Naturalness, weak scale supersymmetry, and the prospect for the observation of supersymmetry at the Fermilab Tevatron and at the CERN LHC

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Naturalness bounds on weak scale supersymmetry in the context of radiative breaking of the electroweak symmetry are analyzed. In the case of minimal supergravity it is found that for low $\tan\beta$ and for low values of fine-tuning Φ, where Φ is defined essentially by the ratio μ^2/M_Z^2 where μ is the Higgs mixing parameter and M_Z is the Z boson mass, the allowed values of the universal scalar parameter m_0, and the universal gaugino mass $m_{1/2}$ lie on the surface of an ellipsoid with radii fixed by Φ leading to tightly constrained upper bounds $\sim\sqrt{\Phi}$. Thus for $\tan\beta\leqslant 2(\leqslant 5)$ it is found that the upper limits for the entire set of sparticle masses lie in the range <700 GeV (<1.5 TeV) for any reasonable range of fine-tuning ($\Phi\leqslant 20$). However, it is found that there exist regions of the parameter space where the fine-tuning does not tightly constrain m_0 and $m_{1/2}$. Effects of nonuniversalities in the Higgs boson sector and in the third generation sector on naturalness bounds are also analyzed and it is found that nonuniversalities can significantly affect the upper bounds. It is also found that achieving the maximum Higgs boson mass allowed in supergravity unified models requires a high degree of fine-tuning. Thus a heavy sparticle spectrum is indicated if the Higgs boson mass exceeds 120 GeV. The prospect for the discovery of supersymmetry at the Fermilab Tevatron and at the CERN LHC in view of these results is discussed. [S0556-2821(98)01819-0]

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I. INTRODUCTION

One of the important elements in supersymmetric model building is the issue of the mass scale of the supersymmetric particles. There is the general expectation that this scale should be of the order of the scale of the electroweak physics, i.e., in the range of a TeV. This idea is given a more concrete meaning in the context of supergravity unification [1] where one has spontaneous breaking of the electroweak symmetry by radiative corrections [2]. Radiative breaking of the electroweak symmetry relates the scale of supersymmetry soft breaking terms directly to the Z boson mass. This relationship then tells us that the soft supersymmetry (SUSY) breaking scale should not be much larger than the scale of the Z boson mass otherwise a significant fine-tuning will be needed to recover the Z boson mass. The above general connection would be thwarted if there were large internal cancellations occurring naturally within the radiative breaking condition which would allow m_0 and $m_{1/2}$ disproportionately large for a fixed fine-tuning. We shall show that precisely such a situation does arise in certain domains of the supergravity parameter space.

The simplest fine-tuning criterion is to impose the constraint that $m_0, m_{\tilde{g}}<1$ TeV where m_0 is the universal soft SUSY breaking scalar mass in minimal supergravity and $m_{\tilde{g}}$ is the gluino mass. The above criterion is easy to implement and has been used widely in the literature (for a review see Ref. [3]). A more involved fine-tuning criterion is given in Ref. [4]. However, it appears that the criterion of Ref. [4] is actually a measure of the sensitivity rather than of fine-tuning [5,6]. Another naturalness criterion is proposed in Ref. [6] and involves a distribution function. Although the distribution function is arbitrary the authors show that different choices of the function lead numerically to similar fine-tuning limits.

In the analysis of this paper we use the fine-tuning criterion introduced in Ref. [7] in terms of the Higgs boson mixing parameter μ which has several attractive features. It is physically well motivated, free of ambiguities, and easy to implement. Next we use the criterion to analyze the upper limits of sparticle masses for low values of $\tan\beta$, i.e., $\tan\beta\leqslant 5$. In this case one finds that m_0 and $m_{1/2}$ allowed by radiative breaking lie on the surface of an ellipsoid, and hence the upper limits of the sparticle masses are directly controlled by the radii of the ellipsoid which in turn are determined by the choice of fine-tuning. For instance, one finds that if one is in the low $\tan\beta$ end of $b-\tau$ unification [8] with the top quark mass in the experimental range, i.e., $\tan\beta\approx 2$, then for any reasonable range of fine-tuning the sparticle mass upper limits for the entire set of SUSY particles lie within the mass range below 1 TeV. Further, one finds that the light Higgs boson mass lies below 90 GeV under the same constraints. Thus in this case discovery of supersymmetry at the LHC is guaranteed according to any reasonable fine-tuning criterion. Next the paper explores larger values of $\tan\beta$, i.e., $\tan\beta\geqslant 10$ and here one finds that m_0 and $m_{1/2}$ for moderate values of fine-tuning do not lie on the surface of an ellipsoid; rather one finds that they lie on the surface of a hyperboloid. In this case m_0 and $m_{1/2}$ are not bounded by the μ constraint equation and large values of m_0 and $m_{1/2}$ can result with a fixed fine-tuning.

Effect of nonuniversalities on naturalness is also analyzed. Again one finds phenomena similar to the ones discussed above, although the domains in which these phenomena occur are shifted relative to those in the universal case. One of the important results that emerges is that the upper limits of sparticle masses can be dramatically affected by nonuniversalities. These results have important implications for the discovery of supersymmetry at the Fermilab Tevatron

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and the CERN Large Hadron Collider (LHC).

Our analysis is carried out in the framework of supergravity models with gravity mediated breaking of supersymmetry [9,1,3]. This class of models possesses many attractive features. One of the more attractive features of these models is that with R parity invariance the lightest neutralino is also the lightest supersymmetric particle over most of the parameter space of the theory and hence a candidate for cold dark matter. Precision renormalization group analyses show [10] that these models can accommodate just the right amount of dark matter consistent with the current astrophysical data [11,12]. However, in this work we shall not impose the constraint of dark matter.

The outline of the paper is as follows. In Sec. II we give a brief discussion of the fine-tuning measure used in the analysis. In Sec. III we use this criterion to discuss the upper limits on the sparticle masses in minimal supergravity for low $\tan\beta$, i.e., $\tan\beta\leqslant 5$ and show that the allowed solutions to radiative breaking lie on the surface of an ellipsoid. In Sec. IV we discuss naturalness in beyond the low $\tan\beta$ region. Here we show that radiative breaking of the electroweak symmetry leads to the soft SUSY breaking parameters lying on the surface of a hyperboloid. In Sec. V we discuss the effects of nonuniversalities on the upper limits. In Sec. VI we show that a high degree of fine-tuning is needed to have the light Higgs boson mass approach its maximum upper limit. The limits on Φ from the current data are discussed in Sec. VII. Implications of these results for the discovery of supersymmetric particles at colliders are also discussed in Secs. III–VI. Conclusions are given in Sec. VIII.

II. MEASURE OF NATURALNESS

We give below an improved version of the analysis of the fine-tuning criterion given in Ref. [7]. The radiative electroweak symmetry breaking condition is given by

$$\frac{1}{2}M_Z^2=\lambda^2-\mu^2, \tag{1}$$

where λ^2 is defined by

$$\lambda^2=\frac{\overline{m}_{H_1}^2-\overline{m}_{H_2}^2\tan^2\beta}{\tan^2\beta-1}. \tag{2}$$

Here $\overline{m}_{H_i}^2=m_{H_i}^2+\Sigma_i$ $(i=1,2)$ where Σ_i arise from the one loop corrections to the effective potential [13]. The issue of fine-tuning now revolves around the fact that a cancellation is needed between the λ^2 term and the μ^2 term to arrange the correct experimental value of M_Z. Thus a large value of λ^2 would require a large cancellation from the μ^2 term resulting in a large fine-tuning. This idea can be quantified by defining the fine-tuning parameter Φ so that

$$\Phi^{-1}=4\frac{\lambda^2-\mu^2}{\lambda^2+\mu^2}. \tag{3}$$

[The factor of 4 on the right hand side in Eq. (3) is just a convenient normalization.] The expression for Φ can be simplified by inserting in the radiative breaking condition Eq. (1). We then get

$$\Phi=\frac{1}{4}+\frac{\mu^2}{M_Z^2}. \tag{4}$$

The result above is valid with the inclusion of both the tree and the loop corrections to the effective potential. (Φ is related to the fine-tuning parameter δ defined in Ref. [7] by $\Phi=\delta^{-1}$). For large μ one has $\Phi\sim\mu^2/M_Z^2$, a result which has a very direct intuitive meaning. A large μ implies a large cancellation between the λ^2 term and the μ^2 term in Eq. (1) to recover the Z boson mass and thus leads to a large fine-tuning. Typically a large μ implies large values for the soft supersymmetry breaking parameters m_0 and $m_{1/2}$ and thus large values for the sparticle masses. However, large cancellation can be enforced by the internal dynamics of radiative breaking itself. In this case a small μ and hence a small fine-tuning allows for relatively large values of m_0 and of $m_{1/2}$. We show that precisely such a situation arises for certain regions of the parameter space of both the minimal model as well as for models with nonuniversalities.

III. UPPER BOUNDS ON SPARTICLE MASSES IN MINIMAL SUPERGRAVITY

We discuss now the upper bounds on the sparticle masses that arise under the criterion of fine-tuning we have discussed above. Using the radiative electroweak symmetry breaking constraint and ignoring the b-quark couplings, justified for small $\tan\beta$, we may express the fine-tuning parameter Φ_0 in the form

$$\Phi_0=-\frac{1}{4}+\left(\frac{m_0}{M_Z}\right)^2C_1+\left(\frac{A_0}{M_Z}\right)^2C_2+\left(\frac{m_{1/2}}{M_Z}\right)^2C_3+\left(\frac{m_{1/2}A_0}{M_Z^2}\right)C_4+\frac{\Delta\mu_{\rm loop}^2}{M_Z^2}, \tag{5}$$

where

$$C_1=\frac{1}{t^2-1}\left(1-\frac{3D_0-1}{2}t^2\right),\quad C_2=\frac{t^2}{t^2-1}k, \tag{6}$$

$$C_3=\frac{1}{t^2-1}(g-t^2e),\quad C_4=-\frac{t^2}{t^2-1}f,$$

$$\Delta\mu_{\rm loop}^2=\frac{\Sigma_1-t^2\Sigma_2}{t^2-1}. \tag{7}$$

Here $t\equiv\tan\beta,e,f,g,k$ and the sign conventions of A_0 and μ are as defined in Ref. [14], D_0 is defined by

$$D_0=1-(m_t/m_f)^2,\quad m_f\simeq 200\sin\beta\ \ \text{GeV}, \tag{8}$$

and Σ_1 and Σ_2 are as defined in Ref. [13].

