Nonfactorizable effects in spectator and penguin amplitudes of hadronic charmless B decays

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Nonfactorizable effects in hadronic charmless $B \to PP, VP$ decays can be parametrized in terms of the effective number of colors, N_c^{eff}, in the effective parameters a_i^{eff} that are linear combinations of Wilson coefficients. It is shown that $N_c^{\text{eff}}(V+A)$ in the penguin amplitudes induced by the $(V-A)(V+A)$ four-quark operators is different from $N_c^{\text{eff}}(V-A)$ in the decay amplitudes arising from the $(V-A)(V-A)$ operators. Central values of the branching ratios for $B^\pm \to \omega \pi^\pm$ and $B \to \pi \pi$ decays favor $N_c^{\text{eff}}(V-A) \approx 2$, in accordance with the nonfactorizable effect observed in $B \to D^{(*)} \pi(\rho)$. Measurements of the interference effects in $B^- \to \pi^-(\rho^-) \pi^0(\rho^0)$ decays will provide a more decisive test of the parameter $N_c^{\text{eff}}(V-A)$. However, $N_c^{\text{eff}}(V+A) \sim 2$ is ruled out by $B^\pm \to \phi K^\pm$. We find that the current bound on $B^\pm \to \phi K^\pm$ implies $N_c^{\text{eff}}(V+A) \gtrsim 4.3$, which is subject to the corrections from W-annihilation and spacelike penguin effects. With $N_c^{\text{eff}}(V-A) \approx 2$ we show that the branching ratio of $B \to \eta' K$ is enhanced considerably at small values of $1/N_c^{\text{eff}}(V+A)$ so that it is compatible with experiment. In particular, the measurement of $B^0 \to \eta' K^0$ is now well explained without resorting to any new mechanism or new physics beyond the standard model. It is crucial to measure the charged and neutral decay modes of $B \to \phi K$ and $B \to \phi K^*$ to test the generalized factorization hypothesis. Finally, we point out that it is difficult to understand the observed large branching ratio of $B^\pm \to \omega K^\pm$ within the present framework. Inelastic final-state interactions may alleviate the difficulty with this decay mode. [S0556-2821(98)06117-7]

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I. INTRODUCTION

To describe the hadronic weak decays of mesons, the mesonic matrix elments are customarily evaluated under the factorization hypothesis so that they are factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. In the naive factorization approach, the relevant Wilson coefficient functions for color-allowed external W-emission (or so-called ''class-I'') and color-suppressed (class-II) internal W-emission amplitudes are given by $a_1 = c_1 + c_2/N_c$, $a_2 = c_2 + c_1/N_c$, respectively, with N_c the number of colors. In spite of its tremendous simplicity, naive factorization encounters two major difficulties. First, it never works for the decay rate of class-II decay modes, though it usually operates for a class-I transition. For example, the predicted decay rate of the color-suppressed decay $D^0 \to \bar{K}^0 \pi^0$ in the naive approach is too small when compared with experiment (for a review, see [1]). Second, the hadronic matrix element under factorization is renormalization scale μ independent as the vector or axial-vector current is partially conserved. Consequently, the amplitude $c_i(\mu)\langle O \rangle_{\text{fact}}$ is not truly physical as the scale dependence of Wilson coefficients does not get compensation from the matrix elements. The first difficulty indicates that it is inevitable and mandatory to take into account nonfactorizable contributions, especially for class-II decays, to render the color suppression of internal W emission ineffective. The second difficulty also should not occur since the matrix elements of four-quark operators ought to be evaluated in the same renormalization scheme as that for Wilson coefficients

and renormalized at the same scale μ.

Because there is only one single form factor (or Lorentz scalar) involved in the class-I or class-II decay amplitude of $B(D) \to PP, PV$ decays (P is the pseudoscalar meson; V is the vector meson), the effects of nonfactorization can be lumped into the effective parameters a_1 and a_2 [2]:[1]

$$a_1^{\text{eff}} = c_1 + c_2\left(\frac{1}{N_c} + \chi_1\right), \quad a_2^{\text{eff}} = c_2 + c_1\left(\frac{1}{N_c} + \chi_2\right), \tag{1.1}$$

