CP-odd interaction of axion with matter

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In a world with an axion the precise measurement of gravitational forces and checks of the equivalence principle at small distances, $\lambda \sim 1$ cm and less, could provide an additional test of CP symmetry. Using the chiral approach, we connect the strength of the CP-odd axion-nucleon vertex with the CP-odd pion-nucleon coupling for color electric dipole moments of quarks as a low-energy source of CP violation. Strong limits on CP-odd coupling $g_{\pi NN}$ coming from atomic electric dipole moments give the best limits on the long range force mediated by axions, $F_{\rm axion}/F_{\rm gravity} < 2\cdot 10^{-10}(1\ {\rm cm}/\lambda)^2$. [S0556-2821(98)02419-9]

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The strong CP problem is one of the most intriguing issues of modern particle physics. The additional term in the QCD Lagrangian $\mathcal{L} = \theta(g^2/32\pi^2)G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$ violates P and CP symmetries [1]. In electroweak theory, the diagonalization of quark mass matrices M_u and M_d involves chiral rotations and brings additional contributions to the theta term $\bar\theta = \theta + \arg(\det M_u M_d)$. Current experimental limits on the electric dipole moment (EDM) of the neutron [2] put a severe constraint on $\bar\theta$ parameter [3], $\bar\theta < 3\times 10^{-10}$.

The puzzling smallness of $\bar\theta$ in comparison with a naive expectation $\theta \sim 1$ is usually referred to as the strong CP problem. It could be solved theoretically in different manners. One can postulate that the starting point for the θ parameter is zero and mass matrices are Hermitean at some high-energy scale due to some symmetry reason [4]. The evaluation down to the electroweak scale must keep radiative corrections to $\bar\theta$ under the level of 10^{-9}. This scenario can hardly accommodate any sizable CP-violating phase beyond CP violation coming from Yukawa matrices (except, maybe, the lepton sector). In contrast, the existence of an axion mechanism allows the dynamical relaxation of $\bar\theta$ [5,6] *and* leaves room for large CP-violating effects induced by the effective operators dim$\geqslant 5$. The combination of CP violation and Peccei-Quinn (PQ) symmetry leads to an interesting phenomenon, the long-range interaction mediated by an axion [7]. This is shown schematically in Fig. 1. The axion mass of 10^{-5} eV corresponds to a characteristic length for this interaction of about 2 cm. It mimics gravity at these distances and therefore could be traceable in the measurement of gravitational forces at small distances [8]. Moreover, this new interaction needs not respect the equivalence principle and can show up in a precise Eötvös-Braginsky type of experiment in the sub-cm region [9].

In the standard model, though, these effects are known to be negligibly small [7,10]. In different modifications of supersymmetric models with new CP-violating sources besides the conventional Kobayashi-Maskawa phase there is a certain hope that the new long-range interaction can be detected by new precision measurements of gravitational forces at small distances [11,12]. Using the naive dimensional analysis (NDA), the authors of Ref. [12] connected the strength of the axion mediated force with maximal CP violation allowed by the limit on the electric dipole moment of neutron. Their result suffers from a rather large uncertainty (four orders of magnitude) and do not include the analysis of possible equivalence principle violation. In this paper we calculate the strength of the axion-matter coupling and the dynamically induced theta term using chiral perturbation theory and some elements of the QCD sum rules. We connect this vertex with the CP-odd pion-nucleon coupling constant, strongly limited from the experiments searching for EDMs of diamagnetic atoms [13]. (For a detailed analysis of this coupling, see Ref. [14].)

Motivated by different supersymmetric models, we chose color EDMs (CEDMs) as the most important effective operators and neglect possible effects from Weinberg three gluon operator and four-fermion type interactions. Then the relevant part of the effective Lagrangian at the scale of 1 GeV is

$$\mathcal{L}_{eff} = +\frac{a}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + i\sum_{i=u,d,s}\frac{\tilde{d}_i}{2}\bar{q}_i g t^a G^a_{\mu\nu}\sigma_{\mu\nu}\gamma_5 q_i, \quad (1)$$

were a is the axion field and f_a is the axion decay constant. In the absence of CP violation, nonremovable by PQ transformation, PQ symmetry sets theta parameter to zero [5]. The situation is different in the presence of extra CP-violating sources, communicated by the operators dim $\geqslant 5$. Thus, CEDMs of quarks drive the theta parameter from zero to a value given by a ratio of two correlators:

$$\theta_{\rm eff} = -\frac{K_1}{|K|}, \quad \text{where}$$

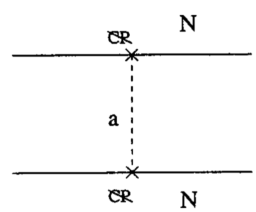

FIG. 1. The symbolic picture of axion exchange between two nucleons with CP violation in both vertices.

