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# SU(3) model for octet baryon and meson fragmentation

D. Indumathi* and H. S. Mani
*Mehta Research Institute of Mathematics and Mathematical Physics, Jhusi, Allahabad 211 019, India*

Anubha Rastogi
*Mehta Research Institute of Mathematics and Mathematical Physics, Jhusi, Allahabad 211 019, India and Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India*

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The production of the octet of baryons and mesons in $e^+e^-$ collisions is analyzed, based on considerations of SU(3) symmetry and a simple model for SU(3) symmetry breaking in fragmentation functions. All fragmentation functions, $D_q^h(x,Q^2)$, describing the fragmentation of quarks into a member of the baryon octet (and similarly for fragmentation into members of the meson octet) are expressed in terms of three SU(3) symmetric functions, $\alpha(x,Q^2)$, $\beta(x,Q^2)$, and $\gamma(x,Q^2)$. With the introduction of an SU(3) breaking parameter, λ, the model is successful in describing hadroproduction data at the $Z$ pole. The fragmentation functions are then evolved using leading order evolution equations and good fits to currently available data at various energies involving both photon and $Z^0$ exchange are obtained.
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## I. INTRODUCTION

The formation of hadrons from the fragmentation of partons is of considerable current interest [1,2]. While parton-level interactions in any process—be it deep inelastic scattering or $e^+e^-$ annihilation—can be calculated, perturbative QCD can only predict the scale ($Q^2$) dependence of the process of fragmentation of quarks and gluons into hadrons; the fragmentation functions themselves are not perturbatively calculable and can only be modelled. Various models exist which attempt to explain the process of fragmentation [3–5]. Many computer simulations [6,7] also exist and are in popular use. However, the role of strangeness suppression as well as isospin in hadroproduction is not yet clearly established. A clean channel to study such phenomena is provided by $e^+e^-$ annihilation experiments due to the fact that the initial interacting vertex is purely electroweak in nature. These experiments have been performed at different energies [8–11].

We propose a simple model for a light quark ($u,d,s$) to fragment into an octet baryon or a pseudoscalar meson, using SU(3) symmetry of quarks and octet hadrons. All fragmentation functions are described in terms of three SU(3) symmetric functions $\alpha(x,Q^2)$, $\beta(x,Q^2)$ and $\gamma(x,Q^2)$ (one set for baryons and another set for mesons) and an SU(3) breaking parameter λ, which have been determined by comparison with data. The model is able to predict the $x$-dependence of the production rates of all octet baryons and mesons. There is good agreement with data at different energies (corresponding to $Z^0$ and photon exchange) over most of the $x$ range of available data. Hence, the overall success of the model does seem to indicate the existence of an underlying SU(3) symmetry between members of a hadron octet.

The paper is organized as follows: in the next section, we present the cross section and kinematics needed for the process under consideration. In Sec. III, we develop the model for quark fragmentation into octet baryons and mesons. In Sec. IV, we fix our model parameters using data on some hadrons at the $Z^0$ pole and use the resulting fits to predict production rates for other hadrons. We find good agreement with data. In Sec. V, therefore, we use leading order evolution equations to simultaneously fit data at two different energies corresponding to hadroproduction via $Z^0$ and photon exchange. We use both the standard Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [12] as well as the modified leading log approximation (MLLA) [13] to evolve the fragmentation functions to different energies and compare the model with data. Section VI contains discussions on our results and concludes the paper. Some details for the expressions of the different quark fragmentation functions in terms of $\alpha$, $\beta$ and $\gamma$ are given in the Appendix.

## II. CROSS SECTION AND KINEMATICS

We consider the production of hadrons in $e^+e^-$ annihilation via $\gamma$ and $Z$ exchange. To leading order, the cross section for producing a hadron $h$ can be expressed [14] in terms of the unknown fragmentation functions, $D_q^h(x_E,Q)$, as

$$\frac{1}{\sigma_{tot}}\frac{\mathrm{d}\sigma^h}{\mathrm{d}x_E}=\frac{\Sigma_q c_q D_q^h(x_E,Q)}{\Sigma_q c_q}. \tag{1}$$

Here $c_q$ are the charge factors associated with a quark $q_i$ of flavor $i$ and can be expressed [14] in terms of the electromagnetic charge, $e_i$, and the vector and axial vector electroweak couplings, $v_i=T_{3i}-2e_i\sin^2\theta_w$ and $a_i=T_{3i}$ as

*Present address: The Institute of Mathematical Sciences, Madras 600 113, India.

 

TABLE I. (a) Members of the baryon octet and (b) members of the meson octet.

$$\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda}{\sqrt{6}} \end{pmatrix} \qquad \begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & \frac{-2\eta}{\sqrt{6}} \end{pmatrix}$$

$$c_q = c_q^V + c_q^A,$$

$$c_q^V = \frac{4\pi\alpha^2}{s}[e_q^2 + 2e_q v_e v_q \rho_1(s) + (v_e^2 + a_e^2) v_q^2 \rho_2(s)],$$

$$c_q^A = \frac{4\pi\alpha^2}{s}(v_e^2 + a_e^2) a_q^2 \rho_2(s),$$

$$\rho_1(s) = \frac{1}{4\sin^2\theta_w \cos^2\theta_w} \frac{s(m_Z^2 - s)}{(m_Z^2 - s)^2 + m_Z^2\Gamma_Z^2},$$

$$\rho_2(s) = \left(\frac{1}{4\sin^2\theta_w \cos^2\theta_w}\right)^2 \frac{s^2}{(m_Z^2 - s)^2 + m_Z^2\Gamma_Z^2}. \tag{2}$$

In Eq. (1), a sum over quarks as well as anti-quarks is implied. Here $x_E$ is the energy fraction, $x_E = E_{\rm hadron}/E_{\rm beam} = 2E_h/\sqrt{s}$. We shall also use the momentum variable, $x_p = P_{\rm hadron}/P_{\rm beam} = 2P_h/\sqrt{s}$; $x_E^2 = x_p^2 + 4m_h^2/s$, where $m_h$ is the mass of the hadron $h$ and $Q$ is the energy scale of the interaction and is equal to $\sqrt{s}$. We shall normally use $x$ to mean $x_E$ unless otherwise specified.

The fragmentation function, $D_q^h(x_E, Q)$, associated with the quark $q$ is the probability for a quark $q$ to hadronise to a hadron $h$ carrying a fraction $x_E$ of the energy of the fragmenting quark. This is not perturbatively calculable from theory, although the scale $(Q)$ dependence of these functions is given by QCD. Data from different experiments at different energies from $\sqrt{s} = 12$–$91.2$ GeV exists on $p$, $\Lambda$, $\Sigma$ and $\Xi$ octet baryon production as well as on $\pi$, $K$, $\eta$ octet meson production [8,9,10,11]. All available data measures the production rate of hadron plus antihadron. Due to the symmetric nature of the process $e^+e^- \to q\overline{q}$, the resulting hadron and antihadron yields are equal. We therefore present results for the sum of hadron and antihadron yields in what follows.

We can also re-express the cross section in terms of the octet and singlet fragmentation function combinations, as

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^h}{dx_E} = \frac{a_0 \Sigma^h(x_E, t) + a_3 D_3^h(x_E, t) + a_8 D_8^h(x_E, t)}{\Sigma_q c_q}, \tag{3}$$

where $\Sigma$, $D_3$ and $D_8$ refer to the singlet, and the two octet $((u-d)$ and $(u+d-2s))$ combinations respectively, with $a_0 = (c_u + c_d + c_s)/3$; $a_3 = (c_u - c_d)/2$ and $a_8 = (c_u + c_d - 2c_s)/6$.

We now present our model for quark fragmentation functions.

## III. THE MODEL

We study semi-inclusive production of the octet baryons and the pseudo-scalar octet mesons in $e^+e^-$ annihilation processes using SU(3) symmetry of the quarks and of the hadrons in their respective octets. The production of the entire meson and baryon octet is described in terms of SU(3) symmetric quantities. We study only light quark $(u,d,s)$ fragmentation where the fragmenting quark $(q_i,\ i=1,\ldots,3)$ is a member of the quark triplet $(q_1 = u,\ q_2 = d,\ q_3 = s)$ and the hadron under study $(h_i^j,\ i,j = 1,\ldots,3)$ is a member of the baryon (or meson) octet (see Table I), so that the process is

$$q \to h + X.$$

In SU(3) language, this can be expressed as

$$3 \to 8 + X.$$

Group theoretical considerations of the corresponding underlying SU(3) flavor symmetry allow $X$ to be either a triplet, antisixplet or fifteenplet. (See Appendix for details.) No other configuration for $X$ is allowed. Note that the fragmenting quark could become part of the valence quark or the sea of the hadron. The flavor multiplet nature of $X$ is independent of this and is determined purely by the assumption of flavor SU(3) conservation. Gluons being flavor singlets do not change the flavor content of $X$ so that $X$ includes any number of additional gluons besides the minimum quark content required by SU(3) symmetry; hence, this is an inclusive fragmentation process.