where $c_{1,2}$ are the Wilson coefficients of the spectator four-quark operators, and nonfactorizable contributions are characterized by the parameters χ_1 and χ_2. Taking the decay $B^- \to D^0 \pi^-$ as an example, we have [4–6]

$$\chi_1 = \varepsilon_8^{(BD, \pi)} + \frac{a_1}{c_2}\varepsilon_1^{(BD, \pi)}, \quad \chi_2 = \varepsilon_8^{(B\pi, D)} + \frac{a_2}{c_1}\varepsilon_1^{(B\pi, D)}, \tag{1.2}$$

where

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[1]As pointed out in [3], the general amplitude of $B(D) \to VV$ decay consists of three independent Lorentz scalars, corresponding to S-, P-, and D-wave amplitudes. Consequently, it is in general not possible to define an effective a_1 or a_2 unless nonfactorizable terms contribute in equal weight to all partial-wave amplitudes.

$$\varepsilon_1^{(BD,\pi)} = \frac{\langle D^0\pi^-|(\bar{d}u)_{V-A}(\bar{c}b)_{V-A}|B^-\rangle_{nf}}{\langle D^0\pi^-|(\bar{d}u)_{V-A}(\bar{c}b)_{V-A}|B^-\rangle_f}$$

$$= \frac{\langle D^0\pi^-|(\bar{d}u)_{V-A}(\bar{c}b)_{V-A}|B^-\rangle}{\langle \pi^-|(\bar{d}u)_{V-A}|0\rangle\langle D^0|(\bar{c}b)_{V-A}|B^-\rangle} - 1,$$

$$\varepsilon_8^{(BD,\pi)} = \frac{1}{2}\frac{\langle D^0\pi^-|(\bar{d}\lambda^a u)_{V-A}(\bar{c}\lambda^a b)_{V-A}|B^-\rangle}{\langle \pi^-|(\bar{d}u)_{V-A}|0\rangle\langle D^0|(\bar{c}b)_{V-A}|B^-\rangle} \quad (1.3)$$

are nonfactorizable terms originated from color-singlet and color-octet currents, respectively, $(\bar{q}_1 q_2)_{V-A} \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ and $(\bar{q}_1\lambda^a q_2)_{V-A} \equiv \bar{q}_1\lambda^a\gamma_\mu(1-\gamma_5)q_2$. The subscripts f and nf in Eq. (1.3) stand for factorizable and nonfactorizable contributions, respectively, and the superscript (BD,π) in Eq. (1.2) means that the pion is factored out in the factorizable amplitude of $B \to D\pi$ and likewise for the superscript $(B\pi, D)$. In the large-N_c limit, $\varepsilon_1 = \mathcal{O}(1/N_c^2)$ and $\varepsilon_8 = \mathcal{O}(1/N_c)$ [6]. Therefore, the nonfactorizable term χ in the $N_c \to \infty$ limit is dominated by color octet-octet operators. Since $|c_1/c_2| \gg 1$, it is evident from Eq. (1.1) that even a small amount of nonfactorizable contributions will have a significant effect on the color-suppressed class-II amplitude. If $\chi_{1,2}$ are universal (i.e., process independent) in charm or bottom decays, then we still have a new factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters $a_{1,2}^{\text{eff}}$. (For $B \to VV$ decays, new factorization implies that nonfactorizable terms contribute in equal weight to all partial-wave amplitudes so that $a_{1,2}^{\text{eff}}$ *can* be defined.) The first systematical study of nonleptonic weak decays of heavy mesons within the framework of the generalized factorization was carried out by Bauer, Stech, and Wirbel [7]. Phenomenological analyses of two-body decay data of D and B mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters $a_{1,2}^{\text{eff}}$ do show some variation from channel to channel, especially for the weak decays of charmed mesons [2,5,8]. An eminent feature emerging from the data analysis is that a_2^{eff} is negative in charm decay, whereas it becomes positive in the two-body decays of the B meson [2,9,6]:

$$a_2^{\text{eff}}(D \to \bar{K}\pi) \sim -0.50, \quad a_2^{\text{eff}}(B \to D\pi) \sim 0.26. \quad (1.4)$$

It should be stressed that since the magnitude of $a_{1,2}$ depends on the model results for form factors, the above values of a_2 should be considered as representative ones. The sign of a_2^{eff} is fixed by the observed destructive interference in $D^+ \to \bar{K}^0\pi^+$ and constructive interference in $B^- \to D^0\pi^-$. Equation (1.4) then leads to

$$\chi_2(\mu \sim m_c; D \to \bar{K}\pi) \sim -0.36,$$

$$\chi_2(\mu \sim m_b; B \to D\pi) \sim 0.11. \quad (1.5)$$

In general the determination of χ_2 is easier and more reliable than χ_1. The observation $|\chi_2(B)| \ll |\chi_2(D)|$ is consistent with the intuitive picture that soft gluon effects become stronger when the final-state particles move slower, allowing more time for significant final-state interactions after hadronization [2].

