$SU(3)$ symmetry breaking in hyperon semileptonic decays

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Flavor $SU(3)$ symmetry breaking in hyperon semileptonic decay form factors is analyzed using the $1/N_c$ expansion. A detailed comparison with experimental data shows that corrections to f_1 are approximately 10%, which agrees with theoretical expectations. Corrections to g_1 are compatible with first-order symmetry breaking. A fit to the experimental data allows one to predict the g_1 form factor for $\Xi^0 \to \Sigma^+$ decay. The proton matrix element of the T^8 component of the axial vector current [which is equal to $3F-D$ in the $SU(3)$ symmetry limit] is found to be $\approx 0.34-0.46$.

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I. INTRODUCTION

In order to determine the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix from hyperon semileptonic decays (HSD), it is important to understand flavor $SU(3)$ symmetry breaking effects in the hyperon β-decay form factors. At present, V_{ud} can be precisely obtained from superallowed $0^+ \to 0^+$ β decays, but V_{us} can be more reliably determined from K_{e3} decays than HSD [1] because there are larger uncertainties due to first-order symmetry breaking corrections in the HSD axial-vector form factors than in kaon matrix elements. Quark model calculations including symmetry breaking corrections [2,3] predict that the vector form factor f_1 is smaller than its $SU(3)$ symmetric value. The value for V_{us} obtained using this prediction is incompatible with the one obtained from K_{e3} decays [4], $V_{us}=0.2196\pm 0.0023$. However, a recent analysis of the data favored f_1 larger than its $SU(3)$ symmetric value, which yields a V_{us} value consistent with the K_{e3} extraction [4].

In this paper we incorporate $SU(3)$ symmetry breaking corrections into the HSD form factors within the framework of the $1/N_c$ expansion of QCD. The HSD form factors are analyzed in a combined expansion in $1/N_c$ and $SU(3)$ flavor symmetry breaking. We will base our analysis on the formalism described in Ref. [5]. The organization of this paper is as follows. Section II gives a brief introduction to the weak form factors relevant for HSD. The $1/N_c$ expansion of the HSD form factors is derived in Sec. III. In order to make this article self-contained, a brief description of the basics of the $1/N_c$ expansion is given as well. In Sec. IV we perform a detailed comparison of the theoretical expressions with the available experimental data [1] for the decay rates, angular correlations and angular spin-asymmetry coefficients of the octet baryons, and for the widths (converted to axial-vector couplings through the Goldberger-Treiman relation) of the decuplet baryons. Results and conclusions are presented in Sec. V. We find that the best fit values for f_1 are larger than the $SU(3)$ symmetric values and yield a V_{us} value consistent with that obtained from K_{e3} decays. The fit also gives a good description of $SU(3)$ symmetry breaking for the axial form factor g_1. The Fermilab KTeV Collaboration will soon publish their initial results for $\Xi^0 \to \Sigma^+$ β-decay [6]. Our fit predicts $f_1 \sim 1.1$ and $g_1 \sim 1.02-1.07$ for this decay.

II. HYPERON SEMILEPTONIC DECAYS

The low-energy weak interaction Hamiltonian for semileptonic decays is given by

$$H_W = \frac{G}{\sqrt{2}} J_\mu L^\mu + \text{H.c.}, \tag{2.1}$$

where the leptonic current

$$L^\mu = \overline{\psi}_e \gamma^\mu (1-\gamma_5) \psi_{\nu_e} + \overline{\psi}_\mu \gamma^\mu (1-\gamma_5) \psi_{\nu_\mu}, \tag{2.2}$$

and the hadronic current expressed in terms of the vector (V_μ) and axial-vector (A_μ) currents is

$$\begin{aligned} J_\mu &= V_\mu - A_\mu, \\ V_\mu &= V_{ud} \overline{u} \gamma_\mu d + V_{us} \overline{u} \gamma_\mu s, \\ A_\mu &= V_{ud} \overline{u} \gamma_\mu \gamma_5 d + V_{us} \overline{u} \gamma_\mu \gamma_5 s. \end{aligned} \tag{2.3}$$

G is the weak coupling constant, and V_{ud} and V_{us} are elements of the CKM matrix. For definiteness, the notation and conventions of Ref. [7] are adopted in the present work.

The matrix elements of the hadronic current between spin-1/2 states can be written as

$$\begin{aligned} \langle B' | V_\mu | B \rangle = V_{\text{CKM}} \overline{u}_{B'}(p_2) & \\ \times \left[f_1(q^2) \gamma_\mu + \frac{f_2(q^2)}{M_1} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{M_1} q_\mu \right] & \\ \times u_B(p_1), & \end{aligned} \tag{2.4}$$

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$$\langle B'|A_\mu|B\rangle = V_{\rm CKM}\overline{u}_{B'}(p_2) \times\left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{M_1}q_\mu\right] \times\gamma_5 u_B(p_1), \tag{2.5}$$

where $u_B(p_1)$, p_1, M_1 $[\overline{u}_{B'}(p_2),p_2,M_2]$ are the Dirac spinor, the four-momentum, and the mass of the initial [final] hyperon, $q=p_1-p_2$ is the four-momentum transfer and $V_{\rm CKM}$ stands for either V_{ud} or V_{us}. The quantities f_1 and g_1 are the vector and axial-vector form factors, f_2 and g_2 are the weak magnetism and electricity form factors, while f_3 and g_3 are the induced scalar and pseudoscalar form factors, respectively. Time reversal invariance requires that the form factors be real. The six form factors are functions of q^2 and, unless explicitly noted otherwise, their values at $q^2=0$ are discussed. f_3 and g_3 may be safely ignored in decays to an electron, because their contributions to the different observables are suppressed by the electron mass.

In the limit of exact flavor $SU(3)$ symmetry, the hadronic weak vector and axial-vector currents belong to $SU(3)$ octets, so the form factors of different HSD are related by $SU(3)$ flavor symmetry,

$$f_k(q^2)=C_F^{B'B}F_k(q^2)+C_D^{B'B}D_k(q^2), \tag{2.6}$$

$$g_k(q^2)=C_F^{B'B}F_{k+3}(q^2)+C_D^{B'B}D_{k+3}(q^2), \tag{2.7}$$

where $F_i(q^2)$ and $D_i(q^2)$ are reduced form factors and $C_F^{B'B}$ and $C_D^{B'B}$ are well-known Clebsch-Gordan coefficients. The weak currents and the electromagnetic current are members of the same $SU(3)$ octet, so all the vector form factors for HSD are related at $q^2=0$ to the electric charges and the anomalous magnetic moments of the nucleons $\kappa_{p,n}$. In particular, $F_1(0)=1$, $D_1(0)=0$, $F_2(0)=\kappa_p+\frac{1}{2}\kappa_n$, $D_2(0)=-\frac{3}{2}\kappa_n$. Additionally, the conservation of the electromagnetic current implies $F_3(q^2)=D_3(q^2)=0$ so that the form factor $f_3(q^2)$ vanishes for all HSD in the $SU(3)$ symmetry limit.