We can express all the quark fragmentation functions for the hadron $h$ as linear combinations of the fragmentation probabilities $\alpha(x,Q)$, $\beta(x,Q)$, and $\gamma(x,Q)$ where $\alpha$ $(\beta,\gamma)$ is the fragmentation probability when $X = 3$ $(\overline{6}, 15)$. The corresponding probabilities for antiquark fragmentation into octet baryons are similarly expressed in terms of $\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$. (In this case, there is an antitriplet fragmenting into an octet through $\overline{q} \to h + X$, with the $X$ being a $\overline{3}$, 6 or $\overline{15}$). These probabilities are also functions of $(x,Q)$. In the case of mesons, which have a quark and an antiquark in the valence, the probability of fragmentation of an antiquark into a meson is the same as that for the corresponding quark to fragment into

TABLE II. Quark fragmentation functions for members of the baryon and meson octet in terms of the SU(3) functions, $\alpha$, $\beta$ and $\gamma$. The antiquark fragmentation functions for the case of baryons are obtained by replacing $\alpha$, $\beta$, $\gamma$ by $\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$ respectively. Those for mesons are obtained by noting that $D_{\bar{q}}^{M}(x,Q) = D_{q}^{\overline{M}}(x,Q)$ for quarks, $q$, in a meson, $M$.

| fragmenting quark | | $p/K^+$ | fragmenting quark | | $n/K^0$ |
|---|---|---|---|---|---|
| $u$ | : | $\alpha+\beta+\frac{3}{4}\gamma$ | $u$ | : | $2\beta+\gamma$ |
| $d$ | : | $2\beta+\gamma$ | $d$ | : | $\alpha+\beta+\frac{3}{4}\gamma$ |
| $s$ | : | $2\gamma$ | $s$ | : | $2\gamma$ |
| fragmenting quark | | $\Lambda^0/\eta$ | fragmenting quark | | $\Sigma^0/\pi^0$ |
| $u$ | : | $\frac{1}{6}\alpha+\frac{9}{6}\beta+\frac{9}{8}\gamma$ | $u$ | : | $\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{11}{8}\gamma$ |
| $d$ | : | $\frac{1}{6}\alpha+\frac{9}{6}\beta+\frac{9}{8}\gamma$ | $d$ | : | $\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{11}{8}\gamma$ |
| $s$ | : | $\frac{4}{6}\alpha+\frac{9}{6}\gamma$ | $s$ | : | $2\beta+\gamma$ |
| fragmenting quark | | $\Sigma^+/\pi^+$ | fragmenting quark | | $\Sigma^-/\pi^-$ |
| $u$ | : | $\alpha+\beta+\frac{3}{4}\gamma$ | $u$ | : | $2\gamma$ |
| $d$ | : | $2\gamma$ | $d$ | : | $\alpha+\beta+\frac{3}{4}\gamma$ |
| $s$ | : | $2\beta+\gamma$ | $s$ | : | $2\beta+\gamma$ |
| fragmenting quark | | $\Xi^0/\overline{K^0}$ | fragmenting quark | | $\Xi^-/K^-$ |
| $u$ | : | $2\beta+\gamma$ | $u$ | : | $2\gamma$ |
| $d$ | : | $2\gamma$ | $d$ | : | $2\beta+\gamma$ |
| $s$ | : | $\alpha+\beta+\frac{3}{4}\gamma$ | $s$ | : | $\alpha+\beta+\frac{3}{4}\gamma$ |

the charge conjugate hadron, that is, $D_{\bar{q}}^{M}=D_{q}^{\overline{M}}$ for a meson $M$. The quark and antiquark fragmentation into hadrons, $h_i^j$, $i,j=1,\ldots,3$, in terms of the SU(3) probability functions, $\alpha$, $\beta$ and $\gamma$ (and $\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$ as well, for baryons), is given in Table II. Note that the functions $\alpha$, $\beta$ and $\gamma$ for the baryon and meson octets are unrelated; they just correspond to the same underlying symmetry. Also, since $X$ can be only a triplet, antisixplet or fifteenplet, only three functions are required to explain quark fragmentation (and three more to explain antiquark fragmentation into baryons) into any member of the hadron octet.

The $s$ quark resides only in the sea of the proton; hence, the function $\gamma$, which parametrizes $s$ quark fragmentation into a proton (see Table II), describes sea quark fragmentation for any member of the baryon octet. For the meson octet, such an identification of a sea quark fragmentation function is not immediately evident. We shall detail the fragmentation into mesons in the next section.

Since the (more) massive strange quark is known to break SU(3) symmetry, we introduce symmetry breaking effects as follows: the fragmentation function is suppressed by an $x$-independent factor λ whenever a strange quark belonging to the valence of the hadron is produced. This means that all non-strange fragmentation functions of strange hadrons are suppressed by λ. For example, $D_u^K$ or $D_u^\Lambda$ are suppressed by a factor λ compared to $D_s^K$ or $D_s^\Lambda$. Note that all (strange and nonstrange) sea fragmentation functions corresponding to a given hadron come with the *same* factor of λ.

There are 3 quark- and 3 antiquark-fragmentation functions for each hadron. Since there are eight baryons (mesons) in the octet, this corresponds to a total of forty eight (twenty four) unknown functions for the baryon (meson) octet, which, in principle, need to be fitted to data. However, in our model, fragmentation into all baryons (mesons) in a given octet can be described in terms of 6 (3) functions alone, along with an SU(3) breaking parameter, λ, thus leading to an enormous simplification in the analysis as well as dramatically increasing the predictive power of the model.

We now go on to detail the model, first for the case of octet baryons and then for the octet mesons.

## IV. COMPARISON WITH DATA AT THE Z POLE

We have constructed a model using SU(3) flavor symmetry that describes octet hadron (baryon as well as meson) fragmentation in terms of a few SU(3) symmetric functions and an SU(3) symmetry breaking parameter, λ. We shall now determine these functions by comparison with data. We choose to study the $Z^0$ exchange process, $e^+e^-\to Z^0\to q\bar{q}$ at the CERN $e^+e^-$ collider LEP [8,9], since the data sample available at this energy is the largest; in particular, data exists at this energy for $p$, $\Lambda$, $\Sigma$ and $\Xi$ octet baryons and for $\pi$, $K$, and $\eta$ octet mesons. For the baryons, this is just adequate to determine the individual functions; for the case of mesons, only a certain valence and sea combination can be determined, as we shall see below. In the next section, we shall use this knowledge to determine hadroproduction rates at different energies where photon exchange dominates (and where insufficient data exists to determine the various model parameters).

### A. Baryon fragmentation

First of all, we observe that our model predicts equal rates of production of $\Sigma^0$ and $(\Sigma^++\Sigma^-)/2$ (see Table II) due to

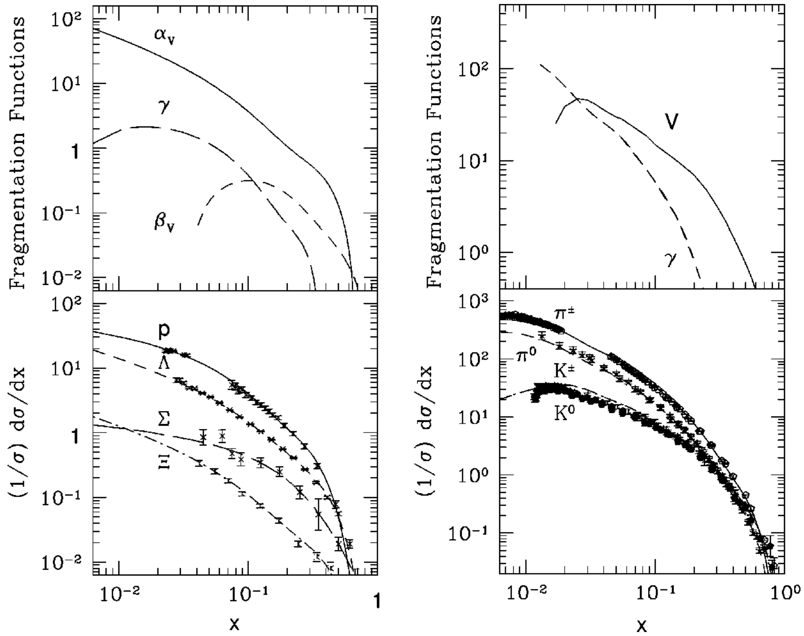


FIG. 1. The functions $\alpha_V$, $\beta_V$ and $\gamma$ for baryons, fitted using $p$, $\Lambda$ and $\Sigma$ data (also shown below) at $\sqrt{s}=91$ GeV [8,9] are shown in the figure on the left. The lines through the data are our best fits to them. These are used to predict the $\Xi$ production rate, which is shown as a dashed-dotted curve, in comparison with data. On the right are shown the mesonic probabilities, $V$ and $\gamma$, fitted using $\pi^\pm$ and $K^\pm$ data (also shown below) at $\sqrt{s}=91$ GeV; the resulting predictions of the $\pi^0$ and $K^0$ ($\simeq K^\pm$) rates are shown as dashed lines, compared with data.

