d = 1 - \frac{r_0^2}{r^2}, \tag{12}$$

with $r_i^2 = r_0^2 \sinh^2\sigma_i$, $i = 1, 5, K$. Q_1, Q_5, and Q_K are related to the characteristic radii r_1, r_5, r_K, and the radius of horizon r_0 as

$$Q_i = \frac{1}{2} r_0^2 \sinh 2\sigma_i, \quad Q_i^2 = r_i^2 (r_i^2 + r_0^2),$$

$$r_i^2 = \sqrt{Q_i^2 + \frac{r_0^4}{4}} - \frac{r_0^2}{2}. \tag{13}$$

The background metric (9) is just a 5D Schwarzschild one with time and space components rescaled by different powers of f. The event horizon (outer horizon) is clearly at $r = r_0$. When all three charges are nonzero, the surface of $r = 0$ becomes a smooth inner horizon (Cauchy horizon). If one of the charges is zero, the surface of $r = 0$ becomes singular. The extremal case corresponds to the limit of $r_0 \to 0$ with the boost parameters $\sigma_i \to \pm\infty$ keeping Q_i fixed. Here one has $Q_1 = r_1^2$, $Q_5 = r_5^2$, and $Q_K = r_K^2$. In this work we are very interested in the limit of $r_0, r_K \ll r_1, r_5$, which is called the dilute gas approximation. This corresponds to the near-extremal black hole and its thermodynamic quantities are given by

$$M_{next} = \frac{2\pi^2}{\kappa_5^2} \left[r_1^2 + r_5^2 + \frac{1}{2} r_0^2 \cosh 2\sigma_K \right], \tag{14}$$

$$S_{next} = \frac{4\pi^3 r_0}{\kappa_5^2} r_1 r_5 \cosh\sigma_K, \tag{15}$$

$$\frac{1}{T_{H,next}} = \frac{2\pi}{r_0} r_1 r_5 \cosh\sigma_K. \tag{16}$$

The above energy and entropy are actually those of a gas of massless 1D particles. In this case the effective temperatures of the left and right moving string modes are given by

$$T_L = \frac{1}{2\pi}\left(\frac{r_0}{r_1 r_5}\right)e^{\sigma_K}, \quad T_R = \frac{1}{2\pi}\left(\frac{r_0}{r_1 r_5}\right)e^{-\sigma_K}. \quad (17)$$

The Hawking temperature is given by their harmonic average

$$\frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R}. \quad (18)$$

III. PERTURBATION ANALYSIS

Here we start with the perturbation around the black hole background [15]

$$F_{tr}^{(K)} = \bar{F}_{tr}^{(K)} + \mathcal{F}_{tr}^{(K)} = \bar{F}_{tr}^{(K)}[1 + \mathcal{F}^{(K)}(t,r,\chi,\theta,\phi)], \quad (19)$$

$$F_{tr} = \bar{F}_{tr} + \mathcal{F}_{tr} = \bar{F}_{tr}[1 + \mathcal{F}(t,r,\chi,\theta,\phi)], \quad (20)$$

$$H_{tr} = \bar{H}_{tr} + \mathcal{H}_{tr} = \bar{H}_{tr}[1 + \mathcal{H}(t,r,\chi,\theta,\phi)], \quad (21)$$

$$\lambda = \bar{\lambda} + \delta\lambda(t,r,\chi,\theta,\phi), \quad (22)$$

$$\nu = \bar{\nu} + \delta\nu(t,r,\chi,\theta,\phi), \quad (23)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (24)$$

Here $h_{\mu\nu}$ is given by

$$h^\mu{}_\nu = \begin{bmatrix} h_1 & h_3 & 0 & 0 & 0 \\ -d^2 h_3/f & h_2 & 0 & 0 & 0 \\ 0 & 0 & h^\chi{}_\chi & h^\chi{}_\theta & h^\chi{}_\phi \\ 0 & 0 & h^\theta{}_\chi & h^\theta{}_\theta & h^\theta{}_\phi \\ 0 & 0 & h^\phi{}_\chi & h^\phi{}_\theta & h^\phi{}_\phi \end{bmatrix}. \quad (25)$$

This seems to be general for the s-wave calculation.

One has to linearize Eqs. (2)–(7) in order to obtain the equations governing the perturbations as

$$\delta R_{\mu\nu}(h) - \frac{4}{3}(\partial_\mu \bar{\lambda}\partial_\nu \delta\lambda + \partial_\mu \delta\lambda \partial_\nu \bar{\lambda}) - 4(\partial_\mu \bar{\nu}\partial_\nu \delta\nu + \partial_\mu \delta\nu \partial_\nu \bar{\nu}) + \frac{1}{2}e^{8/3\bar{\lambda}}\bar{F}_{\mu\rho}^{(K)}\bar{F}_{\nu\alpha}^{(K)}h^{\rho\alpha} - e^{8/3\bar{\lambda}}\bar{F}_{\mu\rho}^{(K)}\mathcal{F}^{(K)}{}_\nu{}^\rho$$

$$-\frac{4}{3}e^{8/3\bar{\lambda}}\bar{F}_{\mu\rho}^{(K)}\bar{F}^{(K)}{}_\nu{}^\rho \delta\lambda + \frac{1}{6}e^{8/3\bar{\lambda}}\bar{F}_{\rho\sigma}^{(K)}\mathcal{F}^{(K)\rho\sigma}\bar{g}_{\mu\nu} - \frac{1}{6}e^{8/3\bar{\lambda}}\bar{F}_{\rho\kappa}^{(K)}\bar{F}^{(K)}{}_\eta{}^\kappa h^{\rho\eta}\bar{g}_{\mu\nu} + \frac{2}{9}e^{8/3\bar{\lambda}}\bar{F}^{(K)2}\bar{g}_{\mu\nu}\delta\lambda + \frac{1}{12}e^{8/3\bar{\lambda}}\bar{F}^{(K)2}h_{\mu\nu}$$

$$+\frac{1}{2}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\rho}\bar{F}_{\nu\alpha}h^{\rho\alpha} - e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\rho}\mathcal{F}_\nu{}^\rho - e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\rho}\bar{F}_\nu{}^\rho\left(-\frac{2}{3}\delta\lambda + 2\delta\nu\right) + \frac{1}{6}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\rho\sigma}\mathcal{F}^{\rho\sigma}\bar{g}_{\mu\nu}$$

$$-\frac{1}{6}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\rho\kappa}\bar{F}_\eta{}^\kappa h^{\rho\eta}\bar{g}_{\mu\nu} + e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}^2\bar{g}_{\mu\nu}\left(-\frac{1}{9}\delta\lambda + \frac{1}{3}\delta\nu\right) + \frac{1}{12}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}^2 h_{\mu\nu} + \frac{1}{2}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\rho}\bar{H}_{\nu\alpha}h^{\rho\alpha}$$