The leading axial-vector g_1 form factor is given in terms of two reduced form factors, D and F. The g_2 form factor for diagonal matrix elements of hermitian currents (e.g. $\langle B|\overline{u}\gamma^\mu\gamma_5 u-\overline{d}\gamma^\mu\gamma_5 d|B\rangle$) vanishes by hermiticity and time-reversal invariance. $SU(3)$ symmetry then implies that $g_2=0$ in the symmetry limit.

For the decuplet baryons, we will follow a formalism consistent with chiral symmetry adopted in Ref. [8] and originally introduced by Peccei [9]. In this formalism, the width of a decuplet baryon B' decaying to an octet baryon B is given by

$$\Gamma_{B'}=\frac{g^2C(B,B')^2(E_B+M_B){\rm q}_\pi^3}{24\pi f_\pi^2 M_{B'}}, \tag{2.8}$$

where E_B and ${\rm q}_\pi$ are the octet baryon energy and the pion three-momentum in the rest frame of B', $f_\pi=93$ MeV is the pion decay constant, g is the axial-vector coupling and $C(B,B')$ is a Clebsch-Gordan coefficient $[C(B,B')=1$, $1/\sqrt{2}$, $1/\sqrt{3}$, $1/\sqrt{2}$ for $\Delta\to N\pi$, $\Sigma^*\to\Lambda\pi$, $\Sigma^*\to\Sigma\pi$, $\Xi^*\to\Xi\pi$, respectively].

III. OPERATOR ANALYSIS

In the $N_c\to\infty$ limit, it has been shown that the baryon sector has a contracted $SU(2F)$ spin-flavor symmetry, where F is the number of light quark flavors [10,11]. Corrections to the large N_c limit can be expressed in terms of $1/N_c$-suppressed operators with well-defined spin-flavor transformation properties [10]. Recently, the $1/N_c$ expansion has yielded predictions for properties of baryons such as axial-vector couplings and magnetic moments [12,5,8] which are in good agreement with the experimental data. The $1/N_c$ expansion of QCD using quark operators as the operator basis [13,14,5] provides a framework for studying the spin-flavor structure of baryons. In the case of three flavors, the lowest lying baryon states fall into a representation of the spin-flavor group $SU(6)$. When $N_c=3$, this corresponds to the very well-known **56** dimensional representation of $SU(6)$.

A complete set of operators can be constructed using the zero-body operator $\mathbb{1}$ and the one-body operators

$$J^i=q^\dagger\left(\frac{\sigma^i}{2}\otimes\mathbb{1}\right)q \quad (1,1),$$

$$T^a=q^\dagger\left(\mathbb{1}\otimes\frac{\lambda^a}{2}\right)q \quad (0,8),$$

$$G^{ia}=q^\dagger\left(\frac{\sigma^i}{2}\otimes\frac{\lambda^a}{2}\right)q \quad (1,8), \tag{3.1}$$

where J^i are the baryon spin generators, T^a are the baryon flavor generators, and G^{ia} are the baryon spin-flavor generators. The transformation properties of these generators under $SU(2)\times SU(3)$ are given explicitly in Eq. (3.1) as (j,d), where j is the spin and d is the dimension of the $SU(3)$ flavor representation.

Any QCD one-body operator transforming according to a given $SU(2)\times SU(3)$ representation has a $1/N_c$ expansion of the form

$$\mathcal{O}_{\rm QCD}=\sum_{n=0}^{N_c} c_n\frac{1}{N_c^{n-1}}\mathcal{O}_n, \tag{3.2}$$

where $c_n(1/N_c)$ are unknown coefficients which have power series expansions in $1/N_c$ beginning at order unity. The sum in Eq. (3.2) is over all possible independent n-body operators $\mathcal{O}_n$ with the same spin and flavor quantum numbers as $\mathcal{O}_{\rm QCD}$. The use of operator identities [5] reduces the operator basis to independent operators. In this analysis we are concerned with the $1/N_c$ expansions of the QCD vector and axial vector currents, whose matrix elements between $SU(6)$ symmetric states give the HSD form factors.

The $1/N_c$ expansion for the HSD amplitudes is derived to first order in flavor symmetry breaking, and to leading order in $1/N_c$ for most of the form factors. For the f_1 form factor, however, we include second-order flavor symmetry breaking corrections, since the Ademollo-Gatto theorem states that there are no first order corrections, so that the leading symmetry breaking correction to f_1 is of second order. A chiral perturbation theory calculation shows that (formally) second-order symmetry breaking effects actually contribute at first order in symmetry breaking [15,16],[1] so we have included these effects. The f_2 form factor is multiplied by q, and so makes a small contribution to the HSD amplitude. Since q is of order the hyperon mass differences, the contribution of the first order $SU(3)$ symmetry breaking correction in f_2 to the HSD amplitude is comparable to a second-order symmetry breaking effect, and is neglected. In the symmetry limit f_2 can be determined from the baryon anomalous magnetic moments, and that is what we do here. The axial form factor g_1 is computed to first order in symmetry breaking. The g_2 form factor vanishes in the symmetry limit, so its contribution is comparable to symmetry breaking terms in f_2, and is neglected. Finally, f_3 and g_3 contributions are proportional to the electron mass, and also will be neglected.

A. Vector form factor f_1

We begin by deriving the $1/N_c$ expansion for the baryon vector current in the $SU(3)$ flavor symmetry limit. At $q^2=0$, the hyperon matrix elements for the vector current are given by the matrix elements of the associated charge or $SU(3)$ generator. Let V^{0a} denote the flavor octet baryon charge[2]

$$V^{0a}=\left\langle B'\left|\left(\bar{q}\gamma^0\,\frac{\lambda^a}{2}q\right)_{\rm QCD}\right|B\right\rangle, \tag{3.3}$$

whose matrix elements between $SU(6)$ symmetric states give the values of the leading vector form factor f_1. V^{0a} is spin-0 and a flavor octet, so it transforms as (0,8) under $SU(2)\times SU(3)$.

