Unified description of light- and strange-baryon spectra

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We present a chiral constituent-quark model for light and strange baryons providing a unified description of their ground states and excitation spectra. The model relies on constituent quarks and Goldstone bosons arising as effective degrees of freedom of low-energy QCD from the spontaneous breaking of chiral symmetry. The spectra of the three-quark systems are obtained from a precise variational solution of a Schrödinger-type equation with a semirelativistic Hamiltonian. The theoretical predictions are found to be in close agreement with experiment. [S0556-2821(98)02619-8]

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An intricate question of low-energy quantum chromodynamics (QCD) involves the effective degrees of freedom that govern the physics of light and strange baryons. The early (naive) quark model [1] was successful in classifying hadrons and describing some gross properties of their spectra but no firm evidence on the dynamics of the valence quarks was achieved. Even when motivated by QCD, the concept of one-gluon exchange (OGE) [2] was introduced as an interaction between confined constituent quarks, a number of delicate problems remained unsolved. In this context up until now one has not been able to explain, e.g., the correct level orderings in light- and strange-baryon spectra [3,4], the spin content of the nucleon [5], or the flavor asymmetry of the sea in the nucleon [6,7]. The shortcomings essentially stem from the fact that the implications of the spontaneous breaking of chiral symmetry (SBχS) are not properly taken into account in such a model, and as a consequence the pertinent interactions between constituent quarks turn out to be inadequate. Evidently, if one assumes constituent quarks of flavors u,d,s with masses considerably larger than the corresponding current-quark masses, this already means that the underlying chiral symmetry of QCD is spontaneously broken. As a consequence of SBχS, at the same time Goldstone bosons appear, which couple directly to the constituent quarks [8–10]. Hence, beyond the scale of SBχS one is left with constituent quarks with dynamical masses related to $\langle\bar{q}q\rangle$ condensates and with Goldstone bosons as the effective degrees of freedom. This feature, that in the Nambu-Goldstone mode of chiral symmetry constituent-quark and Goldstone-boson fields prevail together, is also well supported, e.g., by the σ model [11] or the Nambu–Jona-Lasinio model [12]. In the same framework also problems with the spin and flavor content of the nucleon are naturally resolved [13]. As a consequence, baryons are to be considered as systems of three constituent quarks that interact by Goldstone-boson exchange (GBE) and are subject to confinement [3,4].

Goldstone bosons manifest themselves in the octet of pseudoscalar mesons (π,K,η). In the large-N_C limit, when the axial anomaly vanishes [14], the spontaneous breaking of chiral symmetry $U(3)_L\times U(3)_R\rightarrow U(3)_V$ implies a ninth Goldstone boson [15], which corresponds to the flavor singlet η'. Under real conditions, for $N_C=3$, a certain contribution from the flavor singlet remains and the η' must thus be included in the GBE interaction.

In view of these considerations we propose a semirelativistic chiral constituent quark model that is based on the following three-quark Hamiltonian:

$$H=\sum_{i=1}^{3}\sqrt{\vec{p}_i^{\,2}+m_i^2}+\sum_{i<j=1}^{3}V_{ij}\,. \tag{1}$$

Here the relativistic form of the kinetic-energy operator is employed, with $\vec{p}_i$ the three-momenta and m_i the masses of the constituent quarks, and the dynamical part consists of the quark-quark interaction

$$V_{ij}=V_{conf}+V_{\chi}\,. \tag{2}$$

We take the color-electric confinement interaction in linear form

$$V_{conf}(r_{ij})=V_0+Cr_{ij}\,, \tag{3}$$

with the color factor included in the strength parameter (string tension). This represents a very good approximation of the regular Y-shape string configuration. The chiral potential is derived from GBE. By far the most dominant contribution to the hyperfine interaction in baryons is provided by its spin-spin component, which is manifested by the sum of octet and singlet pseudoscalar meson-exchange potentials [3,4]:

