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New gauge-invariant formulation of the Chern-Simons gauge theory

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A new gauge invariant formulation of the relativistic scalar field interacting with Chern-Simons gauge fields is considered. This formulation is consistent with the gauge fixed formulation. Furthermore, we find that canonical (Noether) Poincaré generators are not gauge invariant even on the constraints surface and do not satisfy the (classical) Poincaré algebra. It is the improved generators, constructed from the symmetric energy-momentum tensor, which are (manifestly) gauge invariant and obey the classical Poincaré algebra. [S0556-2821(98)50122-1]

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Recently, a gauge invariant pertubative analysis using Dirac's dressed matter fields [1] has been of considerable interest in QED and QCD, especially in relation to the infrared divergence and quark confinement problems [2]. In general, there are two approaches in quantum field theory: the gauge invariant formulation (GIF) and the gauge fixed formulation (GFF). The latter is the conventional one, where one chooses a gauge; in the former, on the other hand, one does not fix the gauge, but works with gauge invariant quantities. However as for the formalism itself, it is not clear how the results in the GIF can be matched to the GFF even though this matching is considered in several recent analyzes [3].

Furthermore, a similar gauge independent Hamiltonian analysis [5] in the manner of Dirac [1,4] has been recently considered for the Chern-Simons (CS) gauge theory with matter fields [5]. Actually, after the CS gauge theory was invented, there arose several debates about the gauge dependence of the spin and statistics transmutation phenomena for the charged matter fields, since the analysis was carried out with specific gauge fixing [6–8]. So, with the formulation without gauge fixing, one can expect to resolve this debate, since one is not confined to a specific gauge. But the result of the recent gauge independent analysis for this problem in Ref. [5] is questionable, since there is no room for spin transmutation. This is in sharp contrast to the well-known spin transmutation of GFF [6–8].

In this paper, we shall provide a new gauge invariant Hamiltonian formulation which is consistent with GFF. By introducing a physically plausible assumption, we find a new set of equations for Dirac's dressing function $c_k(\mathbf{x},\mathbf{y})$. Furthermore, we provide, for the first time, a simple interpretation of how the dressing $c_k(\mathbf{x},\mathbf{y})$ is related to the gauge fixing and how GIF is matched to GFF. As a by-product, we find that canonical (Noether) Poincaré generators are not gauge invariant ''even on the constraints surface'' and do not satisfy the (classical) Poincaré algebra. It is the improved generators, constructed from the symmetric (Belinfante) energy-momentum tensor [9], which are (manifestly) gauge invariant and obey the classical Poincaré algebra. This effect is essentially due to the CS term and is important for genuine spin transmutation in the relativistic CS gauge theory. Furthermore, the fact that only the symmetric energy-momentum tensor, not the canonical one, is meaningful is consistent with Einstein's theory of gravity. All results in this paper are at the classical level, but not at the quantum level.

Our model is the Abelian CS gauge theory with massive relativistic complex scalars [5,6]

$$\mathcal{L}=\frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho+(D_\mu\phi)^*(D^\mu\phi)-m^2\phi^*\phi, \quad (1)$$

where $\epsilon^{012}=1$, $g_{\mu\nu}=\mathrm{diag}(1,-1,-1)$, and $D_\mu=\partial_\mu+iA_\mu$. $\mathcal{L}$ is invariant up to the total divergence under the gauge transformations $\phi\to\exp[-i\Lambda]\phi$, $A_\mu\to A_\mu+\partial_\mu\Lambda$, where Λ is a well-behaved function such that $\epsilon^{\mu\nu\lambda}\partial_\mu\partial_\nu\Lambda=0$. By using the basic (equal time) brackets (called Faddeev-Jackiw (FJ) brackets [10])

$$\{A^i(\mathbf{x}),A^j(\mathbf{y})\}=\frac{1}{\kappa}\epsilon^{ij}\delta^2(\mathbf{x}-\mathbf{y}),$$

$$\{\phi(\mathbf{x}),\pi(\mathbf{y})\}=\{\phi^*(\mathbf{x}),\pi^*(\mathbf{y})\}=\delta^2(\mathbf{x}-\mathbf{y}),\ \text{others vanish} \quad (2)$$