isospin symmetry. This is borne out by data [9]; for instance, the multiplicities of $(\Sigma^+ + \Sigma^-)/2$ and $\Sigma^0$ at $Q=91$ GeV are $0.091\pm0.019$ and $0.071\pm0.018$ respectively and are compatible[1] within $1\sigma$.

To obtain predictions for the other baryons, we separate the quark fragmentation functions into valence (V) and sea (S) parts by defining, as usual,

$$\alpha=\alpha_V+\alpha_S;\quad \beta=\beta_V+\beta_S;\quad \gamma=\gamma_V+\gamma_S;$$

$$\overline{\alpha}=\alpha_S;\quad \overline{\beta}=\beta_S;\quad \overline{\gamma}=\gamma_S. \tag{4}$$

There is no $s$ quark in the valence of the proton; hence, we obtain $\gamma_V=0$ or $\gamma=\overline{\gamma}=\gamma_S$ (see Table II). Thus, $\gamma$ describes the sea fragmentation. Furthermore, using an SU(3) symmetric sea, $D^\Lambda_{u_s}=D^\Lambda_{d_s}=D^\Lambda_{s_s}$, leads to the constraint, $\overline{\beta}=\gamma/4+\overline{\alpha}/3$. The final simplifying assumption of $D_{\overline{u}}=D_{\overline{d}}$ for all baryons allows us to express all antiquark fragmentation functions in terms of $\gamma$ alone:

$$\overline{\gamma}=\gamma;$$

$$\overline{\alpha}=0.75\gamma;$$

$$\overline{\beta}=0.5\gamma. \tag{5}$$

[1]Our model does not account for isospin breaking effects which are small compared to SU(3) breaking effects but are known to exist; hence we are looking for agreement only to within this approximation.

The model for octet baryons therefore has three unknown functions $\alpha_V(x_E,Q^2)$, $\beta_V(x_E,Q^2)$, $\gamma(x_E,Q^2)$ and an unknown parameter, $\lambda$, that characterizes SU(3) symmetry breaking. We now attempt to evaluate these by suitable comparison with data.

We expect valence fragmentation functions (leading quark fragmentation) to dominate in the large $x_E$ region and sea fragmentation functions to dominate at small $x_E$. We use the large $x_E$ data to fix the value of $\lambda$. Since $\Xi^-$ has two $s$ quarks in its valence, its $s$-valence fragmentation function is suppressed by a factor of $\lambda$ compared to $p$, $\Lambda$ and $\Sigma^{\pm,0}$. Therefore, we expect $\Xi^-$ production cross sections to be smaller than for other baryons even in the large $x_E$ range, where only the valence contribution survives. Indeed, as can be seen from Fig. 1, data [8] show that the large $x_E$ cross sections are similar for all octet baryons, $p$, $\Lambda^0$, $\Sigma^{\pm,0}$, within errorbars, while $\Xi^-$ data is smaller in that region. Furthermore, assuming that $u$ quark fragmentation dominates the large $x_E$ proton data, we see from Table II that the ratio of $\Xi^-$ to proton production rates is just $\lambda$ (the expressions for the two are otherwise the same). Using the data at $\sqrt{s}=91.2$ GeV, we find that $\lambda=0.07$. This result can be corrected due to the fact that $D^p_d$ is not small at large $x_E$; however, we shall see that this value for $\lambda$ gives good agreement with data.

Since $\lambda$ turns out to be quite small, the production rate of strange baryons is dominated by $s$-quark fragmentation. This means that the $\Lambda^0$ rate is sensitive to $\alpha_V$ while the $\Sigma$ baryons give information on $\beta_V$ (see Table II). On the other hand, $p$ and $\Xi$ depend on the combination $(\alpha_V+\beta_V)$; it is therefore possible to separately determine $\alpha_V$ and $\beta_V$ from the data, rather accurately, especially in the valence-dominated region of large $x, x \gtrsim 0.1$. The (SU(3) symmetric) sea is the same for all members of the octet, upto overall powers of λ.

Note that the measured hadron spectrum is an inclusive one. We take this into account, especially for the case of $p$ and Λ, by defining inclusive fragmentation functions in terms of the exclusive ones used so far:

$$\Lambda_{\rm inclusive} = \Lambda_{\rm exclusive} + 1.0\Sigma^0 + 1.0\Xi^- + 1.0\Xi^0;$$

$$p_{\rm inclusive} = p_{\rm exclusive} + 0.52\Sigma^+ + 0.64\Lambda.$$

The multiplying fractions indicate the branching fractions into $p$ and Λ of the various baryons [15]. Here $\Sigma^*$ and $\Lambda_c$ decays to the various baryons have been ignored since they are very small. $\Lambda_c$ data is between 15 to 100 times smaller than Λ and proton data in the overlapping $x_E$ range of about 0.3–0.8 [16]. The $\Sigma^\pm$ and $\Xi^-$ data is considered to be purely exclusive for this reason. The energy ($x_E$) of the daughter-baryon is taken to be the same as that of the parent because of the small difference in masses of $p$, Λ, Σ and Ξ.

For the case of the 91.2 GeV $Z$-exchange process, data is available for $p$, Λ, Σ and Ξ baryons [8]. We can therefore use any three of the data sets (along with the estimated value of λ), to evaluate the three unknown functions, $\alpha_V$, $\beta_V$ and $\gamma$. In particular, we have evaluated these functions at different $x_E$ values using parametrized fits to the $p$, Λ and Σ data [8,9] and the value of $\lambda = 0.07$ as already estimated; these were then used to predict the cross section for $\Xi^-$. We find that the Ξ data is predicted very well by our model at all $x_E$ as is shown in Fig. 1, where the data used as well as the resulting fits to $\alpha_V$, $\beta_V$ and $\gamma$ are also shown. Note that a change in λ can alter the overall normalization but not the shape of the distribution, which is a prediction of this model.

### B. Meson fragmentation

The pseudo-scalar meson-octet is a self conjugate octet, i.e., the same octet contains mesons as well as their antiparticles. The mesons and their antiparticles are related by charge conjugation. This means that $D_q^h = D_{\bar{q}}^{\bar{h}}$. As a consequence of this fact we immediately see that $\alpha$, $\bar{\beta}$ and $\bar{\gamma}$ are not independent of $\alpha$, $\beta$ and $\gamma$ (see Table II). Of these six quantities, only three are independent. We choose these to be $\alpha$, $\beta$ and $\gamma$. Due to mixing with the singlet sector, we do not consider the $\eta$ meson here.

Just as in the case of Σ, here we predict equal rates for $(\pi^+ + \pi^-)/2$ and $\pi^0$, due to isospin invariance. This is borne out by data [8]; see also Fig. 1. We therefore consider only the combinations $m^\pm \equiv (m^+ + m^-)$ for $m = \pi, K$, and the combination $(K^0 + \overline{K^0}) \equiv 2K_s^0$, which we shall refer to as simply $K^0$.

We make the usual separation into valence and sea fragmentation functions. As before, we reduce the number of unknown functions through various symmetry considerations. We assume that the sea is SU(3) flavor symmetric, so that $D_u^{\pi^-} = D_s^{\pi^-}$ and so on. Using this, we have $\beta = \gamma/2$ and all sea fragmentation functions are equal to $S = 2\gamma$.