$$-e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\rho}\mathcal{H}_\nu{}^\rho + e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\rho}\bar{H}_\nu{}^\rho\left(\frac{2}{3}\delta\lambda + 2\delta\nu\right) + \frac{1}{6}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\rho\sigma}\mathcal{H}^{\rho\sigma}\bar{g}_{\mu\nu} - \frac{1}{6}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\rho\kappa}\bar{H}_\eta{}^\kappa h^{\rho\eta}\bar{g}_{\mu\nu}$$

$$-e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}^2\bar{g}_{\mu\nu}\left(\frac{1}{9}\delta\lambda + \frac{1}{3}\delta\nu\right) + \frac{1}{12}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}^2 h_{\mu\nu} = 0, \quad (26)$$

$$\bar{\nabla}^2\delta\nu - h^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu\bar{\nu} - \bar{g}^{\mu\nu}\delta\Gamma_{\mu\nu}^\rho(h)\partial_\rho\bar{\nu} - \frac{1}{4}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{4}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\nu}\bar{F}_\rho{}^\nu h^{\mu\rho} - \frac{1}{8}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}^2\left(-\frac{4}{3}\delta\lambda + 4\delta\nu\right)$$

$$+\frac{1}{4}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\nu}\mathcal{H}^{\mu\nu} - \frac{1}{4}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\nu}\bar{H}_\rho{}^\nu h^{\mu\rho} - \frac{1}{8}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}^2\left(\frac{4}{3}\delta\lambda + 4\delta\nu\right) = 0, \quad (27)$$

$$\bar{\nabla}^2\delta\lambda - h^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu\bar{\lambda} - \bar{g}^{\mu\nu}\delta\Gamma_{\mu\nu}^\rho(h)\partial_\rho\bar{\lambda} - \frac{1}{2}e^{8/3\bar{\lambda}}\bar{F}_{\mu\nu}^{(K)}\mathcal{F}^{(K)\mu\nu} + \frac{1}{2}e^{8/3\bar{\lambda}}\bar{F}_{\mu\nu}^{(K)}\bar{F}_\rho^{(K)\nu}h^{\mu\rho} - \frac{2}{3}e^{8/3\bar{\lambda}}\bar{F}^{(K)2}\delta\lambda + \frac{1}{4}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$$

$$-\frac{1}{4}e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}_{\mu\nu}\bar{F}_\rho{}^\nu h^{\mu\rho} + e^{-4/3\bar{\lambda}+4\bar{\nu}}\bar{F}^2\left(-\frac{1}{6}\delta\lambda + \frac{1}{2}\delta\nu\right) + \frac{1}{4}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\nu}\mathcal{H}^{\mu\nu} - \frac{1}{4}e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}_{\mu\nu}\bar{H}_\rho{}^\nu h^{\mu\rho}$$

$$-e^{-4/3\bar{\lambda}-4\bar{\nu}}\bar{H}^2\left(\frac{1}{6}\delta\lambda + \frac{1}{2}\delta\nu\right) = 0, \quad (28)$$

$$\left(\bar{\nabla}_\mu+\frac{8}{3}\partial_\mu\bar{\lambda}\right)(\mathcal{F}^{(K)\mu\nu}-\bar{F}^{(K)}{}_\alpha{}^\nu h^{\alpha\mu}-\bar{F}^{(K)}{}_\beta{}^\mu h^{\beta\nu})+\bar{F}^{(K)\mu\nu}\left(\delta\Gamma^\sigma_{\sigma\mu}(h)+\frac{8}{3}\partial_\mu\delta\lambda\right)=0,\tag{29}$$

$$\left(\bar{\nabla}_\mu-\frac{4}{3}\partial_\mu\bar{\lambda}+4\partial_\mu\bar{\nu}\right)(\mathcal{F}^{\mu\nu}-\bar{F}_\alpha{}^\nu h^{\alpha\mu}-\bar{F}^\mu{}_\beta h^{\beta\nu})+\bar{F}^{\mu\nu}\left(\delta\Gamma^\sigma_{\sigma\mu}(h)-\frac{4}{3}\partial_\mu\delta\lambda+4\partial_\mu\delta\nu\right)=0,\tag{30}$$

$$\left(\bar{\nabla}_\mu-\frac{4}{3}\partial_\mu\bar{\lambda}-4\partial_\mu\bar{\nu}\right)(\mathcal{H}^{\mu\nu}-\bar{H}_\alpha{}^\nu h^{\alpha\mu}-\bar{H}^\mu{}_\beta h^{\beta\nu})+\bar{H}^{\mu\nu}\left(\delta\Gamma^\sigma_{\sigma\mu}(h)-\frac{4}{3}\partial_\mu\delta\lambda-4\partial_\mu\delta\nu\right)=0,\tag{31}$$

where

$$\delta R_{\mu\nu}(h)=-\frac{1}{2}\bar{\nabla}^2 h_{\mu\nu}-\frac{1}{2}\bar{\nabla}_\mu\bar{\nabla}_\nu h^\rho{}_\rho+\frac{1}{2}\bar{\nabla}^\rho\bar{\nabla}_\nu h_{\rho\mu}$$
$$+\frac{1}{2}\bar{\nabla}^\rho\bar{\nabla}_\mu h_{\nu\rho},\tag{32}$$

$$\delta\Gamma^\rho_{\mu\nu}(h)=\frac{1}{2}\bar{g}^{\rho\sigma}(\bar{\nabla}_\nu h_{\mu\sigma}+\bar{\nabla}_\mu h_{\nu\sigma}-\bar{\nabla}_\sigma h_{\mu\nu}).\tag{33}$$

Since we start with full degrees of freedom (25), we choose a gauge to study the propagation of fields. For this purpose $\delta R_{\mu\nu}$ can be transformed into the Lichnerowicz operator [16]

$$\delta R_{\mu\nu}=-\frac{1}{2}\bar{\nabla}^2 h_{\mu\nu}+\bar{R}_{\sigma(\nu}h^\sigma{}_{\mu)}-\bar{R}_{\sigma\mu\rho\nu}h^{\sigma\rho}+\bar{\nabla}_{(\nu}\bar{\nabla}_{|\rho|}\hat{h}^\rho{}_{\mu)}.\tag{34}$$

We have to examine whether there exists any choice of gauge which can simplify Eqs. (27) and (28). Conventionally we choose the harmonic (transverse) gauge ($\bar{\nabla}_\mu\hat{h}^{\mu\rho}=\bar{g}^{\mu\nu}\delta\Gamma^\rho_{\mu\nu}=0$) if one concentrates on the propagation of gravitons.