$$V_{\chi}(\vec{r}_{ij})=\left[\sum_{F=1}^{3}V_{\pi}(\vec{r}_{ij})\lambda_i^F\lambda_j^F+\sum_{F=4}^{7}V_K(\vec{r}_{ij})\lambda_i^F\lambda_j^F\right.$$
$$\left.+V_{\eta}(\vec{r}_{ij})\lambda_i^8\lambda_j^8+\frac{2}{3}V_{\eta'}(\vec{r}_{ij})\right]\vec{\sigma}_i\cdot\vec{\sigma}_j\,, \tag{4}$$

where $\vec{\sigma}_i$ and λ_i^F represent the quark spin and flavor matrices, respectively. In the simplest derivation, when pseudoscalar or pseudovector couplings are employed at pointlike meson-quark vertices and the boson fields satisfy the linear Klein-Gordon equation, one obtains, in a static approximation, the well-known meson-exchange potentials

$$V_\gamma(\vec{r}_{ij}) = \frac{g_\gamma^2}{4\pi}\frac{1}{12m_i m_j}\left\{\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\}, \quad (5)$$

with μ_γ ($\gamma = \pi, K, \eta, \eta'$) being the individual phenomenological meson masses and $g_\gamma^2/4\pi$ the meson-quark coupling constants. In general, the structure of this potential in a momentum-space representation is

$$V_\gamma(\vec{q}) \sim \vec{\sigma}_i \cdot \vec{q}\,\vec{\sigma}_j \cdot \vec{q} D(q^2) F^2(q^2), \quad (6)$$

where $D(q^2)$ is the dressed Green function for the chiral field, including both nonlinear terms of the chiral Lagrangian and fermion loops, and $F(q^2)$ is a meson-quark form factor, which takes into account the extended structure of the quasiparticles. In the limit $\vec{q} \to 0$, one has $D(q^2) \to -(\vec{q}^2 + \mu^2)^{-1} \neq \infty$ and $F(q^2) \to 1$, and consequently $V_\gamma(\vec{q} = 0) = 0$. Therefore the pseudoscalar meson-exchange interaction has to satisfy the requirement of the volume integral to vanish: $\int d^3 r V_\gamma(\vec{r}) = 0$. Since at large distances $V_\gamma \sim \mu_\gamma^2 e^{-\mu_\gamma r}/r$, there must be a strong short-range part of opposite sign in order to guarantee the volume integral constraint. Inside baryons this short-range part dominates over the Yukawa tail and it becomes of crucial importance to reproduce the baryon spectra.

A suitable parametrization of the GBE potential, preserving a zero volume integral, is thus given by

$$V_\gamma(\vec{r}_{ij}) = \frac{g_\gamma^2}{4\pi}\frac{1}{12m_i m_j}\left\{\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}}\right\}. \quad (7)$$

It involves the parameters Λ_γ corresponding to the individual exchanged mesons. Clearly the values of Λ_γ should vary with the magnitudes of the meson masses μ_γ: with a larger meson mass μ_γ also Λ_γ should become larger. Otherwise the individual meson-exchange potentials in Eq. (7) could receive unwarranted contributions (e.g., a certain meson-exchange contribution could become attractive instead of repulsive or vice versa at short distances). In order to avoid a proliferation of free parameters, by assuming four independent values of Λ_γ (for each $\gamma = \pi, K, \eta, \eta'$), we adopt the linear scaling prescription

$$\Lambda_\gamma = \Lambda_0 + \kappa\mu_\gamma, \quad (8)$$

which involves only the two free parameters Λ_0 and κ.

Because of the explicit chiral symmetry breaking in QCD, the various quark-meson coupling constants could naturally be different. Again, we try to keep the number of free parameters as small as possible and assume a single octet-quark coupling $g_8^2/4\pi$ for all octet mesons (π, K, η). Its value can be extracted from the phenomenological pion-nucleon coupling constant as $g_8^2/4\pi = 0.67$ [3]. Because of the particular character of the η' meson (cf. the discussion above), the flavor-singlet coupling constant may well be different from the octet one. This assumption is also supported by the successful explanation of the flavor and spin content of the nucleon [13]. Therefore we treat the ratio $(g_0/g_8)^2$ as a free parameter.

TABLE I. Parameters of the semirelativistic constituent-quark model based on GBE.

