CP-odd tadpole renormalization of Higgs scalar-pseudoscalar mixing

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We consider an Abelian model with a CP-conserving Higgs potential spanned by two complex Higgs fields. The CP invariance of the Higgs potential is then broken explicitly beyond the Born approximation by introducing soft-CP-violating Yukawa interactions. Based on the non-renormalization theorem, we derive the consistency conditions under which a CP-odd counterterm exists and, at the same time, renders the one-loop-induced mixing of a CP-even Higgs boson with a CP-odd Higgs scalar ultra-violet finite. The novel CP-odd tadpole renormalization is uniquely determined from the minimization constraints on the Higgs potential. The main phenomenological consequences of the so-generated CP-violating scalar-pseudoscalar mixing are briefly discussed. [S0556-2821(98)03821-1]

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I. INTRODUCTION

Despite physicists' continuous effort since the discovery of CP violation in the $K^0\overline{K}^0$ system in 1964 [1], a deep understanding of the origin of CP asymmetry in nature remains still elusive thus far. This fact has rendered the whole issue of CP nonconservation from the theoretical point of view even more challenging. In the existing literature, two generically different scenarios have been proposed at the Lagrangian level to explain the observed CP asymmetry. In the first scenario, complex parameters, such as complex Yukawa couplings, are introduced in the Lagrangian which break explicitly CP invariance. Such a scenario of explicit CP violation is realized by the well-known standard model (SM) and many of its minimal extensions. Another appealing scheme arises when the ground state of the Higgs potential is not invariant under CP. To make such a scheme work, one needs to extend the Higgs sector of the SM by adding more than one Higgs field. In addition, one requires that the complete tree-level Lagrangian be CP symmetric. After the spontaneous symmetry breaking (SSB) of the Higgs potential, the resulting vacuum state is no longer a CP eigenstate, thereby leading to a CP-noninvariant theory. Such a mechanism is called spontaneous CP violation. Technically, this is manifested by the fact that one of the vacuum expectation values (VEV's) of the Higgs fields becomes complex [2]. In this context, it is also worth mentioning the variant, in which the spontaneous breakdown of the CP symmetry occurs beyond the Born approximation of the Higgs potential through quantum mechanical effects. This mechanism is known as radiative CP violation [3].

Here, we shall study another very interesting possibility of explicit CP violation which may naturally take place in models with an extended Higgs sector, such as the two-Higgs-doublet model (2HDM) and/or the minimal supersymmetric SM (MSSM). For our illustrations, we consider an Abelian model with a CP-conserving Higgs potential formed by two complex Higgs fields. It is worth emphasizing that such a scenario coincides with the *neutral* sector of the respective two-Higgs-doublet model. The Abelian two-Higgs-doublet model predicts four neutral (real) scalars: two physical CP-even Higgs bosons, denoted as H and h, one physical CP-odd Higgs boson, A, and the CP-odd Goldstone boson G^0 which becomes the longitudinal component of the massive gauge boson Z. In this model, all the CP-violating mixings HA, hA, HG^0 and hG^0 are absent at the tree level and to all orders in perturbation theory, if CP is an exact symmetry of the Lagrangian. However, complex Yukawa couplings of the Higgs fields to fermions or charged scalars may explicitly break CP invariance. Depending on the detailed form of Yukawa interactions, one finds in general that Higgs scalar-pseudoscalar transitions induced by one-loop Feynman graphs are *not* ultra-violet (UV) finite. In a sense this may appear paradoxical, since one would expect that the HA-type transitions should be UV safe by themselves, as the tree-level CP-invariant form of the Higgs potential cannot produce the necessary CP-odd counterterms (CT's) to cancel the UV divergences. The latter may even thwart the whole renormalization program of the model. In this paper, we offer a field-theoretic solution to the aforementioned problem, for which we believe that a discussion at a satisfactory level is still missing in the existing literature. By examining carefully the minimization constraints on the Higgs potential, we observe that new CP-odd tadpole CT's do exist which may absorb the above loop-induced UV divergences. Nevertheless, this is not always the case. The non-renormalization theorem dictates the admissible dimensional forms of CP-violating operators which can be introduced in the Lagrangian to break CP without spoiling the renormalizability of the model itself. Based on that theorem, we derive the consistency conditions, such that soft-CP-violating Yukawa couplings together with a Higgs potential invariant under CP at the tree level can co-exist.

Apart from the theoretical interest in providing a self-consistent solution to the problem mentioned above, Higgs scalar-pseudoscalar transitions may directly be probed at present and planned high-energy machines [4,6]. In the SM, HZ mixing is expected to occur at the three-loop level [6], as shown in Fig. 1, and hence must be considered to be not

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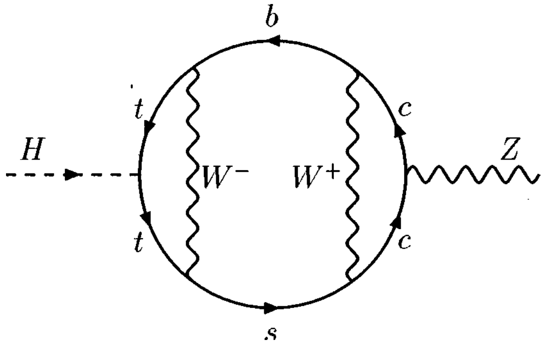

FIG. 1. Typical three-loop graph giving rise to a non-vanishing HZ mixing in the SM. The remaining graphs may be obtained by attaching H and Z in all possible ways to the quark and W-boson lines.

phenomenologically viable. In contrast, HA mixing may be large within our scheme of CP violation, as it can be generated at the one-loop electroweak order. Future e^+e^-, pp and $\mu^+\mu^-$ colliders have the physics capabilities to search for CP-violating signals due to a non-vanishing HA mixing [6]. On the other hand, the chirality enhanced two-loop Barr-Zee mechanism may give rise to a large contribution to the electric dipole moments (EDM's) of neutrons, electrons and muons through a sizable HA mixing. As a consequence, experimental limits on the above EDM's place severe bounds on the size of possible CP-violating HA-type operators.

The paper is organized as follows: In Sec. II we describe the Lagrangian of an Abelian model with two complex Higgs fields, in which the Higgs potential respects CP symmetry, and establish the connection between this scenario and the respective model of phenomenological interest with two Higgs doublets. In the discussion, we also include typical soft-CP-breaking operators which are in agreement with the non-renormalization theorem. In Sec. III we examine the minimization conditions of the Higgs potential and identify the crucial CP-odd tadpole CT's. In Sec. IV we calculate the one-loop induced scalar-pseudoscalar self-energies within a scenario of Yukawa interactions, favored by supersymmetry, and then show how the UV divergences of the self-energy graphs drop out when the CP-odd tadpole renormalizations are taken into account. Finally, in Sec. V we conclude with a short discussion on the phenomenological implications of HA-type mixings for the EDM's of neutrons, electrons and muons.