TABLE I. The scale dependence of $C_1(Q)$–$C_4(Q)$ for minimal supergravity when $m_t = 175$ GeV for $\tan\beta = 2$, 5, 10, and 20.

		Scale dependence of C_1–C_4			
$\tan\beta$	Q(GeV)	C_1	C_2	C_3	C_4
2	91.2	0.7571	0.0711	4.284	0.3119
	2000	0.6874	0.0879	2.851	0.3073
	4000	0.6702	0.0918	2.607	0.3055
	6000	0.6598	0.0941	2.474	0.3043
	8000	0.6523	0.0957	2.384	0.3034
	10000	0.6464	0.0970	2.316	0.3026
5	91.2	0.14212	0.1024	2.871	0.4491
	500	0.09016	0.1099	2.200	0.4245
	1000	0.06843	0.1126	1.973	0.4138
	1500	0.05558	0.1142	1.851	0.4074
	2000	0.04639	0.1152	1.768	0.4028
	2500	0.03924	0.1160	1.706	0.3992
	3000	0.03336	0.1166	1.657	0.3962
	3500	0.02838	0.1172	1.617	0.3937
	4000	0.02406	0.1176	1.583	0.3914
	4500	0.02023	0.1180	1.553	0.3895
	5000	0.01680	0.1184	1.527	0.3877
10	91.2	0.0756	0.1040	2.710	0.4561
	250	0.0446	0.1081	2.305	0.4397
	500	0.0230	0.1108	2.062	0.4280
	750	0.0102	0.1122	1.931	0.4211
	1000	0.0011	0.1132	1.843	0.4160
	1250	−0.0060	0.1140	1.778	0.4121
	1500	−0.0118	0.1146	1.726	0.4089
	1750	−0.0167	0.1151	1.683	0.4061
	2000	−0.0210	0.1155	1.646	0.4037
	2500	−0.0281	0.1162	1.587	0.3997
	3000	−0.0341	0.1167	1.540	0.3964
20	250	0.02850	0.1084	2.269	0.4406
	500	0.00685	0.1109	2.029	0.4286
	750	−0.00592	0.1123	1.899	0.4214
	1000	−0.01504	0.1133	1.812	0.4162
	1250	−0.02213	0.1140	1.747	0.4122
	1500	−0.02795	0.1146	1.695	0.4089
	1750	−0.03288	0.1150	1.653	0.4061
	2000	−0.03716	0.1154	1.617	0.4036
	2500	−0.04433	0.1161	1.558	0.3995
	3000	−0.05020	0.1166	1.511	0.3961

To investigate the upper limits on m_0 and $m_{1/2}$ consistent with a given fine-tuning it is instructive to write Eq. (5) in the form

$$C_1 m_0^2 + C_3 m_{1/2}'^2 + C_2' A_0^2 + \Delta\mu_{\rm loop}^2 = M_Z^2\left(\Phi_0 + \frac{1}{4}\right), \tag{9}$$

where

$$m_{1/2}' = m_{1/2} + \frac{1}{2} A_0 \frac{C_4}{C_3}, \quad C_2' = C_2 - \frac{1}{4}\frac{C_4^2}{C_3}, \tag{10}$$

and $\Delta\mu_{\rm loop}^2$ is the loop correction. Now for the universal case one finds that the loop corrections to μ are generally small for $\tan\beta \leqslant 5$ in the region of fine-tuning of $\Phi_0 \leqslant 20$. Further, using renormalization group analysis one finds that $C_2' > 0$ and $C_3 > 0$ and at least for the range of fine-tuning $\Phi_0 \leqslant 20$, $C_1 > 0$ (see Table I). Thus in this case defining

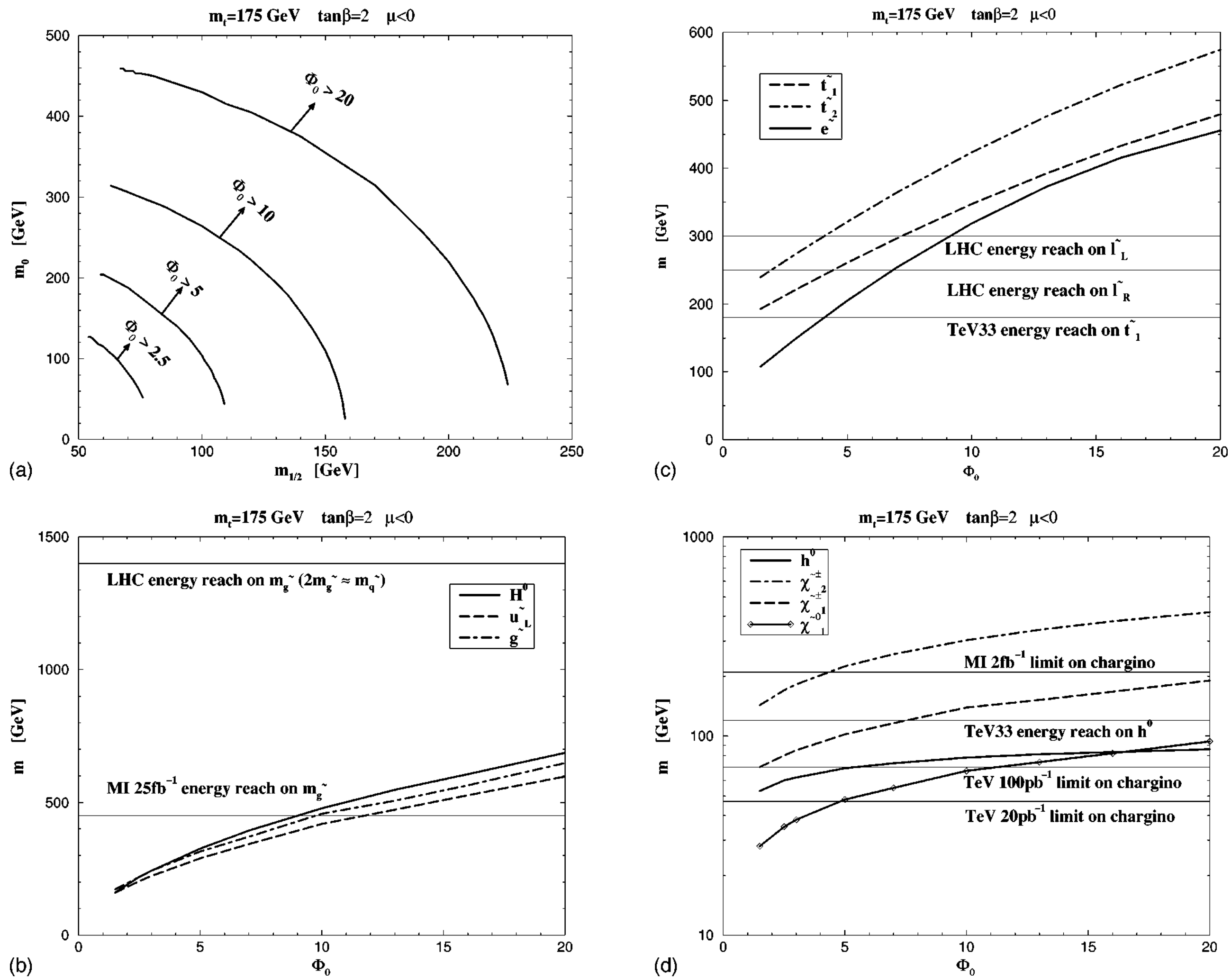

FIG. 1. (a) Contour plot of the upper limit in the $m_0 - m_{1/2}$ plane for different values of Φ_0 when $m_t = 175$ GeV, $\tan\beta = 2$, and $\mu < 0$. The allowed region lies below the curves. (b) Upper bounds on mass of the heavy Higgs boson H^0, of the gluino and of the squark $\tilde{u}_L$ (for the first two generations) for the same parameters as in (a). (c) Upper bounds on mass of the $\tilde{e}_L$, of the light top squark $\tilde{t}_1$, and of the heavy top squark $\tilde{t}_2$ for the same parameters as in (a). (d) Upper bounds on masses of the light Higgs boson h^0, of the light chargino $\tilde{\chi}_1^{\pm}$, of the heavy chargino $\tilde{\chi}_2^{\pm}$, and of the neutralino $\tilde{\chi}_1^0$ for the same parameters as in (a).

$$a^2 = M_Z^2 \frac{\Phi + 1/4}{C_3}, \quad b^2 = M_Z^2 \frac{\Phi + 1/4}{C_1}, \quad c^2 = M_Z^2 \frac{\Phi + 1/4}{C_2'} \tag{11}$$

we find that for $\tan\beta \leq 5$, $\Phi_0 \leq 20$ the radiative breaking condition can be approximated by

$$\frac{m_{1/2}'^2}{a^2} + \frac{m_0^2}{b^2} + \frac{A_0^2}{c^2} \simeq 1 \tag{12}$$

and the renormalization group analysis shows that at the scale $Q = M_Z$ the quantities a^2, b^2, and c^2 are positive. Fixing the fine-tuning parameter Φ_0 fixes a, b, and c and one finds that m_0 and $m_{1/2}$ are bounded as they lie on the boundary of an ellipse. Further Eq. (12) implies that the upper bounds on m_0 and $m_{1/2}$ increase as $\sim \sqrt{\Phi_0}$ for large Φ_0. A similar dependence on fine-tuning was observed in the analysis of Ref. [4].

