## **Flavor changing neutral current processes in** *B* **and** *K* **decays in the supergravity model**

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Flavor changing neutral current processes such as  $b \rightarrow s \gamma$ ,  $b \rightarrow s l^+ l^-$ ,  $b \rightarrow s \nu \bar{\nu}$ ,  $\epsilon_K$ ,  $\Delta m_B$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are calculated in the supersymmetric standard model based on supergravity. We consider two assumptions for the soft supersymmetry breaking terms. In the minimal case soft breaking terms for all scalar fields are taken to be universal at the GUT scale, whereas those terms are different for the squark-slepton sector and the Higgs sector in the nonminimal case. In the calculation we have taken into account the next-to-leading order QCD correction to the  $b \rightarrow s\gamma$  branching ratio, the results from the CERN LEP II superparticles search, and the condition of the radiative electroweak symmetry breaking. We show that  $\Delta m_B$  and  $\epsilon_K$  can be enhanced up to 40% compared to the standard model values in the nonminimal case. In the same parameter region the  $b \rightarrow s \nu \bar{\nu}$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  branching ratios are reduced up to 10%. The corresponding deviation in the minimal case is 20% for  $\Delta m_B$  and  $\epsilon_K$  and within 3% for  $b \rightarrow s \nu \bar{\nu}$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ . For the  $b \rightarrow s l^+ l^-$  process a significant deviation from the standard model is realized only when the  $b \rightarrow s \gamma$  amplitude has an opposite sign to the standard model prediction. The significance of these results from possible future improvements of the  $b \rightarrow s \gamma$  branching ratio measurement and top squark search is discussed.  $[$ S0556-2821(98)04217-9]

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### **I. INTRODUCTION**

Rare processes such as flavor changing neutral current (FCNC) processes have been useful probes for physics beyond the energy scale directly accessible in collider experiments. Among new physics beyond the standard model (SM), supersymmetry (SUSY) is considered to be the most promising candidate. Since FCNCs are absent at the tree level in the minimal supersymmetric standard model (MSSM) as in the SM, these rare processes can give useful constraints on the masses and mixings of the SUSY particles through loop diagrams.

Although squark masses are free parameters within the framework of the MSSM, it is known that too large FCNCs are induced if we allow arbitrary mass splittings and mixings among the squarks with the same quantum numbers  $[1]$ . This suggests that SUSY breaking in the MSSM sector is induced from a generation-independent interaction. A simple realization of the generation-independent SUSY breaking is the minimal supergravity model. In this case the SUSY breaking in the hidden sector is transmitted to the MSSM sector by the gravitational interaction which does not distinguish the generation or other gauge quantum numbers. As a result, induced soft SUSY breaking masses are equal at the Planck scale for all scalar fields in the MSSM sector. FCNC processes have been studied extensively in the supergravity model as well as in the more general context of the SUSY models for the  $K^0$ - $\bar{K}^0$  and the  $B^0$ - $\bar{B}^0$  mixings [2–6], *b*  $\rightarrow$ *s*y [3,4,7],  $b \rightarrow s l^+ l^-$  [4,8,9],  $b \rightarrow s \nu \bar{\nu}$  [4,9], and *K* 

 $\rightarrow \pi \nu \bar{\nu}$  [10,11]. In Ref. [6] the  $B^0 - \bar{B}^0$  mixing and  $\epsilon_K$ *(CP* violating parameter in the  $K^0$ - $\overline{K}^0$  mixing) were calculated in the minimal supergravity model under CERN  $e^+e^$ collider LEP constraints and it was shown that these quantities can be larger than the SM values by 20%. Rare *b* decay processes such as  $b \rightarrow s\gamma$ ,  $b \rightarrow s l^+ l^-$ , and  $b \rightarrow s \nu \overline{\nu}$  are considered in Ref.  $[9]$ , and it was pointed out that, taking account of the LEP 1.5 constraints, there is a parameter region where the  $b \rightarrow s l^+ l^-$  branching ratio can be enhanced by 50% compared to the SM value. Also, the  $b \rightarrow s \nu \bar{\nu}$  branching ratio is shown to be reduced at most by 10% from the SM prediction.

In this way effects of SUSY particles and the charged Higgs boson vary from a few percent to several ten's of a percent depending on various FCNC processes. Since future experiments on *B* and *K* decays may reveal new physics effects of this magnitude, it is important to make quantitative predictions using updated constraints on SUSY parameter space. A recent important theoretical improvement in this aspect is that the complete next-to-leading order formula of the QCD correction to the branching ratio of  $b \rightarrow s \gamma$  becomes available for the SM  $[12]$  and two-Higgs-doublet models [ $13$ ]. As a result, the theoretical uncertainty in the calculation of  $B(b \rightarrow s \gamma)$  has been reduced to a  $\leq 10\%$  level.

In this paper we study the SUSY contributions to FCNC processes under updated constraints. We take account of the next-to-leading order QCD corrections for the evaluation of  $B(b \rightarrow s\gamma)$  as well as the bounds on SUSY particle masses from recent LEP II results  $[14]$  in order to obtain the allowed region in SUSY parameter space. Then we evaluate various FCNC quantities such as  $b \rightarrow s l^+ l^-$ ,  $b \rightarrow s p \overline{v}$ ,  $B^0 \overline{B^0}$  mixing amplitude,  $\epsilon_K$ ,  $K^+\rightarrow \pi^+\nu\bar{\nu}$ , and  $K_L\rightarrow \pi^0\nu\bar{\nu}$  within the al-