Fixed parameters						
Quark masses [MeV]		Meson masses [MeV]				
m_u, m_d	m_s	μ_π	μ_K	μ_η	$\mu_{\eta'}$	$g_8^2/4\pi$
340	500	139	494	547	958	0.67

Free parameters				
$(g_0/g_8)^2$	Λ_0 [fm^{-1}]	κ	V_0 [MeV]	C [fm^{-2}]
1.34	2.87	0.81	-416	2.33

For the constituent-quark masses we take the typical values $m_u = 340$ MeV and $m_s = 500$ MeV. Considering the constituent-quark as well as meson masses and the octet coupling constant as predetermined, the GBE potentials of Eq. (7) involve only three free parameters. Their values, together with the two free parameters of the confinement potential, were determined from a fit to the baryon spectra. The resulting numerical values are given in Table I together with the fixed model parameters. We remark that the parameters given in Table I are only one choice out of a possible set of others that lead to a similar quality of description of the baryon spectra. In case the fixed parameters were chosen differently, e.g., with regard to the specific values of the constituent-quark masses or the octet coupling constant, the free parameters would get slightly changed but a similar fit could be achieved. The baryon spectra alone simply do not guarantee a unique determination of the model parameters. Further studies of other observables are necessary to constrain their values. Nevertheless, it is pleasing to find the present parameter values of reasonable magnitudes. For example, the confinement strength is comparable with the string tension extracted from lattice calculations [16] and it is also consistent with the slopes of Regge trajectories.

The three-quark system with the Hamiltonian of Eq. (1) is treated by solving the Schrödinger equation with the stochastic variational method [17]. This technique has been tested in a number of benchmark cases before. The results prove reliable to an accuracy of better than 1% in the present calculation. In Fig. 1 we show the predictions of our model for all light- and strange-baryon excitation levels up to $M \lesssim 1850$ MeV; the nucleon is normalized to its mass of 939 MeV (which determines the value of the confinement potential parameter V_0). All masses corresponding to three- and four-star resonances in the most recent compilation of the Particle Data Group (PDG) [18] are included.

From the results it is immediately evident that quite a satisfactory description of the spectra of all low-lying light and strange baryons is achieved in a unified framework. In particular, the level orderings of the lowest positive- and negative-parity states in the nucleon spectrum are reproduced correctly, with the $\frac{1}{2}^+$ Roper resonance $N(1440)$ falling well below the negative-parity $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states $N(1535)$ and $N(1520)$, respectively.

Likewise, in the Λ and Σ spectra the positive-parity $\frac{1}{2}^+$ excitations $\Lambda(1600)$ and $\Sigma(1660)$ fall below the negative-