II. ABELIAN TWO-HIGGS-BOSON MODEL

We will consider a $\mathrm{U}(1)_Y$ model with two complex Higgs fields Φ_1 and Φ_2, and require that its complete Lagrangian be CP invariant. As for the discussion of CP symmetry, the results obtained in the Abelian model are also valid for the respective 2HDM. Based on the non-renormalization theorem, we will then investigate the admissible forms of soft-CP-breaking terms which can be added in the Lagrangian, without inducing radiatively new local operators that violate the CP invariance of the Higgs potential and hence the renormalizability of the model itself.

The Lagrangian of the Abelian two-Higgs-boson model may conveniently be expressed as follows:

$$\mathcal{L}_H=\sum_{i=1,2}(D^\mu\Phi_i)^*(D_\mu\Phi_i)+\mathcal{L}_V, \tag{2.1}$$

where

$$D_\mu=\partial_\mu-ig\hat{Y}Z_\mu \tag{2.2}$$

is the covariant derivative, with g and $\hat{Y}$ being the gauge coupling and the hypercharge operator of the $\mathrm{U}(1)_Y$, respectively. Furthermore, $\mathcal{L}_V$ is the part of the Lagrangian containing the Higgs potential. The complex fields Φ_1 and Φ_2 carry the hypercharges $Y(\Phi_1)=Y(\Phi_2)=-1/2$.

It is not very difficult to see that the $\mathrm{U}(1)_Y$ two-Higgs-doublet model is a subgroup of the 2HDM based on $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_{Y'}$, where Y' is the usual SM hypercharge, i.e., $Y'(\Phi_1)=Y'(\Phi_2)=1/2$. In particular, we have

$$\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_{Y'}\supset\mathrm{U}(1)_{Tz}\otimes\mathrm{U}(1)_{Y'}\supset\mathrm{U}(1)_Y, \tag{2.3}$$

where T_z is the third component of the weak isospin with $T_z(\Phi_1)=T_z(\Phi_2)=-1/2$. The covariant derivative for the gauge subgroup $\mathrm{U}(1)_{Tz}\otimes\mathrm{U}(1)_{Y'}$ is

$$\partial_\mu-ig_LT_zW^z_\mu-ig'\hat{Y}'B_\mu, \tag{2.4}$$

where g_L is the gauge coupling of the groups $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_{Tz}$, and g' is the corresponding one for $\mathrm{U}(1)_{Y'}$. In terms of the physical gauge fields

$$A_\mu=\frac{1}{\sqrt{g_L^2+g'^2}}(g'W^z_\mu+g_LB_\mu),$$

$$Z_\mu=\frac{1}{\sqrt{g_L^2+g'^2}}(g_LW^z_\mu-g'B_\mu), \tag{2.5}$$

the covariant derivative (2.4) takes on the form

$$D_\mu=\partial_\mu-ig\hat{Y}Z_\mu-ie\hat{Q}_{em}A_\mu, \tag{2.6}$$

with $g=\sqrt{g_L^2+g'^2}$, $e=g_Lg'/g$ and

$$\hat{Q}_{\mathrm{em}}=T_z+\hat{Y}',\ \hat{Y}=T_z-\frac{g'^2}{g^2}\,\hat{Q}_{\mathrm{em}}. \tag{2.7}$$

In Eq. (2.6), A_μ and $\hat{Q}_{\mathrm{em}}$ may be identified with the known electromagnetic field and the charge generator associated to it, while Z_μ represents the SM Z boson mediating neutral currents with Y quantum charges. The latter gauge field may be related to the Z_μ gauge field of the $\mathrm{U}(1)_Y$ model. It is then obvious that for neutral fields, i.e., $Q_{\mathrm{em}}=0$, the covariant derivative (2.6) coincides with that given in Eq. (2.2). Consequently, the Abelian two-Higgs-boson model under consideration is a realistic scenario of highly phenomenological interest, as it can be realized by the neutral sector of the respective model with two Higgs doublets.

The complex scalars Φ_1 and Φ_2 are responsible for endowing the observed Z boson and fermions with masses through the Higgs mechanism. By the same token, however, they also lead to potentially large flavor-changing neutral current couplings at the tree level. Glashow and Weinberg [7] suggested that natural flavor conservation may be obtained if the whole Lagrangian, including the Yukawa sector $\mathcal{L}_Y$, is invariant under the discrete symmetry D: $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$ and $d_R \to -d_R$, where d_R collectively denotes right-handed down-type quarks and leptons. Under the D symmetry, the field Φ_1 couples to the up-type family u, whereas Φ_2 couples to the down-type one d only.

Imposing the discrete symmetry D on the general form of a $\mathrm{U}(1)_Y$-invariant Higgs potential yields

$$\mathcal{L}_V = \mu_1^2(\Phi_1^*\Phi_1) + \mu_2^2(\Phi_2^*\Phi_2) + \lambda_1(\Phi_1^*\Phi_1)^2 + \lambda_2(\Phi_2^*\Phi_2)^2 + \lambda_{34}(\Phi_1^*\Phi_1\Phi_2^*\Phi_2) + \lambda_5(\Phi_1^*\Phi_2)^2 + \lambda_5^*(\Phi_2^*\Phi_1)^2. \tag{2.8}$$

Note that the term proportional to λ_{34} also comprises the D-symmetric combination $(\Phi_1^*\Phi_2)(\Phi_2^*\Phi_1)$. In addition, we readily see that $\mathcal{L}_V$ remains invariant under the generalized CP transformations compatible with D symmetry,

$$\Phi_1 \to e^{i\phi_1}\Phi_1^*, \quad \Phi_2 \to e^{i\phi_2}\Phi_2^*, \tag{2.9}$$

provided the phases ϕ_1 and ϕ_2 are chosen in a way such that $\phi_1 - \phi_2 = \arg\lambda_5$. As a consequence, the potential $\mathcal{L}_V$ is CP invariant.

We should now notice that the D symmetry of the Higgs potential cannot be promoted to a continuous one of the Peccei-Quinn type [8], unless $\lambda_5 = 0$. To be specific, for vanishing λ_5, one may choose the Q quantum numbers for the fields

$$Q(\Phi_1)=2, \quad Q(\Phi_2)=-1, \; Q(u_L)=Q(d_L)=2,$$
$$Q(u_R)=0, \quad q(d_R)=1, \tag{2.10}$$

and then show that in addition to the gauge symmetry $\mathrm{U}(1)_Y$, the total Lagrangian $\mathcal{L}=\mathcal{L}_H+\mathcal{L}_Y$ is invariant under $\mathrm{U}(1)_Q$, with

$$-\mathcal{L}_Y = \Phi_1 \overline{u_L} h_u u_R + \Phi_2^* \overline{d_L} h_d d_R + \mathrm{H.c.} \tag{2.11}$$