We give now the full analysis without the approximation of Eq. (12). We consider the case of $\tan\beta = 2$ first which lies close to the low end of the $\tan\beta$ region of b-τ unification with the top quark mass taken to lie in the experimental range [8]. In Fig. 1(a) we give the contour plot of the upper limits for the parameters m_0 and $m_{1/2}$ in the $m_0 - m_{1/2}$ plane for the case of $\tan\beta = 2$ and $m_t = 175$ GeV for $2.5 \leq \Phi_0 \leq 20$. As expected, one finds that the contours corresponding to larger values of m_0 and $m_{1/2}$ require larger values of Φ_0. The upper limits of the mass spectra for the same set of parameters as in Fig. 1(a) are analyzed in Figs. 1(b)–1(d). In Fig. 1(b) the upper limits of the mass spectra of the heavy Higgs boson, the first two generation squarks, and the gluino are given. We find that the mass of the squark and of the gluino are very similar over essentially the entire range of

TABLE II. The upper bound on sparticle masses for minimal supergravity when $m_t=175$ GeV and $\mu<0$ for different values of $\tan\beta$ and fine-tuning measure Φ_0. All the masses are in GeV.

						Minimal supergravity $\mu<0$							
$\tan\beta$	Φ	H	$\tilde{u}_l$	$\tilde{e}_l$	$\tilde{e}_r$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{g}$	h	$\tilde{\chi}_1^{\pm}$	$\tilde{\chi}_2^{\pm}$	$\tilde{\chi}_1^0$	m_0
2	5	326	290	212	207	264	325	316	69	102	224	48	204
	10	479	419	320	315	353	429	459	78	139	303	67	315
	20	687	598	463	459	483	579	649	86	190	419	94	459
5	2.5	318	352	292	285	265	352	295	97	77	180	42	282
	5	594	589	560	556	365	507	425	103	114	232	60	556
	10	930	906	888	884	510	744	610	109	167	309	86	886
	20	1417	1381	1368	1365	742	1113	873	116	243	423	123	1368
10	2.5	416	464	403	393	323	431	316	106	73	184	42	395
	5	3702	4089	3914	3887	2311	3311	1272	136	190	382	158	3920
	10	5963	6714	6365	6318	3855	5428	2776	144	283	797	273	6370
	20	8875	10536	9622	9527	6170	8616	4945	150	404	1409	400	9596
20	2.5	1889	2136	2044	2003	1202	1697	566	128	104	214	69	2080
	5	3581	4198	3906	3827	2480	3383	1764	138	194	515	178	3980
	10	5540	6585	6114	5978	3893	5270	3124	145	282	895	274	6210
	20	8007	10092	8954	8734	6078	8167	5322	151	403	1516	399	9060

Φ_0. Upper limits of $\tilde{e}$, $\tilde{t}_1$, $\tilde{t}_2$ are given in Fig. 1(c). In Fig. 1(d) we exhibit the upper limits for the light Higgs boson, the chargino, and the lightest neutralino. We note that except for small values of Φ_0 one finds that the scaling laws [15] (e.g., $m_{\chi_1^0}\simeq\frac{1}{2}m_{\chi_1^{\pm}}$) are obeyed with a high degree of accuracy. We note that the Higgs boson mass upper limit in this case falls below 85–90 GeV for $\Phi_0\leqslant 20$. At the Tevatron in the Main Injector era one will be able to detect charginos using the trileptonic signal [16] with masses up to 230 GeV with 10 fb^{-1} of integrated luminosity [17,18]. Reference to Fig. 1(d) shows that the above implies that the upper limit of chargino masses for the full range of $\Phi_0\leqslant 20$ will be accessible at the Tevatron.

For the gluino the mass range up to 450 GeV will be accessible at the Tevatron in the Main Injector era with 25 fb^{-1} of integrated luminosity. This means that one can explore gluino mass limits up to $\Phi_0=10$ for $\tan\beta=2$. However, at the LHC gluino masses in the range 1.6–2.3 TeV [19]/1.4–2.6 TeV [20] for most values of μ and $\tan\beta$ will be accessible and recent analyses show that the accuracy of the $m_{\tilde{g}}$ mass measurement can be quite good, i.e., to within 1–10% depending on what part of the supergravity parameter space one is in [21]. Thus for $\tan\beta=2$ one will be able to observe and measure with reasonable accuracy the masses of the charginos, the gluino, and the squarks for the full range of values of $\Phi_0\leqslant 20$ at the LHC. It has recently been argued that the Next Linear Collider (NLC), where even more accurate mass measurements [22–24] are possible, will allow one to use this device for the exploration of physics at the post-grand unified theory (GUT) and string scales [25]. The NLC also offers the possibility of testing a good part of the parameter space for the $\tan\beta=2$ model. The analysis given in Table II shows that the full sparticle mass spectrum for $\tan\beta=2$ can be tested at the NLC with $\sqrt{s}=1$ TeV for $\Phi_0\leqslant 10$ and over the entire range $\Phi_0\leqslant 20$ with $\sqrt{s}=1.5$ TeV.

We discuss next the upper limit of sparticle masses for $\tan\beta=5$. In Fig. 2(a) we give the contour plot of m_0 and $m_{1/2}$ upper limits in the $m_0-m_{1/2}$ plane for the same value of the top mass and in the same Φ_0 range as in Fig. 1(a). Here we find that for fixed Φ_0 the contours are significantly further outwards compared to the case for $\tan\beta=2$. Correspondingly the upper limits of the mass spectra for the same value of Φ_0 are significantly larger in Figs. 2(b)–2(d) relative to those given in Figs. 1(b)–1(d). In this case the light chargino mass lies below 243 GeV for $\Phi_0\leqslant 20$ and thus the upper limits for values of $\Phi_0\leqslant 20$ could be probed at the Tevatron in the Main Injector era where chargino masses up to 280 GeV will be accessible with 100 fb^{-1} of integrated luminosity [17,18]. Similarly in this case the gluino mass lies below 873 GeV for $\Phi_0\leqslant 20$ and thus the upper limit for values of $\Phi_0\leqslant 20$ could be probed at the LHC which as mentioned above can probe gluino masses in the mass range of $1.6-2.3$ TeV [19]/$1.4-2.6$ TeV [20]. LHC can probe squark masses up to 2–2.5 TeV, so squark masses of the above size should be accessible at the LHC. A full summary of the results for values of $\tan\beta=2$–20 is given in Table II where the sparticle mass limits in the range $2.5\leqslant\Phi_0\leqslant 20$ are given. The analysis tells us that for a reasonable constraint on Φ_0, i.e., $\Phi_0\leqslant 20$, the gluino and the squarks must be discovered at the LHC for the values of $\tan\beta\leqslant 5$.

IV. REGIONS OF THE HYPERBOLIC CONSTRAINT

In this section we discuss the possibility that in certain regions of the supergravity parameter space the sparticle spectrum can get large even for modest values of the fine-tuning parameter Φ_0. This generally happens in regions