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lowed parameter region. The numerical results depend on the assumption of SUSY breaking terms at the grand unified theory  $(GUT)$  scale. In particular, in the minimal supergravity model soft SUSY breaking terms for all scalar fields are assumed to be the same at the GUT scale. If we would like to suppress too large SUSY contributions to the  $K^0$ - $\bar{K}^0$  mixing, it is sufficient to require the degeneracy of the soft SUSY breaking masses only in the squark sector. Because the strict universality for all scalar masses is not necessarily required in the context of the supergravity model, we study how the allowed deviations of the FCNC quantities change when the universality condition is relaxed. For this purpose we take the soft SUSY breaking term for the Higgs masses as a parameter independent of the universal squark-slepton mass. This kind of assumption was considered in Ref.  $[15]$  in a different context. We will see that the SUSY effects are considerably enhanced in a parameter space which is excluded in the minimal case from the condition of the proper electroweak symmetry breaking. In the nonminimal case, the branching ratios of  $K \rightarrow \pi \nu \bar{\nu}$  can be smaller than the SM values by 10%, and at the same time,  $\epsilon_K$  and the  $B^0$ - $\overline{B}^0$ mixing become larger than the SM values by 40% for tan  $\beta$ =2. The corresponding values in the minimal case are given by 3% and 20%, respectively. For  $b \rightarrow s l^+ l^-$ , the result does not significantly differ from the minimal case: there is a parameter space where the branching ratio becomes larger by 50% than the SM value for a large tan  $\beta$ . We analyze the correlation between the SUSY contributions to the FCNC processes and the  $b \rightarrow s \gamma$  branching ratio. It turns out that the maximal deviation occurs in the case that the  $b \rightarrow s \gamma$  branching ratio is away from the central value of the SM prediction. We also show that a large deviation occurs in a parameter region where the top squark is relatively light. Therefore, the improvement in the  $b \rightarrow s \gamma$  branching ratio measurement and the top squark mass bound will have a great impact on the SUSY search through the various FCNC processes.

The rest of this paper is organized as follows. In the next section, we introduce the supergravity model. In Sec. III we describe the calculation of each FCNC quantity. In Sec. IV, our results of numerical analyses are presented. Section V is devoted to a discussion and conclusions.

#### **II. SUPERGRAVITY MODEL**

In this section we briefly outline calculations of the SUSY particle masses and the mixing parameters in the supergravity model for the minimal and the nonminimal cases. The actual procedure is the same as those discussed in Refs.  $[16,$  $[6, 9]$  except for the choice of the initial soft SUSY breaking parameters for the nonminimal case.

The MSSM Lagrangian is specified by the superpotential and the soft SUSY breaking terms. The superpotential is given by

$$
W_{\text{MSSM}} = f_D^{ij} Q_i D_j H_1 + f_U^{ij} Q_i U_j H_2 + f_L^{ij} E_i L_j H_1 + \mu H_1 H_2,
$$
\n(2.1)

where the chiral superfields  $Q$ ,  $D$ ,  $U$ ,  $L$ ,  $E$ ,  $H_1$ , and  $H_2$ transform under the  $SU(3) \times SU(2) \times U(1)$  group as the following representations:

$$
Q_{i} = \left(3, 2, \frac{1}{6}\right), \quad U_{i} = \left(3, 1, -\frac{2}{3}\right), \quad D_{i} = \left(3, 1, \frac{1}{3}\right),
$$
  
\n
$$
L_{i} = \left(1, 2, -\frac{1}{2}\right), \quad E_{i} = (1, 1, 1),
$$
  
\n
$$
H_{1} = \left(1, 2, -\frac{1}{2}\right), \quad H_{2} = \left(1, 2, \frac{1}{2}\right).
$$
 (2.2)

The suffices  $i, j=1,2,3$  are generation indices.  $SU(3)$  and *SU*(2) indices are suppressed for simplicity. A general form of the soft SUSY breaking terms is given by

$$
-\mathcal{L}_{soft} = (m_Q^2)^i_j \tilde{q}_i \tilde{q}^{\dagger j} + (m_D^2)^j \tilde{d}^{\dagger i} \tilde{d}_j + (m_U^2)^j i \tilde{u}^{\dagger i} \tilde{u}_j + (m_E^2)^i_j \tilde{e}_i \tilde{e}^{\dagger j} + (m_L^2)^j \tilde{l}^{\dagger i} \tilde{l}_j + \Delta_1^2 h_1^{\dagger} h_1 + \Delta_2^2 h_2^{\dagger} h_2 - (B \mu h_1 h_2 + \text{H.c.}) + (A_D^{ij} \tilde{q}_i \tilde{d}_j h_1 + A_U^{ij} \tilde{q}_i \tilde{u}_j h_2 + A_U^{ij} \tilde{e}_i \tilde{l}_j h_1 + \text{H.c.}) + \left(\frac{M_1}{2} \tilde{B} \tilde{B} + \frac{M_2}{2} \tilde{W} \tilde{W} + \frac{M_3}{2} \tilde{G} \tilde{G} + \text{H.c.}\right),
$$
\n(2.3)

where  $\tilde{q}_i$ ,  $\tilde{u}_i$ ,  $\tilde{d}_i$ ,  $\tilde{l}_i$ ,  $\tilde{e}_i$ ,  $h_1$ , and  $h_2$  are scalar components of chiral superfields  $Q_i$ ,  $U_i$ ,  $D_i$ ,  $L_i$ ,  $E_i$ ,  $H_1$ , and  $H_2$ , respectively, and  $\tilde{B}$ ,  $\tilde{W}$ , and  $\tilde{G}$  are  $U(1)$ ,  $SU(2)$ , and  $SU(3)$ gauge fermions, respectively.

In the framework of the supergravity model, the soft SUSY braking parameters are assumed to have a simple structure at the Planck scale. In the present analysis, we take the following initial conditions at the GUT scale  $M_{\text{GUT}}$  ~ 2  $\times$ 10<sup>16</sup> GeV. We neglect the difference between the Planck and GUT scales:

$$
(m_Q^2)^i{}_j = (m_E^2)^i{}_j = m_0^2 \delta^i{}_j,
$$
  

$$
(m_D^2)^i{}_i = (m_U^2)^i{}_j = (m_L^2)^i{}_j = m_0^2 \delta^i{}_i,
$$
 (2.4a)

$$
\Delta_1^2 = \Delta_2^2 = \Delta_0^2,\tag{2.4b}
$$

$$
A_D^{ij} = f_{DX}^{ij} A_X m_0, \quad A_L^{ij} = f_{LX}^{ij} A_X m_0, \quad A_U^{ij} = f_{UX}^{ij} A_X m_0,
$$
\n(2.4c)

$$
M_1 = M_2 = M_3 = M_{gX}.
$$
\n(2.4d)

In the minimal case  $m_0$  and  $\Delta_0$  are assumed to be equal, whereas in the nonminimal case we take  $m_0$  and  $\Delta_0$  as independent parameters. We also assume that  $A_X$ ,  $M_{\varrho}$ , and  $\mu$ are all real parameters to avoid a large electric dipole moment of the neutron [17]. Therefore, no new *CP* violating complex phase [other than that in the Cabibbo-Kobayashi-Maskawa  $(CKM)$  matrix is introduced in the present analysis.