D symmetry is obtained for the choice of the global (phase) parameter $\phi=\pi$. In Eq. (2.11), h_u and h_d are in general (dimensionless) complex Yukawa matrices for the up-type and down-type families, respectively. However, using the freedom of a re-definition of the fermionic fields, one can make both matrices h_u and h_d diagonal with positive entries, without spoiling $\mathrm{U}(1)_Q$ and D symmetries. In fact, after SSB, the Yukawa couplings h_u and h_d get directly related to the observed fermion masses. Obviously, the Lagrangian $\mathcal{L}$ is exactly CP invariant, and so the absence of CP-violating mixing terms of the kind $\Re(\Phi_1)\Im(\Phi_1)$ is guaranteed to all orders in perturbation theory. Also, the above local operator being proportional to $\Im(\Phi_1)^2$ is forbidden because of $\mathrm{U}(1)_Y$ invariance. We reach the same conclusion if $\lambda_5 \neq 0$. Notwithstanding the fact that $\mathrm{U}(1)_Q$ is broken hard by $(\Phi_1^*\Phi_2)^2$, it is however the only four-dimensional operator that can be generated at high orders which is simultaneously invariant under D and $\mathrm{U}(1)_Y$. As it should, the effective potential is CP invariant. In general, this result will hold true even after SSB.[1]

We now examine the consequences for the effective Higgs potential if CP-violating Yukawa interactions are added to the Lagrangian which are invariant under D or $\mathrm{U}(1)_Q$ symmetries. Again, $\mathrm{U}(1)_Y$ invariance forbids the presence of the CP-violating operator $\Re(\Phi_1)\Im(\Phi_1)$. However, after the SSB of the gauge and global symmetries, CP-violating HA-type transitions may become possible. On the other hand, the non-renormalization theorem [9] dictates the form of CT's for a spontaneously broken theory. According to the theorem, the CT's are entirely determined from those given in the symmetric phase of the theory. As a consequence, the loop-induced HA mixings must be UV finite. A scenario of this type has been discussed in the recent literature [6]. Specifically, in addition to the Higgs scalars, the model contains two iso-singlet neutrinos, denoted as N_1 and N_2, and one sequential left-handed neutrino ν_L. Thus, the Yukawa sector of the model allows for the simultaneous presence of Dirac and Majorana mass terms, i.e.,

$$-\mathcal{L}_Y^M = \Phi_1 \overline{\nu_L}(h_1 N_1 + h_2 N_2) + M_1 N_1^T C N_1 + M_2 N_2^T C N_2 + \mathrm{H.c.}, \tag{2.12}$$

where C is the operator of charge conjugation. Lagrangian $\mathcal{L}_Y^M$ has been written in the weak basis, in which M_1 and M_2 are real, while h_1 and h_2 are complex numbers. The model predicts three heavy Majorana neutrinos, which may have masses as low as hundreds of GeV; they may be discovered in the next generation of colliders. For the general case of non-degenerate heavy Majorana neutrinos, the non-vanishing of the rephasing-invariant quantity $\Im(h_1 h_2^*)^2$ signifies CP violation [5]. Notice that $\mathcal{L}_Y^M$ is invariant under the symmetries D and $\mathrm{U}(1)_Q$. Figure 2 shows the flavor diagrams responsible for generating the CP-violating operator $\Re(\Phi_1)\Im(\Phi_1)$ after SSB. As can also be seen by doing a naive power counting in Fig. 2, the resulting HA-type self-energies are UV finite. For more details the reader is referred to [6]. Finally, we should remark that the HZ mixing in the minimal SM is very analogous to the aforementioned case of explicit CP violation (see Fig. 1).

In the following, we will focus our attention on scenarios of explicit CP violation, in which the symmetries D and/or $\mathrm{U}(1)_Q$ are softly broken. Our main interest is to find the consistency conditions of renormalization which ensure the co-existence of a *tree-level* CP-invariant Higgs potential together with soft-CP-breaking Yukawa interactions. The first soft-breaking term one may think of adding to the potential

[1]Exceptions to this case are scenarios of spontaneous or radiative CP violation. Here, we shall not consider such alternatives. More details may be found in [3].

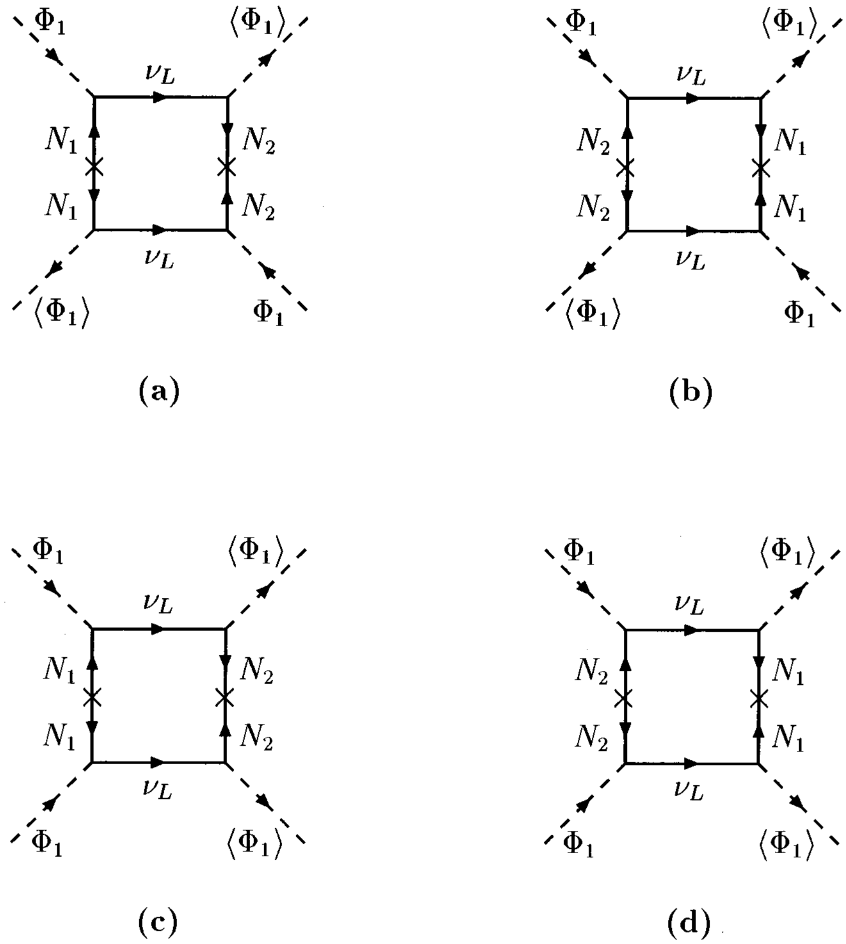

FIG. 2. One-loop flavor graphs responsible for a non-vanishing *HA* mixing in the Majorana-neutrino model.

$\mathcal{L}_V$ in Eq. (2.8) is $\mu^2\Phi_1^*\Phi_2$, where μ is in general a complex parameter. However, the inclusion of such a two-dimensional operator would lead to a CP-violating Higgs potential already at the tree-level, unless

$$\Im(\mu^4\lambda_5^*)=0. \qquad (2.13)$$

Besides the option of fine-tuning, the most natural way to satisfy CP-invariance condition (2.13) is to assume that the Higgs potential has originally a global $\mathrm{U}(1)_Q$ symmetry which is softly broken afterwards by the above μ^2-dependent mass term. In this case, the quartic coupling λ_5 is absent from the potential. There is also another reason advocating for the naturality of the latter scenario. If we had broken $\mathrm{U}(1)_Q$ by trilinear Yukawa operators of dimension 3, we would have been compelled, as a consequence of the non-renormalization theorem, to include all possible operators of lower dimensions, i.e., the two-dimensional mass term μ^2. Again, for $\lambda_5\neq 0$, this would have led to a CP-violating Higgs potential at the tree level. For the Abelian two-Higgs-boson model, we can therefore conclude that in order to have a *consistent* CP-invariant Higgs potential at the tree level, it is sufficient to require for all operators of dimension 4 to be $\mathrm{U}(1)_Q$ symmetric in the complete Lagrangian and allow only for soft-breaking of $\mathrm{U}(1)_Q$ by mass and trilinear Yukawa terms.