with $\pi=(D_0\phi)^*$, $\pi^*=D_0\phi$, there remains the constraint $T\equiv J_0-\kappa B\approx 0$ *á la* FJ, where J_0 is the time component of the conserved matter current $J_\mu=i[(D_\mu\phi)^*\phi-\phi^*D_\mu\phi]$ and $B=\epsilon_{ij}\partial_i A^j$ is the magnetic field. Here, we note that in the FJ brackets method, $T\approx 0$ is the only constraint and the primary constraints of the Dirac brackets method, $\pi_0\approx 0$, $\pi_i-(\kappa/2)\epsilon_{ij}A^j\approx 0$, need not be introduced.

Now in order to develop the manifestly gauge invariant Hamiltonian formulation, we introduce the gauge invariant matter and gauge fields

$$\hat{\phi}(\mathbf{x})\equiv\phi(\mathbf{x})\exp(iW),\quad \hat{\pi}(\mathbf{x})\equiv\pi(\mathbf{x})\exp(-iW),$$

$$\mathcal{A}_\mu(\mathbf{x})\equiv A_\mu(\mathbf{x})-\partial_\mu W, \quad (3)$$

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and their complex conjugates with $W=\int c_k(\mathbf{x},\mathbf{z})A^k(\mathbf{z})d^2z$. The Dirac dressing function $c_k(\mathbf{x},\mathbf{z})$ satisfies

$$\partial_z^k c_k(\mathbf{x},\mathbf{z}) = -\delta^2(\mathbf{x}-\mathbf{z}). \tag{4}$$

Here, we note that there are infinitely many solutions of c_k which satisfy (4) and the gauge invariance of fields in Eq. (3) should be understood on each solution hypersurface, but not on the entire solution space. Furthermore, it should be noted that at the quantum level, due to the commutation relation $[A^i{}_{op}(\mathbf{y}),\hat{\phi}_{op}(\mathbf{x})]=(\hbar/\kappa)\hat{\phi}_{op}(\mathbf{x})\epsilon_{ik}c_k(\mathbf{x},\mathbf{y})$, the gauge invariant operator $\hat{\phi}_{op}(\mathbf{x})$ creates one charged $([J_0(\mathbf{y}),\hat{\phi}_{op}(\mathbf{x})]=\delta(\mathbf{x}-\mathbf{y})\hat{\phi}_{op}(\mathbf{x}))$ particle together with the gauge varying vector field $a^i(\mathbf{y})=(\hbar/\kappa)\epsilon_{ik}c_k(\mathbf{x},\mathbf{y})$, as well as the gauge invariant point magnetic field $b(\mathbf{y})(=\epsilon_{ij}\partial_i a^j)=(1/\kappa)\delta^2(\mathbf{y}-\mathbf{x})$. [This situation is in contrast to the QED case where $\hat{\phi}_{op}(\mathbf{x})$ creates the gauge invariant (physical) electron together with only the gauge invariant electric field [1,2].] Now, returning to the classical level, with Eq. (3), the Poincaré generators which being (manifestly) gauge invariant and satisfying the Poincaré algebra become

$$\begin{aligned}
P_s^0 &= \int d^2x[|\hat{\pi}|^2+|\mathcal{D}^i\hat{\phi}|^2+m^2|\hat{\phi}|^2],\\
P_s^i &= \int d^2x[\hat{\pi}\mathcal{D}^i\hat{\phi}+(\mathcal{D}^i\hat{\phi})^*\hat{\pi}^*],\\
M_s^{12} &= \int d^2x\,\epsilon_{ij}x^i[\hat{\pi}\mathcal{D}^j\hat{\phi}+(\mathcal{D}^j\hat{\phi})^*\hat{\pi}^*],\\
M_s^{0i} &= x^0P_s^i-\int d^2x\,x^i[|\hat{\pi}|^2+|\mathcal{D}^j\hat{\phi}|^2+m^2|\hat{\phi}|^2],
\end{aligned} \tag{5}$$