The $1/N_c$ expansion for a (0,8) operator was obtained in Ref. [17]. Operator reduction rules imply that only n-body operators with a single factor of either T^a or G^{ia} appear. Thus, the allowed one- and two-body operators are

$$\mathcal{O}_1^a=T^a, \tag{3.4}$$

$$\mathcal{O}_2^a=\{J^i,G^{ia}\}. \tag{3.5}$$

The remaining operators are obtained from these operators by anticommuting with J^2, $\mathcal{O}_{n+2}=\{J^2,\mathcal{O}_n\}$. Thus, the $1/N_c$ expansion of V^{0a} has the form

$$V^{0a}=\sum_{n=1}^{N_c} c_n\,\frac{1}{N_c^{n-1}}\,\mathcal{O}_n^a. \tag{3.6}$$

The operator V^{0a} at $q^2=0$ is a special (0,8) operator; it is the generator of $SU(3)$ symmetry transformations. This fixes

$$c_1=1,\quad c_n=0,\quad n>1. \tag{3.7}$$

Thus, the $1/N_c$ expansion of V^{0a} in the limit of exact $SU(3)$ flavor symmetry reduces to

$$V^{0a}=T^a, \tag{3.8}$$

to all orders in the $1/N_c$ expansion. The matrix elements of Eq. (3.8) will be denoted by $f_1^{SU(3)}$ hereafter.

B. Vector form factor with perturbative $SU(3)$ breaking

In QCD, flavor $SU(3)$ symmetry breaking is due to the strange quark mass m_s, and transforms as a flavor octet. In order to construct the most general $1/N_c$ expansion for V^{0a} up to second order in symmetry breaking, we need to consider the spin-0 $SU(2)\times SU(3)$ representations of the quark operators contained in the $SU(6)$ representations **1**, **35**, $\mathbf{\overline{405}}$ and **2695**, i.e. (0,1), (0,8), (0,27), (0,64), and $(0,10+\overline{10})$, since the baryon $1/N_c$ expansion extends only to three-body operators if we restrict ourselves to physical baryon states.[3] The $1/N_c$ expansions for the above representations were computed in Ref. [17]; the results can be summarized as follows.

The $1/N_c$ expansion for a (0,1) QCD operator starts with the zero-body operator $\mathcal{O}_0=\mathbb{1}$. Additional operators are obtained by anticommuting with J^2.

The $1/N_c$ expansion for a (0,8) operator has the same form as Eq. (3.6) and will not be repeated here. The $1/N_c$ expansion for a (0,27) operator contains the two- and three-body operators

$$\mathcal{O}_2^{ab}=\{T^a,T^b\}, \tag{3.9}$$

$$\mathcal{O}_3^{ab}=\{T^a,\{J^i,G^{ib}\}\}+\{T^b,\{J^i,G^{ia}\}\}, \tag{3.10}$$

where the flavor singlet and octet components of the above operators have to be subtracted off. As for a (0,64) operator, the $1/N_c$ expansion starts with a single three-body operator

$$\mathcal{O}_3^{abc}=\{T^a,\{T^b,T^c\}\}, \tag{3.11}$$

where it is understood that the singlet, octet and 27 components are subtracted off in such a way that only the 64 component remains. Finally, for a $(0,10+\overline{10})$ operator, one obtains

$$\mathcal{O}_3^{ab}=\{T^a,\{J^i,G^{ib}\}\}-\{T^b,\{J^i,G^{ia}\}\}. \tag{3.12}$$

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[1]For a more detailed explanation of this seemingly contradictory statement, see Ref. [16].

[2]The subscript QCD emphasizes the fact that $\bar{q}$ and q are QCD quark fields, not the quark creation and annihilation operators of the quark representation.

[3]The $(0,10-\overline{10})$ representation is not allowed by time reversal invariance.

First-order symmetry breaking terms in V^{0a} are given by setting one free flavor index equal to 8 in the operators described above. At second-order in the symmetry breaking, two free flavor indices are set equal to 8. This gives

$$\begin{aligned}V^{0a}+\delta V^{0a}=&(1+\epsilon a_1)T^a+\epsilon a_2\frac{1}{N_c}\{J^i,G^{ia}\}+\epsilon a_3\frac{1}{N_c^2}\{J^2,T^a\}+\epsilon b_1 d^{ab8}T^b+\epsilon b_2\frac{1}{N_c}d^{ab8}\{J^i,G^{ib}\}+\epsilon b_3\frac{1}{N_c^2}d^{ab8}\{J^2,T^b\}\\&+\epsilon a_4\frac{1}{N_c}\{T^a,T^8\}+\epsilon a_5\frac{1}{N_c^2}(\{T^a,\{J^i,G^{i8}\}\}+\{T^8,\{J^i,G^{ia}\}\})+\epsilon a_6\frac{1}{N_c^2}(\{T^a,\{J^i,G^{i8}\}\}-\{T^8,\{J^i,G^{ia}\}\})\\&+\epsilon^2 b_4\frac{1}{N_c}d^{ab8}\{T^b,T^8\}+\epsilon^2 a_7\frac{1}{N_c^2}\{T^a,\{T^8,T^8\}\}+\epsilon^2 b_5\frac{1}{N_c^2}d^{ab8}(\{T^b,\{J^i,G^{i8}\}\}+\{T^8,\{J^i,G^{ib}\}\})\\&+\epsilon^2 b_6\frac{1}{N_c^2}d^{ab8}(\{T^b,\{J^i,G^{i8}\}\}-\{T^8,\{J^i,G^{ib}\}\}),\end{aligned}\tag{3.13}$$

where $\epsilon\sim m_s$ is a (dimensionless) measure of $SU(3)$ breaking. Observe that similar terms with the d symbol replaced by an f symbol are ruled out by time reversal invariance. None of the (0,1) operators contributes to V^{0a} for $\Delta S=0$ and $|\Delta S|=1$ weak decays, so they have been omitted in Eq. (3.13). Note that the coefficients in Eq. (3.13) must be such that there is no symmetry breaking for the $\Delta S=0$ weak decays, since isospin symmetry is not broken by the strange quark mass.

Equation (3.13) can be rewritten in terms of the number of strange quarks, N_s, and the strange quark spin, J_s^i, using [5]

$$T^8=\frac{1}{2\sqrt{3}}(N_c-3N_s),\tag{3.14}$$

$$G^{i8}=\frac{1}{2\sqrt{3}}(J^i-3J_s^i).\tag{3.15}$$

After making use of the identity [8]

$$J^iG^{ia}/J^2=\frac{2}{3}\left(T^a+\frac{1}{2}\{T^a,N_s\}\right),\tag{3.16}$$

valid for $\Delta S=2\Delta I$ transitions, rearranging terms and absorbing factors of N_c^{-1} and N_c^{-2} we obtain a rather compact form for V^{0a}, namely,

$$V^{0a}=T^a,$$

for $\Delta S=0$ decays, and

$$\begin{aligned}V^{0a}=&(1+v_1')T^a+v_2'\{T^a,N_s\}+v_3'\{T^a,N_s^2\}\\&+v_4'\{T^a,J^2\}+v_5'\{T^a,-I^2+J_s^2\},\end{aligned}\tag{3.17}$$

for $|\Delta S|=1$ decays. Here I is the isospin. For the decays we are considering, the v_3' and v_4' terms are not independent, and can be written as linear combinations of the v_1' and v_2' terms. Thus, Eq. (3.17) reduces to

$$V^{0a}=(1+v_1)T^a+v_2\{T^a,N_s\}+v_3\{T^a,-I^2+J_s^2\}.\tag{3.18}$$

The baryons are eigenstates of J^2, I^2, J_s^2, and N_s, so the matrix elements of Eq. (3.18) can be computed straightforwardly. They are listed in Table I for the processes we are concerned with.

