Gödel universes in string theory

John D. Barrow

Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QJ, United Kingdom

Mariusz P. Dąbrowski

Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QJ, United Kingdom
and Institute of Physics, University of Szczecin, 70-451 Szczecin, Poland

We show that homogeneous Gödel spacetimes need not contain closed timelike curves in low-energy-effective string theories. We find exact solutions for the Gödel metric in string theory for the full $O(\alpha')$ action including both dilaton and axion fields. The results are valid for bosonic, heterotic and super-strings. To first order in the inverse string tension α', these solutions display a simple relation between the angular velocity of the Gödel universe, Ω, and the inverse string tension of the form $\alpha' = 1/\Omega^2$ in the absence of the axion field. The generalization of this relationship is also found when the axion field is present. [S0556-2821(98)07920-X]

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I. INTRODUCTION

The relationship between micro and macrophysics is most commonly sought within a framework of unified descriptions of the fundamental forces of nature that is provided by superstring theories or some M theory believed to underlie them [1], [2–4]. The most frequently investigated theory is the bosonic effective action leading to the pre-big-bang superinflationary scenario [5], although some type IIB superstring models [6] and M-theory solutions have also been found [7]. However, the cosmological solutions of low-energy string theory have been confined to solutions of the zero-order equations, [8], [9]. In this paper, we give some new solutions of string theories, including terms up to first order in the inverse string tension α' and both dilaton and axion fields, for homogeneous spacetimes of the form first investigated in general relativity by Gödel in 1949 [10]. We recall that Gödel's solutions attracted considerable interest because they described rotating universes that possessed the completely unexpected property of closed timelike curves (CTC's).[1] This discovery led to a reappraisal of thinking about Mach's ''principle'' [11], inspired the rigorous investigation of the possibility of time travel in relativistic theories of gravity [14], and stimulated the serious investigation of the global structure of spacetimes [15]. However, we find that in string theory, unlike in general relativity, Gödel universes need not contain CTC's. Previously, causal Gödel universes have been found in general relativity in the presence of massless scalar fields by Rebouças and Tiomno [16],

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[1]It is interesting to note that Weyl, in his book *Space, Time and Matter* [12], foresaw that Einstein's theory might permit time travel to occur, remarking that *[in general relativity] it is not impossible for a world-line (in particular, that of my body), although it has a timelike direction at every point, to return to the neighborhood of a point which it has already passed through....[but] the very considerable fluctuations of the [spacetime metric] would be necessary to produce this effect do not occur in the region of the world in which we live* [12], [13].

and in gravity theories derived from an action containing terms quadratic in the Ricci curvature invariants by Accioly [17]. We find that CTC's need not occur in the Gödel universes in heterotic and bosonic string theories containing the full bosonic particle spectrum of axion, dilaton, and graviton.

II. LOW-ENERGY-EFFECTIVE $O(\alpha')$ ACTION FOR STRINGS

We write the general form of the string effective action to α' order in the string frame as, [4],

$$
\begin{aligned}
S = \int d^n x \sqrt{-g}\, e^{-\phi} \bigg\{ & R - 2\Lambda + (\partial\phi)^2 - \frac{1}{12}H^2 \\
& - \alpha'\lambda_0 \bigg[R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - \frac{1}{2}R^{\mu\nu\sigma\rho}H_{\mu\nu\alpha}H_{\sigma\rho}{}^{\alpha} \\
& + \frac{1}{24}H_{\mu\nu\lambda}H^{\nu}{}_{\rho\alpha}H^{\rho\sigma\lambda}H_{\sigma}{}^{\mu\alpha} \\
& - \frac{1}{8}H_{\mu\rho\lambda}H_{\nu}{}^{\rho\lambda}H^{\mu\sigma\alpha}H^{\nu}{}_{\sigma\alpha} \bigg] + O(\alpha'^2) \bigg\},
\end{aligned}
\tag{2.1}
$$