Phenomenologically, the number of colors N_c is often treated as a free parameter and fit to the data. Theoretically, this amounts to defining an effective number of colors, N_c^{eff}, called $1/\xi$ in [7], by

$$1/N_c^{\text{eff}} \equiv (1/N_c) + \chi. \quad (1.6)$$

It is clear from Eq. (1.5) that

$$N_c^{\text{eff}}(D \to \bar{K}\pi) \gg 3, \quad N_c^{\text{eff}}(B \to D\pi) \approx 2. \quad (1.7)$$

Consequently, the empirical rule of discarding subleading $1/N_c$ terms formulated in the large-N_c approach [10] is justified for exclusive charm decay; the dynamical origin of the $1/N_c$ expansion comes from the fact that the Fierz $1/N_c$ terms are largely compensated by nonfactorizable effects in charm decay. Since the large-N_c approach implies $a_2^{\text{eff}} \sim c_2$ and since a_2^{eff} is observed to be positive in $B^- \to D^0(\pi^-, \rho^-)$ decays, one may wonder why the $1/N_c$ expansion is no longer applicable to the B meson. Contrary to common belief, a careful study shows this is not the case. As pointed out in [6], the large-N_c color counting rule for the Wilson coefficient $c_2(\mu)$ is different at $\mu \sim m_b$ and $\mu \sim m_c$ due to the presence of the large logarithm at $\mu \sim m_c$. More specifically, $c_2(m_b) = \mathcal{O}(1/N_c)$ and $c_2(m_c) = \mathcal{O}(1)$. Recalling that $c_1 = \mathcal{O}(1)$, it follows that, in the large-N_c limit [6],

$$a_2^{\text{eff}} = \begin{cases} c_2(m_c) + \mathcal{O}(1/N_c) & \text{for the } D \text{ meson,} \\ c_2(m_b) + c_1(m_b)\left(\dfrac{1}{N_c} + \varepsilon_8(m_b)\right) + \mathcal{O}(1/N_c^3) & \text{for the } B \text{ meson.} \end{cases} \quad (1.8)$$

Therefore, *a priori* the $1/N_c$ expansion does not demand a negative a_2^{eff} for bottom decay and $N_c^{\text{eff}}(B \to D\pi) \sim 2$ is not in conflict with the large-N_c approach. It should be remarked that although χ_2 is positive in two-body decays of the B meson, a theoretical argument suggests that it may become negative in high multiplicity decay modes [6].

Thus far the nonfactorizable effect is discussed at the purely phenomenological level. It is thus important to have a

theoretical estimate of χ_i even approximately. Unfortunately, all existing theoretical calculations based on the QCD sum rule [11], though they confirm the cancellation between the $1/N_c$ Fierz terms and nonfactorizable soft gluon effects [12], tend to predict a negative χ in $\bar{B}^0 \to D^+\pi^-, D^0\pi^0$ and $B \to J/\psi K$ decays. This tantalizing issue should be clarified and resolved in the near future. It is interesting to remark that, relying on a different approach, namely, the three-scale perturbative QCD (PQCD) factorization theorem, to tackle the nonfactorizable effect, Li and one of us [13] are able to explain the sign change of χ_2 from bottom to charm decays.