which are expressed only by the gauge invariant fields and $\mathcal{D}_i\equiv\partial_i+i\mathcal{A}_i$. These are *improved* generators following the terminology of Callan *et al.* [11] constructed from the symmetric (Belinfante) energy-momentum tensor [9]. Note that as far as we are interested in the dynamics of the physically relevant fields of Eq. (3), there are no additional terms proportional to constraints in the Poincaré generators of Eq. (5) [4,5]. Next, let us consider the transformations generated by Poincaré generators (5) for the gauge invariant quantities of Eq. (3). First of all, we consider the spatial translation generated by $\{\hat{\phi}(\mathbf{x}),P_s^j\}=\partial^j\hat{\phi}(\mathbf{x})-i\hat{\phi}(\mathbf{x})\int d^2z(\partial_z^j c_k(\mathbf{x},\mathbf{z})+\partial_x^j c_k(\mathbf{x},\mathbf{z}))A^k(\mathbf{z})$, where we have dropped the terms $\int d^2z\partial_z^i[c_k(\mathbf{x},\mathbf{z})A^k(\mathbf{z})]$ and $\int d^2z\partial_z^k[c_k(\mathbf{x},\mathbf{z})A^j(\mathbf{z})]$, which vanish for sufficiently rapidly decreasing integrand. This shows the translational anomaly (following the terminology of Hagen *et al.* [6–8], ''anomaly'' means an unconventional contribution). However, we assume that this anomaly should not appear in order that $\hat{\phi}$ responds conventionally to translations. This assumption is motivated by the fact that usual local fields have no translational anomaly, regardless of their spin or other properties. With this assumption, we obtain the condition that $c_k(\mathbf{x},\mathbf{z})$ be translationally invariant $\partial_z^i c_k(\mathbf{x},\mathbf{z})=-\partial_x^i c_k(\mathbf{x},\mathbf{z})$, i.e., $c_k(\mathbf{x},\mathbf{z})=c_k(\mathbf{x}-\mathbf{z})$. Furthermore, this condition also guarantees the correct spatial translation law for all other gauge invariant fields in Eq. (3): $\{\mathcal{F}_\alpha(\mathbf{x}),P_s^j\}=\partial^j\mathcal{F}_\alpha(\mathbf{x})$, $\mathcal{F}_\alpha=(\mathcal{A}_\mu,\hat{\phi},\hat{\phi}^*)$.

By applying similar assumption to the time translation, we obtain $\{\mathcal{F}_\alpha(\mathbf{x}),P_s^0\}=\partial^0\mathcal{F}_\alpha(\mathbf{x})$ and using $\int d^2z\partial_z^k[c_k(\mathbf{x}-\mathbf{z})A^0(\mathbf{z})]=0$, we further obtain the condition that $c(\mathbf{x}-\mathbf{z})$ be time independent. However, for the rotation and Lorentz boost, the anomaly is present, since in that case it represents the spin or other properties of $\mathcal{F}_\alpha$. The bracket with the Lorentz generator is expressed as

$$\begin{aligned}
\{\mathcal{F}_\alpha(\mathbf{x}),M_s^{\mu\nu}\} &= x^\mu\partial^\nu\mathcal{F}_\alpha(\mathbf{x})-x^\nu\partial^\mu\mathcal{F}_\alpha(\mathbf{x})+\Sigma_{\alpha\beta}^{\mu\nu}\mathcal{F}_\beta(\mathbf{x})\\
&\quad+\Omega_\alpha^{\mu\nu}(\mathbf{x}),\\
\Omega_{\hat{\phi}}^{\mu\nu}(\mathbf{x}) &= -i\Xi^{\mu\nu}(\mathbf{x})\hat{\phi}(\mathbf{x}),\\
\Omega_{\hat{\phi}^*}^{\mu\nu}(\mathbf{x}) &= i\Xi^{\mu\nu}(\mathbf{x})\hat{\phi}^*(\mathbf{x}),\\
\Omega_{\mathcal{A}_\beta}^{\mu\nu}(\mathbf{x}) &= \partial_\beta\Xi^{\mu\nu}(\mathbf{x}),
\end{aligned} \tag{6}$$