A. Harmonic gauge

Considering the harmonic gauge and $Q_1=Q_5$ case, Eqs. (27) and (28) lead to

$$\bar{\nabla}^2\delta\nu+\frac{Q_1^2}{r^6 f_1^2 f^{1/3}}(2\mathcal{F}-2\mathcal{H}+8\delta\nu)=0,\tag{35}$$

$$\bar{\nabla}^2\delta\lambda-\frac{d}{f^{1/3}}h^{rr}\partial_r^2\bar{\lambda}+\frac{d}{f^{1/3}}h^{\mu\nu}\Gamma^r_{\mu\nu}\partial_r\bar{\lambda}-\frac{2Q_K^2}{r^6 f_K^2 f^{1/3}}$$
$$\times\left(h_1+h_2-2\mathcal{F}^{(K)}-\frac{8}{3}\delta\lambda\right)+\frac{2Q_1^2}{r^6 f_1^2 f^{1/3}}$$
$$\times\left(h_1+h_2-\mathcal{F}-\mathcal{H}+\frac{4}{3}\delta\lambda\right)=0.\tag{36}$$

Now we attempt to disentangle the mixing between $(\delta\nu,\delta\lambda)$ and other fields by using both the harmonic gauge and U(1) field equations in Eqs. (29)–(31). After some calculations, one finds the relations

$$2\mathcal{F}^{(K)}=h_1+h_2-h^{\theta_i}{}_{\theta_i}-\frac{16}{3}\delta\lambda,\tag{37}$$

$$2\mathcal{F}=h_1+h_2-h^{\theta_i}{}_{\theta_i}+\frac{8}{3}\delta\lambda-8\delta\nu,\tag{38}$$

$$2\mathcal{H}=h_1+h_2-h^{\theta_i}{}_{\theta_i}+\frac{8}{3}\delta\lambda+8\delta\nu,\tag{39}$$

where $h^{\theta_i}{}_{\theta_i}=h^\chi{}_\chi+h^\theta{}_\theta+h^\phi{}_\phi$. Using Eqs. (37)–(39), one obtains the linearized equation for $\delta\nu$ and $\delta\lambda$,

$$\bar{\nabla}^2\delta\nu-\frac{8Q_1^2}{r^6 f_1^2 f^{1/3}}\delta\nu=0,\tag{40}$$

$$\bar{\nabla}^2\delta\lambda-\frac{d}{f^{1/3}}h^{rr}\partial_r^2\bar{\lambda}+\frac{d}{f^{1/3}}h^{\mu\nu}\Gamma^r_{\mu\nu}\partial_r\bar{\lambda}+\frac{2}{r^6 f^{1/3}}\left[\frac{Q_1^2}{f_1^2}-\frac{Q_K^2}{f_K^2}\right]h^{\theta_i}{}_{\theta_i}$$
$$-\frac{8}{3r^6 f^{1/3}}\left[\frac{Q_1^2}{f_1^2}+2\frac{Q_K^2}{f_K^2}\right]\delta\lambda=0.\tag{41}$$

We wish to point out that the $\delta\nu$ equation is decoupled completely but the $\delta\lambda$ equation still remains a coupled form.

B. Dilaton gauge

We recognize that it is not enough to decouple $\delta\lambda$ equation from the harmonic gauge condition. But if one introduces the dilaton gauge ($\bar{\nabla}_\mu\hat{h}^{\mu\rho}=h^{\mu\nu}\Gamma^\rho_{\mu\nu}$) [17], the $\delta\lambda$ equation can be reduced to a better simple form. Under this

gauge, one finds the same relations as those in Eqs. (37)–(39) and the same equation for $\delta\nu$ as in Eq. (40). One finds the $\delta\lambda$ equation

$$\bar{\nabla}^2 \delta\lambda - \frac{d}{f^{1/3}} h^{rr} \partial_r^2 \bar{\lambda} + \frac{2}{r^6 f^{1/3}} \left[\frac{Q_1^2}{f_1^2} - \frac{Q_K^2}{f_K^2} \right] h^{\theta_i}{}_{\theta_i} - \frac{8}{3 r^6 f^{1/3}}$$

$$\times \left[\frac{Q_1^2}{f_1^2} + 2 \frac{Q_K^2}{f_K^2} \right] \delta\lambda = 0. \tag{42}$$

In order to decouple the second term (h^{rr}) in Eq. (42), one use the Einstein equation. However, it seems to be a non-trivial task. This is because the (t,r) component of Eq. (26) gives rise to the second order differential equation for h_2. Instead, one may choose $h_2 = h^{\theta_i}{}_{\theta_i} = 0$, which is compatible with the dilaton gauge. Then, in the dilute gas limit, we find a new equation for $\delta\lambda$,

$$\bar{\nabla}^2 \delta\lambda - \frac{8 Q_1^2}{3 r^6 f_1^2 f^{1/3}} \delta\lambda = 0, \tag{43}$$

which is similar to Eq. (40). The situation may be getting better when one introduces the simplest choice such as the K-K setting [5].

C. Krasnitz-Klebanov setting

In this case, the metric perturbation $h_{\mu\nu}$ takes the form

$$h^\mu{}_\nu = \mathrm{diag}[h_1, h_2, 0, 0, 0]. \tag{44}$$

Under this setting, the harmonic gauge condition leads to

$$\frac{1}{2}(h_1 - h_2)' = \left(\frac{1}{2}\frac{d'}{d} + \frac{3}{r} + \frac{1}{6}\frac{f'}{f} \right) h_2 - \left(\frac{1}{2}\frac{d'}{d} - \frac{1}{3}\frac{f'}{f} \right) h_1, \tag{45}$$

where the prime means the differentiation with respect to r. On the other hand, the dilaton gauge condition gives us the relation

$$\frac{1}{2}(h_1 - h_2)' = \left(\frac{d'}{d} + \frac{3}{r} \right) h_2. \tag{46}$$

From now on our calculation will be performed without any gauge choice for $h_{\mu\nu}$ and restriction on charges. Solving Eqs. (29)–(31), one can express the three U(1) fields in terms of $\delta\lambda, \delta\nu, h_1, h_2$ as

$$2\mathcal{F}^{(K)} = h_1 + h_2 - \frac{16}{3} \delta\lambda, \tag{47}$$

$$2\mathcal{F} = h_1 + h_2 + \frac{8}{3} \delta\lambda - 8 \delta\nu, \tag{48}$$

$$2\mathcal{H} = h_1 + h_2 + \frac{8}{3} \delta\lambda + 8 \delta\nu. \tag{49}$$

These are consistent with Eqs. (37)–(39) when $h^{\theta_i}{}_{\theta_i} = 0$. The ten off-diagonal elements of Einstein equation (26) are given by

$$(t,r): \frac{1}{4}\left(\frac{f'}{f} + \frac{6}{r} \right) \partial_t h_2 + \frac{1}{3}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} - 2\frac{f_K'}{f_K} \right) \partial_t \delta\lambda$$

$$- \left(\frac{f_1'}{f_1} - \frac{f_5'}{f_5} \right) \partial_t \delta\nu = 0, \tag{50}$$

$$(t,\chi): \quad \partial_t \partial_\chi h_2 = 0, \tag{51}$$

$$(t,\theta): \quad \partial_t \partial_\theta h_2 = 0, \tag{52}$$

$$(t,\phi): \quad \partial_t \partial_\phi h_2 = 0, \tag{53}$$

$$(r,\chi): \quad -\frac{1}{2}(\partial_r - 3\Gamma_{r\chi}^\chi)\partial_\chi h_1 - \left(\frac{1}{4}\frac{d'}{d} + \frac{1}{r} \right)\partial_\chi(h_1 - h_2)$$