For B meson decay, the effective parameters $a_{1,2}^{\text{eff}}$ have been determined so far only for $B \to D(\pi,\rho)$ and $B \to J/\psi K$ where nonfactorizable effects amount to having $N_c^{\text{eff}} \sim 2$. Recently, several exclusive charmless rare B decay modes have been reported for the first time by CLEO [14–19] and many of them are dominated by the penguin mechanism. It is thus important to know (i) if the constructive interference of tree amplitudes persists in class-III charmless B decay (class-III transitions receive contributions from both external and internal W emissions) and (ii) if N_c^{eff} is the same in spectator and penguin amplitudes. In the literature it is customary to assume that N_c^{eff} behaves in the same way in the penguin and nonpenguin amplitudes. The decay rate of the rare B decays is then studied as a function of $1/N_c^{\text{eff}}$. In the present paper, we shall see that, theoretically and experimentally, $N_c^{\text{eff}}(V+A)$ in the penguin amplitude induced by the $(V-A)(V+A)$ quark operators is different from $N_c^{\text{eff}}(V-A)$ in the tree or penguin amplitude induced by the $(V-A)(V-A)$ operators.[2] We find that $N_c^{\text{eff}}(V+A)$ in penguin-dominated charmless B decays is clearly larger than $N_c^{\text{eff}}(V-A)$ extracted from spectator-dominated processes. Therefore, the nonfactorizable effect in tree and penguin diagrams behaves in a different manner. This observation is the key element for understanding the CLEO measurement of $B^\pm \to \eta' K^\pm$ and $B^0 \to \eta' K^0$. By treating $N_c^{\text{eff}}(V-A)$ and $N_c^{\text{eff}}(V+A)$ differently, the data of $B \to \eta' K$ can be explained in the framework of the standard model without resorting to new mechanisms or new physics beyond the standard model.

This paper is organized as follows. In Sec. II we sketch the starting point of the effective Hamiltonian and emphasize that vertex and penguin corrections to the four-quark operators should be combined together with the Wilson coefficients to render the resulting physical amplitude independent of the choice of the renormalization scheme and scale. Then we extract the information of $N_c^{\text{eff}}(V-A)$ from spectator-dominated charmless B decays $B^\pm \to \omega\pi^\pm$ and $B \to \pi\pi$ in Sec. III and $N_c^{\text{eff}}(V+A)$ from the penguin-dominated process $B \to \phi K$ in Sec. IV. In Sec. V we demonstrate that the measurement of $B \to \eta' K$ also favors a different treatment of $N_c^{\text{eff}}(V-A)$ and $N_c^{\text{eff}}(V+A)$. In Sec. VI we point out a difficulty with $B^\pm \to \omega K^\pm$ within the present framework. Conclusions and discussions are presented in Sec. VII.

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[2]In [20] we have assumed that $N_c^{\text{eff}}(V-A) \approx N_c^{\text{eff}}(V+A) \approx 2$. The present paper is an improved version of [20].

II. CALCULATIONAL FRAMEWORK

We briefly sketch in this section the calculational framework. The relevant effective $\Delta B = 1$ weak Hamiltonian is

$$\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{uq}^*(c_1 O_1^u + c_2 O_2^u) \right.$$

$$+ V_{cb}V_{cq}^*(c_1 O_1^c + c_2 O_2^c)$$

$$\left. - V_{tb}V_{tq}^* \sum_{i=3}^{10} c_i O_i \right] + \text{H.c.}, \qquad (2.1)$$

where $q = d, s,$ and

$$O_1^u = (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, \quad O_2^u = (\bar{q}b)_{V-A}(\bar{u}u)_{V-A},$$

$$O_{3(5)} = (\bar{q}b)_{V-A}\sum_{q'}(\bar{q}'q')_{V-A(V+A)},$$

$$O_{4(6)} = (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A(V+A)},$$

$$O_{7(9)} = \frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'}e_{q'}(\bar{q}'q')_{V+A(V-A)},$$

$$O_{8(10)} = \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A(V-A)},$$

$$(2.2)$$

with $O_3 - O_6$ being the QCD penguin operators and $O_7 - O_{10}$ the electroweak penguin operators. As noted in passing, in order to ensure the renormalization-scale and -scheme independence for the physical amplitude, the matrix elements of four-quark operators have to be evaluated in the same renormalization scheme as that for Wilson coefficients and renormalized at the same scale μ.