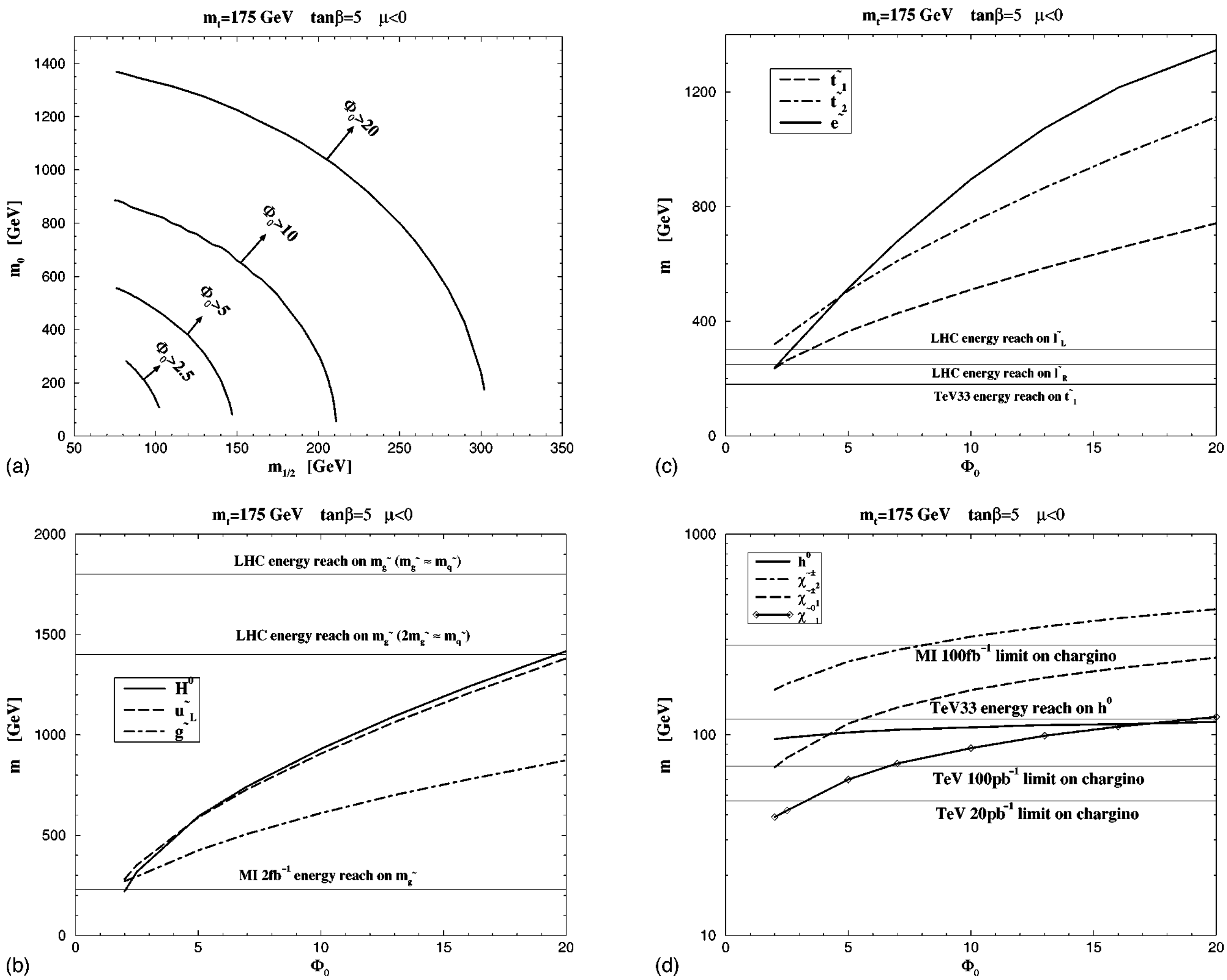

FIG. 2. (a) Contour plot of the upper limit in the $m_0 - m_{1/2}$ plane for different values of Φ_0 when $m_t = 175$ GeV, $\tan\beta = 5$, and $\mu < 0$. The allowed region lies below the curves. (b) Upper bounds on mass of the heavy Higgs boson H^0, of the gluino and of the squark $\tilde{u}_L$ (for the first two generations) for the same parameters as in (a). (c) Upper bounds on mass of the $\tilde{e}_L$, of the light top squark $\tilde{t}_1$, and of the heavy top squark $\tilde{t}_2$ for the same parameters as in (a). (d) Upper bounds on masses of the light Higgs boson h^0, of the light chargino $\tilde{\chi}_1^{\pm}$, of the heavy chargino $\tilde{\chi}_2^{\pm}$, and of the neutralino $\tilde{\chi}_1^0$ for the same parameters as in (a).

where the loop corrections to μ are large. For example, in contrast to the case of small $\tan\beta$ one finds that for the case of large $\tan\beta$ the loop corrections to μ can become rather significant. In this case the size of the loop corrections to μ depends sharply on the scale Q_0 where the minimization of the effective potential is carried out. In fact, in this case there is generally a strong dependence on Q_0 of both the tree and the loop contributions to μ which, however, largely cancel in the sum, leaving the total μ with a sharply reduced but still non-negligible residual Q_0 dependence. An illustration of this phenomenon is given in Fig. 3. The choice of Q_0 where one carries out the minimization of the effective potential is of importance because we can choose a value of Q_0 where the loop corrections are small so that we can carry out an analytic analysis similar to the one in Sec. III. (For example, for the case of Fig. 3 the loop correction to μ is minimized at $Q_0 \approx 1$ TeV.) Generally we find the value Q_0 at which the loop correction to μ is minimized to be about the average of the smallest and the largest sparticle masses, a value not too distant from $\sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, which is typically chosen to minimize the two-loop correction to the Higgs boson mass [26,12]. Choosing a value Q_0 where the loop correction is small (Q_0 is typically greater than 1 TeV here), and following the same procedure as in Sec. III we find that this time $\mathrm{sgn}[C_1(Q_0)] = -1$ [see entries for the case $\tan\beta = 10,20$ in Table I and see also Fig. 5(a)]. There are now two distinct possibilities: case A and case B which we discuss below.

Case A. This case corresponds to

$$\left(\Phi_0 + \frac{1}{4}\right) M_Z^2 - C_2' A_0^2 > 0 \tag{13}$$

and occurs for relatively small values of $|A_0|$. Here the radiative breaking equation takes the form

$$\frac{m_{1/2}'^2}{\alpha^2(Q_0)} - \frac{m_0^2}{\beta^2(Q_0)} \simeq 1, \tag{14}$$

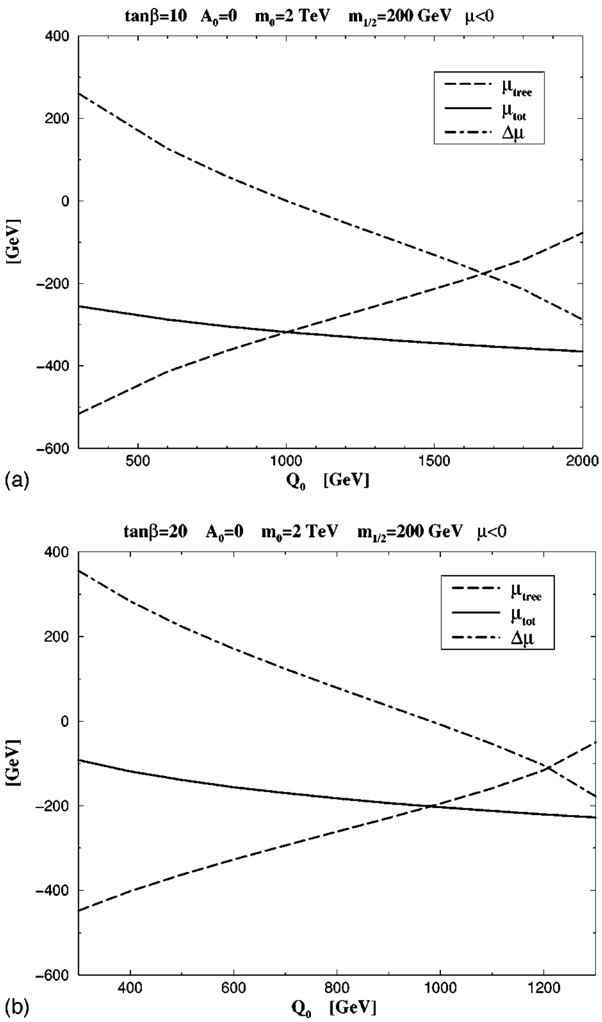

FIG. 3. (a) Variation of μ with the scale Q_0 where the minimization of the potential is carried out for the case when $\tan\beta=10$, $A_0=0$, $m_0=2000$ GeV, $m_{1/2}=200$ GeV, and $\mu<0$. (b) Variation of μ with the scale Q_0 where the minimization of the potential is carried out for the case when $\tan\beta=20$, with the other parameters the same as in (a).

where

$$\alpha^2=\frac{|(\Phi_0+1/4)M_Z^2-C_2'A_0^2|}{|C_3|} \tag{15}$$

and

$$\beta^2=\frac{|(\Phi_0+1/4)M_Z^2-C_2'A_0^2|}{|C_1|}. \tag{16}$$

The appearance of a minus sign changes intrinsically the character of the constraint of the electroweak symmetry breaking. One finds now that unlike the previous case, where m_0 and $m_{1/2}$ lie on the boundary of an ellipse for fixed A_0 [see Figs. 4(a) and also Figs. 1(a) and 2(a)], here they lie on a hyperbola. A diagrammatic representation of the constraint of Eq. (14) is given in Figs. 4(b),4(c). The position of the apex of the hyperbola depends on A_0 as can be seen from Fig. 4(c). The choice of Φ itself does not put an upper bound on m_0 and $m_{1/2}$ and consequently they can get large for a fixed fine-tuning unless other constraints intervene. Thus in this case the rule that the upper bounds are proportional to $\sqrt{\Phi_0}$ breaks down. In fact from Eqs. (14)–(16) we see that for large m_0 and $m_{1/2}$ one has

$$m_0\simeq\sqrt{\frac{|C_3|}{|C_1|}}m'_{1/2} \tag{17}$$

and thus independent of Φ_0. Thus the hyperbolae for different values of fine-tuning have the same asymptote independent of Φ_0 as illustrated in Fig. 4(b).

Case B. This case corresponds to

$$\left(\Phi_0+\frac{1}{4}\right)M_Z^2-C_2'A_0^2<0 \tag{18}$$

and occurs for relatively large values of A_0. Here the radiative breaking equation takes the form

$$\frac{m_0^2}{\beta^2(Q_0)}-\frac{m_{1/2}'^2}{\alpha^2(Q_0)}\simeq 1. \tag{19}$$

A diagrammatic representation of this case is given in Fig. 4(d). As in case A, here also m_0 and $m_{1/2}$ lie on a hyperbola, with the position of the apex determined by the value of A_0. Again here as in case A the choice of Φ_0 itself does not control the upper bound on m_0 and $m_{1/2}$. This can be seen from Fig. 4(d) where the hyperbolae for different values of the fine-tuning have the same asymptote independent of Φ_0 just as in case A. We emphasize that the analytic analysis based on Eqs. (14) and (19) is for illustrative purposes only, and the results presented in this paper are obtained including the b-quark couplings and including the full one loop corrections to μ. In Fig. 5(b) we present a numerical analysis of the allowed region of m_0 and $m_{1/2}$. One finds that the cases $A_0=0$ and $A_0=500$ GeV show that m_0 and $m_{1/2}$ lie on a branch of a hyperbola and simulate the illustration of Fig. 4(c). This is what one expects for the small A_0 case. Similarly for the cases $A_0=-1000$ GeV and $A_0=-2000$ GeV in Fig. 5(b), m_0 and $m_{1/2}$ lie on a branch of a hyperbola and simulate the illustration of the right hyperbola in Fig. 4(d) as is appropriate for a large negative A_0. Similarly for the case $A_0=1000$ GeV in Fig. 5(b), m_0 and $m_{1/2}$ again lie on a branch of a hyperbola and simulate the illustration of the left hyperbola in Fig. 4(d). A similar analysis for $\tan\beta=20$ can be found in Fig. 5(c). Thus one finds that the results of the analytic analysis are supported by the full numerical analysis.