where $\lambda_0 = -\frac{1}{8}$ for heterotic strings, $-\frac{1}{4}$ for bosonic strings and 0 for superstrings; n is the number of spacetime dimensions, α' is the inverse string tension parameter, and the action is truncated at first order. Thus, in the case of superstrings the leading corrections after the zero-order terms will enter at fourth order and will not be investigated in this paper. These higher-order terms lead to a significant increase in algebraic complexity. In Eq. (2.1) ϕ is the dilaton, g is the determinant of the metric, Λ is the cosmological constant, R is the Ricci scalar, $R_{\mu\nu\rho\sigma}$ the Riemann tensor, and $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$ is the axion with $H^2 = H_{\mu\nu\rho}H^{\mu\nu\rho}$ where $B_{\mu\nu}$ is the antisymmetric tensor potential. The field equations for bosonic strings then take the following form:

$$R_{\mu\beta} + \partial_\mu\partial_\beta\phi - \frac{1}{4}H_{\mu\nu\lambda}H_\beta{}^{\nu\lambda}$$

$$+ \frac{1}{2}\alpha'\left[R_{\mu\nu\sigma\rho}R_\beta{}^{\nu\sigma\rho} - R_\mu{}^{\nu\sigma\rho}H_{\beta\nu\alpha}H_{\sigma\rho}{}^\alpha\right.$$

$$+ \frac{1}{8}H_{\mu\nu\lambda}H^\nu{}_{\rho\alpha}H^{\rho\sigma\lambda}H_{\sigma\beta}{}^\alpha$$

$$\left. - \frac{3}{8}H_{\mu\rho\lambda}H_\nu{}^{\rho\lambda}H_\beta{}^{\sigma\alpha}H^\nu{}_{\sigma\alpha}\right] = 0, \qquad (2.2)$$

$$R - 2\Lambda + 2\partial_\mu\partial^\mu\phi - \partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H^2$$

$$+ \frac{1}{4}\alpha'\left[R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - \frac{1}{2}R^{\mu\nu\sigma\rho}H_{\mu\nu\alpha}H_{\sigma\rho}{}^\alpha\right.$$

$$+ \frac{1}{24}H_{\mu\nu\lambda}H^\nu{}_{\rho\alpha}H^{\rho\sigma\lambda}H_\sigma{}^{\mu\alpha}$$

$$\left. - \frac{1}{8}H_{\mu\rho\lambda}H_\nu{}^{\rho\lambda}H^{\mu\sigma\alpha}H^\nu{}_{\sigma\alpha}\right] = 0. \qquad (2.3)$$

These equations are completed by the axion equation of motion

$$\partial_\mu(\delta S/\delta(\partial_\mu B_{\nu\lambda})) = [e^{-\phi}(H^{\mu\nu\lambda} + \alpha' M^{\mu\nu\lambda})]_{;\mu} = 0, \qquad (2.4)$$

where

$$M^{\mu\nu\lambda} = \frac{3}{2}R^{\mu\nu\sigma\rho}H_{\sigma\rho}{}^\lambda + \frac{1}{4}H_\sigma{}^{\mu\alpha}H^\nu{}_{\rho\alpha}H^{\rho\sigma\lambda}$$

$$- \frac{3}{4}H^{\mu\sigma\alpha}H_\rho{}^{\nu\lambda}H^\rho{}_{\sigma\alpha}. \qquad (2.5)$$

To zeroth order in α', Eq. (2.4) is just $(e^{-\phi}H^{\mu\nu\lambda})_{;\nu} = 0$ (see, e.g., [9]) where the semicolon denotes a covariant derivative with respect to the metric. The cosmological constant term Λ is related to the dimension of space and the inverse string tension by [3,5]

$$\Lambda = \frac{n - 26}{3\alpha'}. \qquad (2.6)$$

The relation (2.6) holds for bosonic strings; for superstrings Λ is proportional to $n - 10$.