where $\Xi^{\mu\nu}=-\Xi^{\nu\mu}$, $\Xi^{12}(\mathbf{x})=\epsilon_{ij}x_i\mathcal{A}^j(\mathbf{x})+(1/\kappa)\int d^2z z_k c_k(\mathbf{x}-\mathbf{z})J_0(\mathbf{z})$, $\Xi^{0i}(\mathbf{x})=-x_i\mathcal{A}^0(\mathbf{x})-(1/\kappa)\int d^2z z_i\epsilon_{kj}c_k(\mathbf{x}-\mathbf{z})J^j(\mathbf{z})$. The anomalous term $\Omega_\alpha^{\mu\nu}$ is gauge invariant. At first, it seems odd that the gauge invariant quantities do have the anomaly, but as will be clear later, these quantities are nothing, but the Hagen's rotational anomaly term and other gauge restoring terms in GFF. Before establishing this, it is interesting to note that $\mathcal{A}_\mu$ can be reexpressed completely by the matter currents as $\mathcal{A}_i(\mathbf{x})\approx-(1/\kappa)\int d^2z\epsilon_{ik}c_k(\mathbf{x}-\mathbf{z})J^0(\mathbf{z})$, $\mathcal{A}_0(\mathbf{x})=-(1/\kappa)\int d^2z\epsilon_{kj}c_k(\mathbf{x}-\mathbf{z})J^j(\mathbf{z})$ using the constraint $T\approx0$ and the Euler-Lagrange equation of Eq. (1), $\epsilon_{kj}J^j=F^{0k}$, respectively. These solutions are similar to the Coulomb gauge solution [6] and hence imply the similarity of GIF to GFF with the Coulomb gauge. However it should be noted that the Lorentz anomaly does not occur in the transformation of the current J^μ, even though $\mathcal{A}_\mu$, which is expressed by J^μ as given above, does have the anomaly.

Now, let us consider the basic brackets between the gauge invariant fields

$$\begin{aligned}
\{\mathcal{A}_i(\mathbf{x}),\mathcal{A}_j(\mathbf{y})\} &= \frac{1}{\kappa}[\epsilon_{ij}\delta^2(\mathbf{x}-\mathbf{y})+\xi_{ij}(\mathbf{x}-\mathbf{y})\\
&\quad+\partial_i^x\partial_j^y\Delta(\mathbf{x}-\mathbf{y})],\\
\{\hat{\phi}(\mathbf{x}),\hat{\phi}(\mathbf{y})\} &= -\hat{\phi}(\mathbf{x})\hat{\phi}(\mathbf{y})\frac{1}{\kappa}\Delta(\mathbf{x}-\mathbf{y}),\\
\{\hat{\phi}(\mathbf{x}),\hat{\phi}^*(\mathbf{y})\} &= -\hat{\phi}(\mathbf{x})\hat{\phi}^*(\mathbf{y})\frac{1}{\kappa}\Delta(\mathbf{x}-\mathbf{y}),\\
\{\mathcal{A}_i(\mathbf{x}),\hat{\phi}(\mathbf{y})\} &= -\frac{i}{\kappa}\hat{\phi}(\mathbf{y})[\epsilon_{ik}c_k(\mathbf{y}-\mathbf{x})+\partial_i^x\Delta(\mathbf{x}-\mathbf{y})].
\end{aligned} \tag{7}$$

Here $\Delta(\mathbf{x}-\mathbf{y})=\int d^2z\epsilon^{kj}c_k(\mathbf{x}-\mathbf{z})c_j(\mathbf{y}-\mathbf{z})$ and $\xi_{ij}(\mathbf{x}-\mathbf{y})=\epsilon_{ik}\partial_j^y c_k(\mathbf{y}-\mathbf{x})+\epsilon_{kj}\partial_i^x c_k(\mathbf{x}-\mathbf{y})$. These results, together with the fact that corresponding quantum operator $\hat{\phi}_{op}(\mathbf{x})$

creates the charged scalar particle together with the point magnetic flux at the point $\mathbf{x}$, seem to show resemblance of $\hat{\phi}$ to the *anyon field* [12], but it is found that this is not the case [6,13].