V. EFFECTS OF NONUNIVERSAL SOFT SUSY BREAKING

The analysis of Secs. III and IV above is carried out under the assumption of universal soft supersymmetry boundary conditions at the GUT scale. These universal boundary conditions arise from the assumption of a flat Kahler potential. However, the framework of supergravity unification [1,3] al-

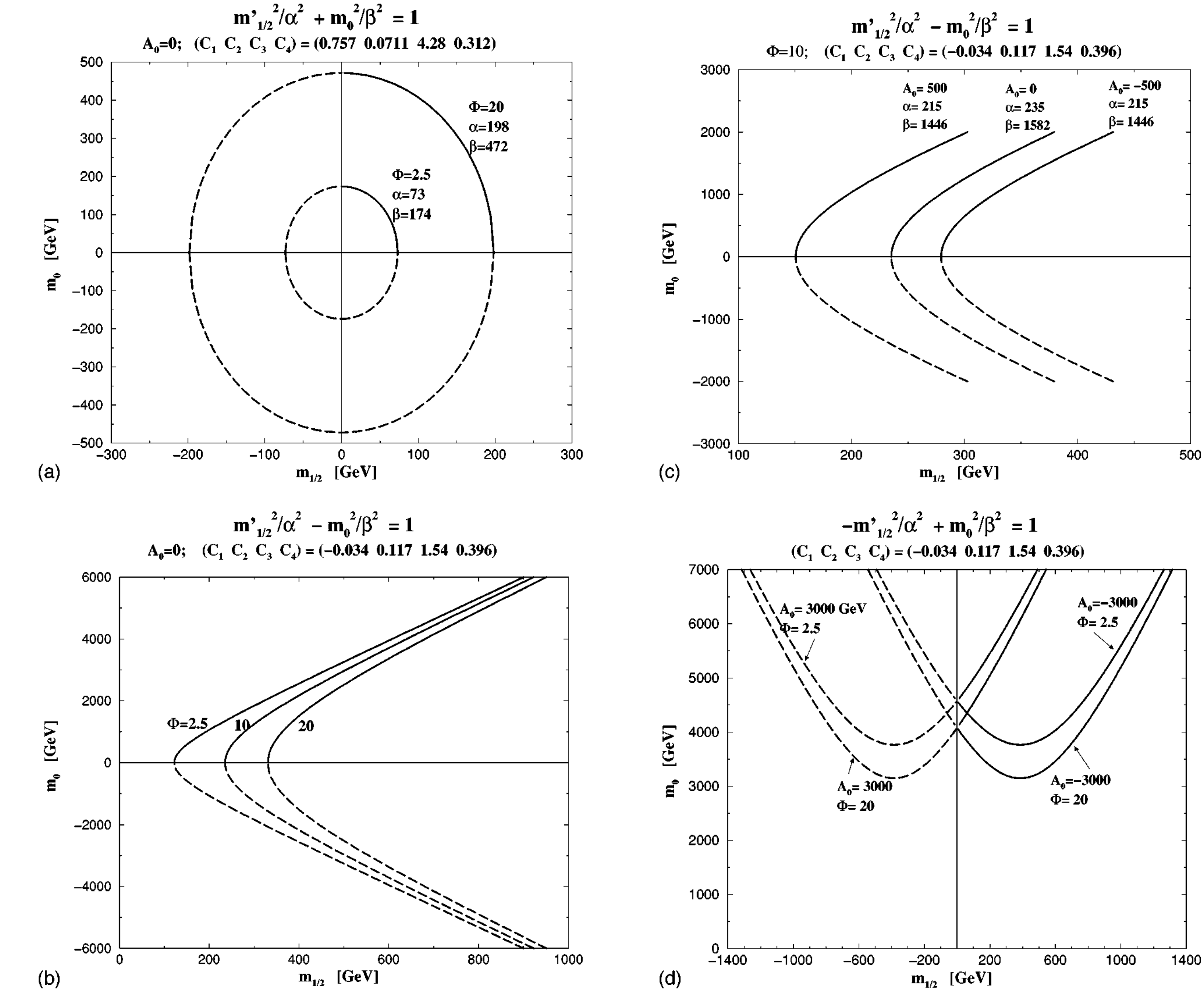

FIG. 4. (a) Diagrammatic illustration of the ellipse represented by Eq. (12), where the values of C_1–C_4 are for $\tan\beta=2$ and $Q=M_Z$ from Table I. The relevant parts of the ellipses are in solid line. (b) Diagrammatic illustration of the hyperbola represented by Eqs. (14) and (28), where the values of C_1–C_4 are for $\tan\beta=10$ and $Q=3000$ GeV from Table I. The relevant parts of the hyperbolae are in solid line. (c) Diagrammatic illustration of the hyperbola represented by Eqs. (14) and (28), where the values of C_1–C_4 are for $\tan\beta=10$ and $Q=3000$ GeV from Table I. The relevant parts of the hyperbolae are in solid line. (d) Diagrammatic illustration of the hyperbola represented by Eqs. (19) and (30), where the values of C_1–C_4 are for $\tan\beta=10$ and $Q=3000$ GeV from Table I. The relevant parts of the hyperbolae are in solid line.

lows for more general Kahler structures and hence for nonuniversalities in the soft supersymmetry breaking parameters [27,25]. In the analysis of this section we shall assume universalities in the soft supersymmetry breaking parameters in the first two generations of matter but allow for nonuniversalities in the Higgs boson sector [25,28–30] and in the third generation of matter [25,30,31]. It is convenient to parametrize the nonuniversalities in the following fashion. In the Higgs boson sector one has

$$m_{H_1}^2=m_0^2(1+\delta_1), \quad m_{H_2}^2=m_0^2(1+\delta_2). \tag{20}$$

Similarly in the third generation sector one has

$$m_{\tilde{Q}_L}^2=m_0^2(1+\delta_3), \quad m_{\tilde{U}_R}^2=m_0^2(1+\delta_4). \tag{21}$$

A reasonable range for the nonuniversality parameters is $|\delta_i|\leqslant 1$ $(i=1\text{–}4)$. Inclusion of nonuniversalities modifies the electroweak symmetry breaking equation determining the parameter μ^2, and leads to corrections to the fine-tuning parameter Φ. One finds that with these nonuniversality corrections Φ is given by

$$\Phi=-\frac{1}{4}+\left(\frac{m_0}{M_Z}\right)^2 C_1'+\left(\frac{A_0}{M_Z}\right)^2 C_2+\left(\frac{m_{1/2}}{M_Z}\right)^2 C_3 +\left(\frac{m_{1/2}A_0}{M_Z^2}\right) C_4+\frac{\Delta\mu_{\rm loop}^2}{M_Z^2}, \tag{22}$$

where

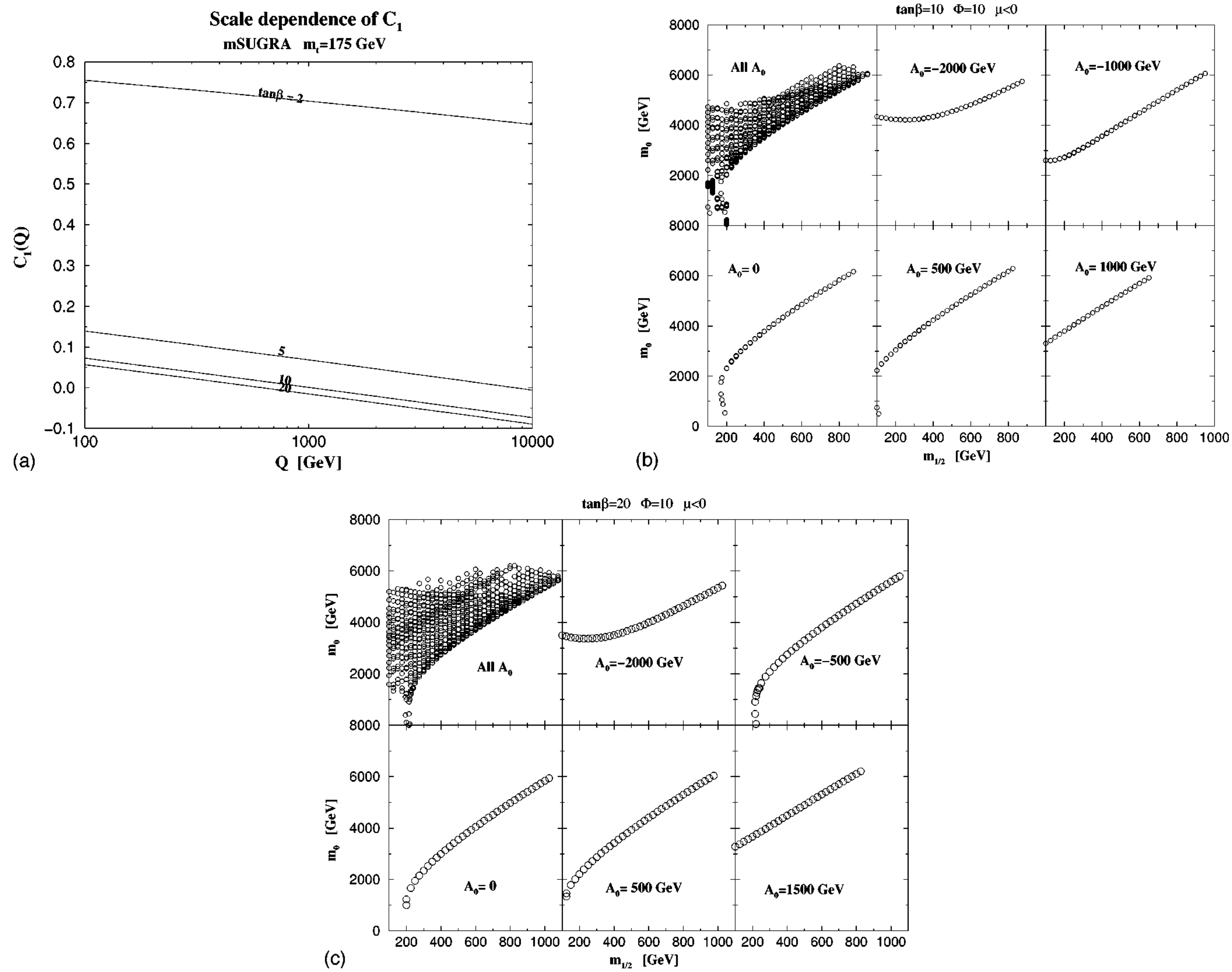

FIG. 5. (a) The scale dependence of $C_1(Q)$ for minimal supergravity when $m_t=175$ GeV for tan $\beta=2$, 5, 10, and 20. (b) Allowed region in the m_0–$m_{1/2}$ plane in the minimal supergravity case for $m_t=175$ GeV, tan $\beta=10$, $\Phi_0=10$, and negative μ. (c) Allowed region in the m_0–$m_{1/2}$ plane in the minimal supergravity case for $m_t=175$ GeV, tan $\beta=20$, $\Phi_0=10$, and negative μ.