In full theory, the leading QCD correction to the weak transition is of the form $\alpha_s \ln(M_W^2/-p^2)$ for massless quarks, where p is the off-shell momentum of external quark lines and its magnitude $-p^2$ depends on the system under consideration. For example, $-p^2 \sim m_b^2$ in the energetic two-body charmless B decays. The merit of the effective Hamiltonian approach is that one can choose a renormalization scale μ so that the leading logarithmic correction $\ln(M_W^2/-p^2) = \ln(M_W^2/\mu^2) + \ln(\mu^2/-p^2)$ is decomposed in such a way that the large logarithmic term $\ln(M_M^2/\mu^2)$ is lumped into the Wilson coefficient function $c(\mu)$ and summed over to all orders in α_s using the renormalization group equation, while the logarithmic correction $\ln(\mu^2/-p^2)$ to the matrix element $\langle O(\mu) \rangle$ is small (for a review, see [26]). Since $O(\mu)$ is the four-quark operator renormalized at the scale μ, its hadronic matrix element is related to the tree level one via

$$\langle O(\mu) \rangle = g(\mu)\langle O \rangle_{\text{tree}}, \qquad (2.3)$$

with

$$g(\mu) \sim 1 + \alpha_s(\mu)\left(\gamma \ln\frac{\mu^2}{-p^2} + c\right) \quad (2.4)$$

for current-current operators, where we have included the nonlogarithmic constant contribution c since the logarithmic contribution $\ln(\mu^2/-p^2)$ is small when $\mu^2 \sim -p^2$ and hence the momentum-independent constant term cannot be neglected. It follows that, schematically,

$$\langle \mathcal{H}_{\text{eff}}\rangle = c(\mu)g(\mu)\langle O\rangle_{\text{tree}} = c^{\text{eff}}\langle O\rangle_{\text{tree}}. \quad (2.5)$$

To the next-to-leading order (NLO), $c(\mu)$ depends on the renormalization scheme chosen and so does the constant c in $g(\mu)$. However, the effective Wilson coefficient c^{eff} is independent of the choice of the renormalization scheme and scale. It should be stressed that, except for the lattice QCD, model calculations of the hadronic matrix elements are actually performed for $\langle O\rangle_{\text{tree}}$ rather than for $\langle O(\mu)\rangle$. (Quark model calculation of $\langle O\rangle_{\text{tree}}$, for example, may involve an implicit low energy scale, but it has nothing to do with the renormalization scale μ.) For example, in the factorization approximation, the matrix element $\langle O\rangle_{\text{fact}}$ is scale independent and hence it cannot be identified with $\langle O(\mu)\rangle$. Therefore, it is important to evaluate $g(\mu)$, the perturbative corrections to the four-quark operators at the scale μ.

As emphasized above, before applying factorization or carrying out any model calculation of hadronic matrix elements, it is necessary to incorporate QCD and electroweak corrections to the operators

$$\langle O_i(\mu)\rangle = \left[I + \frac{\alpha_s(\mu)}{4\pi}\hat{m}_s(\mu) + \frac{\alpha}{4\pi}\hat{m}_e(\mu)\right]_{ij}\langle O_j\rangle_{\text{tree}}, \quad (2.6)$$

so that $c_i(\mu)\langle O_i(\mu)\rangle = c_i^{\text{eff}}\langle O_i\rangle_{\text{tree}}$, where

$$c_i^{\text{eff}} = \left[I + \frac{\alpha_s(\mu)}{4\pi}\hat{m}_s^T(\mu) + \frac{\alpha}{4\pi}\hat{m}_e^T(\mu)\right]_{ij}c_j(\mu). \quad (2.7)$$

Then the factorization approximation is applied to the hadronic matrix elements of the operator O at the tree level. Perturbative QCD and electroweak corrections to the matrices $\hat{m}_s$ and $\hat{m}_e$ from vertex diagrams and penguin diagrams have been calculated in [21–24]. It should be remarked that although the penguin coefficients $c_3 - c_{10}$ are governed by penguin diagrams with t quark exchange, the effective Wilson coefficients do incorporate the effects of the penguin diagrams with internal u and c quarks induced by the current-current operator O_1. For example [24],

$$c_6^{\text{eff}} = c_6(\mu) - \frac{\alpha_s(\mu)}{8\pi}\left[\frac{\lambda_c}{\lambda_t}\tilde{G}(m_c,k,\mu) + \frac{\lambda_u}{\lambda_t}\tilde{G}(m_u,k,\mu)\right]$$

$$\times c_1(\mu) + \cdots, \quad (2.8)$$

where $\lambda_i = V_{ib}V_{iq}^*(q=d,s)$, $\tilde{G}(m_q,k,\mu) = \frac{2}{3}\kappa - G(m_q,k,\mu)$, κ is a constant depending on the renormalization scheme, k is the gluon's virtual momentum, and

$$G(m,k,\mu) = -4\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2-k^2x(1-x)}{\mu^2}\right). \quad (2.9)$$