The soft SUSY breaking parameters at the electroweak scale are calculated by solving the renormalization group equations  $(RGEs)$  of the MSSM [18], and we also impose the radiative electroweak symmetry breaking condition [19]. Taking the quark masses, the CKM matrix, and tan  $\beta$  $=$  $\langle h_2^0 \rangle / \langle h_1^0 \rangle$  as inputs, we first solve one-loop RGEs for the gauge and Yukawa coupling constants to calculate the values at the GUT scale. Then we solve the RGEs for all MSSM parameters downward with initial conditions  $(2.4)$  for each set of the universal soft SUSY breaking parameters  $(m_0,$  $\Delta_0$ ,  $A_X$ ,  $M_{gX}$ ). We include all generation mixings in the RGEs for both Yukawa coupling constants and the soft SUSY breaking parameters. Next, we evaluate the Higgs potential at the  $m_Z$  scale, including the one-loop corrections induced by the Yukawa couplings constants of the third generation  $[20]$ , and require that the minimum of the potential give correct vacuum expectation values of the neutral Higgs fields as  $\langle h_1^0 \rangle = v \cos \beta$  and  $\langle h_2^0 \rangle = v \sin \beta$ , where *v*  $=$  174 GeV. The requirement of radiative electroweak symmetry breaking fixes the magnitude of the SUSY Higgs mass parameter  $\mu$  and the soft SUSY breaking parameter *B*. At this stage, all MSSM parameters at the electroweak scale are determined as functions of the input parameters  $[\tan \beta, m_0,$  $\Delta_0$ ,  $A_X$ ,  $M_{gX}$ , sgn( $\mu$ ). With use of the MSSM parameters at the electroweak scale, we obtain the masses and the mixing parameters (both angles and phases) of all the SUSY particles by diagonalizing the mass matrices. We impose the following phenomenological constraints on the obtained particle spectra.

(1)  $b \rightarrow s\gamma$  constraint from CLEO, i.e.,  $1.0 \times 10^{-4} < B(b)$  $\rightarrow$ *s* $\gamma$ ) $<$ 4.2 $\times$ 10<sup>-4</sup> [21].

(2) The chargino mass is larger than 91 GeV, and all other charged SUSY particle masses should be larger than 80 GeV  $[14]$ .

 $(3)$  All sneutrino masses are larger than 41 GeV  $[22]$ .

 $(4)$  The gluino and squark mass bounds from Fermilab Tevatron experiments [23]. The precise bounds on the gluino and squark masses depend on various SUSY parameters. Here we impose the constraint reported in Ref.  $[23]$  on the parameter space of the gluino mass and the averaged squark mass except for the top squark. Actually, the gluino mass and the squark masses are more strictly constrained in this model from the chargino mass bound and the GUT relation of the gaugino masses, so that these masses are restricted to be larger than about 200 GeV except for the lighter top squark. For the light top squark, the experimental bound is obtained at LEP and Fermilab Tevatron experiments  $[24]$ , which was already taken into account in constraint  $(2)$ .

(5) From the LEP neutralino search [25],  $\Gamma(Z \rightarrow \chi \chi)$  $\langle 8.4 \text{ MeV} \quad \text{and} \quad B(Z \rightarrow \chi \chi') , \quad B(Z \rightarrow \chi' \chi') \langle 2 \times 10^{-5} \rangle$ where  $\chi$  is the lightest neutralino and  $\chi'$  denotes other neutralinos.

~6! The lightest SUSY particle is neutral.

 $(7)$  The condition for not having a charge or color symmetry breaking minimum  $[26]$ .

In the next section we calculate the FCNC and/or *CP* violating quantities such as the branching ratios for *b*

 $\rightarrow$ *sl*<sup>+</sup>*l*<sup>-</sup>, *b* $\rightarrow$ *s* $\nu \bar{\nu}$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ , and the *B*<sup>0</sup>-*B*<sup>0</sup> mixing and  $\epsilon_K$  in the allowed parameter region.

## **III. FCNC PROCESSES IN** *B* **AND** *K* **DECAYS**

# A.  $b \rightarrow s \gamma$ ,  $b \rightarrow s l^+ l^-$ , and  $b \rightarrow s \nu \overline{\nu}$

We basically follow Ref.  $[9]$  for the calculations of *b*  $\rightarrow$ *s* $\gamma$ , *b* $\rightarrow$ *sl*<sup>+</sup>*l*<sup>-</sup></sup>, and *b* $\rightarrow$ *sv*<sup>*v*</sup><sub>*v*</sub><sup>*b*</sup> branching ratios, but we improve the calculation taking into account the next-to-leading order QCD corrections.

The effective Hamiltonian for the *b→s* transition processes is given as  $[27,9,28]$ 

$$
\mathcal{H}_1^{\text{eff}} = \sum_{i=1}^{11} C_i(Q)\mathcal{O}_i(Q) + \text{H.c.},
$$
 (3.1)

where *Q* is the renormalization point. The operators relevant to the present study are

$$
\mathcal{O}_7 = \frac{e}{(4\,\pi)^2} \, m_b(\bar{s}\sigma^{\mu\nu}b_R) F_{\mu\nu},\tag{3.2a}
$$

$$
\mathcal{O}_9 = \frac{e^2}{(4\pi)^2} \left(\overline{s}\,\gamma^\mu b_L\right) (\overline{l}\,\gamma_\mu l),\tag{3.2b}
$$

$$
\mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} \left( \overline{s} \gamma^\mu b_L \right) (\overline{l} \gamma_\mu \gamma_5 l), \tag{3.2c}
$$

for  $b \rightarrow s \gamma$  and  $b \rightarrow s l^+ l^-$ , and

$$
\mathcal{O}_{11} = \frac{e^2}{(4\pi)^2} \left( \bar{s} \gamma^\mu b_L \right) \left[ \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \right],\tag{3.3}
$$

for  $b \rightarrow s \nu \bar{\nu}$ . Other operators (the four-quark operators  $\mathcal{O}_{1,2,\dots,6}$  and the chromomagnetic operator  $\mathcal{O}_8$ ) contribute through the QCD corrections. We first calculate the Wilson coefficients  $C_i$  at the electroweak scale with use of the masses and the mixings of SUSY particles as well as the SM ones. Then we evaluate  $C_i$  at the  $m_b$  scale including the QCD corrections below the electroweak scale in order to obtain the branching ratios of  $b \rightarrow s$  decays.