$$+ \frac{1}{3}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} - 2\frac{f_K'}{f_K} \right)\partial_\chi \delta\lambda - \left(\frac{f_1'}{f_1} - \frac{f_5'}{f_5} \right)\partial_\chi \delta\nu = 0, \tag{54}$$

$$(r,\theta): \quad -\frac{1}{2}(\partial_r - 3\Gamma_{r\theta}^\theta)\partial_\theta h_1 - \left(\frac{1}{4}\frac{d'}{d} + \frac{1}{r} \right)\partial_\theta(h_1 - h_2)$$

$$+ \frac{1}{3}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} - 2\frac{f_K'}{f_K} \right)\partial_\theta \delta\lambda - \left(\frac{f_1'}{f_1} - \frac{f_5'}{f_5} \right)\partial_\theta \delta\nu = 0, \tag{55}$$

$$(r,\phi): \quad -\frac{1}{2}(\partial_r - 3\Gamma_{r\phi}^\phi)\partial_\phi h_1 - \left(\frac{1}{4}\frac{d'}{d} + \frac{1}{r} \right)\partial_\phi(h_1 - h_2)$$

$$+ \frac{1}{3}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} - 2\frac{f_K'}{f_K} \right)\partial_\phi \delta\lambda - \left(\frac{f_1'}{f_1} - \frac{f_5'}{f_5} \right)\partial_\phi \delta\nu = 0, \tag{56}$$

$$(\chi,\theta): \quad (\partial_\chi - \Gamma_{\chi\theta}^\theta)\partial_\theta(h_1 + h_2) = 0, \tag{57}$$

$$(\chi,\phi): \quad (\partial_\chi - \Gamma_{\chi\phi}^\phi)\partial_\phi(h_1 + h_2) = 0, \tag{58}$$

$$(\theta,\phi): \quad (\partial_\theta - \Gamma_{\theta\phi}^\phi)\partial_\phi(h_1 + h_2) = 0. \tag{59}$$

The five diagonal elements of Eq. (26) take the form

$$(t,t): \quad -\frac{1}{2}\frac{f}{d^2}\partial_t^2 h_2 + \frac{1}{2}\partial_r^2 h_1 + \frac{3}{2}\frac{1}{r}h_1' + \frac{1}{2}\frac{1}{d}\frac{1}{r^2}\left[\partial_\chi^2 + 2\cot\chi\partial_\chi + \frac{1}{\sin^2\chi}\left(\partial_\theta^2 + \cot\theta\partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi^2\right)\right]h_1 + \frac{1}{6}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} + \frac{f_K'}{f_K}\right)$$

$$\times (h_2' - h_1') + \frac{1}{3}\frac{d'}{d}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} + \frac{f_K'}{f_K}\right)(h_2 - h_1) - \frac{1}{3}\left(\frac{f_1'^2}{f_1^2} + \frac{f_5'^2}{f_5^2} + \frac{f_K'^2}{f_K^2}\right)(h_2 - h_1) - \frac{1}{4}\frac{d'}{d}(h_2' - 3h_1') + \frac{4}{3}\frac{Q_K^2}{r^6 f_K^2}\frac{1}{d}$$

$$\times\left(h_2 - 2\mathcal{F}^{(K)} - \frac{8}{3}\delta\lambda\right) + \frac{4}{3}\frac{Q_1^2}{r^6 f_1^2}\frac{1}{d}\left(h_2 - 2\mathcal{F} + \frac{4}{3}\delta\lambda - 4\delta\nu\right) + \frac{4}{3}\frac{Q_5^2}{r^6 f_5^2}\frac{1}{d}\left(h_2 - 2\mathcal{H} + \frac{4}{3}\delta\lambda + 4\delta\nu\right) = 0, \tag{60}$$

$$(r,r): \quad -\frac{1}{2}\frac{f}{d^2}\partial_t^2 h_2 + \frac{1}{2}\partial_r^2 h_1 - \frac{3}{2}\frac{1}{r}h_2' + \frac{1}{2}\frac{1}{d}\frac{1}{r^2}\left[\partial_\chi^2 + 2\cot\chi\partial_\chi + \frac{1}{\sin^2\chi}\left(\partial_\theta^2 + \cot\theta\partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi^2\right)\right]h_2 - \frac{1}{12}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} + \frac{f_K'}{f_K}\right)$$

$$\times (5h_1' + h_2') - \frac{2}{3}\left(\frac{f_1'}{f_1} + \frac{f_5'}{f_5} - 2\frac{f_K'}{f_K}\right)\delta\lambda' + 2\left(\frac{f_1'}{f_1} - \frac{f_5'}{f_5}\right)\delta\nu' - \frac{1}{4}\frac{d'}{d}(h_2' - 3h_1') + \frac{4}{3}\frac{Q_K^2}{r^6 f_K^2}\frac{1}{d}\left(h_1 - 2\mathcal{F}^{(K)} - \frac{8}{3}\delta\lambda\right) + \frac{4}{3}\frac{Q_1^2}{r^6 f_1^2}\frac{1}{d}$$

$$\times\left(h_1 - 2\mathcal{F} + \frac{4}{3}\delta\lambda - 4\delta\nu\right) + \frac{4}{3}\frac{Q_5^2}{r^6 f_5^2}\frac{1}{d}\left(h_1 - 2\mathcal{H} + \frac{4}{3}\delta\lambda + 4\delta\nu\right) = 0, \tag{61}$$

$$(\chi,\chi): \quad \frac{1}{r^2}\frac{1}{d}\partial_\chi^2(h_1 + h_2) - \frac{2}{r}\left(\frac{2}{r} + \frac{d'}{d}\right)h_2 - \frac{1}{3}\frac{d'}{d}\frac{f'}{f}h_2 - \frac{1}{6}\left(\frac{f'}{f} + \frac{6}{r}\right)(h_2' - h_1') + \frac{1}{3}\left(\frac{f_1'^2}{f_1^2} + \frac{f_5'^2}{f_5^2} + \frac{f_K'^2}{f_K^2}\right)h_2 - \frac{4}{3}\frac{Q_K^2}{r^6 f_K^2}\frac{1}{d}$$

$$\times\left(h_1 + h_2 - 2\mathcal{F}^{(K)} - \frac{8}{3}\delta\lambda\right) - \frac{4}{3}\frac{Q_1^2}{r^6 f_1^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{F} + \frac{4}{3}\delta\lambda - 4\delta\nu\right) - \frac{4}{3}\frac{Q_5^2}{r^6 f_5^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{H} + \frac{4}{3}\delta\lambda + 4\delta\nu\right) = 0, \tag{62}$$