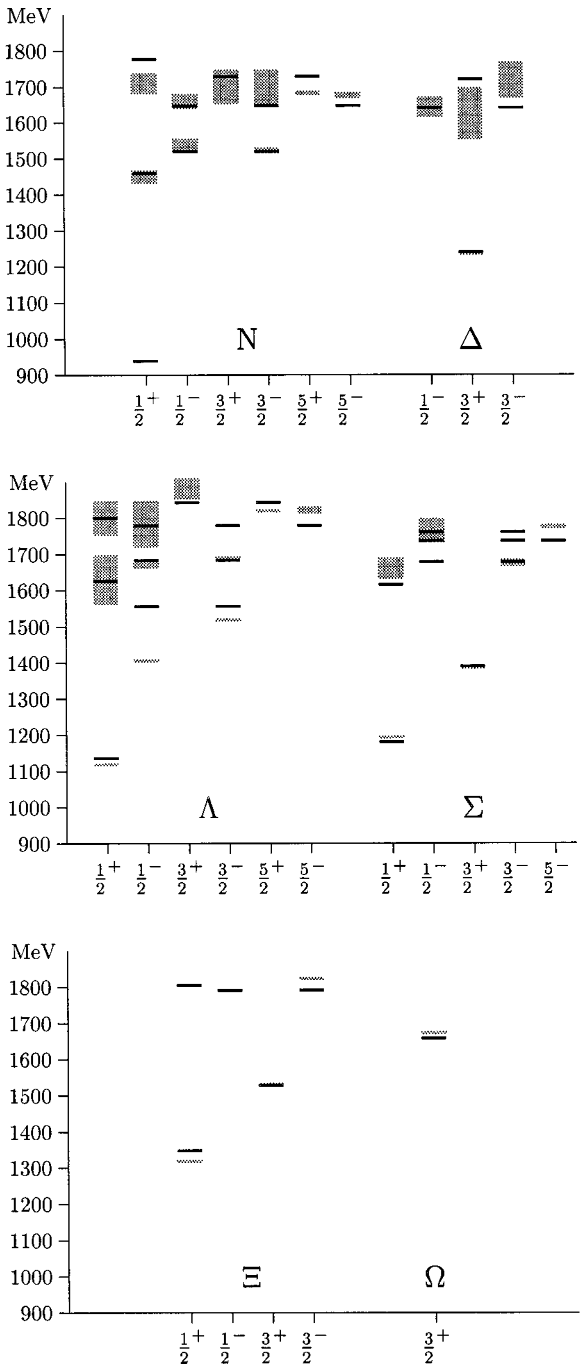

FIG. 1. Energy levels of the lowest light- and strange-baryon states with total angular momentum and parity J^P. The nucleon ground state is 939 MeV. The shadowed boxes represent the experimental values with their uncertainties. The Δ, Σ*, and Ξ* ground-state levels practically fall into their rather tight experimental boxes.

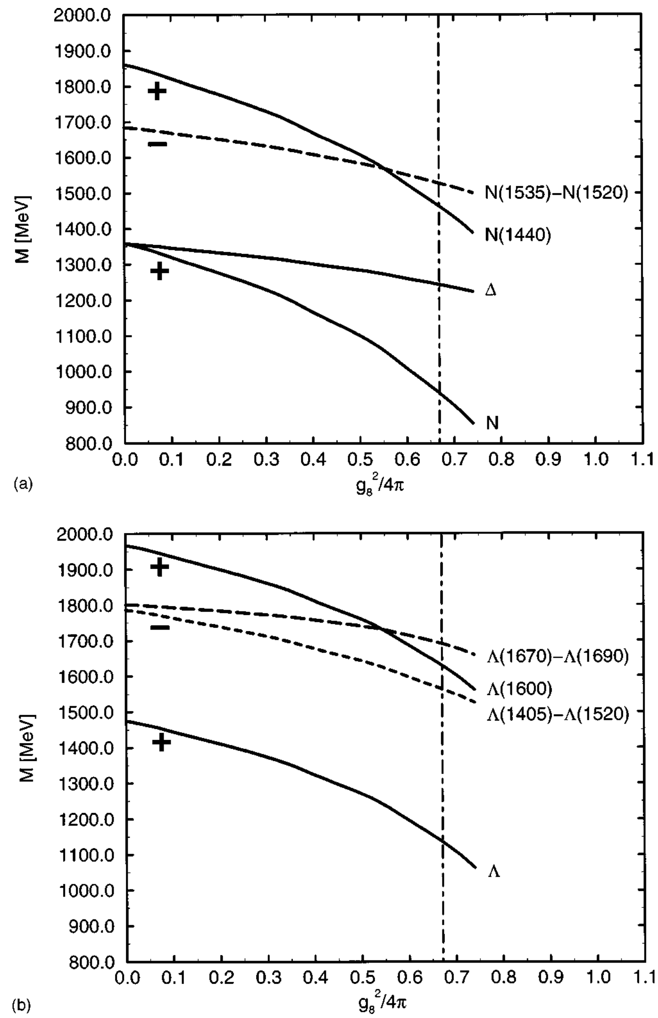

FIG. 2. Level shifts as a function of the strength of the Goldstone-boson-exchange interaction. Solid and dashed lines correspond to positive- and negative-parity states, respectively.

parity $\frac{1}{2}^-$-$\frac{3}{2}^-$ states Λ(1670)-Λ(1690) and the $\frac{1}{2}^-$ state Σ(1750), respectively. In the Λ spectrum, at the same time, the negative-parity $\frac{1}{2}^-$-$\frac{3}{2}^-$ states Λ(1405)-Λ(1520) remain the lowest excitations above the Λ ground state. By correct level orderings of the positive- and negative-parity states a long-standing problem of baryon spectroscopy is resolved. At this stage, only one state is not reproduced in agreement or close to experiment, the flavor singlet Λ(1405); we mention a possible reason below.