III. EXACT STRINGY GÖDEL UNIVERSES

The Gödel metric describes a homogeneous spacetime [10]. Its line element in cylindrical coordinates (t,r,z,ψ) is usually given by either of the two forms [16]

$$ds^2 = -[dt + C(r)d\psi]^2 + D^2(r)d\psi^2 + dr^2 + dz^2, \qquad (3.1)$$

or

$$ds^2 = -dt^2 - 2C(r)dtd\psi + G(r)d\psi^2 + dr^2 + dz^2,$$

where the radial functions have the form

$$C(r) = \frac{4\Omega}{m^2}\sinh^2\left(\frac{mr}{2}\right), \qquad (3.2)$$

$$D(r) = \frac{1}{m^2}\sinh(mr), \qquad (3.3)$$

$$G(r) = \frac{4}{m^2}\sinh^2\left(\frac{mr}{2}\right)\left[1 + \left(1 - \frac{4\Omega^2}{m^2}\right)\sinh^2\left(\frac{mr}{2}\right)\right], \qquad (3.4)$$

with m and Ω constants. In order to *avoid* the existence of CTC's in these spacetimes we require

$$G(r) = D^2(r) - C^2(r) > 0. \qquad (3.5)$$

This is consistent with the "chronology protection conjecture" of Hawking [18]. In a Gödel universe, the four velocity of matter is $u^\alpha = \delta_0^\alpha$ and the rotation vector is $V^\alpha = \Omega\delta_0^3$, while the vorticity scalar is given by $\omega = \Omega/\sqrt{2}$. The original Gödel metric of general relativity, [10], has $m^2 = 2\Omega^2$ and obviously contradicts Eq. (3.5). There has been extensive discussion of the generality and significance of the presence of CTC's in the Gödel metric in general relativity [14], [15].

The only nonvanishing components of the Riemann tensor in an orthonormal frame permitted by the spacetime homogeneity of the Gödel universe are constant, [17], with

$$R_{0101} = R_{0202} = \frac{1}{4}\left(\frac{C'}{D}\right)^2 = \Omega^2,$$

$$R_{1212} = \frac{3}{4}\left(\frac{C'}{D}\right)^2 - \frac{D''}{D} = 3\Omega^2 - m^2, \qquad (3.6)$$

and the prime means the derivative with respect to r. It is interesting to note that with Eq. (3.6) the Gauss-Bonnet term $R_{GB}^2 \equiv R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ vanishes. This term appears in redefined $O(\alpha')$ actions (2.1) (see, e.g., [19]). Because of the metric symmetry we assume that the dilaton depends only on the coordinate along the axis of rotation, z, so

$$\phi = \phi(z) = fz + \phi_0, \qquad (3.7)$$

where f and ϕ_0 are constants. For the axion, a short analysis similar to that given in [9] shows that the only possible *Ansatz* which is consistent with the form (3.7) for the dilaton is

$$H_{012} = -H^{012} = E, \qquad (3.8)$$

with E constant. This can also be expressed in terms of the pseudoscalar axion field, h, [20,9], by

$$h(z) = \frac{E}{f}\exp(-fz - \phi_0) + h_0. \qquad (3.9)$$

The *Ansätze* of Eqs. (3.7) and (3.8) guarantee that the axion's equation of motion is satisfied to zeroth order in α'. Since the axion field's three-form strength is defined by an external derivative of the antisymmetric tensor potential, $B_{\mu\nu}$, one can easily show that the only nonvanishing component of the potential is time dependent

$$B_{12}(t) = Et + t_0. \tag{3.10}$$

This is expected from the discussion of [9,20] since this restriction occurs in every case where the spacetime possesses a distinguished direction. It should then possess $O(n-1,n-1)$ symmetry (despite the fact the metric is not time dependent), which is an example of T duality.

Note the generality of action (2.1): it is given for any spacetime dimension n, and it contains the full spectrum of graviton, axion, and dilaton. It also possesses a general $O(n-1,n-1)$ symmetry [19]. Since this manifests itself in complicated forms in individual solutions, we first discuss two special cases before giving the general Gödel solution for this string theory.

A. Zeroth order in α'

The field equations (2.2) and (2.3) (which are in the string frame) to zeroth order in α', together with Eqs. (3.6)–(3.8) with $n=4$, possess a Gödel solution with metric (3.1)–(3.4) if the following relations hold between the constants Ω, m, E, f, and Λ:

$$\Omega^2 = \frac{m^2}{4} = \frac{E^2}{4} = \frac{-2\Lambda - f^2}{4}. \tag{3.11}$$

When $\Lambda = 0$ the relation (3.11) has no meaning for $m^2 > 0$, but it can be applied for $m^2 \equiv -\mu^2 < 0$, in which case there exists an infinite sequence of both causal and acausal regions (see [16]). We do not consider such solutions here.