$$(\theta,\theta): \quad \frac{1}{r^2 \sin^2\chi}\frac{1}{d}\partial_\theta^2(h_1 + h_2) + \frac{1}{r^2 d}\cot\chi\partial_\chi(h_1 + h_2) - \frac{2}{r}\left(\frac{2}{r} + \frac{d'}{d}\right)h_2 - \frac{1}{3}\frac{d'}{d}\frac{f'}{f}h_2 - \frac{1}{6}\left(\frac{f'}{f} + \frac{6}{r}\right)(h_2' - h_1')$$

$$+ \frac{1}{3}\left(\frac{f_1'^2}{f_1^2} + \frac{f_5'^2}{f_5^2} + \frac{f_K'^2}{f_K^2}\right)h_2 - \frac{4}{3}\frac{Q_K^2}{r^6 f_K^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{F}^{(K)} - \frac{8}{3}\delta\lambda\right) - \frac{4}{3}\frac{Q_1^2}{r^6 f_1^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{F} + \frac{4}{3}\delta\lambda - 4\delta\nu\right)$$

$$- \frac{4}{3}\frac{Q_5^2}{r^6 f_5^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{H} + \frac{4}{3}\delta\lambda + 4\delta\nu\right) = 0, \tag{63}$$

$$(\phi,\phi): \quad \frac{1}{r^2 \sin^2\chi \sin^2\theta}\frac{1}{d}\partial_\phi^2(h_1 + h_2) + \frac{1}{r^2 d}\cot\chi\partial_\chi(h_1 + h_2) + \frac{1}{r^2 d}\frac{1}{\sin^2\chi}\cot\theta\partial_\theta(h_1 + h_2) - \frac{2}{r}\left(\frac{2}{r} + \frac{d'}{d}\right)h_2 - \frac{1}{3}\frac{d'}{d}\frac{f'}{f}h_2$$

$$- \frac{1}{6}\left(\frac{f'}{f} + \frac{6}{r}\right)(h_2' - h_1') + \frac{1}{3}\left(\frac{f_1'^2}{f_1^2} + \frac{f_5'^2}{f_5^2} + \frac{f_K'^2}{f_K^2}\right)h_2 - \frac{4}{3}\frac{Q_K^2}{r^6 f_K^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{F}^{(K)} - \frac{8}{3}\delta\lambda\right)$$

$$- \frac{4}{3}\frac{Q_1^2}{r^6 f_1^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{F} + \frac{4}{3}\delta\lambda - 4\delta\nu\right) - \frac{4}{3}\frac{Q_5^2}{r^6 f_5^2}\frac{1}{d}\left(h_1 + h_2 - 2\mathcal{H} + \frac{4}{3}\delta\lambda + 4\delta\nu\right) = 0. \tag{64}$$

The fixed scalar equations (27) and (28) lead to

$$-\frac{f}{d^2}\partial_t^2\delta\nu+\left[\partial_r^2+\left(\frac{d'}{d}+\frac{3}{r}\right)\partial_r\right]\delta\nu+\frac{1}{d}\frac{1}{r^2}\left[\partial_\chi^2+2\cot\chi\partial_\chi+\frac{1}{\sin^2\chi}\left(\partial_\theta^2+\cot\theta\partial_\theta+\frac{1}{\sin^2\theta}\partial_\phi^2\right)\right]\delta\nu-\frac{1}{4}\frac{d'}{d}\left(\frac{f_1'}{f_1}-\frac{f_5'}{f_5}\right)h_2$$

$$-\frac{1}{8}\left(\frac{f_1'}{f_1}-\frac{f_5'}{f_5}\right)(h_2'-h_1')+\frac{1}{4}\left(\frac{f_1'^2}{f_1^2}-\frac{f_5'^2}{f_5^2}\right)h_2-\frac{Q_1^2}{r^6f_1^2}\frac{1}{d}\left(h_1+h_2-2\mathcal{F}+\frac{4}{3}\delta\lambda-4\delta\nu\right)$$

$$+\frac{Q_5^2}{r^6f_5^2}\frac{1}{d}\left(h_1+h_2-2\mathcal{H}+\frac{4}{3}\delta\lambda+4\delta\nu\right)=0, \tag{65}$$

$$-\frac{f}{d^2}\partial_t^2\delta\lambda+\left[\partial_r^2+\left(\frac{d'}{d}+\frac{3}{r}\right)\partial_r\right]\delta\lambda+\frac{1}{d}\frac{1}{r^2}\left[\partial_\chi^2+2\cot\chi\partial_\chi+\frac{1}{\sin^2\chi}\left(\partial_\theta^2+\cot\theta\partial_\theta+\frac{1}{\sin^2\theta}\partial_\phi^2\right)\right]\delta\lambda+\frac{1}{4}\frac{d'}{d}\left(\frac{f_1'}{f_1}+\frac{f_5'}{f_5}-2\frac{f_K'}{f_K}\right)h_2$$

$$+\frac{1}{8}\left(\frac{f_1'}{f_1}+\frac{f_5'}{f_5}-2\frac{f_K'}{f_K}\right)(h_2'-h_1')-\frac{1}{4}\left(\frac{f_1'^2}{f_1^2}+\frac{f_5'^2}{f_5^2}-2\frac{f_K'^2}{f_K^2}\right)h_2-2\frac{Q_K^2}{r^6f_K^2}\frac{1}{d}\left(h_1+h_2-2\mathcal{F}^{(K)}-\frac{8}{3}\delta\lambda\right)$$

$$+\frac{Q_1^2}{r^6f_1^2}\frac{1}{d}\left(h_1+h_2-2\mathcal{F}+\frac{4}{3}\delta\lambda-4\delta\nu\right)+\frac{Q_5^2}{r^6f_5^2}\frac{1}{d}\left(h_1+h_2-2\mathcal{H}+\frac{4}{3}\delta\lambda+4\delta\nu\right)=0. \tag{66}$$

IV. s-WAVE PROPAGATIONS

From the Bianchi identities (8) one has

$$\partial_\chi\mathcal{F}^{(K)}=\partial_\theta\mathcal{F}^{(K)}=\partial_\phi\mathcal{F}^{(K)}=0,$$

$$\partial_\chi\mathcal{F}=\partial_\theta\mathcal{F}=\partial_\phi\mathcal{F}=0,$$

$$\partial_\chi\mathcal{H}=\partial_\theta\mathcal{H}=\partial_\phi\mathcal{H}=0. \tag{67}$$

This implies either $\mathcal{F}^{(K)}=\mathcal{F}^{(K)}(t,r)$, $\mathcal{F}=\mathcal{F}(t,r)$, $\mathcal{H}=\mathcal{H}(t,r)$ or $\mathcal{F}^{(K)}=\mathcal{F}=\mathcal{H}=0$. The latter together with Eqs. (47)–(49) means that all higher modes of $l\geq1$ are forbidden in this scheme. We wish to study the s-wave propagation with the first case. This case dominates in the absorption of low energies. The important one can be derived from (t,r) component of the Einstein equation. By integrating Eq. (50) over time, we can obtain the relation

$$\left(\frac{f'}{f}+\frac{6}{r}\right)h_2=-\frac{4}{3}\left(\frac{f_1'}{f_1}+\frac{f_5'}{f_5}-2\frac{f_K'}{f_K}\right)\delta\lambda+4\left(\frac{f_1'}{f_1}-\frac{f_5'}{f_5}\right)\delta\nu. \tag{68}$$