Next, we consider how the gauge invariant results are matched to gauge fixed results. Actually, this is connected with the gauge independence of the Poincaré algebra. The master formula for the matching is, as can be easily proved,

$$\{L_a, L_b\} \approx \{L_a, L_b\}_{D_\Gamma}. \tag{8}$$

Here L_a is any gauge invariant quantity, where bracket with the first class constraints T of Eq. (4) vanishes, $\{L_a, T\} \approx 0$. The left-hand side of the formula (8) is the basic bracket of L_a's. The right-hand side of Eq. (8) is the Dirac bracket with gauge fixing function $\Gamma = 0$, $\det|\{\Gamma, T\}| \neq 0$. Moreover, in the latter case, since $\Gamma = 0$ can be strongly implemented, L_a can be replaced by $L_a|_\Gamma$ that represents the projection of L_a onto the surface $\Gamma = 0$. (This formula is implicit already in the recently developed Batalin-Fradkin-Tyutin formalism [14].) Moreover, the left-hand side is gauge independent by construction, since L_a's and the the basic bracket algebra (2) are introduced gauge independently. On the other hand, the Dirac bracket [4] depends explicitly on the chosen gauge Γ in general. But there is one exceptional case, i.e., when the Dirac bracket is considered for the gauge invariant quantities. Our master formula (8) explicitly show this exceptional case: The Dirac bracket for the gauge invariant quantities L_a or their projection $L_a|_\Gamma$ on the surface $\Gamma = 0$ are still gauge invariant and equal to the basic bracket for the corresponding quantities. Another important thing for the matching is to know how the defining equation (4) for $c_k(\mathbf{x} - \mathbf{y})$ is modified in GFF. By considering $\hat{\phi}$ in a specific gauge and the residual gauge transformation of ϕ and A_i, one can find modified (but still make $\hat{\phi}$ be gauge invariant) equation for $c_k(\mathbf{x} - \mathbf{y})$. We provide here three typical cases: (a) Coulomb gauge ($\partial^i A_i \approx 0$): $\int d^2 z c_j(\mathbf{x} - \mathbf{z}) A^j(\mathbf{z}) = 0$, (b) Axial gauge ($A_1 \approx 0$): $\partial_z^2 c_2(\mathbf{x} - \mathbf{z}) = -\delta^2(\mathbf{x} - \mathbf{z})$, (c) Weyl gauge ($A_0 \approx 0$): $\partial_z^j c_j(\mathbf{x} - \mathbf{z}) = -\delta^2(\mathbf{x} - \mathbf{z})$. These results are generally valid for any other gauge theories when they are formulated by our gauge invariant formulation. Note that these results are different from recent claims of Refs. [2,3] except in the case of Coulomb gauge [15]. Moreover, the Weyl gauge does not modify the equation for c_k from Eq. (4). Then, using these relations and Eq. (8), we could consider the gauge fixed results directly from GIF. However, as can be observed in these examples, gauge fixings restrict the solution space in general. Therefore, all the quantities which appear in Eq. (6) are gauge invariant for each solution hypersurface which is selected by gauge fixing, but their functional form may be different depending on the gauges. Here we show the case of Coulomb gauge, which is found to have a special meaning. In this case we find, using $c^j(\mathbf{x} - \mathbf{z}) = -(1/2\pi)(\mathbf{x} - \mathbf{z})^j / |\mathbf{x} - \mathbf{z}|^2$ which solves the equation in "(a)",

$$\Xi^{12}(\mathbf{x}) = \frac{1}{2\pi\kappa} Q,$$

$$\Xi^{0i}(\mathbf{x}) = \frac{1}{2\pi\kappa} \int d^2 z \, \frac{(\mathbf{x} - \mathbf{z})^i (\mathbf{x} - \mathbf{z})^k}{|\mathbf{x} - \mathbf{z}|^2} \epsilon_{jk} J^j(\mathbf{z}), \tag{9}$$