Thus, all valence fragmentation functions can be expressed in terms of the function, $V$, where $V$ is given, for example, by the difference $(D_u - D_{\bar{u}})$ in $\pi^+$, as

$$V = \alpha + \beta - \frac{5}{4}\gamma = \alpha - \frac{3}{4}\gamma,$$

and all sea fragmentation functions are given in terms of $\gamma$ alone. Now $V$ and $\gamma$ can be determined by comparison with ($\pi^\pm$, $K^\pm$ and $K^0$) data; since the two sets of $K$ data are not very different [8], it is not possible to determine $\alpha$ and $\beta$ individually in this case.

The assumption (made in the baryon case) that the daughter hadron carries away the bulk of the energy of the parent, thus contributing to the same $x_E$ bin, enabled us to simply add the various hadron fragmentation functions to arrive at an ''inclusive'' hadron fragmentation function. Here, since both $\pi$'s and $K$'s are very much lighter than their decay sources (mostly $D$ mesons and baryons), this assumption is no longer reasonable. We therefore merely estimate errors arising from the inclusive nature of the data by comparing multiplicities rather than $x_E$ distributions.

The bulk of the contamination of the pion sample which is due to $K_s^0 \to \pi\pi$ decays is estimated to be about 4% from the ratio of the relative multiplicities, $N^\pi/N^{K_s} \sim 10$ [8,17] and a branching fraction $B \sim 0.35$ for the decay (here $\pi$ includes $\pi^+, \pi^-$ and $\pi^0$); hence we ignore this. $K_L^0$ does not decay within the detector. However, there is a substantial contribution from charm meson feed-down for the $K$ data sample (from the decay of all the $D$'s); this is estimated from multiplicity data [8,17] to be about 16% for $K^\pm$ and about 20% for $K^0$ mesons. The contamination from baryons is negligible.

As before, we include SU(3) symmetry breaking effects: the fragmentation probability is suppressed by a factor λ whenever a strange quark belonging to the valence of the meson is produced. Since the fragmenting quark excites a quark pair rather than a diquark pair (as in the case of baryons) the value of λ here is not related to that for the baryonic sector. We can get bounds for λ by using the $\pi^\pm$ and $K^\pm$ multiplicities which are equal to $17.05 \pm 0.43$ and $2.26 \pm 0.18$ respectively [8,17]: The total rates for $\pi^\pm$ and $K^\pm$ production (their multiplicities) are related to the first moment of $V$ and $S$, and the parameter λ; positivity constraints on $V$ and $S$ (since they are probabilities) then require that $\lambda < 0.14$. A tighter constraint on λ will be found in the next section, when we apply the model to data at different energies.

We use a typical value of $\lambda = 0.08$ to fit $V$ and $S$ over the available $x_E$ range of $\pi^\pm$ and $K^\pm$ data. These were then used to predict cross sections for $\pi^0$ and $(K^0 + \overline{K^0})$. Good agreement with the data was obtained, as can be seen from Fig. 1. The fits to $V$ and $\gamma$ as determined from $\pi^\pm$ and $K^\pm$ data are also shown here along with the data that have been used to determine them.

We have been able to explain the production of the entire meson and baryon octet by using SU(3) symmetry and the suppression factor λ at $\sqrt{s} = 91.2$ GeV. Encouraged by this success, we investigate whether the model works for other c.m. energies also. In the next section, we discuss the evolution of these fragmentation functions to different energies. Since backwards evolution to lower energies is usually numerically unstable, instead of using the fits to $\alpha$, $\beta$ and $\gamma$ that we have obtained at the $Z^0$ pole, we parametrize an input set of fragmentation functions at a low starting scale of $Q_0^2=2\ \mathrm{GeV}^2$ and then evolve these upwards to the energies of interest in order to effect a comparison with available data. However, we use the information we have obtained at the $Z^0$ pole, on the relative size and importance of the various contributions at different $x$ values (as seen in Fig. 1) to help parametrize these functions at the starting scale of evolution.

## V. COMPARISON WITH DATA FOR PHOTON EXCHANGE

### A. Leading log evolution

Over the last decade, several experiments, performed over a wide range of c.m. energies ($12\ \mathrm{GeV}\leqslant\sqrt{s}\leqslant 91.2\ \mathrm{GeV}$), have reported measurements of rates of hadron production at $e^+e^-$ colliders. It is known that the cross-section is not a constant over the range of c.m. energies; for instance, this has enabled the extraction of the running coupling constant $\alpha_s$. We use leading order (LO) DGLAP evolution equations [12] to relate the fragmentation functions (and hence the cross-section) at a given energy $Q^2=s$ to those at the starting scale $Q_0^2=2\ \mathrm{GeV}^2$.

Nonsinglet (comprising the valence functions $\alpha_V$ and $\beta_V$ for baryons and $V$ for mesons) and singlet fragmentation functions (including $\gamma$ for baryons and $S$ for mesons) evolve differently under evolution. The singlet, $\Sigma^h(x,t)$ $(=u(x,t)+d(x,t)+s(x,t)+\overline{u}(x,t)+\overline{d}(x,t)+\overline{s}(x,t))$, mixes with the gluon fragmentation function, $g(x,t)$; here we have used $x=x_E$, $t=Q^2$, and $D_q^h=q$ for convenience. Due to the $(1/x)$ pole in the $P_{gg}$ splitting function, the contribution of the sea increases significantly with increasing $Q^2$, at low $x$. However, the sea contribution remains small at larger $x$ values, $x\gtrsim 0.05$, where most of the data is available. (See Fig. 1, for example, for the relative size of the sea contribution at the $Z^0$ pole.)

The symmetry between the singlet sector of different baryons is broken by $\lambda$. However, all singlet combinations for the other baryons can be expressed in terms of the proton singlet fragmentation function, $\Sigma^p$, and strange valence fragmentation functions. We have

$$\Sigma^\Lambda=\lambda\Sigma^p+(1-\lambda)s_V^\Lambda;$$

$$\Sigma^\Sigma=\lambda\Sigma^p+(1-\lambda)s_V^\Sigma;$$

$$\Sigma^{\Xi^-}=\lambda^2\Sigma^p+\lambda(1-\lambda)s_V^{\Xi^-}. \tag{6}$$

Similarly, we construct only the singlet combination $\Sigma^\pi$ for the pion. All other meson singlets can then be recovered from this, and the valence function $V$.

TABLE III. Input values at $Q_0^2=2\ \mathrm{GeV}^2$ for the valence and singlet fragmentation functions for the (a) baryon and (b) meson octet; see Eq. (7) in the text for details.

| Baryons | $\alpha_V$ | $\beta_V$ | $\gamma$ | $g$ |
|---|---|---|---|---|
| $a$ | 3.0 | 25.8 | 3.5 | 2.5 |
| $b$ | 4.8 | 8.0 | 13.6 | 13.4 |
| $c$ | −0.55 | 1.52 | 0.12 | 0.12 |
| $d$ | −3.96 | −5.19 | −7.82 | 0 |
| $e$ | 13.12 | 9.84 | 38.0 | 0 |

| Mesons | $V$ | $\gamma$ | $g$ |
|---|---|---|---|
| $a$ | 2.33 | 3.5 | 0.25 |
| $b$ | 2.15 | 12.76 | 11.4 |
| $c$ | −0.64 | −0.75 | 0.12 |
| $d$ | 5.35 | 3.87 | 0 |
| $e$ | −5.12 | 61.59 | 0 |

The input functions $F_i(x)=\alpha_V,\beta_V,\gamma$ (for the case of baryons) and $F_i(x)=V,\gamma$ (for the case of mesons), at the starting scale $Q_0^2=2\ \mathrm{GeV}^2$, were parametrized as

$$F_i(x)=a_i(1-x)^{b_i}(x^{c_i})(1+d_ix+e_ix^2). \tag{7}$$

The parameters $a,b,c,d,e$ for different input fragmentation functions are given in Table III. Since the gluon is a flavor singlet, we have used a common gluon fragmentation function for all the hadrons in an octet. However, we emphasize that the evolved fragmentation functions are not very sensitive to the choice of the gluon fragmentation function, which is therefore not well-determined in our model.

We tuned the starting parameters to yield a good fit to the 91 GeV hadroproduction data which is essentially via $Z$-exchange [8,9]. We then used the *same* set to predict the rates for lower $Q^2$ values which are dominated by photon exchange. The resulting fits at the $Z$ pole ($\sqrt{s}=91.2$ GeV) and a fit to the available baryon data sample at $\sqrt{s}=34$ GeV [10] and at $\sqrt{s}=14$ GeV [11] for $\lambda=0.07$ are shown in Figs. 2(a), 3(a) and 4(a).