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$$K=i\left\{\int dx e^{ikx}\left\langle 0\left|T\left(\frac{\alpha_s}{8\pi}G\tilde{G}(x),\frac{\alpha_s}{8\pi}G\tilde{G}(0)\right)\right|0\right\rangle\right\}_{k=0},$$

$$K_1=i\left\{\int dx e^{ikx}\left\langle 0\left|T\left(\frac{\alpha_s}{8\pi}G\tilde{G}(x),i\sum_i\frac{\tilde{d}_i}{2}\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\times\bar{q}_i g(G\sigma)\gamma_5 q_i(0)\right)\right|0\right\rangle\right\}_{k=0} \qquad (2)$$

and $G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}\equiv G\tilde{G}$, $(G\sigma)\equiv t^a G^a_{\mu\nu}\sigma_{\mu\nu}$. The nonzero value of $\theta_{\rm eff}$, in its turn, induces CP-violating axion-nucleon coupling [7]:

$$g_{aNN}=\frac{\theta_{\rm eff}}{f_a}\left\langle N\left|\frac{m_d m_s m_u}{m_s m_d+m_s m_u+m_d m_u}(\overline{uu}+\overline{dd}+\overline{ss})\right|N\right\rangle. \qquad (3)$$

Even though we are already able to set a constraint on $\bar{\theta}$, it is still instructive to calculate K_1 and $\theta_{\rm eff}$ along the same lines as K and $m_a^2=-K/f_a^2=(m_\pi^2 f_\pi^2)/f_a^2\ \ m_u m_d/(m_u+m_d)^2$ were calculated in Ref. [15]. For the case of $m_u=m_d$ the explicit derivation of K_1 can be found in Ref. [16]. Using the anomaly equation in the following form:

$$\partial_\mu\frac{m_d m_s\bar{u}\gamma_\mu\gamma_5 u+m_u m_s\bar{d}\gamma_\mu\gamma_5 d+m_u m_d\bar{s}\gamma_\mu\gamma_5 s}{m_s m_d+m_s m_u+m_d m_u}=\frac{2m_u m_d m_s}{m_s m_d+m_s m_u+m_d m_u}(\bar{u}\gamma_5 u+\bar{d}\gamma_5 d+\bar{s}\gamma_5 s)+\frac{\alpha_s}{4\pi}G\tilde{G}, \qquad (4)$$

we apply the standard technique of current algebra. The correlator of interest K_1 can be rewritten in the form of the equal-time commutator, which we calculate easily, plus a term containing the singlet combination of pseudoscalars built from quark fields:

$$K_1=-\frac{1}{2}\left\langle 0\left|\frac{m_d m_s m_u}{m_s m_d+m_s m_u+m_d m_u}\left(\frac{\tilde{d}_u}{m_u}\bar{u}g(G\sigma)u+\frac{\tilde{d}_d}{m_d}\bar{d}g(G\sigma)d+\frac{\tilde{d}_s}{m_s}\bar{s}g(G\sigma)s\right)\right|0\right\rangle$$
$$+\int d^4x\left\langle 0\left|T\left\{\frac{im_u m_d m_s}{m_s m_d+m_s m_u+m_d m_u}(\bar{u}\gamma_5 u+\bar{d}\gamma_5 d+\bar{s}\gamma_5 s)(x),i\sum_i\frac{\tilde{d}_i}{2}\bar{q}_i g(G\sigma)\gamma_5 q_i(0)\right\}\right|0\right\rangle. \qquad (5)$$

The second term here is suppressed by an extra power of light quark masses in the numerator. It would bring a comparable contribution, though, if there were an intermediate hadronic state with a mass, vanishing in the chiral limit $m_i\to 0$. At the same time the flavor structure of this term shows that the lightest intermediate state here is η' which is believed to remain heavy even if quark masses vanish. Thus the contribution from the second term is negligible in the limit $m_\pi\ll m_{\eta'}$.