C. Axial-vector form factor g_1

The $1/N_c$ expansion for the axial-vector current A^{ia} was discussed in great detail in Ref. [5] and we will only state the answer here. The axial current matrix elements can be written as

$$\begin{aligned}\frac{1}{2}A^{ia}=&aG^{ia}+bJ^iT^a+\Delta^a(c_1G^{ia}+c_2J^iT^a)+c_3\{G^{ia},N_s\}\\&+c_4\{T^a,J_s^i\}+\frac{1}{\sqrt{3}}\delta^{a8}W^i\\&-\frac{d}{2}\left(\{J^2,G^{ia}\}-\frac{1}{2}\{J^i,\{J^j,G^{ja}\}\}\right),\end{aligned}\tag{3.19}$$

where

$$W^i=(c_4-2c_1)J_s^i+(c_3-2c_2)N_sJ^i-3(c_3+c_4)N_sJ_s^i,\tag{3.20}$$

TABLE I. Operator matrix elements for the vector form factor f_1.

Transition	$f_1^{SU(3)}$	v_1	v_2	v_3
$n\to p$	1	0	0	0
$\Sigma^{\pm}\to\Lambda$	0	0	0	0
$\Lambda\to p$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	0
$\Sigma^-\to n$	-1	-1	-1	2
$\Xi^-\to\Lambda$	$\sqrt{3/2}$	$\sqrt{3/2}$	$3\sqrt{3/2}$	$\sqrt{6}$
$\Xi^-\to\Sigma^0$	$1/\sqrt{2}$	$1/\sqrt{2}$	$3/\sqrt{2}$	0
$\Xi^0\to\Sigma^+$	1	1	3	0

TABLE II. Operator matrix elements for the axial form factor g_1.

Transition	a	b	d	c_1	c_2	c_3	c_4
$\Delta\to N$	-2	0	9/2	0	0	0	0
$\Sigma^*\to\Lambda$	-2	0	9/2	0	0	-4	0
$\Sigma^*\to\Sigma$	-2	0	9/2	0	0	-4	8
$\Xi^*\to\Xi$	-2	0	9/2	0	0	-8	4
$n\to p$	5/3	1	0	0	0	0	0
$\Sigma^\pm\to\Lambda$	$\sqrt{2/3}$	0	0	0	0	$\sqrt{8/3}$	0
$\Lambda\to p$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	0	$-\sqrt{3/2}$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	$-\sqrt{3/2}$
$\Sigma^-\to n$	1/3	-1	0	1/3	-1	1/3	1/3
$\Xi^-\to\Lambda$	$1/\sqrt{6}$	$\sqrt{3/2}$	0	$1/\sqrt{6}$	$\sqrt{3/2}$	$\sqrt{3/2}$	$7/\sqrt{6}$
$\Xi^-\to\Sigma^0$	$5/\sqrt{18}$	$1/\sqrt{2}$	0	$5/\sqrt{18}$	$1/\sqrt{2}$	$5/\sqrt{2}$	$1/\sqrt{2}$
$\Xi^0\to\Sigma^+$	5/3	1	0	5/3	1	5	1

and $\Delta_a=1$ for $a=4,5,6$, or 7 and equals zero otherwise. A term proportional to d has been added in order for the $SU(3)$ symmetric parameters D, F, and $\mathcal{C}$ [18] to have arbitrary values. Adding this term will avoid the mixing between symmetry breaking effects and $1/N_c$ corrections in the symmetric couplings. Furthermore, the couplings have been parametrized in such a way that only the parameters a, b, and d contribute to processes which take place in the strangeness-zero sector. Including $SU(3)$ breaking, the reduced form factors D and F are defined as

$$D=a, \quad F=\frac{2}{3}a+b, \tag{3.21}$$

so that $g_1/f_1=D+F$ is positive for neutron decay, which fixes all other signs. Thus, for any process, the matrix elements of A^{ia} are given as the sum of the parameters a, b, d, $c_1,\ldots,c_4$ times matrix elements of the operators involved in the expansion (3.19). The operator matrix elements were computed in Ref. [8] and are listed in Table II for the sake of completeness.

D. Weak magnetism form factor f_2

In the limit of exact $SU(3)$ flavor symmetry, the weak magnetism form factors f_2 are directly related to the anomalous magnetic moments of the nucleons, and are given in terms of two invariants m_1 and m_2. Since the magnetic moment is a spin-1 octet operator, it has a $1/N_c$ expansion identical in structure to the axial current. It is convenient to define the two parameters m_1 and m_2 by

$$M^i=m_1G^{iQ}+m_2J^iT^Q,$$

where Q represents the $SU(3)$ generator which is the electric charge, so $G^{iQ}\equiv G^{i3}+G^{i8}/\sqrt{3}$, and $T^Q\equiv T^3+T^8/\sqrt{3}$. The parameters $m_{1,2}$ can be determined from the anomalous magnetic moments of the hyperons.

The contributions of f_2 to the different observables of HSD in the $SU(3)$ limit are first-order symmetry breaking contributions because of the kinematic factor of q. Previous work [4,19] has shown that reasonable shifts from the $SU(3)$ predictions of f_2 do not have any observable effect upon χ^2 or g_1 in a global fit to experimental data. We will use the best fit values [8] $m_1=2.87$ and $m_2=-0.077$ obtained from the baryon anomalous magnetic moments to fix f_2.

E. Weak electricity form factor g_2

In the $SU(3)$ flavor symmetry limit, the form factor g_2 vanishes, so that g_2 is proportional to $SU(3)$ symmetry breaking at leading order. g_2 transforms oppositely to g_1 and f_2 under time reversal, and therefore has a different $1/N_c$ operator expansion.

Let W^{ia} be the operator whose matrix elements give the values of g_2. At first order in $SU(3)$ symmetry breaking, the contribution to g_2 transforms as $(1,8)$ and $(1,10-\overline{10})$ under

TABLE III. Operator matrix elements for the weak electricity form factor g_2.

Transition	b_1	b_2	b_3
$n\to p$	0	0	0
$\Sigma^\pm\to\Lambda$	0	0	1
$\Lambda\to p$	$-\sqrt{3/2}$	$-\sqrt{3/2}$	-1
$\Sigma^-\to n$	1/3	-1	0
$\Xi^-\to\Lambda$	$1/\sqrt{6}$	$\sqrt{3/2}$	1
$\Xi^-\to\Sigma^0$	$5/\sqrt{18}$	$1/\sqrt{2}$	0
$\Xi^0\to\Sigma^+$	5/3	1	0

TABLE IV. Experimental data on three measured $\Delta S=0$ hyperon semileptonic decays. The units of R are 10^{-3} s^{-1} for neutron decay and 10^6 s^{-1} for the remaining decays.