In the meson sector, we find that $\lambda$ is constrained to lie between 0.04–0.12, with $\lambda=0.08$ giving the best fit to the data. The overall shape of the meson data is very well realized at all energies, as can be seen from Figs. 2(b), 3(b) and 4(b) for $\sqrt{s}=91.2$, 34 and 14 GeV respectively. The model parameters hence yield a reasonable fit to all baryon and meson data at these energies. The meson data is better fitted than the baryon data. The discrepancy in overall normalization could be due to the inclusive nature of the measurement (especially acute in the case of $p$, $\Lambda$ and $K$) and possible energy dependence of the suppression factor $\lambda$; note also that there is a substantial contribution to $K$ production rates from charm feed-down, that we have not accounted for. However, we emphasize that our model is fairly simple; its biggest advantage is that it predicts the production rates of several mesons and baryons with relatively few inputs.

Recently, the total inclusive charged hadron cross section has been measured at LEP at $\sqrt{s}=161$ GeV [18]. We know

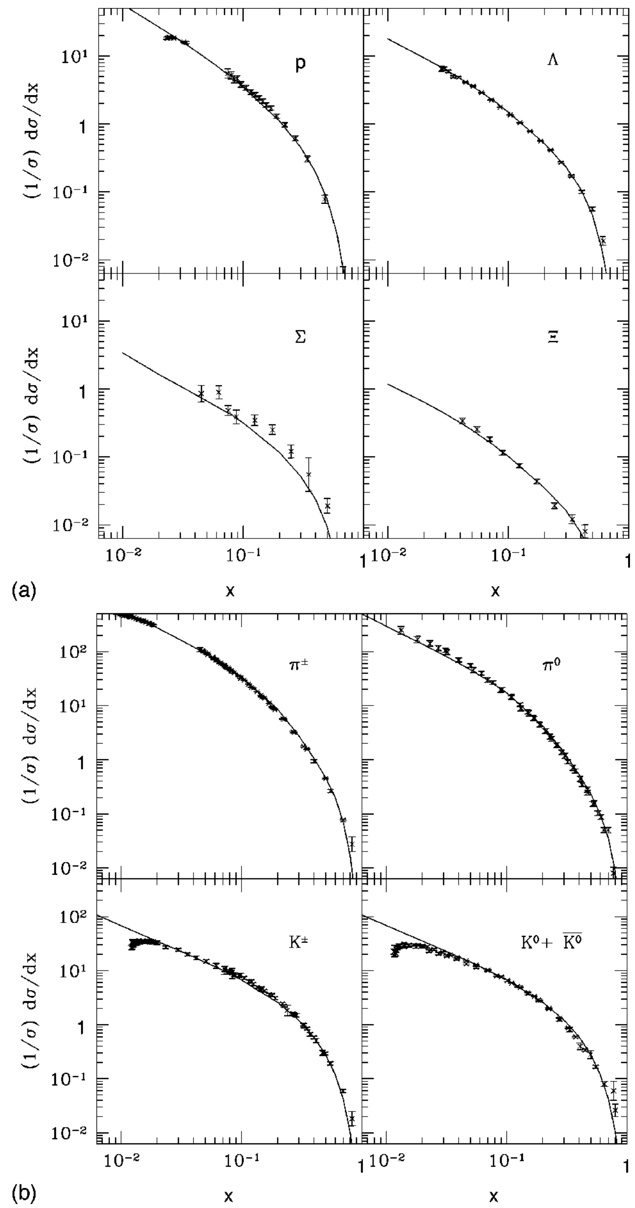


FIG. 2. (a) The figure shows the model fits to the $\sqrt{s}=91$ GeV data for baryons, obtained from DGLAP evolution of the input functions at $Q_0^2=2\ \mathrm{GeV}^2$. (b) The figure shows the model fits to the $\sqrt{s}=91$ GeV data for mesons, obtained from DGLAP evolution of the input functions at $Q_0^2=2\ \mathrm{GeV}^2$.

from the multiplicity data at the $Z^0$ pole that 81% (91%) of the charged particle inclusive cross section is from pions (pions plus kaons). Specifically, the total charged particle multiplicity at $\sqrt{s}=91.2$ GeV is $21.4\pm0.02\pm0.43$ [17], of which $17.05\pm0.43$ are $\pi^{\pm}$ and $2.26\pm0.01\pm0.16\pm0.09$ are $K^{\pm}$ mesons. We therefore compare the charged particle spectrum at 161 GeV (with multiplicity $24.46\pm0.45\pm0.44$) with our predictions for $\pi^{\pm}$ and $(\pi^{\pm}+K^{\pm})$; we expect the latter should saturate the data to within 10%. Our model shows excellent agreement with data, as can be seen from Fig. 5.

We remark that the charge factors $c_q/\Sigma_q c_q$ for quarks $q=u,d,s$, are very different for pure $Z^0$ and photon exchange. For instance, the charge factor for an $s$ quark is 1/6 at 14 and 34 GeV and 13/36 at 91.2 GeV, more than a factor of two larger. On the other hand, that for the $u$ quark is almost a factor of two smaller. This means that the photon exchange data is more sensitive to $u$ quark fragmentation than the $Z^0$ data. That the model predictions for strange hadrons such as

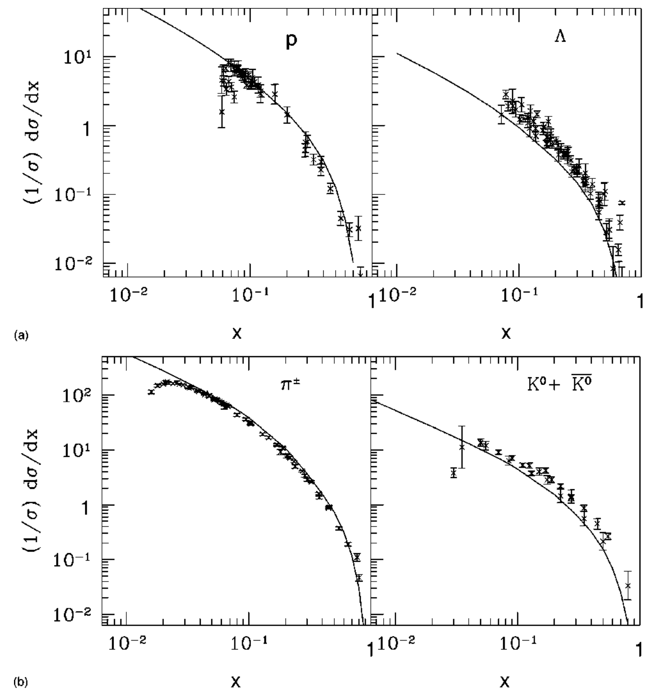


FIG. 3. (a) As in Fig. 2(a) for the $\sqrt{s}=34$ GeV baryon data from DGLAP evolution. (b) As in Fig. 2(b) for the $\sqrt{s}=34$ GeV meson data from DGLAP evolution.

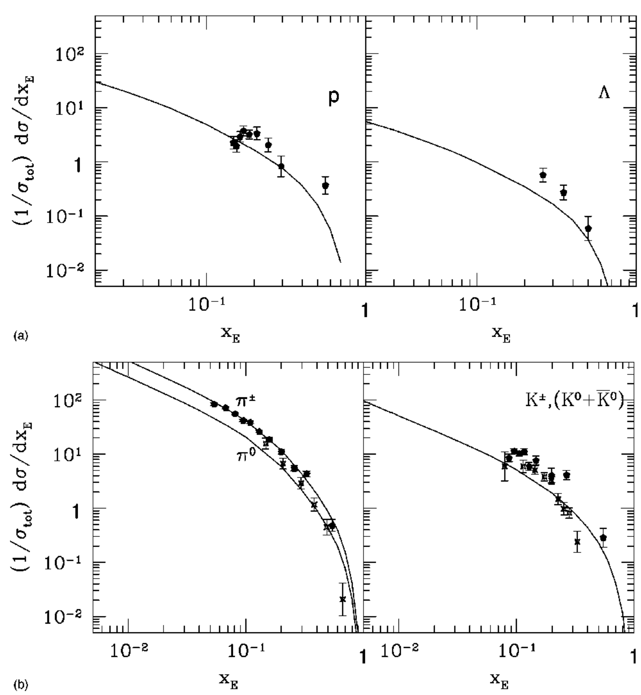


FIG. 4. (a) As in Fig. 2(a) for the $\sqrt{s}=14$ GeV baryon data from DGLAP evolution. (b) As in Fig. 2(b) for the $\sqrt{s}=14$ GeV meson data from DGLAP evolution.