As for the next-to-leading order QCD correction in the calculation of  $B(b \rightarrow s\gamma)$ , we follow Refs. [29, 30, 12, 31,  $32$ ] for the SM contribution and Ref.  $[13]$  for the charged Higgs boson contribution. The QCD correction consists of the  $O(\alpha_s)$  matching at the electroweak scale [29,30,13], the next-to-leading order anomalous dimension [12], two-loop matrix elements  $[31]$ , and bremsstrahlung corrections  $[32]$ . In Ref. [30], the SM value is given as  $B(b\rightarrow s\gamma)_{\text{SM}}^{\text{NLO}}$  $=$ (3.60 $\pm$ 0.33) $\times$ 10<sup>-4</sup> compared to the leading order result  $B(b \rightarrow s \gamma)_{\text{SM}}^{\text{LO}} = (2.8 \pm 0.8) \times 10^{-4}$ . Here  $O(\alpha_s)$  matching conditions for the SUSY loop corrections have not been completed. In Ref. [33], these corrections are given for the case that the ratio of the chargino mass and the top squark mass is large. Since we are mainly interested in the case that both particles are relatively light, we do not include these corrections. Recently, electroweak radiative corrections to  $B(b \rightarrow s \gamma)$  have been considered in Ref. [34]. We will discuss these effects on the numerical results later, although we have not included them explicitly in the calculation. For the next-to-leading order QCD corrections to  $b \rightarrow s l^+ l^-$  and *b*  $\rightarrow$ *sv* $\bar{\nu}$ , we follow Refs. [35, 28].

The main SM contributions to the  $b \rightarrow s$  decays come from the loop diagrams involving the top quark and the relevant CKM matrix element is  $V_{ts}^*V_{tb}$ , which is approximately written as  $V_{ts}^* V_{tb} \approx -V_{cs}^* V_{cb}$  because of the unitarity and the smallness of  $V_{us}^* V_{ub}$ . Also, the charm quark loop contribution has the CKM factor  $V_{cs}^* V_{cb}$ . Consequently, unlike  $B^0$ - $\overline{B}^0$  mixing,  $\epsilon_K$ , and  $K \rightarrow \pi \nu \overline{\nu}$ , the SM values of the branching ratios for above processes are calculable without much uncertainty since the relevant CKM factors are known to a good accuracy.

The SM predictions of the branching ratios for these processes are  $B(b \to s l^+ l^-) \approx 0.8(0.6) \times 10^{-5}$  for  $l = e(\mu)$  and  $B(b \rightarrow s \nu \bar{\nu}) \approx 4.2 \times 10^{-5}$ . These processes have not yet been observed experimentally, and only upper bounds are given by  $B(b\rightarrow s^2 l^+ l^-)$  < 5.7(5.8) × 10<sup>-5</sup> for  $l = e(\mu)$  [36] and  $B(b \to s \nu \bar{\nu})$  < 3.9×10<sup>-4</sup> [37]. The  $b \to s l^+ l^-$  process is expected to be observed in the near future at the *B* factories and hadron machines.

## **B.**  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The branching ratios of  $K \rightarrow \pi \nu \bar{\nu}$  processes are calculated by evaluating the Wilson coefficient  $C_{11}^d$  in the effective Hamiltonian

$$
\mathcal{H}_d^{\text{eff}} = C_{11}^d \mathcal{O}_{11}^d + \text{H.c.},
$$
  

$$
\mathcal{O}_{11}^d = \frac{e^2}{(4\pi)^2} \left( \bar{s} \gamma^\mu d_L \right) \left[ \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \right], \tag{3.4}
$$

in a similar way as  $b \rightarrow s \nu \bar{\nu}$ . The branching ratios normalized to that of the  $K_{e3}$  decay are written as

$$
\frac{B(K^+\to\pi^+\,\nu\,\bar{\nu})}{B(K^+\to\pi^0e^+\,\nu_e)} = \left(\frac{\alpha}{4\,\pi}\right)^2 \frac{\sum_{\nu} |C_{11}^d|^2}{|V_{us}|^2 G_F^2} \, r_{K^+},\tag{3.5a}
$$

$$
\frac{B(K_L \to \pi^0 \nu \bar{\nu})}{B(K^+ \to \pi^0 e^+ \nu_e)} = \left(\frac{\alpha}{4\pi}\right)^2 \frac{\Sigma_{\nu} |\text{Im } C_{11}^d|^2}{|V_{us}|^2 G_F^2} \frac{\tau_{K_L}}{\tau_{K^+}} r_{K_L},
$$
\n(3.5b)

where  $\tau_{K_L}(\tau_{K^+})$  denotes the lifetime for  $K_L(K^+)$  and  $r_{K^+}$ and  $r_{K_L}$  are isospin breaking factors [38].

The SM contributions to  $C_{11}^d$  come from both the top and charm loops with CKM factors  $V_{ts}^* V_{td}$  and  $V_{cs}^* V_{cd}$ , respectively. The dependences on  $V_{td}$  (or  $\rho$  and  $\eta$  in Wolfenstein's parametrization) are different in  $K^+\rightarrow \pi^+\nu\bar{\nu}$  and  $K_L$  $\rightarrow \pi^0 \nu \bar{\nu}$  since only the  $V_{ts}^* V_{td}$  term contributes to  $K_L$  $\rightarrow \pi^0 \nu \bar{\nu}$ , while the sum of both terms contributes to *K*<sup>-</sup>  $\rightarrow \pi^+ \nu \bar{\nu}$ . The details of the calculation of  $K \rightarrow \pi \nu \bar{\nu}$  processes in the SM are available in Ref.  $[28]$ . Following this reference, we have taken into account the next-to-leading order QCD correction to the SM contribution.



FIG. 1. Allowed regions in the space of the lighter chargino mass  $m_{\tilde{\chi}_1^{\pm}}$  and the lighter top squark mass  $m_{\tilde{t}_1}^{\pm}$  for (a) tan  $\beta=2$  and (b) tan  $\beta$ =30. The dots represent the allowed region for the full parameter space, and the squares show the allowed region for the minimal case  $(m_0 = \Delta_0)$ .