$$C_1'=\frac{1}{t^2-1}\left(1-\frac{3D_0-1}{2}t^2\right)+\frac{1}{t^2-1}\left(\delta_1-\delta_2 t^2 - \frac{D_0-1}{2}(\delta_2+\delta_3+\delta_4)t^2\right)+\frac{3}{5}\frac{t^2+1}{t^2-1}\frac{pS_0}{m_0^2}, \quad (23)$$

and C_2, C_3, and C_4 are as defined in Eqs. (6) and (7). Here S_0 is the trace anomaly term

$$S_0=\mathrm{Tr}(Ym^2) \quad (24)$$

evaluated at the GUT scale M_G. It vanishes in the universal case since $\mathrm{Tr}(Y)=0$, but contributes when nonuniversalities are present. p is as defined in Ref. [30]. Numerically for $M_G=10^{16.2}$ GeV and $\alpha_G=1/24$ one has $p\simeq 0.045$. Equation (23) shows how important the effects of nonuniversalities are on Φ. For a moderate value of $m_0=250$ GeV the factor $(m_0/M_Z)^2$ is ~ 7.5 and since $\delta_i \sim \mathrm{O}(1)$, Φ gets a huge shift. This means that the upper limits of the sparticle masses are going to be sensitively dependent on the magnitudes and signatures of δ_i.

It is instructive to write the radiative breaking equation Eq. (22) with nonuniversalities in a form similar to Eq. (9). We get

$$C_1' m_0^2+C_3 m_{1/2}'^2+C_2' A_0^2+\Delta\mu_{\mathrm{loop}}^2=M_Z^2\left(\Phi+\frac{1}{4}\right), \quad (25)$$

where C_1' is defined in Eq. (23), and C_2 and C_3 are defined in Eq. (6), and where $\Delta\mu_{\mathrm{loop}}^2$ is the loop correction. We discuss the case of nonuniversalities in the Higgs boson sector first and consider two extreme examples within the constraint of $|\delta_i|\leqslant 1$ $(i=1-2)$. These are (i) $\delta_1=1$, $\delta_2=-1$ and (ii) $\delta_1=-1$, $\delta_2=1$, with $\delta_3=0=\delta_4$ in both cases. For case (i) we find from Eq. (23) that the nonuniversalities make a positive contribution to C_1', and thus $C_1'>0$ (see Table III). As for the universal case the loop corrections in this case

TABLE III. $C_1'(M_Z)$ for different values of δ_1 and δ_2 when $m_t=175$ GeV for $\tan\beta=2$, 5, 10, and 20.

$\tan\beta$	δ_1	δ_2	C_1'
2	−1.0	1.0	−0.341
	−0.75	0.75	−0.067
	−0.5	0.5	0.208
	−0.25	0.25	0.483
	0.0	0.0	0.757
	0.25	−0.25	1.032
	0.5	−0.5	1.306
	0.75	−0.75	1.581
	1.0	−1.0	1.855
5	−1.0	1.0	−0.572
	−0.75	0.75	−0.393
	−0.5	0.5	−0.215
	−0.25	0.25	−0.036
	0.0	0.0	0.142
	0.25	−0.25	0.321
	0.5	−0.5	0.499
	0.75	−0.75	0.677
	1.0	−1.0	0.856
10	−1.0	1.0	−0.597
	−0.75	0.75	−0.429
	−0.5	0.5	−0.261
	−0.25	0.25	−0.092
	0.0	0.0	0.076
	0.25	−0.25	0.244
	0.5	−0.5	0.412
	0.75	−0.75	0.580
	1.0	−1.0	0.748
20	−1.0	1.0	−0.603
	−0.75	0.75	−0.437
	−0.5	0.5	−0.272
	−0.25	0.25	−0.106
	0.0	0.0	0.060
	0.25	−0.25	0.225
	0.5	−0.5	0.391
	0.75	−0.75	0.556
	1.0	−1.0	0.722

are generally small. Thus in this case one finds that the radiative breaking condition takes the form

$$\frac{m_{1/2}'^2}{a^2}+\frac{m_0^2}{b'^2}+\frac{A_0^2}{c^2}\simeq 1, \tag{26}$$

where a and c are defined by Eq. (11) and b' is defined by

$$b'^2=M_Z^2\frac{(\Phi+1/4)}{|C_1'|}. \tag{27}$$

As in the universal case [see Figs. 1(a), 2(a), and 4(a)] here also for given fine-tuning one finds that m_0 and $m_{1/2}$ are bounded as they lie on the boundary of an ellipse. Further, $C_1'>C_1$ implies that a given Φ corresponds effectively to a smaller Φ_0, and hence admits smaller values of the upper limits of the squark masses relative to the universal case. This is what is seen in Table IV. Here we find that the upper limits are generally decreased over the full range of Φ.

For case (ii) the situation is drastically different. Here the nonuniversalities make a negative contribution driving C_1' negative (see Table III) and further C_1' remains negative in the relevant Q range (see Table V). Thus the radiative breaking solutions no longer lie on the boundary of an ellipse. The analysis in this case is somewhat more complicated in that the loop corrections to μ^2 at the scale $Q=M_Z$ are large. For illustrative purposes one may carry out an analysis similar to the one discussed in Sec. III and go to the scale $Q=Q_0'$, where the loop corrections to μ^2 are negligible. Again there are two cases and we discuss these below.

Case C. This case is defined by Eq. (13) and the radiative symmetry breaking constraint here reads

$$\frac{m_{1/2}'^2}{\alpha^2(Q_0')}-\frac{m_0^2}{\beta'^2(Q_0')}\simeq 1, \tag{28}$$

where

$$\beta'^2=\frac{|(\Phi+1/4)M_Z^2-C_2'A_0^2|}{|C_1'|}. \tag{29}$$

Equation (28) shows that the radiative symmetry breaking constraint in this case is a hyperbolic constraint.

Case D. This case is defined by Eq. (18) and the radiative symmetry breaking constraint here reads

$$\frac{m_0^2}{\beta'^2(Q_0')}-\frac{m_{1/2}'^2}{\alpha^2(Q_0')}\simeq 1. \tag{30}$$

Again the radiative symmetry breaking constraint is a hyperbolic constraint.

Cases C and D are similar to the cases A and B except that here m_0 and $m_{1/2}$ lie on a hyperbola even for small $\tan\beta$ because of the effect of the specific nature of the nonuniversalities in this case. Thus here it is the nonuniversalities which transform the radiative breaking equation from an ellipse to a hyperbola. Of course m_0 and $m_{1/2}$ do not become arbitrarily large, since eventually other constraints set in and limit the allowed values of m_0 and $m_{1/2}$. Results of the analysis are given in Fig. 6. One finds that m_0 and $m_{1/2}$ indeed can become large for a fixed fine-tuning.

To understand the effects of the nonuniversalities in the third generation in comparison to the nonuniversalities in the Higgs boson sector it is useful to express $\Delta\Phi$ in the following alternate form:

TABLE IV. The upper bounds on sparticle masses for the case of nonuniversalities in the Higgs boson sector when $(\delta_1,\delta_2)=(1,-1)$, $\delta_3=0=\delta_4$, $m_t=175$ GeV, and $\mu<0$ for different values of $\tan\beta$ and Φ. All the masses are in GeV.

Nonuniversal case: $(\delta_1,\delta_2,\delta_3,\delta_4)=(1,-1,0,0)$, $\mu<0$													
$\tan\beta$	Φ	H	$\tilde{u}_l$	$\tilde{e}_l$	$\tilde{e}_r$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{g}$	h	$\tilde{\chi}_1^{\pm}$	$\tilde{\chi}_2^{\pm}$	$\tilde{\chi}_1^0$	m_0
2	5	313	291	148	140	267	328	319	70	104	225	49	134
	10	457	419	220	207	354	430	459	79	140	304	68	205
	20	655	601	317	296	488	585	656	87	193	420	95	299
5	2.5	213	274	133	121	243	336	301	95	79	181	44	109
	5	356	391	221	228	324	430	429	103	115	233	62	204
	10	535	569	334	312	462	580	620	110	170	310	89	315
	20	775	841	485	451	677	820	915	116	254	425	130	462
10	2.5	203	285	129	103	246	349	316	104	74	185	43	93
	5	371	406	234	216	333	447	446	110	112	237	62	215
	10	559	583	357	332	471	598	637	116	169	314	90	338
	20	810	843	520	483	680	828	920	122	251	427	130	496
20	2.5	216	286	125	69	242	350	315	105	69	186	40	75
	5	383	409	236	203	330	451	448	111	109	237	60	216
	10	577	590	357	323	472	604	646	117	167	315	89	343
	20	843	854	519	479	687	835	932	122	252	428	130	508

$$\Delta\Phi=\frac{1}{t^2-1}\left\{\delta_1-\left[1-\frac{1}{2}\left(\frac{m_t}{m_f}\right)^2\right]\delta_2 t^2+\frac{1}{2}\left(\frac{m_t}{m_f}\right)^2(\delta_3+\delta_4)t^2\right\}$$
$$\times\left(\frac{m_0}{M_Z}\right)^2+\frac{3}{5}\,\frac{t^2+1}{t^2-1}\,\frac{pS_0}{M_Z^2}. \tag{31}$$

Since $m_t<m_f$ one has $[1-\frac{1}{2}(m_t/m_f)^2]>0$ which implies that the effect of a negative (positive) δ_2 can be simulated by a positive (negative) value of δ_3 or by a positive (negative) value of δ_4. This correlation can be seen to hold by a comparison of Tables IV and VI. As in the case of Table IV where a positive δ_1 and a negative δ_2 leads to lowering of the upper limits on squark masses, we find that a positive δ_3 or a positive δ_4 produces a similar effect. The analysis of Table VI where we choose $(\delta_1,\delta_2,\delta_3,\delta_4)=(0,0,1,0)$ supports this observation. A similar correlation can be made between the case of $\delta_1<0,\delta_2>0$, and the case $\delta_3+\delta_4<0$ by the comparison given above. We note, however, that the effects of nonuniversalities in the Higgs sector and in the third generation sector are not identical in every respect as they enter in different ways in other parts of the spectrum. However, the gross features of the upper limits of squarks in Table VI can be understood by the rough comparison given above.