From three angular equations (62)–(64), one finds the relation

$$\left(\frac{f'}{f}+\frac{6}{r}\right)(h_1'-h_2')=\left[2\frac{d'}{d}\frac{f'}{f}+\frac{12}{r}\left(\frac{2}{r}+\frac{d'}{d}\right)\right.$$

$$-2\left(\frac{f_1'^2}{f_1^2}+\frac{f_5'^2}{f_5^2}+\frac{f_K'^2}{f_K^2}\right)\right]h_2+\frac{32}{3}\frac{1}{r^6d}$$

$$\times\left(2\frac{Q_K^2}{f_K^2}-\frac{Q_1^2}{f_1^2}-\frac{Q_5^2}{f_5^2}\right)\delta\lambda$$

$$+32\frac{1}{r^6d}\left(\frac{Q_1^2}{f_1^2}-\frac{Q_5^2}{f_5^2}\right)\delta\nu. \tag{69}$$

Equations (60) and (61) lead to another relation

$$\left(\frac{f'}{f}+\frac{6}{r}\right)(h_1'+h_2')=-\frac{8}{3}\left(\frac{f_1'}{f_1}+\frac{f_5'}{f_5}-2\frac{f_K'}{f_K}\right)\delta\lambda'$$

$$+8\left(\frac{f_1'}{f_1}-\frac{f_5'}{f_5}\right)\delta\nu'. \tag{70}$$

From Eqs. (69) and (70), one can obtain

$$\left(\frac{f'}{f}+\frac{6}{r}\right)h_2'=-\frac{4}{3}\left(\frac{f_1'}{f_1}+\frac{f_5'}{f_5}-2\frac{f_K'}{f_K}\right)\delta\lambda'+4\left(\frac{f_1'}{f_1}-\frac{f_5'}{f_5}\right)\delta\nu'$$

$$-\left[\frac{d'}{d}\frac{f'}{f}+\frac{6}{r}\left(\frac{2}{r}+\frac{d'}{d}\right)-\left(\frac{f_1'^2}{f_1^2}+\frac{f_5'^2}{f_5^2}+\frac{f_K'^2}{f_K^2}\right)\right]$$

$$\times h_2-\frac{16}{3}\frac{1}{r^6d}\left(2\frac{Q_K^2}{f_K^2}-\frac{Q_1^2}{f_1^2}-\frac{Q_5^2}{f_5^2}\right)\delta\lambda$$

$$-16\frac{1}{r^6d}\left(\frac{Q_1^2}{f_1^2}-\frac{Q_5^2}{f_5^2}\right)\delta\nu. \tag{71}$$

However, this equation is redundant because it can be obtained by differentiating Eq. (68) with respect to r. All information for h_1 and h_2 are thus encoded in Eq. (68) and (69), which say that h_1 and h_2 are not the independent modes and thus only two fixed scalars are propagating in the 5D black hole background. This can be confirmed from the fact that the relevant value of l should be determined by $l\geq|S|$, $S=$spin. Since the gravitons have spin 2, it is not surprising that they are redundant with the $l=0$ (s-wave) case. Similarly, three U(1) modes with $l=0$ are also redundant because the photon has spin 1. This was clearly shown in Eqs. (47)–(49). Inserting Eqs. (68), (69) into Eqs. (65), (66), we obtain

the following equations:

$$[r^{-3}\partial_r dr^3 \partial_r - d^{-1}f\partial_t^2 + f_{\nu\nu}(r)]\delta\nu + f_{\nu\lambda}(r)\delta\lambda = 0, \tag{72}$$

$$[r^{-3}\partial_r dr^3 \partial_r - d^{-1}f\partial_t^2 + f_{\lambda\lambda}(r)]\delta\lambda + f_{\lambda\nu}(r)\delta\nu = 0, \tag{73}$$

where $f_{\nu\nu}(r), f_{\nu\lambda}(r), f_{\lambda\lambda}(r), f_{\lambda\nu}(r)$ are given by

$$
\begin{aligned}
f_{\nu\nu}(r) = & -\frac{8}{r^2[3r^4 + 2r^2(r_1^2 + r_5^2 + r_K^2) + r_1^2 r_K^2 + r_1^2 r_5^2 + r_5^2 r_K^2]^2}\bigg[3r^4\bigg\{r_1^4 + r_5^4 + r_1^2 r_5^2 + \frac{3}{2}r_0^2(r_1^2 + r_5^2)\bigg\} + 3r^2\{r_5^4 r_K^2 + r_1^4 r_K^2 + r_1^4 r_5^2 \\
& + r_5^4 r_1^2 + 2(r_1 r_5 r_K)^2 + 2r_0^2(r_1^2 r_K^2 + r_1^2 r_5^2 + r_5^2 r_K^2)\} + r_1^4 r_5^4 + r_5^4 r_K^4 + r_1^4 r_K^4 + 2r_1^2 r_5^2 r_K^2(r_1^2 + r_5^2 + r_K^2) + r_0^2\bigg\{3r_1^2 r_5^2 r_K^2 \\
& + \frac{1}{2}r_1^4(r_5^2 + r_K^2) + \frac{1}{2}r_5^4(r_1^2 + r_K^2) + 2r_K^4(r_1^2 + r_5^2)\bigg\}\bigg], \tag{74}
\end{aligned}
$$

$$
\begin{aligned}
f_{\nu\lambda}(r) = & \frac{8}{r^2[3r^4 + 2r^2(r_1^2 + r_5^2 + r_K^2) + r_1^2 r_K^2 + r_1^2 r_5^2 + r_5^2 r_K^2]^2}\bigg[r^4\bigg\{r_1^4 - r_5^4 - r_5^2 r_K^2 + r_1^2 r_K^2 + \frac{3}{2}r_0^2(r_1^2 - r_5^2)\bigg\} \\
& + r^2\{r_1^4(r_5^2 + r_K^2) - r_1^2 r_5^4 - r_5^4 r_K^2\} + \frac{1}{2}r_0^2\{r_1^2 r_5^2(r_5^2 - r_1^2) + r_K^2(r_5^4 - r_1^4)\}\bigg], \tag{75}
\end{aligned}
$$