For $b \rightarrow s$ transitions, $|\lambda_u| \ll |\lambda_t|$, $\lambda_c \sim -\lambda_t$, and hence

$$c_6^{\text{eff}} = c_6(\mu) + \frac{\alpha_s(\mu)}{8\pi}\tilde{G}(m_c,k,\mu)c_1(\mu) + \cdots. \quad (2.10)$$

The importance of the so-called "charming" penguin diagrams for $b \rightarrow s$ transition was emphasized recently (and probably overemphasized) in [25].

Using the next-to-leading order $\Delta B = 1$ Wilson coefficients obtained in the 't Hooft–Veltman (HV) scheme and the naive dimension regularization (NDR) scheme at $\mu = m_b(m_b)$, $\Lambda_{\overline{\text{MS}}}^{(5)} = 225$ MeV and $m_t = 170$ GeV in Table 22 of [26], we obtain the effective renormalization-scheme- and -scale-independent Wilson coefficients c_i^{eff} at $k^2 = m_b^2/2$:[3]

$$c_1^{\text{eff}} = 1.149, \quad c_2^{\text{eff}} = -0.325,$$

$$c_3^{\text{eff}} = 0.0211 + i0.0045, \quad c_4^{\text{eff}} = -0.0450 - i0.0136,$$

$$c_5^{\text{eff}} = 0.0134 + i0.0045, \quad c_6^{\text{eff}} = -0.0560 - i0.0136,$$

$$c_7^{\text{eff}} = -(0.0276 + i0.0369)\alpha, \quad c_8^{\text{eff}} = 0.054\alpha,$$

$$c_9^{\text{eff}} = -(1.318 + i0.0369)\alpha, \quad c_{10}^{\text{eff}} = 0.263\alpha. \quad (2.11)$$

Two important remarks are in order. First of all, $c_{1,2}^{\text{eff}}$ are surprisingly very close to the leading order Wilson coefficients, $c_1^{\text{LO}} = 1.144$ and $c_2^{\text{LO}} = -0.308$ at $\mu = m_b(m_b)$ [26], recalling that $c_2^{\text{NDR}} = -0.185$ and $c_2^{\text{HV}} = -0.228$ at NLO [26] deviate substantially from the leading order values. This means that $\langle O_{1,2}(\mu)\rangle \approx \langle O_{1,2}\rangle_{\text{tree}}$. Hence, it explains why the conventional way of applying the Wilson coefficients at leading order and evaluating the matrix elements of current-current operators at the tree level is "accidentally" justified provided that $\mu^2 \sim -p^2$. Second, comparing Eqs. (2.11) with the leading order penguin coefficients [26]

$$c_3^{\text{LO}} = 0.014, \quad c_4^{\text{LO}} = -0.030, \quad c_5^{\text{LO}} = 0.009, \quad c_6^{\text{LO}} = -0.038 \quad (2.12)$$

at $\mu = m_b(m_b)$, we see that $\text{Re}(c_{3-6}^{\text{eff}}) \approx \frac{3}{2}c_{3-6}^{\text{LO}}(\mu)$. This implies that, contrary to the case of current-current operators, penguin corrections to the current-current operators give important contributions to the QCD penguin operators. This means that the decay rates of charmless B decay modes dominated by penguin diagrams will be too small by a factor of $\sim(1.5)^2 = 2.3$ if only leading order penguin coefficients are employed for calculation.

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[3]We use the complete expressions of $\hat{m}_s(\mu)$ given in [24] and $\hat{m}_e(\mu)$ in [22] to evaluate c_i^{eff}. Note that while our c_{1-6}^{eff} are consistent with the numerical results given in [24,27], our values for c_{7-10}^{eff} are different from those shown in [27].