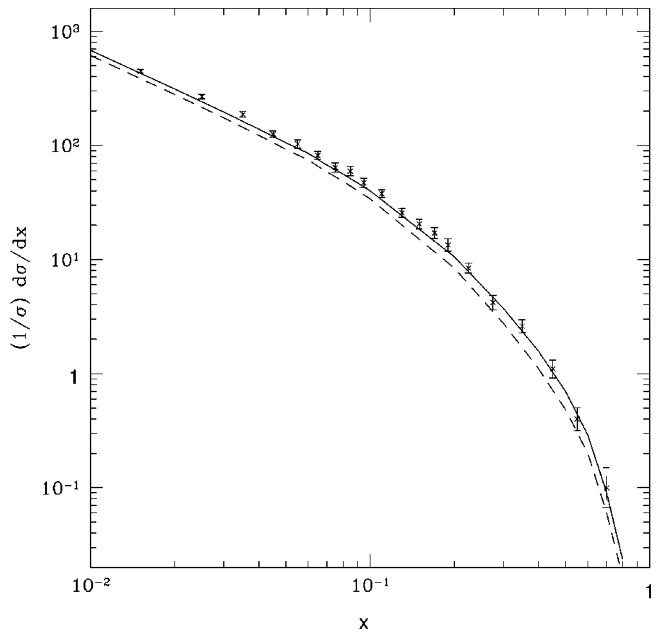


FIG. 5. The model prediction for hadroproduction of $\pi^\pm$ (dashed line) and $\pi^\pm + K^\pm$ (solid line) mesons is compared with the total inclusive charged particle data at $\sqrt{s} = 161$ GeV [18].

$\Lambda$ and $K$ (where the $u$ contribution is suppressed by a factor of $\lambda$) are systematically smaller than data may therefore mean that $D_u$ is actually larger than the model prediction, thus indicating that a single strangeness suppression factor $\lambda$ may not suffice. In other words, our simple model may not completely account for all SU(3) breaking effects. In this context, it would be interesting to obtain data on the $\Sigma$ baryon at a different energy (data is available only on the $Z^0$ pole) and check whether this trend is visible there as well.

Finally, the data (specially for mesons, $p$ and $\Lambda$) show a decreasing trend at low $x$. The usual DGLAP evolution [12] cannot account for such a trend since the pole in the splitting function $P_{gg}$ always drives the gluon, and hence the sea, to larger values at small $x$. In 1988, Dokshitzer, Khoze, and Troyan [13] proposed a model wherein this dip could be accounted for by including gluon coherence effects. The resulting modified leading log approximation (MLLA) then gives a Gaussian distribution for the singlet fragmentation functions. In the next section, we discuss singlet evolution using MLLA and look for improved fits to the low $x$ data.

### B. Modified leading log evolution

The main result of the MLLA evolution [13,19] is that the low-$x$ singlet fragmentation functions have a Gaussian form in the variable $\log(x_p)$:

$$x_p D(x_p, Q) = \frac{N(Q)}{\sqrt{2\pi}\sigma(Q)} \exp[-[\log(x_p) - \log(x_0)]^2/[2\sigma^2(Q)]], \tag{8}$$

where $N(Q)$ is the total multiplicity, $\sigma$ is the width of the Gaussian and $x_0$ is the position of the peak of the Gaussian.

TABLE IV. Multiplying constant factors for description of the singlet fragmentation functions for baryons and mesons in the MLLA approach; see Eq. (10) in the text for details.

| | $p$ | $\Lambda$ | $\Sigma^\pm$ | $\Xi^-$ | | $\pi^\pm$ | $\pi^0$ | $K^\pm$ | $K^0$ |
|---|---|---|---|---|---|---|---|---|---|
| $C_1$ | 0.70 | 0.70 | 0.70 | 0.70 | $C_1$ | 0.92 | 0.92 | 0.75 | 0.75 |
| $C_2$ | 0.4 | 0.4 | 0.4 | 0.4 | $C_2$ | 0.59 | 0.59 | 0.425 | 0.425 |

The $Q^2$ dependence of $N$, $\sigma$ and $x_0$ are computable for total inclusive hadrons within this approach. They are given as an expansion in terms of the scale parameter, $Y = \log(Q/\Lambda)$, $\Lambda = 200$ MeV:

$$N(Q) \propto Y^{-B/2+1/4} \exp\sqrt{16 N_c Y/b};$$

$$\sigma^2 = Y^2/(3z);$$

$$\log(1/x_0) = Y[1/2 + \sqrt{c/Y} - c/Y], \tag{9}$$

where $N_c$ and $n_f$ are the number of colors and flavors, which determine the constants,

$$a = 11N_c/3 + 2n_f/(3N_c^2); \quad b = (11N_c - 2n_f)/3;$$

$$B = a/b; \quad z = \sqrt{(16 N_c Y)/b};$$

$$c = (11/48)[1 + (2n_f)/(11N_c^3)]^2/[1 - 2n_f/(11N_c)].$$

The total multiplicities (at 91.2 GeV, for instance) [8,9,17] can be used to fix the proportionality constant for $N(Q)$; the individual particle multiplicities then determine $N^h(q)$, the multiplicity of the specific hadron, $h$. The values of $\sigma$ and $x_0$ are in good agreement with inclusive data [13]; however, we are here interested in semi-inclusive spectra. In general, the peak shifts to smaller $x$ (here meaning $x_p$) values for heavier hadrons. Also, the semi-inclusive widths are naturally smaller than the total inclusive ones. We therefore parametrize the corresponding semi-inclusive parameters as

$$x_0^h = C_1^h x_0,$$

$$\sigma_h^2 = C_2^h \sigma^2, \tag{10}$$

where $x_0^h$ is the peak position and $\sigma_h$ the width of the data for hadron $h$. Here $C_1^h$ and $C_2^h$ are $Q^2$ independent constants which we fit to the 91.2 GeV data. They are given in Table IV. These are then used to determine the rates at lower energies. The resulting fits are again quite good and are shown in Figs. 6, 7 and 8 for data corresponding to pure $Z^0$ and photon exchange (at 34 and 14 GeV) respectively.

Note that the MLLA is a fit to the singlet fragmentation functions alone; therefore comparison should be made with data for $x_p \lesssim 0.1$, where the valence contribution is expected to be small. In the case of $Z^0$ exchange, this is also a good fit to the entire data. This is because at 91.2 GeV, the cross section is dominated by the singlet term, as can be seen from writing Eq. (3) explicitly:

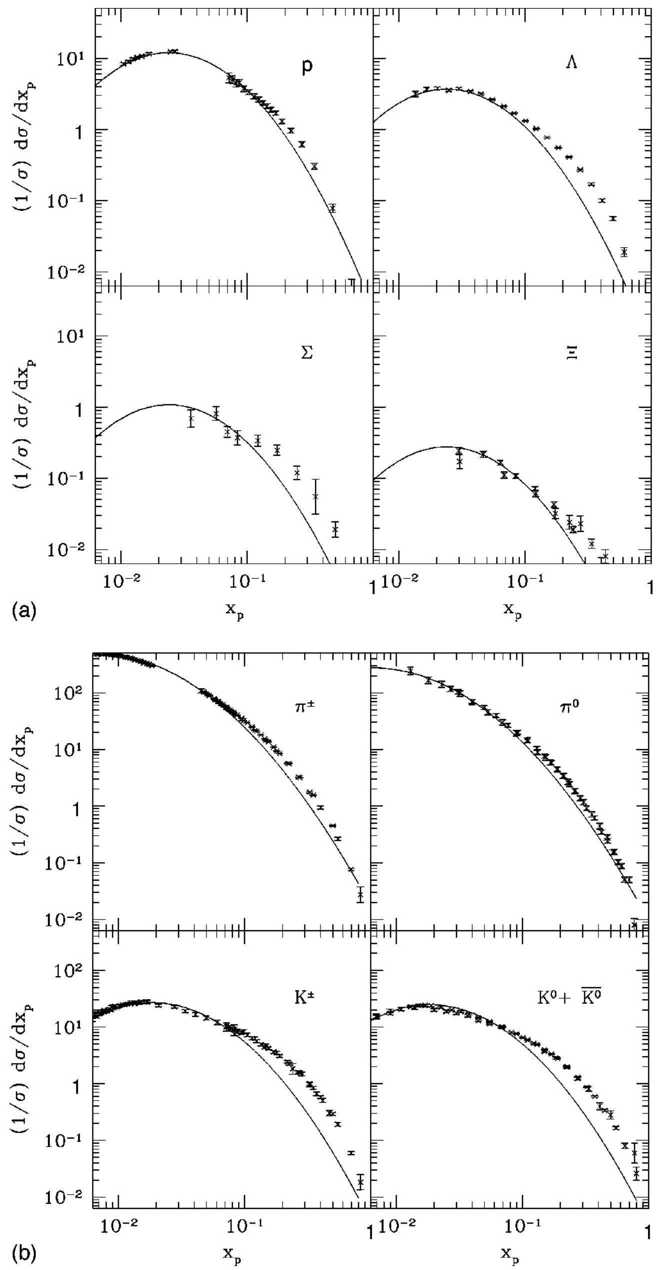


FIG. 6. (a) The figure shows the model fits to the $\sqrt{s}=91$ GeV data for baryons from MLLA evolution of the input fragmentation functions. Note that we have used $x=x_p$ in order to clearly exhibit the small-$x$ data which is of interest here. (b) The figure shows the model fits to the $\sqrt{s}=91$ GeV data for mesons from MLLA evolution of the input fragmentation functions. Note that we have used $x=x_p$ in order to clearly exhibit the small-$x$ data which is of interest here.