Motivated by the MSSM, we now present a specific scenario, in which both symmetries $\mathrm{U}(1)_Q$ and CP are softly broken on the same footing. The model includes the (hyper-)charged scalars $\chi_L^\pm$ and $\chi_R^\pm$, such as left-handed and right-handed scalar quarks, which have trilinear Yukawa couplings to the Higgs fields. To be precise, the interactions of the charged scalars are governed by the Lagrangian

$$-\mathcal{L}_Y^\chi=m_L^2\chi_L^+\chi_L^-+m_R^2\chi_R^+\chi_R^-+(f\Phi_1+h\Phi_2)\chi_L^+\chi_R^-+\text{H.c.}, \qquad (2.14)$$

where f and h are in general complex mass parameters, while m_L and m_R are real. We assign to the newly introduced charged scalars the following hypercharge and Q quantum numbers: $Y(\chi_L^+)=1$, $Y(\chi_R^+)=1/2$, and $Q(\chi_L^-)=2$ and $Q(\chi_R^\pm)=0$. It is then clear that both CP and $\mathrm{U}(1)_Q$ symmetries are softly broken by the operator $\Phi_2\chi_L^+\chi_R^-$ of dimension 3. In fact, the model admits CP violation, unless the rephasing-invariant constraint $\Im(\mu^2fh^*)=0$ holds true.

As we will see in Sec. IV, the above scalar model leads to Higgs scalar-pseudoscalar self-energies which are *not* UV finite. In the next section, we shall show how a CP-invariant Higgs potential can still produce the necessary CP-odd CT's, which can cancel the loop-induced UV divergences.

III. *CP*-ODD TADPOLE RENORMALIZATION

In this section, we shall examine the minimization conditions imposed on a Higgs potential which is invariant under CP at the tree level. We shall then show that CP-odd tadpole CT's do exist which are relevant for the renormalization of Higgs scalar-pseudoscalar mixings. Following the discussion given in Sec. II, we consider the Higgs potential

$$\begin{aligned}\mathcal{L}_V^\mu=&\mu_1^2(\Phi_1^*\Phi_1)+\mu_2^2(\Phi_2^*\Phi_2)+\mu^2(\Phi_1^*\Phi_2)\\&+\mu^{*2}(\Phi_2^*\Phi_1)+\lambda_1(\Phi_1^*\Phi_1)^2\\&+\lambda_2(\Phi_2^*\Phi_2)^2+\lambda_{34}(\Phi_1^*\Phi_1\Phi_2^*\Phi_2)+\cdots .\end{aligned}\tag{3.1}$$

Unlike the mass term μ^2, the Higgs potential $\mathcal{L}_V^\mu$ is symmetric under $U(1)_Q$; μ^2 breaks only softly $U(1)_Q$. The ellipsis in Eq. (3.1) denotes possible new quartic interactions of the type $\Phi_1^*\Phi_1\chi_L^+\chi_L^-$, $(\chi_L^+\chi_L^-)^2$, etc., which are allowed by $U(1)_Q$ and $U(1)_Y$. Since the charged scalars $\chi_L^\pm$ and $\chi_R^\pm$ do not develop VEV's, the presence of these additional quartic operators will not affect the minimization constraints on the Higgs potential. For phenomenological simplicity, we may assume that these extra quartic couplings are rather suppressed. Finally, it is very interesting to notice that the Higgs potential in Eq. (3.1) is analogous to that predicted in the MSSM [10]. So the discussion presented in this section can easily carry over to the latter case as well.

Without any loss of generality, it proves more convenient to perform the minimization of the Higgs potential in the weak basis, in which both VEV's of the Higgs fields are positive. As usual, we use the linear parametrizations

$$\Phi_1=\frac{1}{\sqrt{2}}(v_1+H_1+iA_1),\quad \Phi_2=\frac{1}{\sqrt{2}}(v_2+H_2+iA_2),\tag{3.2}$$

where v_1 and v_2 are the VEV's of the unbroken Higgs fields, H_1 and H_2 are CP-even Higgs bosons, and A_1 and A_2 are CP-odd scalars. The minimization constraints may then be determined by demanding the vanishing of the following tadpole parameters:

$$\begin{aligned}T_{H_1}\equiv&\left.\frac{\partial\mathcal{L}_V^\mu}{\partial H_1}\right|_{\langle H_i\rangle=\langle A_i\rangle=0}\\=&v_1\left(\mu_1^2+\Re\mu^2\frac{v_2}{v_1}+\lambda_1v_1^2+\frac{1}{2}\lambda_{34}v_2^2\right),\end{aligned}\tag{3.3}$$

$$\begin{aligned}T_{H_2}\equiv&\left.\frac{\partial\mathcal{L}_V^\mu}{\partial H_2}\right|_{\langle H_i\rangle=\langle A_i\rangle=0}\\=&v_2\left(\mu_2^2+\Re\mu^2\frac{v_1}{v_2}+\lambda_2v_2^2+\frac{1}{2}\lambda_{34}v_1^2\right),\end{aligned}\tag{3.4}$$

$$T_{A_1}\equiv\left.\frac{\partial\mathcal{L}_V^\mu}{\partial A_1}\right|_{\langle H_i\rangle=\langle A_i\rangle=0}=v_2\Im\mu^2,\tag{3.5}$$

$$T_{A_2}\equiv\left.\frac{\partial\mathcal{L}_V^\mu}{\partial A_2}\right|_{\langle H_i\rangle=\langle A_i\rangle=0}=-v_1\Im\mu^2.\tag{3.6}$$

Clearly, in the Born approximation one has that $\Im\mu^2=0$ and CP is a good symmetry of the Higgs potential. However, at high orders $\Im\mu^2$ no longer vanishes due to CP-violating Yukawa interactions. In fact, the CP-odd tadpoles T_{A_1} and T_{A_2} depend explicitly on $\Im\mu^2$ and are therefore very crucial to render the H_iA_j self-energies UV finite.