$$f_{\lambda\nu}(r) = 3f_{\nu\lambda}, \tag{76}$$

$$
\begin{aligned}
f_{\lambda\lambda}(r) = & -\frac{8}{r^2[3r^4 + 2r^2(r_1^2 + r_5^2 + r_K^2) + r_1^2 r_K^2 + r_1^2 r_5^2 + r_5^2 r_K^2]^2}\bigg[r^4\bigg\{r_1^4 + r_5^4 - r_1^2 r_5^2 + 4r_K^4 + 2r_K^2(r_1^1 + r_5^2) + \frac{3}{2}r_0^2(r_1^2 + r_5^2 + 4r_K^2)\bigg\} \\
& + r^2\{r_5^4 r_K^2 + r_1^4 r_K^2 + r_1^4 r_5^2 + r_1^2 r_5^4 + 4r_K^4(r_1^2 + r_5^2) + 6(r_1 r_5 r_K)^2 + 6r_0^2(r_1^2 r_K^2 + r_1^2 r_5^2 + r_5^2 r_K^2)\} + r_1^4 r_5^4 + r_5^4 r_K^4 + r_1^4 r_K^4 \\
& + 2r_1^2 r_5^2 r_K^2(r_1^2 + r_5^2 + r_K^2) + r_0^2\bigg\{3r_1^2 r_5^2 r_K^2 + \frac{3}{2}r_1^4(r_5^2 + r_K^2) + \frac{3}{2}r_5^4(r_1^2 + r_K^2)\bigg\}\bigg]. \tag{77}
\end{aligned}
$$

Note that for $r_1 = r_5 \equiv R$, one finds $f_{\nu\lambda} = f_{\lambda\nu} = 0$. Then Eqs. (72) and (73) reduce to

$$\left[r^{-3}\partial_r dr^3 \partial_r - d^{-1}f\partial_t^2 - \frac{8R^4}{r^2(r^2 + R^2)^2}\left(1 + \frac{r_0^2}{R^2}\right)\right]\delta\nu = 0, \tag{78}$$

$$
\begin{aligned}
&\left[r^{-3}\partial_r dr^3 \partial_r - d^{-1}f\partial_t^2 - \frac{8(R^2 + 2r_K^2)^2}{r^2[3r^2 + (R^2 + 2r_K^2)]^2}\right. \\
&\left.\times\left(1 + \frac{3r_0^2}{R^2 + 2r_K^2}\right)\right]\delta\lambda = 0. \tag{79}
\end{aligned}
$$

We note that Eq. (78) is exactly the same form as Eq. (40). This is so because for $r_1 = r_5$, there is no mixing between graviton and fixed scalar ($\delta\nu$). However, a mixing between graviton and $\delta\lambda$ is still present and thus we obtain the decoupled equation (79) by using Eqs. (68) and (69). We would like to find the fixed scalar equations for the general

($r_1 \neq r_5 \neq r_K$) case. Equations (72) and (73) can be modified with $3f_{\nu\lambda}(r) = f_{\lambda\nu}(r)$ and $\delta\tilde{\lambda} \equiv \delta\lambda/\sqrt{3}$ to

$$[r^{-3}\partial_r dr^3 \partial_r - d^{-1}f\partial_t^2 + f_{\nu\nu}(r)]\delta\nu + \sqrt{3}f_{\nu\lambda}(r)\delta\tilde{\lambda} = 0, \tag{80}$$

$$[r^{-3}\partial_r dr^3 \partial_r - d^{-1}f\partial_t^2 + f_{\lambda\lambda}(r)]\delta\tilde{\lambda} + \sqrt{3}f_{\nu\lambda}(r)\delta\nu = 0. \tag{81}$$

The above can be decoupled by a rotation of the fields,

$$\delta\tilde{\lambda} = (\cos\alpha)\phi_+ + (\sin\alpha)\phi_-, \tag{82}$$

$$\delta\nu = -(\sin\alpha)\phi_+ + (\cos\alpha)\phi_-, \tag{83}$$

where the rotation angle (α) satisfies the relation

$$\tan\alpha - \frac{1}{\tan\alpha} = \frac{1}{\sqrt{3}}\frac{f_{\lambda\lambda}(r) - f_{\nu\nu}(r)}{f_{\nu\lambda}(r)} = \frac{2}{\sqrt{3}}\frac{r_1^2 + r_5^2 - 2r_K^2}{r_1^2 - r_5^2}. \tag{84}$$

From Eq. (84) one obtains

$$\cos^2\alpha = \frac{1}{2} \pm \frac{1}{4} \frac{r_1^2 + r_5^2 - 2r_K^2}{\sqrt{r_1^4 + r_5^4 + r_K^4 - r_1^2 r_5^2 - r_1^2 r_K^2 - r_5^2 r_K^2}}. \quad (85)$$

Then Eqs. (80) and (81) lead to the decoupled equations for $\phi_\pm$,

$$[r^{-3}\partial_r dr^3\partial_r - d^{-1}f\partial_t^2 + \sin^2\alpha f_{\nu\nu} + \cos^2\alpha f_{\lambda\lambda} \\ - 2\sqrt{3}\cos\alpha\sin\alpha f_{\nu\lambda}]\phi_+ = 0, \quad (86)$$

$$[r^{-3}\partial_r dr^3\partial_r - d^{-1}f\partial_t^2 + \cos^2\alpha f_{\nu\nu} + \sin^2\alpha f_{\lambda\lambda} \\ + 2\sqrt{3}\cos\alpha\sin\alpha f_{\nu\lambda}]\phi_- = 0. \quad (87)$$

Here we consider $\phi_\pm(r,t) = \tilde{\phi}_\pm(r)e^{-i\omega t}$ as a mode with energy ω. Inserting Eqs. (85), (74)–(77) into Eqs. (86),(87), we obtain the equations

$$\left[(dr^3\partial_r)^2 + \omega^2 r^6 f - \frac{8dr^4 r_\pm^4}{(r^2 + r_\pm^2)^2}\left(1 + \frac{r_0^2}{r_\pm^2}\right)\right]\tilde{\phi}_\pm = 0, \quad (88)$$

where the effective radii $r_\pm$ are defined as

$$r_\pm^2 = \frac{1}{3}[r_1^2 + r_5^2 + r_K^2 \pm \sqrt{r_1^4 + r_5^4 + r_K^4 - r_1^2 r_5^2 - r_1^2 r_K^2 - r_5^2 r_K^2}]. \quad (89)$$