The SM predictions for the above branching ratios are given by  $B(K^+\to \pi^+ \nu \bar{\nu})=(0.6-1.5)\times 10^{-10}$  and  $B(K_L)$  $\rightarrow \pi^0 \nu \bar{\nu}$ ) = (1.1–5.0) × 10<sup>-11</sup>, taking into account the ambiguity of unknown CKM parameters  $[28]$ . Recently, one candidate event of  $K^+\rightarrow \pi^+\nu\bar{\nu}$  has been reported and the branching ratio derived from this observation corresponds to  $4.2^{+9.7}_{-3.5} \times 10^{-10}$  [39]. On the other hand, for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  only the upper bound is given by  $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.8 \times 10^{-6}$ [40]. Although the upper bound is still  $10^5$  larger than the SM prediction, dedicated searches for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are planned at KEK  $[41]$ , BNL  $[42]$ , and Fermilab  $[43]$ . The *K*  $\rightarrow \pi \nu \bar{\nu}$  processes are theoretically very clean, and the theoretical errors, such as QCD corrections, are expected to be  $\leq$ 10% for *K*<sup>+</sup>→π<sup>+</sup>νν<sup>*ν*</sup> and a few percent for *K*<sub>*L*→π<sup>0</sup>νν</del><sup>*ν*</sup></sub> [28]. Therefore,  $K \rightarrow \pi \nu \bar{\nu}$  processes may give useful infor-



FIG. 2.  $C_7$ ,  $C_9$ , and  $C_{10}$  normalized to the SM values for (a) the full parameter space with tan  $\beta$ =2, (b) the minimal case with tan  $\beta$ =2, (c) the full parameter space with tan  $\beta$ =30, and (d) the minimal case with tan  $\beta$ =30.

mation on the SUSY parameters if the branching ratios are measured at the 10% level.

## **C.**  $B^0$ - $\overline{B}^0$  mixing and  $\epsilon_K$

The  $B^0$ - $\overline{B}^0$  mixing matrix element  $M_{12}(B)$  is calculated from the effective Hamiltonian

$$
\mathcal{H}_2^{\text{eff}} = \frac{1}{128\pi^2} A(B)(\overline{d}\gamma^\mu b_L)(\overline{d}\gamma_\mu b_L) + \text{H.c.},\qquad(3.6)
$$

with

$$
M_{12}(B) = \frac{1}{2m_B} \langle B^0 | \mathcal{H}_2^{\text{eff}} | \bar{B}^0 \rangle = \frac{\hat{B}_B \eta_B f_B^2 m_B}{384 \pi^2} A(B),
$$
\n(3.7)

where  $m_B$ ,  $f_B$ ,  $\hat{B}_B$ , and  $\eta_B$  are the *B*-meson mass, decay constant, bag parameter, and QCD correction factor, respectively. The  $K^0$ - $\overline{K}^0$  mixing matrix element  $M_{12}(K)$  is obtained in the same way by replacing the external bottom quark with the strange quark, and  $\epsilon_K$  is proportional to Im  $M_{12}(K)$ . We calculate the coefficient  $A(B)$  and  $A(K)$  as described in Ref.  $[6]$  with the inclusion of the next-to-leading order QCD corrections given in Ref. [44]. The experimental values for the  $B^0$ - $\overline{B}^0$  and  $K^0$ - $\overline{K}^0$  mixings are given as  $\Delta m_B$  $=2|M_{12}(B)| = (0.474 \pm 0.031) \text{ ps}^{-1}$  [22,45] and  $|\epsilon_K|$  $=$ (2.280±0.013) $\times$ 10<sup>-3</sup> [22]. At present, these observables do not constrain the SUSY parameters very strongly because the CKM parameters relevant to these quantities are not well determined and considerable hadronic uncertainties still exist in  $\hat{B}_K$ ,  $\hat{B}_B$ , and  $f_B$ .

### **IV. NUMERICAL RESULTS**

In this section we show our numerical results. We scan the soft SUSY breaking parameter space in the range of  $m_0$ 



FIG. 3.  $B(b \rightarrow s \gamma)$  in the supergravity model as a function of the charged Higgs mass for (a) tan  $\beta$ =2 and (b) tan  $\beta$ =30. Each solid line shows the branching ratio in the two-Higgs-doublet model (type II). Each dashed line shows the branching ratio in the SM. Dotted lines denote the upper and lower bounds on the branching ratio given by CLEO. For tan  $\beta=2$  the values in the minimal case are also plotted with circles.

 $\leq 600 \text{ GeV}, \ \Delta_0 \leq 600 \text{ GeV}, \ M_{gX} \leq 600 \text{ GeV}, \text{ and } |A_X| \leq 5$ for each fixed value of tan  $\beta$ . For the CKM matrix, we use the ''standard'' phase convention of the Particle Data Group [22], taking  $V_{us} = 0.2205$ ,  $V_{cb} = 0.041$ ,  $|V_{ub}/V_{cb}| = 0.08$ , and  $\delta_{13}$ =90° as input parameters. We also change the value of  $\delta_{13}$  and comment on the results if necessary. We fix the pole masses of the top, bottom, and charm quarks as 175, 4.8, and 1.4 GeV, respectively. We also take  $\alpha_s(m_Z)=0.118$ .

Let us first discuss the general features of the mass spectrum and the generation mixings of squarks determined by RGEs.

 $(1)$  The first and second generation squarks with the same gauge quantum numbers remain highly degenerate in masses, but the third generation squarks, especially the top



FIG. 4. Branching ratios of  $b \rightarrow s\gamma$  and  $b \rightarrow s\mu^{+}\mu^{-}$  for (a) tan  $\beta$ =2 and (b) tan  $\beta$ =30. Here  $B(b \rightarrow s \mu^+ \mu^-)$  is obtained by integrating in the range  $2m_{\mu} < \sqrt{s} < m_{J/\psi} - 100$  MeV, where  $\sqrt{s}$  is the invariant mass of the  $\mu^+\mu^-$  pair. The dots show the values in the full parameter space, the squares show those in the minimal case, and the circle represents the SM value. The vertical dotted lines show the upper and lower bounds on  $B(b \rightarrow s\gamma)$  given by CLEO.

squark, can be significantly lighter due to the renormalization effect of the top Yukawa coupling constant.

 $(2)$  The squark flavor mixing matrix which diagonalizes the squark mass matrix is approximately the same as corresponding CKM matrix apart from the left-right mixing of the top squarks.