The remarkable successes of the GBE quark-quark interaction of Eqs. (4) and (7) are, of course, brought about by the particular symmetry introduced through the spin-flavor operators $\vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F$ and by the short-range part of the interaction with a proper sign [3,4]. This makes the GBE potential just adequate for the level structures found in experiment, and thus a unified description of all light- and strange-baryon spectra is possible, even though our model in the present simplest version involves only a handful of free parameters. The action of the chiral potential V_χ on the energy levels becomes especially transparent when the coupling constant is gradually increased (see Fig. 2). Starting out from the case with confinement only, one observes that with increasing coupling the inversion of the lowest positive- and negative-parity states $N(1440)$ and $N(1535)$-$N(1520)$ in the N spectrum is achieved. At the same time the level crossing of the corresponding states Λ(1600) and Λ(1405)-Λ(1520) in the Λ spectrum is avoided, just as demanded by phenomenology.

While other existing types of hyperfine interactions, such as the color-magnetic interaction motivated by one-gluon exchange (OGE) or the instanton-induced 't Hooft interaction, can explain the octet-decouplet splittings in baryons [2,19], they usually fail in reproducing the correct orderings of positive- and negative-parity excitations; see, e.g., the corresponding works in Refs. [20,21]. These problems are successfully solved by the flavor-dependent GBE interaction. It is important to realize that this achievement is not a matter of the parametrization of the radial dependence of the quark-quark potential but notably a consequence of the flavor-spin operators in the GBE interaction [3,4]. Their structures are naturally obtained by assuming that beyond the scale of (SBχS) constituent-quark and Goldstone-boson fields are the relevant degrees of freedom in light and strange baryons.

At this instance, a remark is in order about the necessity of employing a relativistic kinetic-energy operator in the three-quark Hamiltonian (1). Certainly, this is only an intermediate step towards a fully covariant treatment but it already allows one to include kinematical relativistic effects. In any nonrelativistic approach these effects get compensated by the potential parameters, which will not only assume unrealistic values (cf., e.g., our previous nonrelativistic model [22]) but one is also faced with such disturbing consequences as $v/c>1$ (where v is the mean velocity of the constituent quark and c is the velocity of light).

At the present stage, tensor forces are not yet included in our model. However, we have already made estimates and numerical tests of their influence. They turn out to be much less important for baryon masses, as compared to the spin-spin part, at least for the states considered in Fig. 1. It is clear also from phenomenology that tensor forces can play only a subordinate role as the splittings of corresponding LS multiplets are generally small.

So far, the constituent-quark model derived from GBE provides a reasonable description of light- and strange-baryon spectra. Nevertheless, it needs further improvement in many respects. For example, the coupling to decay channels should be explicitly included by providing in addition to the QQQ Fock component further ones such as $QQQ\pi$, $QQQK$, $QQQ\eta$, and $QQQ\eta'$. This will affect especially those states lying close to continuum thresholds. One may expect, in particular, that thereby the $\Lambda(1405)$ level will be shifted down since it lies close to the $\bar{K}N$ threshold [23]. Furthermore, such a refinement, leading to a unitary model, will especially influence also high-lying resonances. On the other hand, these resonances are mostly sensitive to the confinement and not so much to the hyperfine interaction. Therefore a reasonable description of high-lying resonances requires in addition a more realistic confinement model, where string breakup is implemented. It is thus premature to consider these states in the context of the present model. However, the GBE quark-quark interaction can and should be tested with regard to observables other than the spectra in order to obtain additional constraints. It will be interesting to see how far the description of light and strange baryons in terms of constituent quarks and Goldstone bosons as effective degrees of freedom can be driven.

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