$$\frac{1}{\sigma_{tot}}\frac{\mathrm{d}\sigma^{h,Z}}{\mathrm{d}x}=\frac{12\Sigma^h-1.5D_3^h-0.5D_8^h}{36}.$$

We see that the singlet contribution is about 10 times larger than either of the octet contributions. We therefore expect the MLLA approach to yield sensible fits to the data at this energy. In the case of photon exchange data, the singlet contribution is still large:

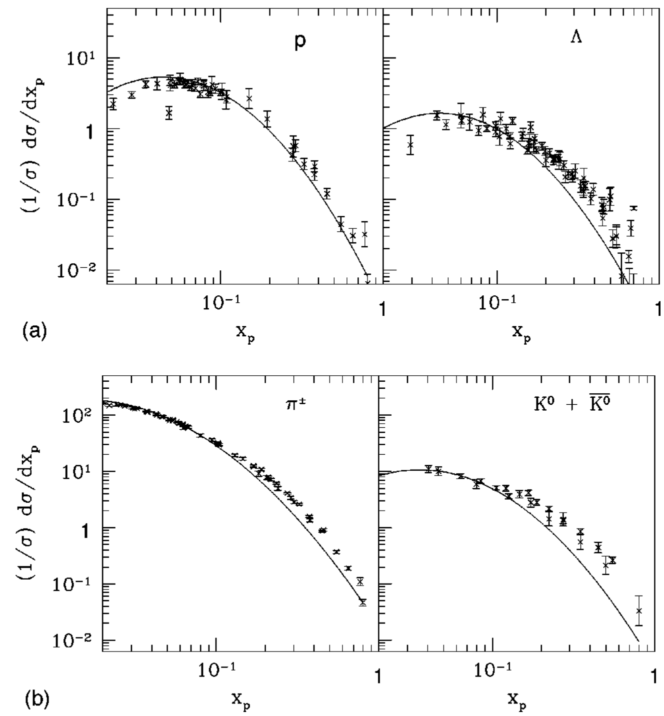


FIG. 7. (a) As in Fig. 6(a) for $\sqrt{s}=34$ GeV for baryons using MLLA evolution. (b) As in Fig. 6(b) for $\sqrt{s}=34$ GeV for mesons using MLLA evolution.

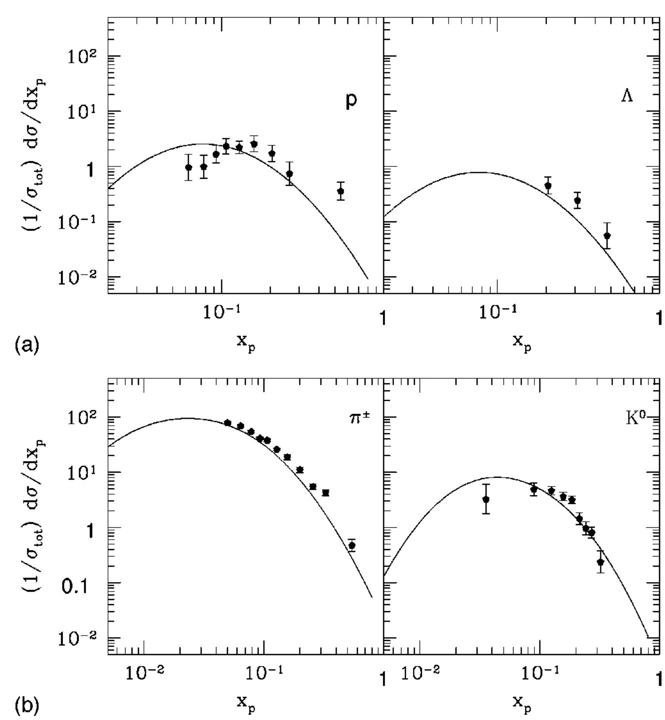


FIG. 8. (a) As in Fig. 6(a) for $\sqrt{s}=14$ GeV for baryons using MLLA evolution. (b) As in Fig. 6(b) for $\sqrt{s}=14$ GeV for mesons using MLLA evolution.

$$\frac{1}{\sigma_{tot}}\frac{\mathrm{d}\sigma^{h,\gamma}}{\mathrm{d}x}=\frac{2\Sigma^h+1.5D_3^h+0.5D_8^h}{6},$$

but the $D_3$ contribution is not small. Hence MLLA may not be a very good description of the data at smaller energies, especially at larger $x$. However, for the case of $\Lambda$ and $\Sigma^0$, $D_3=0$ so that the MLLA singlet term may still saturate the event rate to a good approximation although $D_8$ is negative.

## VI. DISCUSSION AND CONCLUSIONS

We have proposed a simple model for quark fragmentation into an octet baryon or a pseudoscalar meson, using SU(3) symmetry of quarks and octet hadrons. All quark fragmentation functions have been described in terms of three SU(3) symmetric functions $\alpha(x,Q^2)$, $\beta(x,Q^2)$ and $\gamma(x,Q^2)$ and an SU(3) breaking parameter λ. The antiquark fragmentation functions are correspondingly described by $\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$. There are 3 quark (plus 3 antiquark) fragmentation functions corresponding to a given hadron; hence a given hadron octet would involve a total of $(24\times 2)$ fragmentation functions. All these are described in our model by just 6 functions, leading to a very simple model, but with strong predictive power. Leading log evolution of these fragmentation functions has been used to compare the model predictions with data. We find that it is possible to fit the model parameters in such a way as to get a good agreement with the $x$-dependence of all octet baryons and mesons, at three different sample energies (corresponding to $Z^0$ and photon exchange) over most of the $x$ range of available data. These fits were then used to determine the inclusive cross section at $\sqrt{s}=161$ GeV where both photon and $Z^0$ exchange are involved. There was good agreement with data here as well. We have used both DGLAP evolution as well as the modified leading log approximation (MLLA) to evolve the fragmentation functions; the latter has especially been used to explain the decrease in the hadroproduction rates at small-$x$ seen in the data for many hadrons. We have used a small non-zero input gluon distribution (as shown in Table III); however, very little sensitivity to the gluon fragmentation function is seen and this is therefore not well-determined in our analysis.

The model realizes the shape of the $x$-distribution of all available data on octet mesons and baryons very well. It does not describe the $\Lambda$ and $K$ data at 34 and 14 GeV very accurately; however, it is able to give a good agreement with even this data to within $2\sigma$. All other baryon and meson data are fitted very well. However, we note that it is possible to get good fits at all $Q^2$ for each hadron *individually*. The SU(3) symmetry constraint relating the different hadron fragmentation functions worsens the fit in some cases; this reflects the simplicity of our model, which incorporates SU(3) symmetry breaking effects in a very simple way. The goodness-of-fit from the model therefore also indicates the extent to which this symmetry breaking is a universal phenomenon, independent of the type of quark or diquark that is produced [6,7,9].

The parameter λ takes into account the difference in masses of the strange and non-strange quarks and suppresses non-strange quark fragmentation into strange hadrons. This parameter is similar to the suppression factor of the Lund Monte Carlo [3]; however, it is determined by means of a simple comparison of data of strange and non-strange hadrons. The Lund model uses string fragmentation and has a much larger suppression for the case of baryons (suppression factor$=0.06$) as compared to the suppression factor of 0.2–0.3 for mesons. Our model has very similar values for the suppression factors for the two cases ($\lambda=0.07$ for baryons, 0.08 for mesons).