From the relations (3.11) one can easily deduce that the axion field must not vanish if a solution is to exist and act as a source of rotation. This is consistent with the known repulsive behavior of the axion acting as a torsion field, which leads to bouncing solutions within the string theory (e.g., Ref. [21]). However, from Eq. (3.11), there is another constraint,

$$\Lambda < -\frac{f^2}{2}, \tag{3.12}$$

which shows that the cosmological term has to be negative. Formally, the dilaton can vanish without disrupting the causal structure of the solution (3.1), but in that case the axion plays the role of a scalar field minimally coupled to gravity—the case already studied in Ref. [16].

These results are quite different from the situation in general relativity with both a scalar field and electromagnetic field present [16]. It is known [20] that the action (2.1) can be transformed to the Einstein frame where the field equations resemble Einstein gravity with source terms given by axion and dilaton fields [20]. Using the *Ansätze* (3.7)–(3.9) we checked the validity of the formulas (6.8), (6.9) of Ref. [16], to obtain

$$\Omega^2 = \frac{m^2}{4} = \frac{f^2}{4} = -2\Lambda. \tag{3.13}$$

We see also that (in analogy to the electromagnetic field that is present there), we cannot admit the axion and obtain a causal model (3.1) in the Einstein frame because there is no way to fulfill the dilaton equation of motion. This might be an artifact of choosing Gödel geometry as a model of the universe and may disappear once one considers different background geometries (see, e.g., Ref. [22]). Note that Λ has to be negative in both frames. For $\lambda_0 = 0$ in Eq. (2.1), these conclusions also hold for superstrings.

B. First order in α', non-zero dilaton and zero axion

Now we add the α' terms to the Eqs. (2.2), (2.3), but neglect the axion. The resulting equations are reduced to three polynomial constraints:

$$2\Omega^2 - 2\alpha'\Omega^4 = 0, \tag{3.14}$$

$$2\Omega^2 - m^2 + \alpha'(10\Omega^4 - 6\Omega^2 m^2 + m^4) = 0, \tag{3.15}$$

$$2\Omega^2 - 2m^2 - 2\Lambda - f^2 + \alpha'(11\Omega^4 - 6\Omega^2 m^2 + m^4) = 0. \tag{3.16}$$

We have three equations and five constants and so two (say, Λ and f) can be chosen arbitrarily.

From Eq. (3.14) we immediately have the simple relation

$$\alpha' = \frac{1}{\Omega^2}, \tag{3.17}$$

which gives the velocity of rotation of the Gödel universe in terms of the inverse string tension. This relation gives a simple connection between micro- and macrophysics in this spacetime, with a balance between string tension and rotation. Using Eq. (3.17), subtracting Eqs. (3.15) and (3.16), we find

$$m^2 + 2\Lambda + f^2 = \Omega^2. \tag{3.18}$$

From Eqs. (3.15) and (3.17) we calculate the possible values of Ω in terms of m to be

$$\Omega_+^2 = \frac{1}{3}m^2, \tag{3.19}$$

or

$$\Omega_+^2 = \frac{1}{4}m^2. \tag{3.20}$$

The case (3.19) allows CTC's [$G(r) < 0$ in Eq. (3.5)], just as in general relativity. For the case (3.20), after using Eq. (3.16), we obtain the relations

$$\Omega^2 = \frac{1}{\alpha'} = \frac{m^2}{4} = \frac{-2\Lambda - f^2}{3}, \quad (3.21)$$

and the condition (3.12) must still hold. This can also be related to the number of dimensions $n = 4$, using Eq. (2.6) to remove Λ from the relation (3.21), so

$$\Omega^2 = \frac{1}{\alpha'} = \frac{m^2}{4} = \frac{3}{35} f^2, \quad (3.22)$$

and the cosmological constant (2.6) must be negative, in agreement with [17].