	$n\to pe^-\overline{\nu_e}$	$\Sigma^+\to\Lambda e^+\nu_e$	$\Sigma^-\to\Lambda e^-\overline{\nu_e}$
R	1.1274 ± 0.0025	0.250 ± 0.063	0.387 ± 0.018
$\alpha_{e\nu}$	-0.0766 ± 0.0036	-0.35 ± 0.15	-0.404 ± 0.044
α_e	-0.08559 ± 0.00086		
α_ν	0.990 ± 0.008		
A			0.07 ± 0.07
B			0.85 ± 0.07
g_1/f_1	1.2601 ± 0.0025		

TABLE V. Experimental data on four measured $|\Delta S|=1$ hyperon semileptonic decays. The units of R are $10^6\ s^{-1}$.

	$\Lambda\to pe^-\overline{\nu_e}$	$\Sigma^-\to ne^-\overline{\nu_e}$	$\Xi^-\to\Lambda e^-\overline{\nu_e}$	$\Xi^-\to\Sigma^0 e^-\overline{\nu_e}$
R	3.161 ± 0.058	6.876 ± 0.235	3.435 ± 0.192	0.531 ± 0.104
$\alpha_{e\nu}$	-0.019 ± 0.013	0.347 ± 0.024	0.53 ± 0.10	
α_e	0.125 ± 0.066	-0.519 ± 0.104		
α_ν	0.821 ± 0.060	-0.230 ± 0.061		
α_B	-0.508 ± 0.065	0.509 ± 0.102		
A			0.62 ± 0.10	
g_1/f_1	0.718 ± 0.015	-0.340 ± 0.017	0.25 ± 0.05	1.287 ± 0.158

spin and flavor. The (1,8) expansion is given by

$$\delta W_8^{ia}\propto ib_1 f^{ab8}G^{ib}+ib_2 f^{ab8}\frac{J^iT^b}{N_c}, \tag{3.22}$$

which involves the f symbol, rather than the d symbol since g_2 has the opposite time-reversal properties from g_1 and f_2. The $(1,10-\overline{10})$ expansion has not been presented previously in the literature. The operator $\{G^{ig},T^h\}-\{G^{ih},T^g\}$, which contains $(10+\overline{10})$, can be split into 10 and $\overline{10}$ representations by contracting with $f^{acg}d^{bch}$ [5]. The resulting operator contains $i(10-\overline{10})$, which is T-even. This procedure leads to the contribution

$$\delta W_{10-\overline{10}}^{ia}\propto if^{8cg}d^{ach}(\{G^{ig},T^h\}-\{T^g,G^{ih}\})$$
$$=-if^{acg}d^{8ch}(\{G^{ig},T^h\}-\{T^g,G^{ih}\}). \tag{3.23}$$

For HSD, the three operators (3.22) and (3.23) are linear combinations of the three allowed invariants

$$\mathrm{Tr}[T^a,T^8]\overline{B}B,\quad \mathrm{Tr}\ \overline{B}[T^a,T^8]B,$$
$$\mathrm{Tr}\ \overline{B}T^a\ \mathrm{Tr}\ BT^8-\mathrm{Tr}\ \overline{B}T^8\ \mathrm{Tr}\ BT^a \tag{3.24}$$

given by a general $SU(3)$ analysis [7] neglecting isospin breaking. We choose as independent parameters b_1 and b_2 in Eq. (3.22), and b_3 that multiplies the third invariant in Eq. (3.24). The matrix elements are listed in Table III. For any process, the matrix elements of W^{ia} can be given as a sum of the parameters b_{1-3} times the operator matrix elements listed in Table III.

The parameters b_{1-3} are proportional to ϵ. As mentioned in the introductory remarks to this section, g_2 should be neglected for a consistent analysis. Nevertheless, we tried to see if we could obtain some information on g_2 from the experimental data using the above formulas for g_2. However, the data are not accurate enough for an extraction of the small g_2-dependence of the decay amplitudes. One expects that g_2 should be about 25% of f_2, so that g_2 is $\lesssim 0.5$.

TABLE VI. Experimental values of axial-vector couplings in decuplet-to-octet processes.

	$\Delta\to N\pi$	$\Sigma^*\to\Lambda\pi$	$\Sigma^*\to\Sigma\pi$	$\Xi^*\to\Xi\pi$
g	-2.04 ± 0.01	-1.71 ± 0.03	-1.60 ± 0.13	-1.42 ± 0.04

IV. FITTING THE DATA

The experimentally measured quantities [1] in HSD are the total decay rate R, angular correlation coefficients $\alpha_{e\nu}$, and angular spin-asymmetry coefficients α_e, α_ν, α_B, A, and B. Often, the data is presented in terms of R and the ratio g_1/f_1 for the decay. This information is displayed in Tables IV and V for the measured decays. The theoretical expressions for the total decay rates and angular coefficients can be found in Ref. [7]. The radiative corrections and the four-momentum-transfer contribution to the form factors are also discussed in this reference. In the present analysis, we will take these corrections into account.[4] The experimentally measured quantity for the decuplet baryons is the decay width, which has been converted to an axial-vector coupling for each decay using the Goldberger-Treiman relation and Eq. (2.8). This information is displayed in Table VI.

In this section we perform a number of different fits to the experimental data. The experimental data which are used are the decay rates and the spin and angular correlation coefficients. The value of g_1/f_1 is not included, since it is determined from the other quantities and is not an independent measurement. For $\Xi^-\to\Sigma^0$ decay, we have used g_1/f_1, however, since the spin and angular correlation coefficients have not been measured. The parameters to be fitted are those arising from the $1/N_c$ expansions for the couplings, namely, v_{1-3} for f_1 introduced in Eq. (3.18) and a, b, d, c_{1-4} for g_1 given in Eq. (3.19). We also attempted to fit b_{1-3} for g_2, but the experimental data is not sufficiently accurate to determine the small g_2 contribution to the decay amplitude. We therefore neglect g_2 in the rest of the analysis. Finally, in the first stage of the analysis we use as inputs the PDG values of V_{ud} and V_{us} [1] (which are primarily obtained from nuclear β decay and K_{e3} decay, respectively). We later proceed to fit for them as well.

A. $SU(3)$ fit

The simplest possible fit is an $SU(3)$ symmetric fit to HSD (ignoring the decuplet decays) which involves only two

[4] In this work we have adopted a dipole form for the leading vector and axial-vector form factors, with masses $M_V=0.84$ GeV and $M_A=0.96$ GeV for $\Delta S=0$ transitions, and $M_V=0.97$ GeV and $M_A=1.11$ GeV for $|\Delta S|=1$ processes [7]. In Ref. [8] somewhat different values of M_V and M_A were used. Our results are insensitive to this difference.