where $Q = \int d^2 z J_0$ [16]. This is exactly Hagen's rotational anomaly and Coulomb gauge restoring term in the Lorentz transformation, respectively [6]. Furthermore, the basic brackets defined in Eq. (7) is found to be the usual Dirac brackets in the Coulomb gauge by noting $\Delta(\mathbf{x} - \mathbf{y}) = 0$ and $\xi_{ij}(\mathbf{x} - \mathbf{y}) = -\epsilon_{ij} \delta^2(\mathbf{x} - \mathbf{y})$ in this case. [Upon using $c^j(\mathbf{x} - \mathbf{z}) = \partial_z^j (1/2\pi) \ln|\mathbf{x} - \mathbf{z}|$ and performing the integration by parts, we obtain $\Delta(\mathbf{x} - \mathbf{y}) = -(1/2\pi) \oint_{S^1_{R \to \infty}} d\theta \hat{\theta} \cdot \hat{r} \ln R = 0$, where the integration is evaluated on a circle with infinite radius R, polar angle θ, and their corresponding (orthogonal) unit vectors $\hat{r}, \hat{\theta}$. Moreover, using the antisymmetry $c^j(\mathbf{x} - \mathbf{z}) = -c^j(\mathbf{z} - \mathbf{x})$ and Eq. (4), the expression for ξ_{ij} given above can be verified.] This implies that the gauge invariant operator $\hat{\phi}_{op}$ satisfies the boson commutation relation, $[\hat{\phi}_{op}(\mathbf{x}), \hat{\phi}_{op}(\mathbf{y})] = 0$ in this case. Here, we note the special importance of the Coulomb gauge in that the original fields ϕ, ϕ^*, A_μ themselves are already gauge invariant fields such that they already have the full anomaly structures of Eq. (6). Furthermore, this gauge is the simplest one to obtain the anomalous spin of the original matter field ϕ as Ξ^{12} of Eq. (9), since this does not have other gauge restoring terms as in the rotationally nonsymmetric gauge. This is made clear by noting the relation [5,6] $M_s^{12} \approx M_c^{12} - (\kappa/2) \int d^2 z \partial^k (z^k A^l A^l - z^l A^l A^k)$, where M_c^{12} is the canonical angular momentum

$$M_c^{12} = \int d^2 z \{ \epsilon_{lk} z^l [\pi \partial^k \phi + (\partial^k \phi)^* \pi^*] - \kappa z^l A^l (\partial^k A^k) + (\kappa/2) \partial^k (z^l A^l A^k) \}.$$

The surface terms in $M_s^{12} - M_c^{12}$ and M_c^{12}, which are gauge invariant for the rapidly decreasing gauge transformation function Λ, give the gauge independent spin terms "$(1/4\pi\kappa) Q^2$" [6–8] (unconventional) and "0" (conventional) in M_s^{12}, respectively. [Explicit manipulations of the gauge independence of the unconventional term have been established only for limited class of gauges [6–8]. But these results are generalized to the case of general gauges due to the gauge invariance of the term.] From which the anomalous spin $Q/2\pi\kappa$ of Eq. (9) for the matter field is readily seen to follow for general gauges [17]. On the other hand, the second term of M_c^{12}, which vanishes only in the Coulomb gauge, gives for the general gauges the gauge restoring contribution to the rotation transformation for the matter field.

Before completing our analysis, we note that the canonical (Noether) Poincaré generators cannot be considered as the physical ones, since the canonical boost generators $M_c^{0i} \approx M_s^{0i} + (\kappa/2) \int d^2 z A^0 \epsilon_{ij} A^j$ are not gauge invariant due to the last gauge variant term "even on the constraint surface" and do not satisfy the (classical) Poincaré algebra:

$$\{M_c^{0i},M_c^{12}\}\approx-\epsilon_{ij}M_c^{0j}+\frac{\kappa}{2}\epsilon_{ij}\int d^2z\,\partial^l(\epsilon_{kl}z^kA^0A^j),$$

$$\{M_c^{0i},M_c^{0j}\}\approx-\epsilon_{ij}M_c^{12}-\epsilon_{ij}\frac{1}{4\pi\kappa}Q^2+\epsilon_{ij}\frac{\kappa}{2}\int d^2z\left(\frac{5}{2}A_0^2+A^kA^k+\partial^0(z^kA^0A^k)\right). \tag{10}$$