Finally we get the dynamically induced theta term in the following compact form:

$$\theta_{\rm eff}=-(m_0^2/2)[(\tilde{d}_u/m_u)+(\tilde{d}_d/m_d)+(\tilde{d}_s/m_s)]. \qquad (6)$$

m_0^2 here is the ratio of the quark-gluon condensate to the quark condensate, known to sufficient accuracy from the baryon sum rules [17], $m_0^2=\langle 0|g\bar{q}(G\sigma)q|0\rangle/\langle 0|\bar{q}q|0\rangle\simeq 0.8\ \text{GeV}^2$.

Now we turn to the calculation of the axion-nucleon CP-violating coupling not related to $\bar{\theta}$. This coupling is given by the matrix element over the nucleon state of structure similar to K_1:

$$i\int dx\left\langle N\left|\left\{e^{ikx}T\left(\frac{\alpha_s}{8\pi}G\tilde{G}(x),i\sum_{i=u,d,s}\frac{\tilde{d}_i}{2}\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\times\bar{q}_i t^a(G^a\sigma)\gamma_5 q_i(0)\right)\right\}_{k=0}\right|N\right\rangle. \qquad (7)$$

Using Eq. (4), we go along the same lines as in the calculation of $\theta_{\rm eff}$ and connect Eq. (7) with the matrix elements of dipole-type operators over the nucleon. The new thing here is the existence of additional contributions related to the diagrams of Fig. 2. Taking into account all contributions, after some algebra, we arrive at the following formula:

$$\frac{m_d m_s m_u}{m_s m_d+m_s m_u+m_d m_u}$$
$$\times\left[-\frac{1}{2}\left\langle N\left|\left(\frac{\tilde{d}_u}{m_u}\bar{u}(G\sigma)u+\frac{\tilde{d}_d}{m_d}\bar{d}(G\sigma)d\right.\right.\right.\right.$$
$$\left.\left.\left.+\frac{\tilde{d}_s}{m_s}\bar{s}(G\sigma)s\right)\right|N\right\rangle+m_0^2\left\langle N\left|\frac{1}{2}(\bar{u}u-\bar{d}d)\frac{\tilde{d}_u-\tilde{d}_d}{m_u+m_d}\right.\right.$$

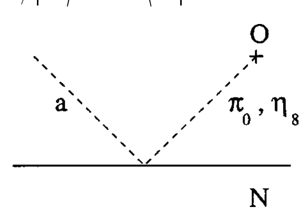

FIG. 2. Additional contribution to g_{aNN}, not related to θ or $\langle N|\bar{q}g(G\sigma)q|N\rangle$.

$$+(\overline{u}u+\overline{d}d-2\overline{s}s)\frac{\tilde{d}_u+\tilde{d}_d-2\tilde{d}_s}{m_u+m_d+4m_s}\left|N\right\rangle\Bigg]. \tag{8}$$

It should be noted here that the CP-odd nature of the interaction $a\overline{N}N$ was crucial for the transformation of Eq. (7) to the series of matrix elements from local operators (8). It allows us to drop all ''nonlocal'' terms proportional to $k_\mu\langle N|T(\overline{q}\gamma_\mu\gamma_5 q(x), i\overline{q}g(G\sigma)\gamma_5 q(0)|N\rangle$ as leading to k^2 in the numerator.

Further progress is related to the application of the low-energy theorem [18] in the 0^+ channel to the matrix element of the quark-gluon operator

$$\langle N|\overline{q}g(G\sigma)q|N\rangle\simeq\tfrac{5}{3}m_0^2\langle N|\overline{q}q|N\rangle. \tag{9}$$

For $\langle N|\overline{q}q|N\rangle$ we have the following numbers for certain flavors [19]: $\langle p|\overline{u}u|p\rangle\simeq 4.8$, $\langle p|\overline{d}d|p\rangle\simeq 4.1$, $\langle p|\overline{s}s|p\rangle\simeq 2.8$, with the obvious generalization on the neutron. From these relations we are able to learn one unfortunate thing. The dynamics of strong interactions makes the CP-violating coupling of an axion with a neutron and proton nearly equal even if CP violation at small distances is isospin asymmetric (except, maybe, some very specific cases), $g_{app}-g_{ann} \ll g_{app}+g_{ann}\equiv 2g_{aNN}$. The strength of isospin-symmetric coupling g_{aNN} follows from Eq. (8):

$$g_{aNN}\simeq\frac{1}{f_a}\frac{m_u m_d}{m_u+m_d}m_0^2\left[3.8\left(\frac{\tilde{d}_u}{m_u}+\frac{\tilde{d}_d}{m_d}\right)-0.7\frac{\tilde{d}_s}{m_s}\right]. \tag{10}$$