TABLE VII. Best fitted parameters for the vector and axial-vector form factors. V_{ud} and V_{us} in fits A and B are inputs. Errors are from the χ^2 fit only, and do not include any theoretical uncertainties.

	Fit A	Fit B	Fit C	Fit D
V_{ud}	0.9736±0.0010	0.9736±0.0010	0.9743±0.0009	0.9743±0.0009
V_{us}	0.2196±0.0023	0.2196±0.0023	0.2194±0.0023	0.2194±0.0023
v_1		−0.03±0.04	−0.03±0.04	−0.02±0.04
v_2		0.05±0.03	0.05±0.03	0.05±0.03
v_3		−0.01±0.01	−0.01±0.01	−0.01±0.01
a	0.86±0.02	0.87±0.02	0.87±0.02	0.84±0.02
b	−0.16±0.03	−0.18±0.03	−0.18±0.03	−0.12±0.03
d	−0.07±0.01	−0.07±0.01	−0.07±0.01	−0.03±0.01
c_1	−0.03±0.02	−0.03±0.02	−0.03±0.02	−0.01±0.02
c_2	0.09±0.04	0.10±0.04	0.10±0.04	0.05±0.04
c_3	−0.06±0.01	−0.07±0.01	−0.07±0.01	−0.05±0.01
c_4	0.04±0.01	0.03±0.01	0.03±0.01	0.02±0.01
F	0.41±0.02	0.40±0.02	0.40±0.02	0.43±0.02
D	0.86±0.02	0.87±0.02	0.87±0.02	0.84±0.02
$3F-D$	0.37±0.08	0.34±0.08	0.34±0.08	0.46±0.08

parameters a, b for g_1; it corresponds to a fit using only F and D. The results are $a=0.797\pm0.006$, $b=-0.059\pm0.009$, which yield $F=0.47\pm0.01$, $D=0.80\pm0.01$, $3F-D=0.62\pm0.03$, with $\chi^2=62.3$ for 23 degrees of freedom. The large χ^2 of the fit is clear evidence for $SU(3)$ breaking. A similar fit using the rates and g_1/f_1 ratios was performed in Ref. [8]. Both results are in very good agreement. We also followed this reference in order to make a preliminary study of $\Delta S=0$ decays only. Our fits produce similar results and there is no need to show them here.

B. First-order symmetry breaking

The next step is to see how the results are modified once first-order symmetry breaking is taken into account. To this order, f_2 will be kept at its $SU(3)$ symmetric value and g_2 is set to zero. f_1 is also kept at its symmetry-limit value, $f_1^{SU(3)}$, because of the Ademollo-Gatto theorem.[5] Thus, only the order ϵ terms in g_1 introduced in Eq. (3.19) will be considered. Fitting a, b, d, c_{1-4} leads to the results listed as fit A of Table VII, with $\chi^2=51.6$ for 22 degrees of freedom. Comparing the values of F and D with those of the previous section, we see that the change due to symmetry breaking corrections is in fact small [compared to a naive estimate of $SU(3)$ breaking]. The leading parameter a is order unity, b is order $1/N_c$, d is order $1/N_c^2$, and the values of c_{1-4} are small or smaller than expected from first-order symmetry breaking ($\epsilon\sim30\%$, which is a measure of symmetry breaking) and factors of $1/N_c$. These results agree with the ones presented in Ref. [8], which were obtained by using the total decay rates and g_1/f_1 ratios as experimental inputs.

Notice that the quantity $3F-D$, which is relevant for the analysis of spin-dependent deep inelastic scattering, is smaller than its value determined in the $SU(3)$ limit, and is considerably smaller than its $SU(6)$ symmetric value of 1 [20,21,8,22]. Before drawing any conclusions, however, we will study in the next section the effect of symmetry breaking in f_1 on the different observables, and in particular upon the reduced form factors F and D.

C. Symmetry breaking in f_1

In the previous sections f_1 was fixed at its $SU(3)$ symmetric value, $f_1^{SU(3)}$. We now proceed to incorporate symmetry breaking corrections into the f_1 form factors in $|\Delta S|=1$ decays. Formally, one expects that these corrections should be second order in symmetry breaking, due to the Ademollo-Gatto theorem. However, we know from explicit computations of chiral loops [15,16] that there are, in fact, corrections which can be considered to be first order in symmetry breaking. These were not included in Ref. [8].

The best fit parameters v_{1-3} for f_1 and a, b, d, c_{1-4}, for g_1 are displayed as fit B in Table VII. The resulting form factors are given in Tables VIII and IX. The theoretical predictions for the different observables are listed in Tables X, XI and XII for the sake of completeness. The fit has $\chi^2=39.2$ with 19 degrees of freedom.

From Table IX, we observe that $SU(3)$ breaking corrections to the leading vector form factors f_1 are as much as 12%, depending on the strange-quark content of the decaying and emitted baryons. Furthermore, we can observe that the natural trend is $f_1/f_1^{SU(3)}>1$, as was pointed out in Refs. [4, 19]. Additionally, the ratios g_1/f_1 of fit B (in Tables XI and XII) agree with the experimental ones listed in Tables IV and V. As for the axial-vector couplings of the decuplet baryons, we can see in Table X that the theoretical predictions are in good agreement with their experimental values. The highest contribution to χ^2 comes from $\Sigma^*\to\Lambda\pi$ decay.

In Tables XI and XII, the predictions for the different observables are in reasonable agreement with their experimental counterparts displayed in Tables IV and V, respectively. The highest contributions to the total χ^2 arise mainly from α_e ($\Delta\chi^2=2.7$) in $n\to pe^-\overline{\nu}_e$, α_e ($\Delta\chi^2=2.4$), α_ν ($\Delta\chi^2=6.6$) in $\Lambda\to pe^-\overline{\nu}_e$, α_ν ($\Delta\chi^2=3.8$) in Σ^-

[5] We have mentioned earlier that chiral corrections to f_1 are not necessarily second order. They are included in the next section.

TABLE VIII. Predicted form factors. Errors are from the χ^2 fit only, and do not include any theoretical uncertainties. f_2 has the same values for fits A–D, and f_1 has the same values for fits B–D.