C. First order in α', non-zero axion and dilaton

In the case of the bosonic string ($\lambda_0 = -1/4$), using Eqs. (2.5), (2.6), the field equations (2.2), (2.3) for the Gödel metric reduce to the three polynomials,

$$2\Omega^2 - \frac{1}{2}E^2 + \frac{1}{2}\alpha'\left[-4\Omega^4 + 4\Omega^2 E^2 + \frac{5}{4}E^4\right] = 0, \quad (3.23)$$

$$2\Omega^2 - m^2 + \frac{1}{2}E^2 + \frac{1}{2}\alpha'\left[20\Omega^4 - 12\Omega^2 m^2\right.$$

$$\left. + 2m^4 - 2E^2(m^2 - 2\Omega^2) - \frac{5}{4}E^4\right] = 0, \quad (3.24)$$

$$2\Omega^2 - 2m^2 - 2\Lambda - f^2 + \frac{1}{2}E^2$$

$$+ \frac{1}{4}\alpha'\left[44\Omega^4 - 24\Omega^2 m^2 + 4m^4\right.$$

$$\left. - 2E^2(m^2 - \Omega^2) - \frac{5}{4}E^4\right] = 0. \quad (3.25)$$

The axion equation of motion (2.4) is also fulfilled in this general case. We now have three equations and six constants, leaving three (say, Λ, f and E) arbitrary.

From Eqs. (3.23) and (3.24) we obtain

$$\Omega^2 = \frac{m^2}{4}. \quad (3.26)$$

This confirms that it is possible to obtain a Gödel solution with no CTC's which fulfills Eq. (3.5) in the general case. The value of α' can now be expressed in terms of the velocity of rotation of the universe, Ω, and the strength of axion field, E, from Eq. (3.23), which gives,

$$\alpha' = \frac{4\Omega^2 - E^2}{(4\Omega^2 + E^2)[\Omega^2 E - (5/4)E^2]}. \quad (3.27)$$

The relation between other constants can be obtained from Eq. (3.25). Similar calculations can also be performed for heterotic strings [with $\lambda_0 = -\frac{1}{8}$ in Eq. (2.1)] which demonstrate the existence of Gödel solutions without CTC's within that theory also. The superstring case ($\lambda_0 = 0$) does not generate a solution of this type to this order in α' because the quadratic correction term vanishes in the action (2.1) and the dominating term in the action is quartic in the Riemann curvature tensor [23].

IV. DISCUSSION

We have found a class of Gödel universes without closed timelike curves within the framework of low-energy-effective string theories. First, to zeroth order in the inverse string tension α', we investigated the situation in both the Einstein frame (as studied already in general relativity in [16]) and in the string frame. We found that the axion cannot be introduced in the Einstein frame but plays a crucial role in the string frame, where it cannot be neglected. Then we extended the analysis to include the full $O(\alpha')$ action with both dilaton and axion taken into account. By including terms of the first order in α' in the field equations, we found that Gödel universes without closed timelike curves are also possible in the general case. Our solutions display a simple relation between the inverse string tension parameter α' and the velocity of rotation of the universe [Eq. (3.17)] which provides a direct link between micro and macrophysics. This is the first class of exact solutions that has been found for a string theory where terms of first order in α' are admitted in the field equations (a method for generating exact solutions in Bianchi type I universes was given by Mueller in Ref. [4]). All the Gödel universes we have found require negative cosmological constant which, in general, can be related to the number of spacetime dimensions if multidimensional cases are also investigated. Our results, obtained for bosonic strings, are also valid for heterotic and superstrings, as is evident from the form of the action, although the numerical values of the constants will change with the value of the parameter λ_0. The simplicity of the final forms of the solutions we have found for Gödel universes suggests that the symmetries of string theory have the power to exclude unwanted peculiarities in the causal structure of spacetime that are permitted by general relativity. They also suggest a path towards finding further exact solutions at higher order in the string tension parameter. Although the Gödel universe does not describe our universe, it does isolate a general feature that rotation can introduce into relativistic spacetimes. In this sense it is physically relevant. Historically, the Gödel universe revealed the possibility of time travel in general relativity. Our analysis does not prove that there cannot be time travel in string theory. However, its absence from Gödel metrics at the order we have examined should act as a stimulus to further investigations at higher order.

ACKNOWLEDGMENTS

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