A comparison of Tables II, IV, and VI shows that the nonuniversalities have a remarkable effect on the upper limits of sparticle masses. One finds that the upper limits on the sparticle masses can increase or decrease dramatically depending on the type of nonuniversality included in the analysis. The prospects for the observation of sparticles at colliders are thus significantly affected. For the case of Tables IV and VI one finds that the sparticle spectrum falls below 1 TeV in the range $\tan\beta\leqslant 5,\Phi\leqslant 20$. Thus in this case the gluino and the squarks should be discovered at the LHC and all of the other sparticles should also be discovered over most of the mass ranges in Table IV. In contrast for the case of nonuniversality of Table VI we find that the nature of nonuniversal contribution is such that squark masses can exceed the discovery potential of even the LHC. The analysis given above is for $\mu<0$. A similar analysis holds with essentially the same general conclusions for the $\mu>0$ case.

VI. UPPER LIMIT ON THE HIGGS BOSON MASS

One of the most interesting parts of our analysis concerns the dependence of the Higgs boson mass upper limits on Φ_0. For the analysis of the Higgs boson mass upper limits we have taken account of the one loop corrections to the masses and further chosen the scale Q which minimizes the two loop corrections [26,12]. For $\tan\beta=2$ the upper limit on the Higgs boson mass increases from 60 GeV at $\Phi_0=2.5$ to 86 GeV at $\Phi_0=20$. Further from the successive entries in this case in Table II we observe that in each of the cases where an increment in the Higgs boson mass occurs, one requires a significant increase in the value of Φ_0. The same general pattern is repeated for larger values of $\tan\beta$. Thus for $\tan\beta=5$ the Higgs boson mass increases from 97 to 116 GeV as Φ_0 increases from 2.5 to 20. In Fig. 7 we exhibit the upper bound on the Higgs boson mass as a function of $\tan\beta$. From the analysis of Table II and Fig. 7 one can draw the general conclusion that the Higgs boson mass upper limit is a sensitive function of $\tan\beta$ and Φ_0. For values of $\tan\beta$ near the low end, i.e., $\tan\beta\approx 2$, the upper limit of the Higgs boson mass lies below 85–90 GeV for any reasonable range of fine-tuning, i.e., $\Phi_0\leqslant 20$. This is a rather strong result.

TABLE V. The scale dependence of $C_1'(Q), C_2(Q)–C_4(Q)$ for $(\delta_1,\delta_2,\delta_3,\delta_4)=(-1,1,0,0)$ for $m_t=175$ GeV and $\tan\beta=2$, 5, 10, and 20.

$(\delta_1,\delta_2,\delta_3,\delta_4)=(-1,1,0,0)$					
$\tan\beta$	Q(GeV)	C_1'	C_2	C_3	C_4
2	91.2	−0.341	0.071	4.284	0.312
	250	−0.363	0.0765	3.742	0.3110
	500	−0.378	0.0802	3.415	0.3101
	750	−0.387	0.0825	3.239	0.3094
	1000	−0.394	0.0841	3.119	0.3089
	1250	−0.399	0.0853	3.030	0.3084
	1500	−0.404	0.0863	2.959	0.3080
	1750	−0.408	0.0872	2.901	0.3077
	2000	−0.411	0.0879	2.851	0.3073
5	91.2	−0.572	0.1024	2.871	0.4491
	250	−0.602	0.1069	2.452	0.4348
	500	−0.624	0.1099	2.200	0.4245
	750	−0.636	0.1115	2.064	0.4183
	1000	−0.645	0.1126	1.973	0.4138
	1250	−0.652	0.1135	1.905	0.4103
	1500	−0.658	0.1142	1.851	0.4074
	1750	−0.663	0.1147	1.806	0.4049
	2000	−0.667	0.1152	1.768	0.4028
10	91.2	−0.597	0.1040	2.710	0.4561
	250	−0.628	0.1081	2.305	0.4397
	500	−0.649	0.1108	2.062	0.4280
	750	−0.662	0.1122	1.931	0.4211
	1000	−0.671	0.1132	1.843	0.4160
	1250	−0.678	0.1140	1.778	0.4121
	1500	−0.684	0.1146	1.726	0.4089
	1750	−0.689	0.1151	1.683	0.4061
	2000	−0.693	0.1155	1.646	0.4037
20	91.2	−0.603	0.1043	2.671	0.4575
	250	−0.634	0.1084	2.269	0.4406
	500	−0.655	0.1109	2.029	0.4286
	750	−0.668	0.1123	1.899	0.4214
	1000	−0.677	0.1133	1.812	0.4162
	1250	−0.684	0.1140	1.747	0.4122
	1500	−0.690	0.1146	1.695	0.4089
	1750	−0.695	0.1150	1.653	0.4061
	2000	−0.699	0.1154	1.617	0.4036

Thus if the low $\tan\beta$ region of $b-\tau$ unification turns out to be the correct scenario then our analysis implies the existence of a Higgs boson mass below 85–90 GeV for any reasonable range of fine-tuning. This scenario will be completely tested at LEPII which can allow coverage of the Higgs boson mass up to $m_h \approx 95$ GeV with $\sqrt{s}=192$ GeV. If no Higgs boson is seen at LEPII then a high degree of fine-tuning, i.e., $\Phi_0>20$, is indicated on the low $\tan\beta$ end of $b-\tau$ unification.

Further, the analysis also indicates that in order to approach the maximum allowed Higgs boson mass one needs to have a high degree of fine-tuning. In particular from Table II and Fig. 7 we see that going beyond 120 GeV in the Higgs boson mass requires a value of Φ_0 on the high side, preferably 10 and 20. The strong correlation of the Higgs boson mass upper limits with the value of Φ_0 has important implications for sparticle masses. Thus if the Higgs boson mass turns out to lie close to its allowed upper limit then a larger value of Φ_0 would be indicated. In turn a large Φ_0 would point to a heavy sparticle spectrum. At TeV33 with 25 fb^{-1} of integrated luminosity Higgs boson mass up to 120 GeV will be probed. A nonobservation of the light Higgs boson in this mass range will imply that one needs a high degree of fine-tuning which would point in the direction of heavy spar-

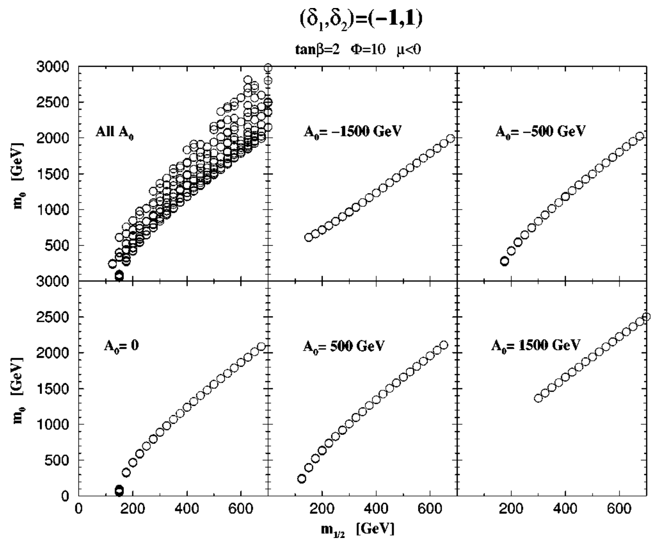

FIG. 6. Allowed region in the m_0–$m_{1/2}$ plane under the nonuniversal boundary condition of $(\delta_1,\delta_2)=(-1,1)$ for $m_t=175$ GeV, $\tan\beta=2$, $\Phi=10$, and negative μ.

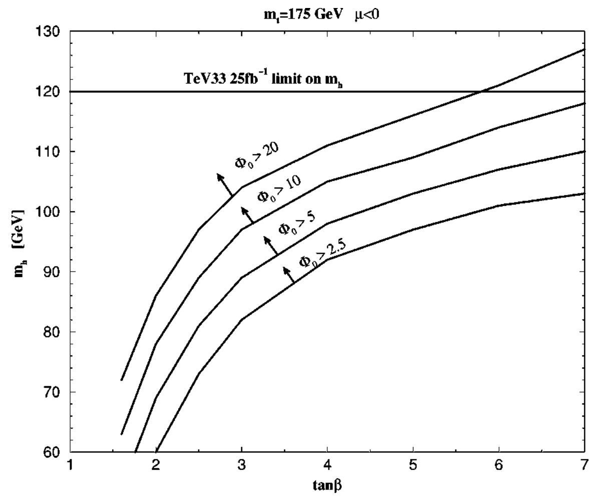

FIG. 7. Upper bounds on the light Higgs boson h^0 mass for different values of Φ_0 as a function of $\tan\beta$ when $m_t=175$ GeV and $\mu<0$.

ticle masses. These results are in general agreement with the analysis of Ref. [32] which arrived at much the same conclusion using a very different criterion of fine-tuning. In particular the analysis of Ref. [32] also found that the nonobservation of the Higgs boson mass below 120 GeV will imply a heavy spectrum.