Another approach [4] uses an SU(6) analysis of fragmentation functions using a quark and diquark model. Our SU(3) symmetric functions $\alpha$ and $\beta$ are analogous to the SU(6) symmetric functions $\hat{S}(z)$ and $\hat{T}(z)$ defined therein. The function $\gamma$ (not included in their model) describes sea fragmentation, for instance, $s$ fragmenting to a proton. We find that $\gamma$ is large in the small $x$ region, so that its contribution is significant and cannot be ignored.

We find that the strange quark fragmentation dominates strange hadroproduction over almost the entire $x$ range. This is especially true for $\Lambda$, which has recently been of much interest [1,20]. It is possible to extend the model to include spin-dependent fragmentation functions; the unpolarized result then indicates that polarized $\Lambda$ fragmentation will be dominated by its strange fragmentation function, which can then be readily parametrized and studied.

Finally, our results suggest that there is indeed an underlying symmetry among the baryons and mesons in an octet, which can be tested further by extending the model to decuplet baryons and other hadrons.

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## APPENDIX

We briefly detail the calculations leading to the results in Table II for the quark fragmentation functions in terms of $\alpha$, $\beta$ and $\gamma$.

Let $q_i$ represent a quark triplet and $h_i^j$ the hadron octet, $i,j=1,2,3$.

**Case 1.** $X$ **is a triplet,** $X_i$: Then the invariant amplitude for the process $q\to h+X$ is $q_i h_j^i X^j$, where $X_i$ and $q_i$ are normalized. Here $h_i^j$ are the elements of the meson/baryon matrix (see Table I). Thus, the rate for $u\to p+X$ is $\alpha|u h_3^1 X^3|^2$ which is equal to $\alpha$. Similarly, the rate for $u\to\Lambda+X$ is $\alpha|u h_1^1 X^1|^2$ which is equal to $\alpha/6$ and so on.

**Case 2.** $X$ **is a sixplet,** $X_{ij}$: Now $X_{ij}$ is symmetric in $i$ and $j$ and is expressed in terms of triplets as $(q_i q_j+q_j q_i)/\sqrt{2}$, where each $q$ is normalized. The invariant amplitude is $\epsilon^{imj}q_i h_j^k X_{km}$. Thus, $d\to p+X$ as $\beta|\sqrt{2}|^2$ and so on.

**Case 3.** $X$ **is a fifteenplet,** $X_i^{jk}$: Then $X_i^{jk}$ is symmetric in $j,k$

and is antisymmetric in $i,j$. In terms of triplets, the normalized $X$ can be re-expressed as

$$X_i^{jk}=\frac{1}{\sqrt{2}}\left[q^jq^kq_i+q^kq^jq_i-\frac{1}{4}\,\delta_i^j(q^lq^kq_l+q^kq^lq_l)-\frac{1}{4}\,\delta_i^k(q^jq^lq_l+q^lq^jq_l)\right],$$

where each of the $q_i$'s is normalized.

The invariant amplitude is $X_j^{ik}h_i^jq_k$. In this case, we shall have to take the interference terms for the diagonal elements of the meson/baryon matrix also into account. Note that $X_i^{ij}=0$ (sum over $i$ is implied). Thus, the rate for $u\to\Lambda+X$ is $\gamma|(1/\sqrt{6})X_1^{11}+(1/\sqrt{6})X_2^{21}-(2/\sqrt{6})X_3^{31}|^2$ which is equal to $\gamma|(3/\sqrt{6})(X_1^{11}+X_2^{21})|^2$. On evaluating this expression, we find that this is equal to $9\gamma/8$. The other rates can also be found in a similar manner. The final results are given in Table II.


[1] D. de Florian, M. Stratmann, and W. Vogelsang, hep-ph/9710410; Phys. Rev. D **57**, 5811 (1997).

[2] J. Binnewies, B. A. Kniehl, and G. Kramer, Z. Phys. C **65**, 471 (1995).

[3] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rep. **97**, 31 (1983).

[4] M. Nzar and P. Hoodbhoy, Phys. Rev. D **51**, 32 (1995).

[5] S. Chun and C. Buchanan, Phys. Rep. **292**, 239 (1998).

[6] T. Sjöstrand, Comput. Phys. Commun. **82**, 74 (1994).

[7] G. Marchesini *et al.*, Comput. Phys. Commun. **67**, 465 (1992).

[8] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C **66**, 355 (1995); **64**, 361 (1994); ALEPH Collaboration, R. Barate *et al.*, Phys. Rep. **294**, 1 (1998); Z. Phys. C **74**, 451 (1997); DELPHI Collaboration, P. Abreu *et al.*, Nucl. Phys. **B444**, 3 (1995); Z. Phys. C **67**, 543 (1995); **65**, 587 (1995); Phys. Lett. B **318**, 249 (1993); DELPHI Collaboration, D. Adam *et al.*, Z. Phys. C **69**, 561 (1995); L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. B **328**, 223 (1994); OPAL Collaboration, R. Akers *et al.*, Z. Phys. C **67**, 389 (1995); **63**, 181 (1994); OPAL Collaboration, G. Alexander *et al.*, *ibid.* **73**, 569 (1997); **73**, 587 (1997); see also [9] below.

[9] OPAL Collaboration, G. Alexander *et al.*, Z. Phys. C **73**, 587 (1997).

[10] TASSO Collaboration, M. Althoff *et al.*, Z. Phys. C **27**, 27 (1985); Phys. Lett. **130B**, 340 (1983); TASSO Collaboration, R. Brandelik *et al.*, *ibid.* **108B**, 71 (1982); **105B**, 75 (1981); **94B**, 444 (1980); TASSO Collaboration, W. Braunschweig *et al.*, Z. Phys. C **47**, 167 (1990); **45**, 209 (1990); **42**, 189 (1989); **33**, 13 (1986); TPC Collaboration, H. Aihara *et al.*, Phys. Rev. Lett. **61**, 1263 (1988); Z. Phys. C **27**, 187 (1985); JADE Collaboration, D. D. Pitzl *et al.*, *ibid.* **46**, 1 (1990); JADE Collaboration, W. Bartel *et al.*, *ibid.* **28**, 343 (1985); HRS Collaboration, T. L. Geld *et al.*, Phys. Rev. D **45**, 3949 (1992); HRS Collaboration, M. Derrick *et al.*, *ibid.* **35**, 2639 (1987); HRS Collaboration, P. Baringer *et al.*, Phys. Rev. Lett. **56**, 1346 (1986); CELLO Collaboration, H. J. Behrend *et al.*, Z. Phys. C **47**, 1 (1990); **46**, 397 (1990); Mark-II Collaboration, S. R. Klein *et al.*, Phys. Rev. Lett. **58**, 644 (1987); Mark-II Collaboration, C. de la Vaissiere *et al.*, *ibid.* **54**, 2071 (1985).

[11] TASSO Collaboration, M. Althoff *et al.*, Z. Phys. C **17**, 5 (1983); **27**, 27 (1985); CELLO Collaboration, H. J. Behrend *et al.*, *ibid.* **20**, 207 (1983).

[12] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972); Yu. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977); G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).

[13] Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller, and S. I. Troyan, Rev. Mod. Phys. **60**, 373 (1988); Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller, and S. I. Troyan, *Basics Of Perturbative QCD* (Editions Frontières, Gif-sur-Yvette, France, 1991).

[14] See, for instance, P. Nason and B. R. Webber, Nucl. Phys. **B421**, 473 (1994).

[15] The baryon branching fractions into $p$ and $\Lambda$ have been taken from the Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[16] The data were taken from the Durham Database, 1997.

[17] L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. B **328**, 223 (1994); TASSO Collaboration, W. Braunschweig *et al.*, Z. Phys. C **45**, 193 (1989); OPAL Collaboration, P. D. Acton *et al.*, *ibid.* **53**, 539 (1992); Phys. Lett. B **291**, 503 (1992); TPC Collaboration, H. Aihara *et al.*, Phys. Rev. Lett. **52**, 577 (1984); JADE Collaboration, W. Bartel *et al.*, Phys. Lett. **104B**, 325 (1981).

[18] OPAL Collaboration, K. Ackerstaff *et al.*, Z. Phys. C **75**, 193 (1997).

[19] A. Bassetto, M. Ciafaloni, and G. Marchesini, Phys. Rep. **100**, 201 (1983).

[20] V. Ravindran, Nucl. Phys. **B490**, 272 (1997).