It is the improved generators (5), constructed from the symmetric energy-momentum tensor, which are (manifestly) gauge invariant and obey the classical Poincaré algebra. Hence, the improved generators (5) have a unique meaning consistently with Einstein's theory of gravity [18]. From this fact, it is seen that the anomalous spin of the relativistic matter, which comes only from M_s^{12}, is not artificial, contrary to recent claim of Graziano and Rothe [6]. Furthermore, this uniqueness of anomalous spin is in contrast to the anomalous statistics, which has only artificial meaning in this case. This is because we can obtain in any field theories [13] any *arbitrary* statistics by constructing gauge invariant exotic operator of the form of Semenoff and its several variations [6]. In this sense the relativistic CS gauge theory does not respect the *spin-statistics* relation [12] in agreement with Hagen's result [6,13]. Here, we add that the situation of nonrelativistic CS gauge theory is not better than this relativistic case. This is because even though the anomalous statistics is uniquely defined by removing the gauge field (in this case the gauge field is pure gauge due to point nature of the sources in nonrelativistic quantum field theory) the anomalous spin has no unique meaning [6,7].

In summary, we have considered a new GIF consistent with GFF. Our formalism is new in the following three points. (A) We introduced the assumption that there be no translation transformation anomaly for gauge invariant quantities $\mathcal{F}_\alpha$. From this assumption, we obtained several new conditions for the dressing function $c_k(\mathbf{x},\mathbf{z})$, which are crucial in our development. (B) We introduced the master formula (8), which allowed matching to the gauge fixed system. (C) We found and used the manner how the equation of the dressing function $c_k(\mathbf{x},\mathbf{z})$ are modified after gauge fixing. Using this formulation, we have obtained a novel GIF, which is consistent with the conventional GFF: The former formulation provides exactly the rotational anomaly of the latter. Hence, in our formulation there is no inconsistency, as in the previous gauge independent formulation of Ref. [5]. As a byproduct, we explicitly found that the anomalous spin of the charged matter has a unique meaning [6]. This is due to the uniqueness of the Poincaré generators when constructed from the symmetric energy-momentum tensor.

We would like to conclude with three additional comments. First, in our formulation, there is no gauge noninvariance problem of Poincaré generators on the physical states. This is essentially due to absence of additional terms proportional to constraints in the generators of Eq. (5), in contrast to the old formulation of Dirac [1]. Second, the master formula (8), which guarantees the classical Poincaré covariance of our CS gauge theory in all gauges, also works in all other gauge theories. Hence, as far as the *gauge dependent* operator ordering problem does not occur, the *quantum* Poincaré covariance for one gauge guarantees also the covariance for all other gauges. The gauge independent proof of quantum covariance has been an old issue in quantum field theory and now it is reduced to the solvability of the problem of the gauge dependent operator ordering. Finally, it has been recently reported that canonical Poincaré generators in QED or QCD also do not satisfy the Poincaré algebra. But it's origin is different from ours [2].

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[15] The authors of Refs. [2,3] considered $\int d^2z c_j(\mathbf{x}-\mathbf{z})A^j(\mathbf{z})=0$ even ''(b)'' and ''(c)'' cases. But then, the manifestly gauge invariant fields in Eq. (3) are not gauge invariant under the residual gauge symmetries $\phi \rightarrow e^{-i\Lambda}\phi$, $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$ with x^1 and x^0 independent Λ for ''(b)'' and ''(c),'' respectively.

[16] In the Weyl gauge, the simplest solution is $c_1=0$, $c_2(\mathbf{x}) = \delta(x^1)\epsilon(x^2)$ with a step function $\epsilon(x)$. This corresponds to a different solution hypersurface to the Coulomb gauge and hence, it is related anomalous term s in Eq. (6) have different functional form to Eq. (9) even though they are gauge invariant on its own hypersurface: $\Xi^{12}=(1/\kappa)\int_{-\infty}^{\infty} dy^2 J_0(x^1,y^2)$, $\Xi^{02}=(1/\kappa)\int_{-\infty}^{\infty} dy^2 J^1(x^1,y^2)$, $\Xi^{01}=0$.

[17] This is because the commutation relation $\{Q,\phi(x)\}= -i\,\phi(x)$ is gauge independent in the general gauges $\int d^2z K_\mu(\mathbf{x},\mathbf{z})A^\mu(\mathbf{z})\approx 0$ with kernel $K_\mu(\mathbf{x},\mathbf{z})$.

[18] There may be other differently improved generators depending on what gravity theory is chosen like as in Ref. [11]. But we do not consider this possibility in this paper.