VII. FINE-TUNING LIMITS FROM THE CURRENT EXPERIMENTAL DATA

One may put limits on the fine-tuning parameter using the current experimental data on sparticle searches at colliders [33,34]. The result of this analysis is presented in Table VII. For low $\tan\beta$ the strongest lower limits on the fine-tuning parameter arise from the lower limits on the Higgs boson mass. In Table VII we have used the experimental lower limits on the Higgs boson mass from the four detectors at LEP, i.e., L3, OPAL, ALEPH, and DELPHI [33], to obtain lower limits on Φ for values of $\tan\beta$ from 2 to 20. As expected one finds that the strongest limit on Φ arises for the smallest $\tan\beta$, and the constraint on Φ falls rapidly for larger $\tan\beta$. Thus for $\tan\beta$ greater than 5 the lower limit on Φ already drops below 2 which is not a stringent fine-tuning constraint. Lower limits on Φ from the current data on the

TABLE VI. The upper bound on sparticle masses for nonuniversalities in the third generation when $(\delta_3,\delta_4)=(1,0)$, $m_t=175$ GeV, and $\mu<0$ for different values of $\tan\beta$ and Φ. All the masses are in GeV.

$(\delta_1,\delta_2,\delta_3,\delta_4)=(0,0,1,0)$, $\mu<0$													
$\tan\beta$	Φ	H	$\tilde{u}_l$	$\tilde{e}_l$	$\tilde{e}_r$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{g}$	h	$\tilde{\chi}_1^{\pm}$	$\tilde{\chi}_2^{\pm}$	$\tilde{\chi}_1^0$	m_0
2	5	290	292	162	140	266	326	319	70	104	225	49	147
	10	418	419	242	211	351	429	456	79	139	304	68	226
	20	597	601	349	306	486	583	656	87	193	420	95	331
5	2.5	198	275	151	125	244	337	301	95	79	181	44	126
	5	325	391	258	223	325	430	429	103	115	233	62	239
	10	485	559	389	340	453	573	611	110	168	310	87	368
	20	702	807	571	501	652	791	880	116	245	424	125	546
10	2.5	194	291	144	110	247	349	316	104	74	185	43	110
	5	342	422	279	239	333	445	446	110	112	237	62	259
	10	514	606	428	371	471	599	637	116	169	314	90	407
	20	747	870	629	549	680	828	920	122	251	427	130	604
20	2.5	213	293	134	77	245	352	316	106	70	187	41	88
	5	360	431	281	229	331	452	449	112	110	238	61	261
	10	541	621	433	364	473	605	646	118	168	316	90	416
	20	804	894	638	543	688	836	932	123	252	429	131	620

TABLE VII. Current experimental lower bounds on masses of the lightest Higgs boson and various sparticles from LEP and the Tevatron. Corresponding fine-tunings ($\mu<0$) are also shown.

tan β	$\sqrt{s}=183$ GeV LEP 95% C.L. lower bound on m_h mass lower bdd (GeV)	Lower bound on $\Phi(\mu<0)$
2	86 (L3)	20
	74 (OPAL scan B)	8
	88 (ALEPH)	23
	84 (DELPHI)	18
5	72 (L3)	0.25
	71 (OPAL scan B)	
	73 (ALEPH)	
	76 (DELPHI)	
10	72 (L3)	0.25
	70 (OPAL scan B)	
	76 (ALEPH)	
	75 (DELPHI)	
20	71 (L3)	0.25
	70 (OPAL scan B)	
	76 (ALEPH)	
	76 (DELPHI)	
Particle	**$\sqrt{s}=183$ GeV LEP 95% C.L. lower bounds on various sparticles masses mass lower bdd (GeV)**	**Lower bound on $\Phi(\mu<0)$**
	24 independent of m_0 (DELPHI)	
χ^0	14 any m_0 (ALEPH)	0.25 for tan $\beta\geqslant 2$
	27 for tan $\beta=2$ (L3)	0.25
$\chi^{\pm}$	51 (ALEPH)	0.25 for tan $\beta\geqslant 2$
$\tilde{t}$	$\tilde{t}\to c\chi$ $m_{\tilde{t}}>74$ (ALEPH)	0.25 for tan $\beta\geqslant 2$
	$\tilde{t}\to bl\nu\chi$ $m_{\tilde{t}}>82$ (ALEPH)	
Particle	**95% C.L. lower bounds on various sparticles masses from Ref. [34] mass lower bdd (GeV)**	**Lower bound on $\Phi(\mu<0)$**
$\chi^{\pm}$	$m_{\chi^{\pm}}>45$, 0.66 pb	0.25
	$m_{\chi^{\pm}}>124$, 0.01 pb	$\Phi>8$, tan $\beta=2$
		$\Phi>5.8$, tan $\beta=5$
$\tilde{q}$ $\tilde{g}$	$m_{\tilde{g}}>230$, heavy squarks	$\Phi>2.7$, tan $\beta=2$
	$m_{\tilde{q},\tilde{g}}>260$, $m_{\tilde{q}}=m_{\tilde{g}}$	$\Phi>4.0$, tan $\beta=2$
		0.25, tan $\beta\geqslant 5$
	$m_{\tilde{q}}>219$, heavy gluinos	$\Phi>2.8$, tan $\beta=2$
		0.25, tan $\beta\geqslant 5$

lower limits on the neutralino, the chargino, the top squark, the heavy squarks, and the gluino are also analyzed in Table VII. One finds that here the current lower limits on the chargino mass produce the stongest lower limit on Φ. For tan β of 2, the lower limit on Φ from the Higgs boson sector is still more stringent constraint than the lower limit constraint from the chargino sector. However, for tan $\beta=5$ the constraint from the chargino sector becomes more stringent than the constraint from the Higgs sector. These constraints on the fine-tuning will become even more stringent after the

CERN e^+e^- collider LEP II completes its runs and if supersymmetric particles do not become visible.

VIII. CONCLUSIONS

In this paper we have analyzed the naturalness bounds on sparticle masses within the framework of radiative breaking of the electroweak symmetry for minimal supergravity models and for nonminimal models with nonuniversal soft SUSY breaking terms. For the case of minimal supergravity it is found that for small values of $\tan\beta$, i.e., $\tan\beta \leqslant 5$ and a reasonable range of fine-tuning, i.e., $\Phi \leqslant 20$, the allowed values of m_0 and $m_{1/2}$ lie on the surface of an ellipsoid with the radii determined by the value of fine-tuning. Specifically for the case $\tan\beta = 2$ it is found that the upper limits on the gluino and squark masses in minimal supergravity lie within 1 TeV and the light Higgs boson mass lies below 90 GeV for $\Phi_0 \leqslant 20$. For $\tan\beta \leqslant 5$ the upper limits of the sparticle masses all still lie within the reach of the LHC for the same range of Φ_0. The analysis shows that the upper limits of sparticle masses are very sensitive functions of $\tan\beta$. As values of $\tan\beta$ become large the loop corrections to μ become large and the nature of the radiative breaking equation can change, i.e., m_0 and $m_{1/2}$ may not lie on the surface of an ellipsoid. Thus it is found that there exist regions of the parameter space for large $\tan\beta$ where the upper bounds on the sparticle masses can get very large even for reasonable values of fine-tuning.

We have also analyzed the effects of nonuniversalities in the Higgs boson sector and in the third generation sector on the upper limits on the sparticle masses. It is found that nonuniversalities have a very significant effect on the overall size of the sparticle mass upper limits. Thus we find that the case (i) $\delta_1 > 0$ or $\delta_2 < 0$ and $\delta_3 = 0 = \delta_4$ has the effect of decreasing the upper limits on the squark masses, and in contrast the case (ii) $\delta_1 < 0$ or $\delta_2 > 0$ and $\delta_3 = 0 = \delta_4$ has the effect of increasing the upper limits on the squark masses. Remarkably for $\delta_1 = 1$, $\delta_2 = -1$, and $\delta_3 = 0 = \delta_4$ all of the sparticle masses lie below 1 TeV for $\tan\beta \leqslant 5$ and $\Phi \leqslant 20$ because of the nonuniversality effects. In this case the sparticles would not escape detection at the LHC. However, for the case $\delta_1 = -1$, $\delta_2 = 1$, and $\delta_3 = 0 = \delta_4$ there is an opposite effect and the nonuniversalities raise the upper limits of the sparticle masses. Here for the same range of $\tan\beta$, i.e., $\tan\beta \leqslant 5$ the first and second generation squark masses can reach approximately 3 TeV for $\Phi \leqslant 10$ (4–5 TeV for $\Phi \leqslant 20$) and consequently these sparticles may escape detection even at the LHC. Similar effects occur for the nonuniversalities in the third generation sector. Thus nonuniversalities have important implications for the detection of supersymmetry at colliders.

Finally, it is found that the upper limit on the Higgs boson mass is a very sensitive function of $\tan\beta$ in the region of low $\tan\beta$ and moving the upper limit beyond 120 GeV towards its maximally allowed value will require a high degree of fine-tuning. In turn large fine-tuning would result in a corresponding upward movement of the upper limits of other sparticle masses. Thus a nonobservation of the Higgs boson at the upgraded Tevatron with an integrated luminosity of 25 fb^{-1}, would imply a high degree of fine-tuning and point to the possibility of a heavy sparticle spectrum.

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