It is now important to identify the true Goldstone boson, G^0, of the theory, which is associated with the SSB of $U(1)_Y$. According to the Goldstone theorem, G^0 should remain massless and have a pure pseudoscalar coupling to like-flavor fermions to all orders in perturbation theory. If the Abelian $U(1)_Y$ symmetry is gauged, G^0 becomes the longitudinal degree of freedom of the gauge boson Z. Apart from obtaining the massless eigenstate from the Higgs boson mass matrix, the easiest way to find G^0 is to look for the flat direction of the Higgs potential. This amounts to finding the field configuration G^0 for which

$$\left.\frac{\partial\mathcal{L}_V^\mu}{\partial G^0}\right|_{\langle H_i\rangle=\langle A_i\rangle=0}\equiv T_{G^0}=0.\tag{3.7}$$

Since G^0 is CP-odd, it must be a linear combination of the CP-odd fields A_1 and A_2. Thus, we are seeking a solution to the equation

$$\begin{aligned}T_{G^0}=&\frac{\partial A_1}{\partial G^0}T_{A_1}+\frac{\partial A_2}{\partial G^0}T_{A_2}\\=&c_\beta T_{A_1}+s_\beta T_{A_2}=0,\end{aligned}\tag{3.8}$$

where the usual shorthand notation $s_x=\sin x$ and $c_x=\cos x$ is employed. Equation (3.8) implies that

$$\tan\beta=-\frac{T_{A_1}}{T_{A_2}}=\frac{v_2}{v_1}.\tag{3.9}$$

As a result, the two physical CP-odd states G^0 and A are related to A_1 and A_2 through the orthogonal transformation

$$\begin{pmatrix}A_1\\A_2\end{pmatrix}=\begin{pmatrix}c_\beta&-s_\beta\\s_\beta&c_\beta\end{pmatrix}\begin{pmatrix}G^0\\A\end{pmatrix}.\tag{3.10}$$

The very same result would have been obtained if we had diagonalized the mass matrix for the pseudoscalar bosons. Likewise, the Higgs scalars H_1 and H_2 are related to the physical CP-even Higgs particles, denoted as h and H, through an analogous orthogonal rotation of angle θ, i.e.,

$$\begin{pmatrix}H_1\\H_2\end{pmatrix}=\begin{pmatrix}c_\theta&-s_\theta\\s_\theta&c_\theta\end{pmatrix}\begin{pmatrix}h\\H\end{pmatrix}.\tag{3.11}$$

Details on the diagonalization of the Higgs boson mass matrices and a discussion of stability conditions for the Higgs potential are given in the Appendix.

Because of the above orthogonal transformation of the CP-odd fields, it is obvious that the number of independent tadpole parameters in Eqs. (3.3)–(3.6) has been reduced to 3. However, there still exists one non-trivial CP-odd tadpole CT, given by

$$T_A=\frac{\partial A_1}{\partial A}\,T_{A_1}+\frac{\partial A_2}{\partial A}\,T_{A_2}$$
$$=-s_\beta T_{A_1}+c_\beta T_{A_2}=-v\Im\mu^2, \tag{3.12}$$

with $v=\sqrt{v_1^2+v_2^2}$. In particular, we find that all Higgs scalar-pseudoscalar mixings in the Higgs potential are proportional to the tadpole renormalization constant T_A of the pseudoscalar A. More explicitly, we obtain the HA-type CT's

$$\mathcal{L}_V^{HA}=\frac{T_A}{v}\,[(c_\theta h-s_\theta H)(s_\beta G^0+c_\beta A)$$
$$-(s_\theta h+c_\theta H)(c_\beta G^0-s_\beta A)]. \tag{3.13}$$

From the above discussion, it became clear that CP-violating quantum effects will shift the Higgs ground states to a CP-odd direction which should be re-adjusted by requiring that the CTT_A should cancel the loop-induced tadpole graph of the A boson. In this way, a novel CP-odd renormalization is obtained which must be included in the calculation of HA-type self-energies. In the next section, we will illuminate this point further.

IV. LOOP-INDUCED SCALAR-PSEUDOSCALAR MIXING

To elucidate the necessity of a CP-odd tadpole renormalization, we shall calculate HA-type self-energies within an extension of the two-Higgs-doublet model, in which the charged scalars $\chi_L^\pm$ and $\chi_R^\pm$ are introduced. As has been discussed in Sec. II, such a scenario can consistently accommodate a CP-invariant Higgs potential at the tree level together with a CP-violating Yukawa sector described by the Lagrangian $\mathcal{L}_Y^\chi$ in Eq. (2.14).

From Eq. (2.14), we may now determine the mass eigenstates $\chi_1^\pm$ and $\chi_2^\pm$ for the charged scalars, i.e.,

$$-\mathcal{L}_{\rm mass}^\chi=(\chi_L^+,\chi_R^+)\begin{pmatrix} m_L^2 & a^2\\ a^{*2} & m_R^2\end{pmatrix}\begin{pmatrix}\chi_L^-\\ \chi_R^-\end{pmatrix}$$
$$=(\chi_1^+,\chi_2^+)\begin{pmatrix} M_1^2 & 0\\ 0 & M_2^2\end{pmatrix}\begin{pmatrix}\chi_1^-\\ \chi_2^-\end{pmatrix}, \tag{4.1}$$

with $a^2=(fv_1+hv_2)/\sqrt{2}$ and

$$\begin{pmatrix}\chi_L^-\\ \chi_R^-\end{pmatrix}=\begin{pmatrix} c_\phi & s_\phi e^{i\delta}\\ -s_\phi e^{-i\delta} & c_\phi\end{pmatrix}\begin{pmatrix}\chi_1^-\\ \chi_2^-\end{pmatrix}. \tag{4.2}$$

The requirement of having positive masses for charged scalars leads to the inequality $m_L m_R-|a|^2>0$. In Eq. (4.2), the phase δ is trivial, since it can always be eliminated by the judicious phase re-definition of the field χ_R^-, e.g., $\chi_R^-\to e^{-i\delta}\chi_R^-$. In this weak basis, a^2 is a real parameter. The other mass parameters in Eq. (4.1) are related to the physical masses of the charged scalars, M_1 and M_2, and the mixing angle ϕ as follows:

$$m_L^2=M_1^2c_\phi^2+M_2^2s_\phi^2,$$
$$m_R^2=M_1^2s_\phi^2+M_2^2c_\phi^2,$$
$$a^2=(M_2^2-M_1^2)s_\phi c_\phi. \tag{4.3}$$

As has been mentioned in Sec. II, the model violates CP through trilinear Yukawa interactions in $\mathcal{L}_Y^\chi$ for $\Im(\mu^2 fh^*)\neq 0$. In particular, one finds that the CP-even Higgs bosons H_1 and H_2 (or equivalently h and H) couple to the same bilinear operators of charged scalars as the CP-odd bosons G^0 and A do. These couplings are precisely those which induce HA-type transitions at the one-loop level. To be specific, the interaction Lagrangian of interest, obtained from $\mathcal{L}_Y^\chi$, reads

$$\mathcal{L}_{\rm int}^\chi=-\frac{i}{v}\,a^2G^0\chi_1^+\chi_2^-+\frac{2s_\phi c_\phi}{vs_\beta c_\beta}\,\Im b^2A(\chi_1^+\chi_1^--\chi_2^+\chi_2^-)-\frac{i}{vs_\beta c_\beta}\,(a^2c_\beta^2-b^2c_\phi^2-b^{*2}s_\phi^2)A\chi_1^+\chi_2^-$$
$$+\frac{2s_\phi c_\phi}{vc_\beta}\,\Re b^2H_1(\chi_1^+\chi_1^--\chi_2^+\chi_2^-)+\frac{2s_\phi c_\phi}{vs_\beta}\,(a^2-\Re b^2)H_2(\chi_1^+\chi_1^--\chi_2^+\chi_2^-)-\frac{1}{vc_\beta}\,(b^2c_\phi^2-b^{*2}s_\phi^2)H_1\chi_1^+\chi_2^-$$
$$-\frac{1}{vs_\beta}\,[a^2(c_\phi^2-s_\phi^2)-b^2c_\phi^2+b^{*2}s_\phi^2]H_2\chi_1^+\chi_2^-+\text{H.c.}, \tag{4.4}$$