At this point we should plug in the limits on $\tilde{d}_i$ coming from the EDM type of experiments. As was shown in Ref. [14], the data on atomic EDMs set better limits on $\tilde{d}_u$ and $\tilde{d}_d$ than the neutron EDM does. For us, however, it is more important that g_{aNN} can be connected with the pion-nucleon CP-odd vertex with a small degree of uncertainty. This vertex $g_{\pi NN}$ leads to the T-odd nucleon-nucleon interaction $\overline{N}N\overline{N}'i\gamma_5 N'$, to Schiff moment of nuclei, and ultimately to an atomic EDM. This calculation was done in Ref. [14] by means of the same technique that we exploit here and we simply quote this result:

$$g_{\pi NN}=\frac{1}{2f_\pi}\langle N|\tilde{d}_u\overline{u}g(G\sigma)u-\tilde{d}_d\overline{d}g(G\sigma)d|N\rangle$$
$$\simeq\frac{1}{f_\pi}\frac{5}{6}m_0^2(\tilde{d}_u-\tilde{d}_d)\left\langle N\left|\frac{\overline{u}u+\overline{d}d}{2}\right|N\right\rangle. \tag{11}$$

Again, $\langle N|\overline{u}u-\overline{d}d|N\rangle$ was neglected in comparison with $\langle N|\overline{u}u+\overline{d}d|N\rangle$. Formulas (10),(11) solve, in principle, the problem of correspondence between $g_{\pi NN}$ and g_{aNN}. However, instead of writing a general formula connecting these two quantities, it is convenient to go to some specific and more instructive physical cases making some assumptions about underlying CP-violating physics.

Case 1: Proportionality. By ''proportionality'' we understand a certain situation when $\tilde{d}_u/m_u=\tilde{d}_d/m_d=\tilde{d}_s/m_s$. It happens, for example, in the minimum supersymmetric model when the relative CP-violating phase between trilinear soft-breaking parameter and gaugino mass is mediated by gluino exchange diagram.

In this case the relation between the two couplings is given by the following formula:

$$g_{aNN}=g_{\pi NN}\frac{f_\pi}{f_a}\frac{2m_u m_d}{m_d^2-m_u^2}. \tag{12}$$

As to the potential violation of the equivalence principle by this interaction, it is given by the following relation:

$$\frac{g_{ann}-g_{app}}{g_{ann}+g_{app}}=\frac{\langle p|\overline{u}u-\overline{d}d|p\rangle}{\langle p|\overline{u}u+\overline{d}d|p\rangle}\simeq 0.08. \tag{13}$$

Case 2: One-flavor dominance. This is opposite to the previous case when $\tilde{d}_i/m_i$ is enhanced for certain flavor. This may happen within different grand unification scenarios [20].

For $u(d)$ dominance the connection between g_{aNN} and $g_{\pi NN}$ is unambiguous; it follows from general formulas (8) and (11) *before* taking matrix elements, so that major QCD-related uncertainties are gone. The resulting relation takes the simplest possible form:

$$g_{aNN}=g_{\pi NN}\frac{f_\pi}{f_a}\frac{m_{d(u)}}{m_u+m_d}, \tag{14}$$

where the subscripts $d(u)$ correspond to the case of up(down)-flavor dominance.

Equation (14) is held to a rather good accuracy (to the same accuracy at which chiral perturbation theory works for the interaction of nucleons and pions). Equation (12) is somewhat less accurate because it involves Eq. (14), and therefore can be valid only within the accuracy of 30–50%.

If $\tilde{d}_u-\tilde{d}_d$ is close to zero [i.e., $m_u\simeq m_d$ in Eq. (12)], the coupling $g_{\pi NN}$ becomes small. This does not mean, of course, that g_{aNN} can grow uncontrollably on account of the limit from neutron EDM which is sensitive to $\tilde{d}_u+\tilde{d}_d$ combination [14]. In this case the limits are relaxed by a factor of 2–3 and the related error is larger.