Equation (88) takes the same form as in Eq. (78). Since it is difficult to find an analytic solution to (88), we patch together a solution between the near region (region I, $r \ll r_1, r_5$), the intermediate region (region II, $r_0 \ll r \ll \omega^{-1}$), and the far region (region III, $r \gg r_1, r_5$). Region II overlaps each of the other two because of $r_0 \ll r_1$, $r_5 \ll \omega^{-1}$. In the dilute gas regime ($r_0, r_K \ll r_1, r_5$), we write down the dominant terms and their approximate solutions in the three regions as

$$\text{I:}[(dr^3\partial_r)^2 + r_1^2 r_5^2(r^2 + r_K^2)\omega^2 - 8r^4 d]\tilde{\phi}_\pm^{\text{I}} = 0,$$

$$\tilde{\phi}_\pm^{\text{I}} = E\frac{r^2}{r_0^2} + G, \quad (90)$$

$$\text{II:}\left[(r^3\partial_r)^2 - 8\frac{r_\pm^4}{(1 + r_\pm^2/r^2)^2}\right]\tilde{\phi}_\pm^{\text{II}} = 0,$$

$$\tilde{\phi}_\pm^{\text{II}} = \frac{C_\pm}{(1 + r_\pm^2/r^2)} + D_\pm\left(1 + \frac{r_\pm^2}{r^2}\right)^2, \quad (91)$$

$$\text{III:}[(r^3\partial_r)^2 + r^6\omega^2]\tilde{\phi}_\pm^{\text{III}} = 0,$$

$$\tilde{\phi}_\pm^{\text{III}} = \alpha_\pm\frac{J_1(\omega r)}{\omega r} + \beta_\pm\frac{N_1(\omega r)}{\omega r}, \quad (92)$$

where $C_\pm, D_\pm, \alpha_\pm, \beta_\pm$ are the unknown constants. The full solution in region I can be expressed in terms of the hypergeometric functions [4], and we present here the limiting form for $r \gg r_0$. E is obtained by the requirement that the solution be purely ingoing at the horizon as

$$E = \frac{2\Gamma(1 - ia - ib)}{\Gamma(2 - ia)\Gamma(2 - ib)}. \quad (93)$$

Here a and b are related to the left and right moving temperatures as

$$a = \frac{\omega}{4\pi T_L}, \quad b = \frac{\omega}{4\pi T_R}. \quad (94)$$

The quantity G may be similarly fixed, but its value is not relevant to us. A matching procedure leads to the relation

$$\alpha_\pm = 2C_\pm = 2E\frac{r_\pm^2}{r_0^2}. \quad (95)$$

The absorption probability is given by the ratio of the incoming fluxes at the horizon ($r = r_0$) and at spatial infinity ($r = \infty$) [2]. The flux per unit solid angle for a field f is given by

$$F = \frac{1}{2i}(f^* dr^3\partial_r f - \text{c.c.}). \quad (96)$$

The absorption probability of $\phi_\pm$ is given by

$$P_{\text{abs}}^{\phi_\pm} = \frac{F_{r_0}}{F_\infty} = 2\pi r_1 r_5\sqrt{r_0^2 + r_K^2}\,\omega^3 \frac{r_0^4}{4|E|^2 r_\pm^4}. \quad (97)$$

Then the absorption cross section is given by

$$\sigma_{\text{abs}}^{\phi_\pm} = \frac{4\pi}{\omega^3}P_{\text{abs}}^{\phi_\pm} = \frac{\pi^3 r_1^6 r_5^6}{64r_\pm^4}\omega(\omega^2 + 16\pi^2 T_L^2) \\ \times (\omega^2 + 16\pi^2 T_R^2)\frac{e^{\omega/T_H} - 1}{(e^{\omega/T_L} - 1)(e^{\omega/T_R} - 1)}, \quad (98)$$

which is the same form as in Ref. [5]. When $r_1 = r_5 = r_+ = R$, one finds the absorption cross section for ν. For $r_1 = r_5 = R$, $r_-^2 = R^2/3$, one gets the cross section for λ.

V. DISCUSSIONS

Let us first discuss the role of a fixed scalar ν. Although ν is related to the scale of T^4, it turns out to be the 10D dilaton (ϕ_{10}) when $\phi_6 = \phi_{10} - 2\nu = 0$. For the $Q_1 = Q_5$ case, one finds the same linearized equation for the harmonic, dilaton gauge, and K-K setting. This means that the fixed scalar (ν)

gives us a gauge-invariant result. In the low energy limit ($\omega \to 0$), the s-wave absorption cross section takes the form

$$\sigma_{\text{abs}}^{\nu} = C \mathcal{A}_H \left(\frac{r_0}{R} \right)^4 , \qquad (99)$$

where $C = 1/4$ for the semiclassical approach from Eq. (98), 1/4 for the effective string method [4], 1/12 for the three-dimensional anti–de Sitter space (AdS$_3$) calculation [18], and 1/4 for the boundary conformal field theory (CFT) calculation [19]. This means that all calculation methods lead to the same result, upto the numerical factors. In the dilute gas limit ($R \ll r_0$), one finds $\sigma_{\text{abs}}^{\nu} \to 0$, whereas $\sigma_{\text{abs}}^{\Phi} = \mathcal{A}_H$ for a minimally decoupled scalar Φ.

On the other hand, $\lambda (= \nu_5 - \phi_6/2)$ is entirely determined by the scale (ν_5) of the KK circle (S^1) when ϕ_6 is turned off. The semiclassical result (98) with $r_-^2 = R^2/3$ takes the form

$$\sigma_{\text{abs}}^{\lambda} = \frac{9}{4} \mathcal{A}_H \left(\frac{r_0}{R} \right)^4 . \qquad (100)$$

On the effective string side, the λ coupling is [5]

$$-\frac{T_{\text{eff}}}{8} \lambda [\partial_+ X \partial_- X \{ (\partial_+ X)^2 + (\partial_- X)^2 \} + (\partial_+ X)^2 (\partial_- X)^2] \qquad (101)$$

plus the fermionic terms. Here $T_{\text{eff}} (= 1/2\pi^2 R^2)$ is the effective string tension. The last term is an operator of dimension (2,2) which also couples to ν-fixed scalar. This gives $\sigma_{\text{abs}}^{\lambda} = (\mathcal{A}_H/16)(r_0/R)^4$. Also there are additional contributions to the cross section which arise from the first two terms.

They have dimensions (3,1) and (1,3). The presence of these gives rise to some disagreement between the semiclassical and effective string cross sections even for $Q_1 = Q_5$.

On the semiclassical calculation, this discrepancy originates from a complicated mixing between λ and other fields. Hence it may depend on the decoupling procedure. In this work we find that λ depends on the gauge choice. For example, one obtains Eq. (41) for the harmonic gauge, Eq. (43) for the dilaton gauge together with $h_2 = h^{\theta_i}{}_{\theta_i} = 0$, and Eq. (79) for K-K setting. Furthermore, substituting Eq. (68) into Eq. (69) leads to

$$\frac{1}{2}(h_1 - h_2)' = \left[\frac{d'}{d} + \frac{12/r^2 - (2f'{}_1^2/f_1^2 + f'{}_K^2/f_K^2)}{f'/f + 6/r} - \frac{4}{r^6 d} \frac{Q_K^2/f_K^2 - Q_5^2/f_1^2}{f'_1/f_1 - f'_K/f_K} \right] h_2 . \qquad (102)$$

This is a result purely from the Einstein's equation. However, it is shown that Eq. (102) is not compatible with either the harmonic gauge condition Eq. (45) or the dilaton gauge condition Eq. (46). Although the K-K setting is a convenient choice for obtaining the decoupled equations, it does not always guarantee a consistent solution.

In conclusion, the fixed scalar ν is clearly understood to be a good test field. However, the role of λ as a test field is obscure because it is a gauge-dependent field and gives rise to some disagreement for the cross section.

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