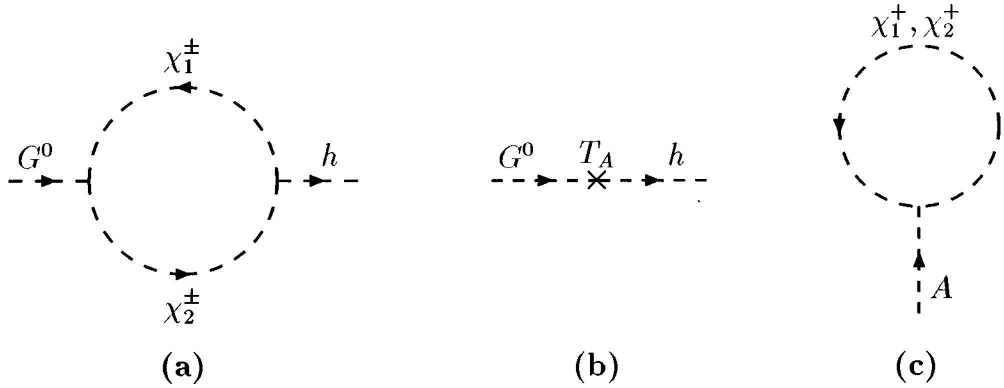

FIG. 3. Diagrams pertinent to the G^0h mixing: (a) one-loop self-energy graph, (b) CP-odd tadpole renormalization, (c) tadpole graph of the A boson.

where the mass parameter $b^2=fv_1/\sqrt{2}$ may take complex values. To avoid double counting in Eq. (4.4), the Hermitian conjugate (H.c.) term is understood to be included only for couplings which are not self-conjugate by themselves. Moreover, the weak eigenstates H_1 and H_2 may be expressed in terms of the physical states h and H by virtue of Eq. (3.11). Nevertheless, we can simplify our calculations by assuming that the unknown Higgs-scalar mixing θ is very small. In the limit of $\theta\to 0$, we then have $h\equiv H_1$ and $H\equiv H_2$.

Since our interest is to study the UV behavior of scalar-pseudoscalar transitions, we calculate the loop-induced HA-type self-energies at vanishing external momentum, i.e., $q\to 0$. This may also be justified by the effective potential formalism [11], in which the charged scalars are integrated out as being heavy degrees of freedom. As an example, let us consider the one-particle-irreducible (1PI) self-energy graph shown in Fig. 3(a), which gives rise to the G^0h mixing at the one-loop level. It is then straightforward to obtain

$$\Pi^{G^0h}_{(a)}(q^2=0)=-2\,\frac{s_\phi c_\phi}{c_\beta}\,\frac{\Im b^2}{v^2}\,(M_1^2-M_2^2)I(M_1,M_2), \tag{4.5}$$

where the loop function

$$\begin{aligned}I(M_1,M_2)&=\mu^{4-n}\int\frac{d^nk}{(2\pi)^n i}\,\frac{1}{(k^2-M_1^2)(k^2-M_2^2)}\\&=\frac{1}{16\pi^2}\left[\frac{1}{\varepsilon}-\gamma_E+1+\ln 4\pi-\ln\left(\frac{M_1M_2}{\mu^2}\right)\right.\\&\quad\left.+\frac{M_1^2+M_2^2}{2(M_1^2-M_2^2)}\ln\left(\frac{M_2^2}{M_1^2}\right)\right],\end{aligned} \tag{4.6}$$

is defined in $n=4-2\varepsilon$ dimensions. The parameter μ in Eq. (4.6) denotes the 't Hooft mass. As a consequence of the Goldstone theorem mentioned above, the G^0h self-energy must vanish for zero momentum transfer. Otherwise, one would find that the true Goldstone boson G^0 receives a nonzero radiative mass in contradiction with the Goldstone theorem, since $\Pi^{G^0G^0}(0)=[\Pi^{G^0h}_{(a)}(0)]^2/M_h^2\neq 0$ through G^0h mixing at two loops. In fact, for non-degenerate charged scalars, the self-energy $\Pi^{G^0h}_{(a)}$ does not vanish; it is even plagued by an UV infinity.

Evidently, one is faced with the fact that some CP-odd CT must be included in the calculation, as shown in Fig. 3(b), so as to render the G^0h self-energy UV finite. Fortunately, Lagrangian $\mathcal{L}_Y^{HA}$ in Eq. (3.13) provides the necessary renormalization constant for that purpose. Indeed, for zero Higgs-scalar mixing $(\theta=0)$, we have

$$\Pi^{G^0h}_{(b)}(0)=\frac{s_\beta T_A}{v}. \tag{4.7}$$

The CP-odd tadpole parameter T_A may be determined by the usual renormalization condition

$$\Gamma^A(0)+T_A=0; \tag{4.8}$$

i.e., quantum corrections must not shift the true ground state of the effective potential. In Eq. (4.8), $\Gamma^A(0)$ is the 1PI tadpole graph of the CP-odd Higgs scalar A, which is also depicted in Fig. 3(c). In this way, we find

$$T_A=2\,\frac{s_\phi c_\phi}{s_\beta c_\beta}\,\frac{\Im b^2}{v}\,(M_1^2-M_2^2)I(M_1,M_2). \tag{4.9}$$

It is now easy to verify that

$$\Pi^{G^0h}_{(a)}(0)+\Pi^{G^0h}_{(b)}(0)=0, \tag{4.10}$$

as it should on account of the Goldstone theorem.