The experimental bound on EDM of ^{199}Hg [13] translates into the following stringent limit on $g_{\pi NN}$:

$$g_{\pi pp}\simeq g_{\pi NN}<2\times 10^{-11}. \tag{15}$$

In the most interesting case of proportionality we get the following bound on the ratio of axion-mediated long-range interaction to gravitational force for the range of new force $\lambda=m_a^{-1}$:

$$\frac{F_{\text{axion}}}{F_{\text{gravity}}}=\frac{g_{aNN}^2}{4\pi\gamma m_N^2}=5\times 10^{11}g_{\pi NN}^2\left(\frac{1\ \text{cm}}{\lambda}\right)^2$$
$$<2\times 10^{-10}(1\ \text{cm}/\lambda)^2, \tag{16}$$

where $\gamma=6.7\times 10^{-39}\,\text{GeV}^2$ is the gravitational constant. This ratio is tremendously small as compared to the current experimental sensitivity at the level of 10^{-4}.

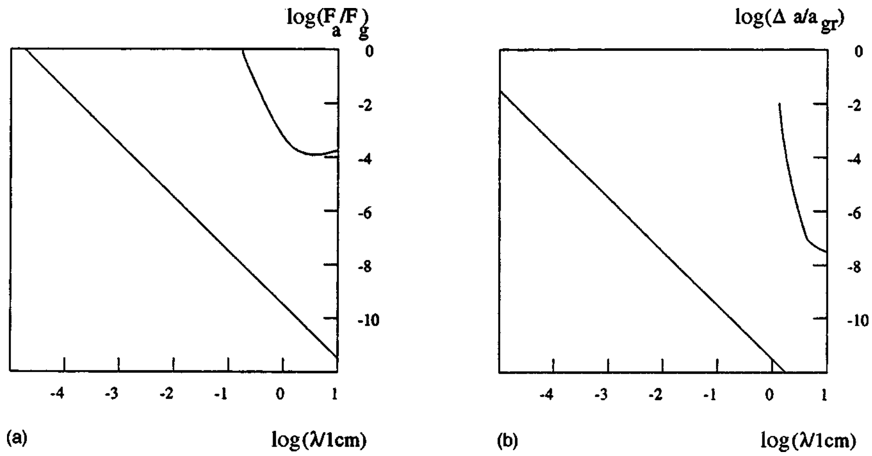

FIG. 3. (a) The comparison of maximal strength for $F_{\text{axion}}/F_{\text{gravity}}$ (straight line) with current experimental sensitivity (curve in the upper corner) [8]. (b) The comparison of predicted maximal strength for $\Delta a/a$ with the limits from Ref. [9].

More promising in terms of experimental accuracy is a high-precision check of the equivalence principle at small distances. The existing experiment sets the limit on the relative accelerations of Cu and Pb toward a uranium attractor as precise as 10^{-8} at distances of $\lambda \geqslant 10$ cm [9]. What can we expect from the interaction mediated by the axion in this case? For the case of maximal isospin-asymmetric CP violation ($\tilde{d}_u$ dominance) the estimate of the equivalence principle violation by axion mediated forces takes the following form:

$$\frac{a_{\text{Cu}}-a_{\text{Pb}}}{a_{\text{gravity}}} \simeq \frac{(N-Z)_{\text{Pb}}}{2A_{\text{Pb}}} \frac{\langle p|\overline{u}u-\overline{d}d|p\rangle}{\langle p|\overline{u}u+\overline{d}d|p\rangle} \frac{F_{\text{axion}}}{F_{\text{gravity}}} < 2\times 10^{-12}(1\ \text{cm}/\lambda)^2. \tag{17}$$

The results (16),(17) are summarized in Fig. 3.

We have shown that the sensitivity of current gravitational experiments at small, ~ 1 cm, distances needs to be improved by six orders of magnitude to probe the CP-violating interaction of an axion with matter. The limits obtained here are firm and held within the factor of 2. They are by far more precise than the limits reported in the previous analysis [12]. Our result is based on the relations between g_{aNN} and $g_{\pi NN}$, Eqs. (12)–(14), which are held to better accuracy than the calculations of m_a, g_{aNN}, and $g_{\pi NN}$. Although color EDMs are, perhaps, the most interesting CP-violating operators, one can implement a similar procedure for all varieties of CP-odd operators dim=6 built from the quark fields. Our result for $F_{\text{axion}}/F_{\text{gravity}}$, Eq. (16), is significantly lower than the central value of the prediction based on NDA and neutron EDM data [12]. Once again it shows the usefulness of the limits on CP violation extracted from EDM of ^{199}Hg [13].

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