As a result, SUSY contributions to the  $b \rightarrow s [s \rightarrow d]$  transition amplitudes and  $M_{12}(B)$   $[M_{12}(K)]$  are proportional to  $V_{tb}V_{ts}^*$  [ $V_{ts}V_{td}^*$ ] and  $(V_{tb}V_{td}^*)^2$  [ $(V_{ts}V_{td}^*)^2$ ], respectively. Therefore, the CP violating phase of  $M_{12}(B(K))$  is equal to that in the SM. These features are the same as those in the minimal case  $[16,6,9]$ .

The quantitative difference between the minimal and non-



FIG. 5. The branching ratio for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  normalized to the SM value for tan  $\beta = 2$  (a) as a function of the lighter chargino mass, (b) as a function of the lighter top squark mass, and (c) as a function of  $B(b \rightarrow s\gamma)$ . Each dot represents the value in the full parameter space, and each square shows the value for the minimal case. The vertical dotted lines in (c) show the upper and lower bounds on  $B(b \rightarrow s\gamma)$  given by CLEO. In (a) and (b) the CLEO bound is imposed (see text).



FIG. 6. Correlation between  $B(K^+\to \pi^+\nu\bar{\nu})/B(K^+$  $\rightarrow \pi^+ \nu \bar{\nu}$ <sub>SM</sub> and  $B(K_L \rightarrow \pi^0 \nu \bar{\nu})/B(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM}$  for tan  $\beta=2$ . Here  $m_0$  is fixed to 150 GeV and  $\delta_{13}$  is taken as 30°, 90°, and 150°.

minimal choices of the soft SUSY breaking parameters appears in the mass spectrum. In Fig. 1 we show the allowed region in the space of the lighter chargino and the lighter top squark masses for a different assumption on  $m_0$  and  $\Delta_0$ , for tan  $\beta$ =2 and 30. We present the allowed region for the full parameter space and the minimal case  $(m_0 = \Delta_0)$ . Contrary to the minimal case, we see that a relatively light top squark and chargino with masses  $m_{\tilde{t}_1}^2 \sim 100 \text{ GeV}$  and  $m_{\tilde{x}_1^{\pm}}$  $\sim$  100 GeV are simultaneously realized especially for tan  $\beta$  $=$  2. This difference of the allowed mass spectrum leads to a quantitative change in the prediction of the FCNC observables for the minimal and nonminimal cases.

# A.  $b \rightarrow s \gamma$ ,  $b \rightarrow s l^+ l^-$ , and  $b \rightarrow s \nu \overline{\nu}$

As discussed above, the SUSY contribution to the  $b \rightarrow s$ transition amplitudes is proportional to the  $V_{tb}V_{ts}^*$  element just as the SM and charged Higgs boson contributions. As discussed in the Sec. III A, the  $V_{tb}V_{ts}^*$  element is well constrained from the unitarity of the CKM matrix so that there is little ambiguity associated with this input parameter. The Wilson coefficients  $C_7$ ,  $C_9$ , and  $C_{10}$  are relevant to the *b*  $\rightarrow$ *s* $\gamma$  and *b* $\rightarrow$ *sl*<sup>+</sup>*l*<sup>-</sup> decays. The values of *C*<sub>7</sub>, *C*<sub>9</sub>, and *C*<sub>10</sub> in the supergravity model are shown in Fig. 2. Each coefficient is evaluated at the bottom quark mass scale and is normalized by the corresponding SM value. The SUSY contribution to  $C_7$  can be as large as or even larger than the SM contribution especially for a large tan  $\beta$ . We can see that the sign of  $C_7$  can be opposite to that of the SM prediction. On the other hand, the SUSY contributions to  $C_9$  and  $C_{10}$  are relatively small and interfere with SM ones, constructively in  $C_9$  and destructively in  $C_{10}$ . These features are the same as those in the minimal case discussed in Ref.  $[9]$ .

In Fig. 3 we show the branching ratio of  $b \rightarrow s \gamma$  as a function of the charged Higgs boson mass for tan  $\beta=2$ (minimal and nonminimal cases) and tan  $\beta$ =30 (nonminimal



FIG. 7. Correlation between  $\epsilon_K / (\epsilon_K)_{\text{SM}}$  and  $\Delta m_B / (\Delta m_B)_{\text{SM}}$  for tan  $\beta$ =2. Here  $m_0$  is fixed to 150 GeV and  $\delta_{13}$  is taken as 30°, 90°, and 150°.

case). For tan  $\beta$ =30, the plot looks the same even if the parameter space is restricted to the minimal case. Here we fix the renormalization point  $\mu_b$  as  $\mu_b = m_b$ . In the calculation of  $B(b \rightarrow s\gamma)$  we use the electromagnetic coupling constant  $\alpha_{EM}$  at the  $m_b$  scale, which is given by  $\alpha_{EM}^{-1}(m_b) \approx 132.3$ . Considering that the next-to-leading order formulas still contain theoretical ambiguities due to the  $\mu_b$  dependence and the choice of the various input parameters, we should allow a theoretical uncertainty at the 10% level for each point. It is interesting to notice that for the minimal case with tan  $\beta$ =2 there are two branches for  $B(b \rightarrow s \gamma)$ . In one branch the branching ratio is close to the two-Higgs-doublet model  $(type II)$  prediction; therefore, the contributions from SUSY particles are small. In the other branch it is consistent with the SM value, so that the charged Higgs boson contribution is canceled by the SUSY contributions.

In Fig. 4 we show the correlation between the branching ratios of  $b \rightarrow s \gamma$  and  $b \rightarrow s \mu^+ \mu^-$ . In this figure, in order to avoid the  $J/\psi$  resonance, we use the branching ratio for *b*  $\rightarrow$ *s* $\mu^{+}\mu^{-}$  integrated in the region  $2m_{\mu} < \sqrt{s} < m_{J/\psi}$  $-100$  MeV, where  $\sqrt{s}$  is the invariant mass of the  $\mu^+\mu^$ pair. As discussed in Ref.  $[9]$ , the branching ratio in this region depends on the phase of the  $b-s-J/\psi$  coupling  $\kappa$ through the interference effect. Although the branching ratio can change by  $\pm 15\%$ , this ambiguity will be reduced if we can measure the lepton invariant mass spectrum near the  $J/\psi$ resonance region. As an example, we take  $\kappa$  as  $+1$  here. We can see a strong correlation between the two branching ratios since only  $C_7$  receives the large SUSY contribution. In the present supergravity model, therefore, a large deviation of  $B(b \rightarrow s l^+ l^-)$  from the SM prediction is expected only when the sign of  $C_7$  is opposite to that in the SM, which is realized for a large tan  $\beta$ . This situation is similar to the minimal case  $[9]$ .