	Fit A			Fit B		Fit C	Fit D
Transition	f_1	f_2	g_1	f_1	g_1	g_1	g_1
$n\rightarrow p$	1.00	1.85	1.269±0.001	1.00	1.269±0.001	1.268±0.002	1.268±0.002
$\Sigma^{\pm}\rightarrow\Lambda$	0.00	1.17	0.60±0.01	0.00	0.60±0.01	0.60±0.01	0.60±0.01
$\Lambda\rightarrow p$	−1.22	−1.10	−0.89±0.01	−1.25±0.02	−0.89±0.01	−0.89±0.01	−0.89±0.01
$\Sigma^{-}\rightarrow n$	−1.00	1.02	0.34±0.01	−1.04±0.02	0.34±0.01	0.34±0.01	0.34±0.01
$\Xi^{-}\rightarrow\Lambda$	1.22	−0.07	0.27±0.03	1.35±0.05	0.25±0.03	0.25±0.03	0.25±0.03
$\Xi^{-}\rightarrow\Sigma^{0}$	0.71	1.31	0.73±0.02	0.79±0.03	0.72±0.02	0.72±0.02	0.76±0.02
$\Xi^{0}\rightarrow\Sigma^{+}$	1.00	1.85	1.03±0.02	1.12±0.05	1.02±0.02	1.02±0.03	1.07±0.03

TABLE IX. Symmetry breaking for f_1. The ratio $f_1/f_1^{SU(3)}$ is displayed. Errors are from the χ^2 fit only, and do not include any theoretical uncertainties.

Transition	Fit B, C, D	Anderson and Luty [16]	Donoghue *et al.* [2]	Krause [15]	Schlumpf [3]
$\Lambda\rightarrow p$	1.02±0.02	1.024	0.987	0.943	0.976
$\Sigma^{-}\rightarrow n$	1.04±0.02	1.100	0.987	0.987	0.975
$\Xi^{-}\rightarrow\Lambda$	1.10±0.04	1.059	0.987	0.957	0.976
$\Xi^{-}\rightarrow\Sigma^{0}$	1.12±0.05	1.011	0.987	0.943	0.976
$\Xi^{0}\rightarrow\Sigma^{+}$	1.12±0.05				

TABLE X. Theoretical predictions for decuplet-to-octet axial-vector couplings g. Errors are from the χ^2 fit only, and do not include any theoretical uncertainties.

Transition	Fit A	χ^2	Fit B	χ^2	Fit C	χ^2
$\Delta\rightarrow N$	−2.03±0.01	0.3	−2.04±0.01	0.2	−2.04±0.01	0.2
$\Sigma^{*}\rightarrow\Lambda$	−1.78±0.02	5.7	−1.77±0.02	3.7	−1.77±0.02	3.7
$\Sigma^{*}\rightarrow\Sigma$	−1.49±0.07	0.7	−1.55±0.07	0.1	−1.55±0.07	0.1
$\Xi^{*}\rightarrow\Xi$	−1.38±0.04	0.8	−1.39±0.04	0.5	−1.39±0.04	0.5

TABLE XI. Theoretical predictions for three $\Delta S=0$ hyperon semileptonic decays and their contributions to the total χ^2. The units of R are 10^{-3} s^{-1} for neutron decay and 10^{6} s^{-1} for the remaining decays. The predictions for fits C and D are the same as for fit B.

	$n\rightarrow pe^{-}\overline{\nu}_e$				$\Sigma^{+}\rightarrow\Lambda e^{+}\nu_e$				$\Sigma^{-}\rightarrow\Lambda e^{-}\overline{\nu}_e$			
	Fit A	χ^2	Fit B	χ^2	Fit A	χ^2	Fit B	χ^2	Fit A	χ^2	Fit B	χ^2
R	1.13	0.9	1.13	0.9	0.23	0.1	0.23	0.1	0.39	0.0	0.39	0.0
$\alpha_{e\nu}$	−0.08	0.3	−0.08	0.3	−0.41	0.1	−0.41	0.1	−0.41	0.1	−0.41	0.1
α_e	−0.09	2.7	−0.09	2.7								
α_ν	0.99	0.1	0.99	0.1								
A									0.05	0.1	0.05	0.1
B									0.88	0.2	0.88	0.2
g_1/f_1	1.27		1.27									

TABLE XII. Theoretical predictions for five $|\Delta S|=1$ hyperon semileptonic decays and their contributions to the total χ^2. The units of R are $10^6\ \mathrm{s}^{-1}$. The values for fits C and D are the same as for fit B for the first three decay modes. For $\Xi^-\to\Sigma^0$ decay, fit C gives the same values as fit B, and fit D gives $R=0.4$ and $g_1/f_1=0.96$.

	$\Lambda\to pe^-\overline{\nu}_e$				$\Sigma^-\to ne^-\overline{\nu}_e$				$\Xi^-\to\Lambda e^-\overline{\nu}_e$				$\Xi^-\to\Sigma^0 e^-\overline{\nu}_e$				$\Xi^0\to\Sigma^+ e^-\overline{\nu}_e$		
	Fit A	χ^2	Fit B	χ^2	Fit A	χ^2	Fit B	χ^2	Fit A	χ^2	Fit B	χ^2	Fit A	χ^2	Fit B	χ^2	Fit A	Fit B	Fit D
R	3.14	0.1	3.19	0.2	6.43	3.6	6.83	0.0	2.87	8.7	3.32	0.4	0.36	2.7	0.38	2.2	0.65	0.68	0.73
$\alpha_{e\nu}$	−0.03	1.3	−0.02	0.0	0.34	0.2	0.36	0.3	0.60	0.5	0.65	1.5					−0.14	−0.07	−0.10
α_e	0.01	2.9	0.02	2.4	−0.63	1.2	−0.61	0.8									−0.11	−0.05	−0.07
α_ν	0.98	6.9	0.97	6.6	−0.35	4.0	−0.35	3.8									1.00	1.00	1.00
α_B	−0.59	1.5	−0.59	1.7	0.67	2.4	0.65	1.9									−0.52	−0.56	−0.55
A									0.53	0.8	0.46	2.6					0.71	0.75	0.73
B																	0.61	0.56	0.59
g_1/f_1	0.73		0.71		−0.34		−0.33		0.22		0.18		1.03		0.91		1.03	0.91	0.96

$\to ne^-\overline{\nu}_e$, and R ($\Delta\chi^2=2.2$) and g_1/f_1 ($\Delta\chi^2=5.7$) in $\Xi^-\to\Sigma^0 e^-\overline{\nu}_e$. If α_ν in $\Lambda\to pe^-\overline{\nu}_e$ decay is left out, there are small readjustments of the parameters and predicted observables, some of them almost imperceptible, so that one can draw the same conclusions as above. This fact suggests that there is an experimental inconsistency in the value of this α_ν.

One can redo the above fit including g_2. The best fit parameters are $b_1=0.9\pm0.5$, $b_2=1.0\pm0.2$, $b_3=-0.0\pm0.4$, and χ^2 is reduced to 27 for 16 degrees of freedom. The reduction in χ^2 suggests that there is a non-zero g_2, but the large error bars indicate that the experimental data is not sufficiently accurate to determine g_2.[6] The fit including g_2 gives $3F-D=0.3\pm0.1$, and $f_1=1.4\pm0.1$, $g_1=1.4\pm0.1$ for $\Xi^0\to\Sigma^+$ decay.