By analogy, we can calculate the hA self-energy. The diagrams contributing to such a CP-violating Higgs-scalar transition are displayed in Fig. 4. The crucial difference with the G^0h mixing in Fig. 3 is that in addition to the off-diagonal couplings $h\chi_1^\pm\chi_2^\mp$ and $A\chi_1^\pm\chi_2^\mp$, the transition hA can also proceed via the diagonal couplings $h\chi^+_{1(2)}\chi^-_{1(2)}$ and $A\chi^+_{1(2)}\chi^-_{1(2)}$. The individual contributions shown in Figs. 4(a)–4(c) are given by

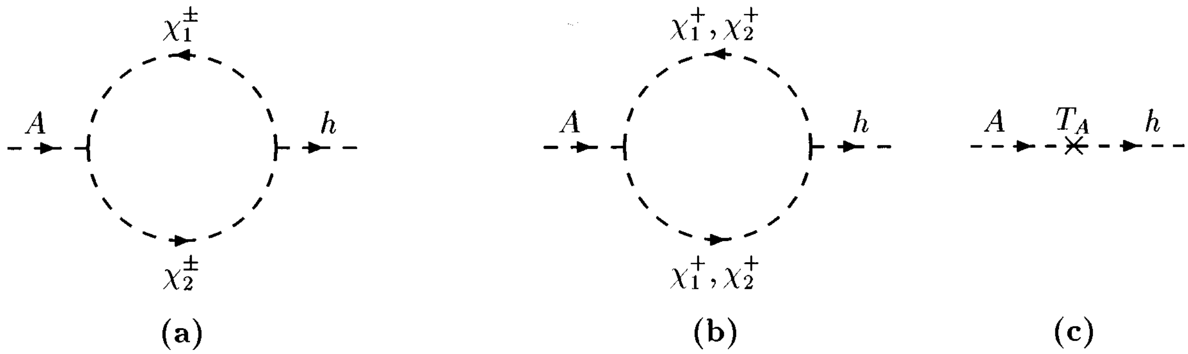

FIG. 4. Diagrams pertinent to the hA mixing.

$$\Pi^{hA}_{(a)}(0) = -2\left[\frac{s_\phi c_\phi}{s_\beta}\frac{\Im b^2}{v^2}(M_1^2 - M_2^2) + 4\frac{s_\phi^2 c_\phi^2}{s_\beta c_\beta^2}\frac{\Im b^2 \Re b^2}{v^2}\right] I(M_1,M_2), \quad (4.11)$$

$$\Pi^{hA}_{(b)}(0) = 4\frac{s_\phi^2 c_\phi^2}{s_\beta c_\beta^2}\frac{\Im b^2 \Re b^2}{v^2} \times [I(M_1,M_1) + I(M_2,M_2)], \quad (4.12)$$

$$\Pi^{hA}_{(c)}(0) = \frac{c_\beta T_A}{v} = 2\frac{s_\phi c_\phi}{s_\beta}\frac{\Im b^2}{v^2}(M_1^2 - M_2^2) I(M_1,M_2). \quad (4.13)$$

Adding the self-energy expressions in Eqs. (4.11)–(4.13), we find

$$\Pi^{hA}(0) = 4\frac{s_\phi^2 c_\phi^2}{s_\beta c_\beta^2}\frac{\Im b^2 \Re b^2}{v^2} \times [I(M_1,M_1) + I(M_2,M_2) - 2I(M_1,M_2)] = \frac{s_\phi^2 c_\phi^2}{s_\beta c_\beta^2}\frac{\Im b^2 \Re b^2}{2\pi^2 v^2}\left[\frac{M_1^2 + M_2^2}{2(M_1^2 - M_2^2)}\ln\left(\frac{M_1^2}{M_2^2}\right) - 1\right]. \quad (4.14)$$

It is obvious that the hA self-energy becomes UV finite only after the CP-odd tadpole graph in Fig. 4(c) has been included. Similarly, one may calculate the CP-violating HA transition and arrive at an analogous UV-safe analytic expression. Finally, we should emphasize that $\Pi^{hA}(0)$ exhibits a strong non-decoupling behavior for large mass differences of the charged scalars $\chi_1^\pm$ and $\chi_2^\pm$, i.e., $\Pi^{hA}(0) \propto \Im f^2 \ln(M_2/M_1)$ for $M_2/M_1 \gg 1$ [12]. The phenomenological consequences of the Abelian two-Higgs-boson model will be discussed in the next section.

V. DISCUSSION

The existence of a sizable Higgs scalar-pseudoscalar mixing may have profound implications for experiments of CP violation both at collider and lower energies. In particular, a large CP-violating hA mixing may give rise to substantial contributions to the EDM's of neutron, electron and muon. The experimental upper bounds on the EDM's of these light fermions are very tight[2] [13]: $d_n = 1.1\times 10^{-25}$ e cm, $d_e = (-0.3\pm 0.8)\times 10^{-26}$ e cm and $d_\mu = (3.7\pm 3.4)\times 10^{-19}$ e cm. On the theoretical side, Barr and Zee suggested a mechanism [15] which is found to play a key role in enhancing the EDM prediction in models with CP violation in the Higgs sector. Even though the Barr-Zee mechanism occurs at two loops, it is still very sensitive to the mixing of scalar and pseudoscalar Higgs particles. Shortly afterwards, Gunion and Wyler extended this idea by contemplating an analogous quark chromo-EDM operator [16], which may dominate in the neutron EDM over other CP-violating operators such as the CP-odd three-gluon moment introduced by Weinberg [17].

Taking the aforementioned contributions into account, Hayashi *et al.* [18] have constrained the parameter space of a two-Higgs-doublet model with maximal CP violation [17]. They found that the mass splitting ΔM between a scalar and a pseudoscalar Higgs boson due to a tree-level hA or HA mixing should not be too large. Adapting their results to the Abelian two-Higgs-boson model, we may estimate the upper limits

$$\frac{\Delta M}{M} < 0.10,\ 0.13,\ 0.24, \quad (5.1)$$

for $M = (M_h + M_A)/2 = 200$, 400, and 600 GeV, respectively. In the above estimate, we have assumed that $(M_h^2 - M_A^2)/M^2 \ll 1$, and $M_h, M_A \ll M_H$. The proposed Brookhaven experiment searching for the muon EDM [14] might lower further the upper bounds in Eq. (5.1), even up to one order of magnitude.

Since we are interested in confronting our theoretical predictions with the phenomenological limits on a hA mixing in Eq. (5.1), it is more convenient to do so by reducing the large number of independent parameters present in the Abelian

[2]There has been a recent proposal that next-round experiments at the Brookhaven National Laboratory may improve the accuracy of present measurements on the muon EDM by six orders of magnitude [14].

two-Higgs-boson model. As we have made explicit in Sec. II, the results obtained in this model for the scalar-pseudoscalar mixing apply equally well to the respective 2HDM. For definiteness, we choose

$$\tan\beta=\tan\phi=1,\ \ \Re\mu^2=-2\lambda_1 v_1^2=-\frac{1}{2}\lambda_{34}v^2. \quad (5.2)$$

With the above choice of parameters, we have a degenerate hA system, namely $M_A^2=M_h^2=-2\lambda_1 v^2$, whereas the H-boson mass is in general not fixed, i.e., $M_H^2=-(\lambda_1+\lambda_2)v^2$. The mass degeneracy of h and A is then lifted after integrating out the charged scalar states $\chi_1^\pm$ and $\chi_2^\pm$. The self-energy $\Pi^{hA}(0)$ in Eq. (4.14) will then quantify the loop-induced mass difference ΔM between h and A. To a good approximation, we obtain

$$\frac{\Delta M}{M}\approx\frac{\Pi^{hA}(0)}{M^2}=(0.56,2.05,3.20,4.10,4.83)\times10^{-2}\times\left(\frac{\Im b^2\Re b^2}{v^2M^2}\right), \quad (5.3)$$

for ratios of charged scalar masses $M_2/M_1=2$, 4, 6, 8, and 10, respectively. Furthermore, in order to retain the perturbative nature of the Higgs potential, the mass parameter $|b|$ should be of order v and the quartic couplings should not be much larger than unity. These two facts allow us to deduce the qualitative limit $|\Im b^2\Re b^2/(v^2M^2)|<10$. We then find that the parameter $\Delta M/M$ measuring the radiative mass splitting between h and A may adequately reach the observable level of 10% for modest values of $|b|$ and charged-scalar-mass ratios, e.g., for $|\Im b^2\Re b^2/(v^2M^2)|=5$ and $M_2/M_1=4$. Clearly, more accurate constraints on the Abelian model may be derived if a global analysis of all sensitive low-energy observables, such as the electroweak oblique parameters S, T and U [19], is performed. For the case of a 2HDM, such an analysis will also depend on further model details in the gauge sector. We shall not embark upon this topic here. Instead, it may be worth stressing that scenarios with $M_h\approx M_A$ may lead to spectacular phenomena of resonant CP violation through scalar-pseudoscalar mixing at high-energy pp, e^+e^- and $\mu^+\mu^-$ colliders. More details may be found in [6].