The amplitude of  $b \rightarrow s \nu \bar{\nu}$  is determined by the Wilson coefficient  $C_{11}$ . Apart from the CKM matrix element, the



FIG. 8.  $\Delta m_B$  normalized by the SM value for tan  $\beta$ =2 (a) as a function of the lighter chargino mass, (b) as a function of the lighter top squark mass, and (c) as a function of  $B(b \rightarrow s\gamma)$ . Each dot represents the value in the full parameter space, and each square shows the value for the minimal case. The vertical dotted lines in (c) show the upper and lower bounds on  $B(b \to s\gamma)$  given by CLEO. In (a) and (b) the CLEO bound is imposed.



FIG. 9. Correlation between  $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ and  $\Delta m_B / (\Delta m_B)_{\text{SM}}$  for (a) tan  $\beta = 2$  and (b) tan  $\beta = 30$ .

SUSY contribution to  $C_{11}$  is the same as the SUSY contribution to  $C_{11}^d$ . The branching ratio for  $b \rightarrow s \nu \bar{\nu}$  normalized by the SM prediction  $[B(b \rightarrow s \nu \bar{\nu})/B(b \rightarrow s \nu \bar{\nu})_{SM}]$  is practically the same as a similar ratio for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  [ $B(K_L$  $\rightarrow \pi^0 \nu \bar{\nu}/B(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\rm SM}$ , which is discussed in the next subsection.

## **B.**  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$

As shown in Eq.  $(3.5)$ , the branching ratios for  $K^+$  $\rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are proportional to  $|C_{11}^d|^2$  and  $\left|\text{Im } C_{11}^d\right|^2$ , respectively. In the SM,  $C_{11}^d$  is divided into two parts according to the relevant CKM matrix elements as follows:

$$
C_{11}^d = V_{td} V_{ts}^* C_{11}^d(\text{top}) + V_{cd} V_{cs}^* C_{11}^d(\text{charm}). \tag{4.1}
$$

As discussed before, the SUSY contribution is proportional to  $V_{td}V_{ts}^*$ , therefore, we can write

$$
C_{11}^{d} \simeq V_{td} V_{ts}^{*} [C_{11}^{d}(\text{top}) + C_{11}^{d}(\text{SUSY})] + V_{cd} V_{cs}^{*} C_{11}^{d}(\text{charm}),
$$
\n(4.2)

where  $C_{11}^d$  (SUSY) is the SUSY contribution including the charged Higgs boson contribution. This kind of parametrization for  $K \rightarrow \pi \nu \bar{\nu}$  is considered in Ref. [11].

In Fig. 5 we show the branching ratio for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ normalized by the SM prediction as a function of the lighter chargino mass and the lighter top squark mass for tan  $\beta$ =2. Also, the correlation with the  $B(b \rightarrow s\gamma)$  is shown. In Figs. 5(a) and 5(b), we use the CLEO bound on  $B(b \rightarrow s \gamma)$  as a constraint on the SUSY parameter space. In order to take into account the theoretical ambiguity in a simple way, we allow a 10% uncertainty in the branching ratio and use (1.0  $\times 10^{-4}$ ) $\times$ 0.9 and  $(4.2\times10^{-4})$  $\times$ 1.1 as lower and upper bounds, respectively. Note that the ratio  $B(K_L)$  $\rightarrow \pi^0 \nu \bar{\nu}$ /*B*( $K_L \rightarrow \pi^0 \nu \bar{\nu}$ )<sub>SM</sub> does not depend on the CKM parameters because only the first term in Eq.  $(4.1)$  contributes to this process. We see that the branching ratio for  $K_L$  $\rightarrow \pi^0\nu\bar{\nu}$  becomes smaller than the SM prediction by 10%. In the minimal case the maximal deviation is within 3%. We investigated in which parameter region the maximal deviation is realized. We found that a large deviation occurs in the  $m_0 \approx 150$  GeV and  $\Delta_0 \approx 400$  GeV region which corresponds to the parameter region with  $m_{\chi_1^{\pm}}$ ,  $m_{\tilde{t}_1}^{\pm} \approx 100 \text{ GeV}$  shown in Fig. 1. From Fig.  $5(c)$  we can see that a sizable reduction of  $B(K_L \to \pi^0 \nu \bar{\nu})$  occurs when  $B(b \to s \gamma)$  becomes larger than the SM value. We also calculated  $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$  for different tan  $\beta$  and found that the deviation becomes smaller for a large tan  $\beta$ . For example, the maximal deviation is about 5% for tan  $\beta$ =30. As we can see in Eq. (4.2), the branching ratios of  $K^+\rightarrow \pi^+\nu\bar{\nu}$  and  $K_L\rightarrow \pi^0\nu\bar{\nu}$  have a strong correlation. We show the correlation for three different values of  $\delta_{13}$  in Fig. 6. In this figure we fix  $m_0 = 150$  GeV, but the correlation does not depend on the value of  $m_0$ . The deviation from the SM value for  $B(K^+\rightarrow \pi^+\nu\bar{\nu})$  is about 20% smaller than that for  $B(K_L \to \pi^0 \nu \bar{\nu})$ .