Finally, we fit the data with the CKM matrix elements V_{ud} and V_{us} as free parameters (neglecting g_2). Unfortunately, there is not enough information on the $|\Delta S|=1$ decays to make a detailed analysis and extract a value of V_{us} from these data only. We will content ourselves with performing a global fit to data allowing both V_{ud} and V_{us} to be free parameters. The best fit values for the CKM parameters are

$$V_{ud}=0.9743\pm0.0009,\quad V_{us}=0.2194\pm0.0023. \quad (4.1)$$

These values have to be compared to their Particle Data Group (PDG) counterparts [1] which are $V_{ud}=0.9736\pm0.0010$ and $V_{us}=0.2196\pm0.0023$. The latter is the one quoted from K_{e3} decays. The best fit values for the other parameters are listed as fit C in the tables. The values for the HSD parameters in fit C are indistinguishable from fit B, and have not been listed separately in Tables XI and XII.

D. Errors

The fits to the experimental data have used theoretical expressions for $SU(3)$ breaking in the f_1 and g_1 form factors at leading order in the $1/N_c$ expansion. The theoretical errors are of order ϵ/N_c, where ϵ is a measure of $SU(3)$ breaking. In most hadronic quantities, $SU(3)$ breaking is of order 20–30%, so ϵ/N_c is of order 5–10%. One can also get a measure of the uncertainty in the results from the fit itself. One can use the PDG procedure [1] for rescaled errors to reduce χ^2 to one per degree of freedom. This multiplies all the errors on fit C by 1.4. It is important to keep in mind that the tables list only the errors obtained from χ^2 fits to the data, and do not include theory errors or any rescaling factors.

The quantity $3F-D$ is not well determined. Small changes in the fit tend to move F and D in opposite directions, so that there are large changes in $3F-D$. As an example of a theoretical uncertainty, consider using Eq. (2.8) for the decuplet widths with the factor $(E_B+M_B)/M_{B'}$ omitted. This modified formula is what is obtained [23] if one computes the decuplet decay widths using the baryon chiral perturbation theory formalism of Ref. [18]. The modification of Eq. (2.8) is equivalent to changes of order ϵ/N_c in the theoretical formulas used. The best fit value for $3F-D$ changes to 0.46, and for g_1 in $\Xi^0\to\Sigma^+$ β-decay becomes 1.07. The fitted parameters are listed as fit D in Table VIII. The difference between these numbers and those in fit C can be regarded as an estimate of the theoretical uncertainty in the fits.

One can redo the fits using a (fixed) non-zero value of g_2 with b_{1-3} of the estimated theoretical size of $\lesssim 0.5$. This changes the value of g_1 in $\Xi^0\to\Sigma^+$ by 5–10%, which is consistent with the estimated theoretical uncertainty.

V. CONCLUSIONS

In this work we have analyzed the pattern of $SU(3)$ symmetry breaking in the HSD form factors within the $1/N_c$ expansion. We have incorporated second-order symmetry-breaking corrections to the leading vector form factor f_1; f_2 was kept at its value predicted by $SU(3)$ symmetry, and g_2 was kept at its $SU(3)$ symmetry value of zero. Additionally, we have corrected the axial-vector form factors g_1 to first order in symmetry breaking. In the several different fits to the experimental data we found that symmetry breaking corrections to f_1 increase their magnitudes over their $SU(3)$

[6] For example, fitting to all the experimental data except g_1/f_1 in $\Xi^-\to\Sigma^0$ decay gives completely different values: $b_1=-0.7\pm1.2$, $b_2=0.6\pm0.4$, $b_3=0.0\pm0.4$.

symmetric predictions by up to 12%, and that corrections to g_1 are consistent with expectations.

We can predict the form factors for $\Xi^0 \to \Sigma^+$ β-decay, which will soon be measured by KTeV [6]. Isospin symmetry relates this decay to $\Xi^- \to \Sigma^0$, $z(\Xi^0 \to \Sigma^+) = \sqrt{2} z(\Xi^- \to \Sigma^0)$, where z is any of the form factors f_i or g_i. This can be seen explicitly from the tables. The measurement of Ξ^0 β-decay will provide some very important information on $SU(3)$ breaking in HSD. An $SU(3)$ symmetric fit predicts that g_1 for $\Xi^0 \to \Sigma^+$ decay is about 1.27. Directly using the measured value of g_1/f_1 for $\Xi^- \to \Sigma^0$ decay and the $SU(3)$ symmetry value for f_1 predicts that g_1 for Ξ^0 decay should be 1.29 ± 0.16. The $SU(3)$ breaking analysis of this paper predicts that g_1 should have a smaller value, in the range 1.02–1.07. This number was obtained from a combined fit of HSD and pionic decays of the decuplet baryons, which are related in the $1/N_c$ approach. The fit is not entirely satisfactory, and it appears that some of the experimental inputs are not consistent. Nevertheless, the result that g_1 for Ξ^0 decay (and also $3F-D$) is smaller than its $SU(3)$ symmetric value is robust. An $SU(3)$ breaking fit using only HSD data (without including decuplet decays) would give a value for g_1 that is larger than the $SU(3)$ symmetric value of 1.27. As noted in Ref. [8], there is clear evidence for $SU(3)$ breaking in the decuplet decays. At leading order in the $1/N_c$ expansion, this necessarily implies $SU(3)$ breaking in the hyperon β decays, and leads to smaller values for g_1 in the Ξ β decays than an $SU(3)$ symmetric fit.

Before closing, let us stress the fact that the pattern of flavor symmetry breaking lowers the values of F/D and $3F-D$ with respect to their $SU(6)$ predictions of 2/3 and 1, respectively, as was observed previously in Refs. [20, 21, 8, 22]. A further improvement on the parameters obtained in the present work can come from additional or better measurements on the several observables in HSD and decuplet baryons. However, with the current available data, the $1/N_c$ expansion provides a reasonable framework to analyze flavor $SU(3)$ breaking in HSD in a model-independent way. One can regard fits C and D as best fits to $SU(3)$ breaking with the current data, and the difference between fits C and D as an estimate of the theoretical uncertainty in the results.

The nucleon matrix element of the T^8 component of the axial vector current, i.e., $3F-D$, is needed to extract the value of the strange quark spin and of the total quark spin from the measured value of the spin-dependent deep inelastic structure functions of the proton and neutron. The results of this paper show that there is a significant correction to these extracted values due to $SU(3)$ symmetry breaking effects in HSD. One can determine the strange quark spin in the proton from elastic neutrino-nucleon scattering without having to use $SU(3)$ symmetry[24]. This measurement is currently being performed by the LSND Collaboration. When their results are available, they can be used to test the pattern of $SU(3)$ breaking in HSD found in this paper.

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