It appears rather difficult to determine experimentally the origin of a non-vanishing Higgs scalar-pseudoscalar mixing which generically occurs in models with extended Higgs sectors. A CP-violating Higgs-scalar mixing could arise either spontaneously, due to the spontaneous breakdown of CP either at the tree level or through quantum mechanical effects, or explicitly due to CP-violating Yukawa interactions. Here, we have concentrated on the latter alternative. Within the framework of an Abelian two-Higgs-boson model which constitutes a realistic and phenomenologically relevant subgroup of the 2HDM, we have examined the conditions under which a tree-level CP-invariant Higgs potential can consistently co-exist with other CP-violating couplings, without spoiling renormalizability at high orders. Based on the non-renormalization theorem, we have reached the conclusion that for a tree-level Higgs potential, CP invariance can be enforced by a global $U(1)_Q$ symmetry of the Peccei-Quinn type, which can only be broken softly, namely by CP-violating operators having dimensionality less than 4. Within such a CP-violating scenario, the CP-odd tadpole renormalization induced by the tadpole graph of the pseudoscalar Higgs boson A has been found to be very important to render the radiatively generated hA and HA transitions UV finite. The formalism developed in this paper may apply equally well to more elaborate theories, such as the MSSM. We defer the study of scalar-pseudoscalar mixing in the MSSM to a forthcoming communication [20].

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APPENDIX: HIGGS-BOSON MASS MATRIX

Here, we will discuss the diagonalization of the Higgs-boson mass matrix and the stability conditions for the Higgs potential in Eq. (3.1). In general, the Higgs-boson masses in this Abelian model are obtained by diagonalizing the 4×4 matrix

$$\mathcal{M}_H^2=\begin{pmatrix} M_S^2 & M_{SP}^2 \\ M_{PS}^2 & M_P^2\end{pmatrix}, \quad (A1)$$

where M_S^2, M_P^2 and M_{SP}^2 are all 2×2 sub-matrices. In Eq. (A1), the general mass matrix $\mathcal{M}_H^2$ is written in the weak basis spanned by the fields H_1, H_2, A_1 and A_2. Thus, the matrices M_S^2 and M_P^2 describe the CP-conserving mass transitions $H_i\to H_j$ and $A_i\to A_j$, respectively, whereas M_{SP}^2 and M_{PS}^2 contain the CP-violating $H_i\to A_j$ and $A_i\to H_j$ transitions. Since $\mathcal{M}_H^2$ is symmetric, the matrices M_S^2 and M_P^2 must be symmetric as well, while $M_{SP}^2=(M_{PS}^2)^T$. As we have seen in Sec. III, all entries of M_{SP}^2 are proportional to the CP-odd tadpole parameter T_A. At the tree level or in the CP-invariant limit of the theory, we have $T_A=0$ and the diagonalization of M_S^2 and M_P^2 then proceeds independently.

To leading order, we adopt the limit of $T_A\to0$ in the mass-matrix diagonalization. We start considering the mass matrix for the CP-even Higgs scalars:

$$M_S^2=\begin{pmatrix} -2\lambda_1 v_1^2+\tan\beta\Re\mu^2-T_{H_1}/v_1 & -\lambda_{34}v_1v_2-\Re\mu^2 \\ -\lambda_{34}v_1v_2-\Re\mu^2 & -2\lambda_2 v_2^2+\cot\beta\Re\mu^2-T_{H_2}/v_2 \end{pmatrix}, \tag{A2}$$

where the tadpole parameters T_{H_1} and T_{H_2} are defined in Eqs. (3.3) and (3.4), respectively. After diagonalizing M_S^2 through the orthogonal transformation in Eq. (3.11), we obtain the mass eigenstates h and H. Their physical masses and the respective Higgs-scalar mixing are related to the weak parameters of the Higgs potential as follows:

$$-2\lambda_1 v_1^2+\tan\beta\Re\mu^2-T_{H_1}/v_1=M_h^2c_\theta^2+M_H^2s_\theta^2,$$

$$-2\lambda_2 v_2^2+\cot\beta\Re\mu^2-T_{H_2}/v_2=M_h^2s_\theta^2+M_H^2c_\theta^2,$$

$$-\lambda_{34}v_1v_2-\Re\mu^2=(M_H^2-M_h^2)s_\theta c_\theta. \tag{A3}$$

From Eq. (A3), we see that stability of the Higgs potential can naturally be achieved for negative values of the quartic couplings λ_1 and λ_2, whereas λ_{34} may have either sign in compliance with the constraint $\det M_S^2>0$. From an analogous analysis of M_P^2, we find that the parameter $\Re\mu^2$ must always be positive. More explicitly, we have, for the mass matrix of pseudoscalars,

$$M_P^2=\begin{pmatrix} \tan\beta\Re\mu^2-T_{H_1}/v_1 & -\Re\mu^2 \\ -\Re\mu^2 & \cot\beta\Re\mu^2-T_{H_2}/v_2 \end{pmatrix}. \tag{A4}$$

The mass matrix M_P^2 can be diagonalized via the orthogonal transformation of the weak fields A_1 and A_2 given in Eq. (3.10). In the mass basis, M_P^2 reads

$$\hat{M}_P^2=\begin{pmatrix} -\dfrac{c_\beta T_{H_1}+s_\beta T_{H_2}}{v} & \dfrac{s_\beta T_{H_1}-c_\beta T_{H_2}}{v} \\ \dfrac{s_\beta T_{H_1}-c_\beta T_{H_2}}{v} & \dfrac{\Re\mu^2}{s_\beta c_\beta}-\dfrac{s_\beta\tan\beta T_{H_1}+c_\beta\cot\beta T_{H_2}}{v} \end{pmatrix}. \tag{A5}$$

It is now easy to see that at the tree level, $\hat{M}_P^2$ has a massless eigenvalue corresponding to the true Goldstone boson G^0 and a massive one related to the CP-odd Higgs boson A, i.e.,

$$M_A^2=\frac{\Re\mu^2}{s_\beta c_\beta}. \tag{A6}$$

Since M_A^2 should always be positive for a stable theory, the latter implies that $\Re\mu^2>0$.

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