# **C.**  $B^0$ - $\overline{B}^0$  mixing and  $\epsilon_K$

Just as in the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  case, the  $B^0$ - $\overline{B}^0$  mass splittings  $\Delta m_B$  and  $\epsilon_K$  normalized to SM values are linearly correlated with each other as noted in  $[5,6]$ . We show the correlation for  $\delta_{13} = 30^{\circ}$ , 90°, and 150° in Fig. 7. We see that the deviation from the SM in  $\epsilon_K$  is about 80% of that in  $\Delta m_B$ . In the following, we only show the results for  $\Delta m_B$ , but the corresponding results on  $\epsilon_K$  can be easily obtained from Fig. 7. In Fig. 8 we show  $\Delta m_B$  normalized by the SM value as a function of the lighter chargino mass, the lighter top squark mass, and  $B(b \rightarrow s\gamma)$  for tan  $\beta=2$ . The deviation can be as large as 40% in the nonminimal case, whereas  $20\%$  in the minimal case. From Fig.  $8(b)$  we can see that a deviation larger than 20% is realized only in the nonminimal case when the top squark mass is smaller than 200 GeV. In this region  $B(b \rightarrow s\gamma)$  also deviates from the SM value significantly as shown in Fig. 8 $(c)$ . This result indicates the importance of further improvement of the  $B(b \rightarrow s\gamma)$ measurement and the top squark search. If the lower bound for the top squark mass is raised to 200 GeV, the maximal deviation of  $\Delta m_B$  is reduced to 25%. On the other hand, if the  $b \rightarrow s \gamma$  branching ratio turns out to be close to the present upper or lower bound,  $\Delta m_B$  and  $\epsilon_K$  might be significantly enhanced. We should notice that because the theoretical uncertainty is already reduced to the 10% level the experimental determination of  $B(b \rightarrow s\gamma)$  at that level will put a strong constraint on the SUSY parameter space. We also calculated  $\Delta m_B$  for tan  $\beta$ =30 and found that the deviation from the SM value is less than 10%. In Fig. 9 we show the correlation between  $B(K_L \to \pi^0 \nu \bar{\nu})$  and  $\Delta m_B$ . For tan  $\beta = 2$  we see a strong correlation between these two quantities: *B*(*KL*  $\rightarrow \pi^0 \nu \bar{\nu}$ ) is reduced by 10% when  $\Delta m_B$  is enhanced by 40%. We can also see the correlation for tan  $\beta$ =30. In this case  $\Delta m_B$  can be enhanced by 10% in the region where  $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$  is reduced by 5%.

#### **V. CONCLUSIONS AND DISCUSSION**

In this paper we have studied the FCNC processes of *B* and *K* mesons in the minimal supergravity model and in the supergravity model with an extended parameter space of the soft SUSY breaking parameters. We take into account the recent mass bounds for SUSY particles at LEP II and the next-to-leading order QCD corrections to various processes including  $b \rightarrow s \gamma$ .

We find that the branching ratio for  $b \rightarrow s l^+ l^-$  can be enhanced by about 50% compared to the SM value for a large tan  $\beta$  when the sign of  $C_7$  becomes opposite to that of the SM. For tan  $\beta=2$ , the  $b\rightarrow s\nu\bar{\nu}$ ,  $K^+\rightarrow \pi^+\nu\bar{\nu}$ , and  $K_L$  $\rightarrow \pi^0 \nu \bar{\nu}$  processes have similar SUSY contributions and it turns out that these branching ratios are reduced at most by 10% in the nonminimal case, whereas less than 3% in the minimal case. The  $B^0$ - $\overline{B}^0$  mixing and  $\epsilon_K$  are enhanced up to 40% from the SUSY contributions in the nonminimal case, whereas 20% in the minimal case. We investigate the correlation among  $\Delta m_B$ ,  $\epsilon_K$ , and  $B(K \rightarrow \pi \nu \bar{\nu})$  and found that a large deviation occurs when the chargino is lighter than 150 GeV and the top squark is lighter than 200 GeV. In the same parameter region,  $B(b \rightarrow s \gamma)$  is close to the upper or lower bound of the presently allowed region. For a large tan  $\beta$ , the deviations of  $\Delta m_B$ ,  $\epsilon_K$ , and  $B(K \rightarrow \pi \nu \bar{\nu})$  are smaller. In the minimal case these deviations are somewhat smaller than the previous calculation [6,9] especially for  $b \rightarrow s \nu \bar{\nu}$ . This is because the mass bounds for chargino, etc., have been improved by the LEP II experiments. We note that the maximal deviation depends on the light top squark mass bound. Therefore, the light top squark search in Tevatron experiments can reduce the possible parameter space where a large deviation from the SM value in FCNC processes is realized.

In this paper we extend the minimal supergravity model by introducing an additional parameter for the soft SUSY breaking term in the Higgs sector. This is not a unique way to extend the soft SUSY breaking terms. In order to avoid too large FCNCs, we only require that the squarks and sleptons in the same quantum numbers should have a common mass term at the Planck scale. Since the main difference is the change of the SUSY mass spectrum, a deviation with a similar magnitude is expected to be realized in a more general case as long as a light top squark and light chargino mass region is allowed.

In Ref.  $\begin{bmatrix} 34 \end{bmatrix}$  electroweak radiative corrections to *B*(*b*)  $\rightarrow$ *s* $\gamma$ ) are computed. They found that the fermion and photonic loop effects reduce the branching ratio by  $9\pm 2$  %. It is argued that the dominant contribution is due to the electric charge renormalization, and as a result, the electromagnetic coupling constant should be evaluated at  $q^2=0$ , i.e.,  $\alpha_{EM}^{-1}(0) = 137.036$ . Since we use  $\alpha_{EM}(m_b)$ , this correction reduces  $B(b \rightarrow s \gamma)$  by 3%.

Let us finally discuss the implications of these results when various information is obtained in future *B* and *K* decay experiments. First, since no new phase appears in  $M_{12}(B)$ , the CP asymmetry measured in the  $B^{0}(\overline{B}^{0})$  $\rightarrow$ *J*/ $\psi$ K<sub>s</sub> decay is directly related to the angle  $\phi_1$  $= \arg(-V_{td}^*V_{tb}/V_{cd}^*V_{cb})$  of the unitarity triangle. CP asymmetries in other *B* decay modes and the ratio of the  $\Delta m_B$ 's for  $B_s$  and  $B_d$  also provide information on the CKM matrix elements as in the SM. On the other hand,  $\lq\lq V_{td}\rq$  obtained from  $\Delta m_B$  and  $\epsilon_K$  may be different from that obtained above if we assume the SM analysis. In the same way " $|V_{td}|$ " from the branching ratios of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ may be different. As shown in Fig. 9, the SUSY contributions are constructive to the SM contribution in  $\Delta m_B$  ( $\epsilon_K$ ) and destructive in  $B(K \to \pi \nu \bar{\nu})$  so that the deviations of "  $|V_{td}|$ " from the true value become opposite. Therefore, combining CP asymmetry in *B* decay,  $\Delta m_{B_s}$ , and various FCNC observables in *B* and *K* decays, we may obtain a hint of the existence of SUSY particles.

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