We shall see later that running quark masses appear in the matrix elements of $(S-P)(S+P)$ penguin operators through the use of equations of motion. The running quark mass should be applied at the scale $\mu \sim m_b$ because the energy released in the energetic two-body charmless decays of the B meson is of order m_b. Explicitly, we use [28]

$$m_u(m_b) = 3.2 \text{ MeV}, \quad m_d(m_b) = 6.4 \text{ MeV},$$

$$m_s(m_b) = 105 \text{ MeV}, \quad m_c(m_b) = 0.95 \text{ GeV},$$

$$m_b(m_b) = 4.34 \text{ GeV}, \quad (2.13)$$

in the ensuing calculation, where we have applied $m_s = 150$ MeV at $\mu = 1$ GeV.

It is convenient to parametrize the quark mixing matrix in terms of the Wolfenstein parameters A, λ, ρ, and η [29], where $A = 0.804$ and $\lambda = 0.22$. In the present paper we employ two representative values for ρ and η: (i) $\rho = 0.16$, $\eta = 0.34$ and (ii) $\rho = -0.12$, $\eta = 0.35$. Both of them satisfy the constraint $\sqrt{\rho^2 + \eta^2} = 0.37$. A recent analysis of all available experimental constraints imposed on the Wolfenstein parameters yields [30]

$$\bar{\rho} = 0.156 \pm 0.090, \quad \bar{\eta} = 0.328 \pm 0.054, \quad (2.14)$$

where $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$, and it implies that the negative ρ region is excluded at 93% C.L.

III. NONFACTORIZABLE EFFECTS IN SPECTATOR AMPLITUDES

The combinations of the effective Wilson coefficients $a_{2i} = c_{2i}^{\text{eff}} + (1/N_c) c_{2i-1}^{\text{eff}}$, $a_{2i-1} = c_{2i-1}^{\text{eff}} + (1/N_c) c_{2i}^{\text{eff}}$ ($i = 1, \ldots, 5$) appear in the decay amplitudes. As discussed in the Introduction, nonfactorizable effects in the decay amplitudes of $B \to PP, VP$ can be absorbed into the parameters a_i^{eff}. This amounts to replacing N_c in a_i by $(N_c^{\text{eff}})_i$. (It must be emphasized that the factor of N_c appearing in any place other than a_i should *not* be replaced by N_c^{eff}.) Explicitly,

$$a_{2i}^{\text{eff}} = c_{2i}^{\text{eff}} + \frac{1}{(N_c^{\text{eff}})_{2i}} c_{2i-1}^{\text{eff}}, \quad a_{2i-1}^{\text{eff}} = c_{2i-1}^{\text{eff}} + \frac{1}{(N_c^{\text{eff}})_{2i-1}} c_{2i}^{\text{eff}}. \quad (3.1)$$

It is customary to assume in the literature that $(N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \cdots \approx (N_c^{\text{eff}})_{10}$ so that the subscript i can be dropped. A closer investigation shows that this is not the case. Consider an operator of the form $O = \bar{q}_1^\alpha \Gamma q_2^\beta \, \bar{q}_3^\beta \Gamma' q_4^\alpha$ which arises from the Fierz transformation of a singlet-singlet operator with Γ and Γ' being some combinations of Dirac matrices. Applying the identity

$$O = \frac{1}{3} \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 + \frac{1}{2} \bar{q}_1 \lambda^a \Gamma q_2 \, \bar{q}_3 \lambda^a \Gamma' q_4 \quad (3.2)$$

to the matrix element of $M \to P_1 P_2$ leads to (assuming the quark content $\bar{q}_1 q_2$ for P_1)

$$\langle P_1 P_2 | O | M \rangle = \frac{1}{3} \langle P_1 | \bar{q}_1 \Gamma q_2 | 0 \rangle \langle P_2 | \bar{q}_3 \Gamma' q_4 | M \rangle$$

$$+ \frac{1}{3} \langle P_1 P_2 | \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 | M \rangle_{nf}$$

$$+ \frac{1}{2} \langle P_1 P_2 | \bar{q}_1 \lambda^a \Gamma q_2 \, \bar{q}_3 \lambda^a \Gamma' q_4 | M \rangle. \quad (3.3)$$

The nonfactorizable effects due to octet-octet and singlet-singlet operators are characterized by the parameters ε_8 and ε_1, respectively, as shown in Eq. (1.3):

$$\varepsilon_1 = \frac{\langle P_1 P_2 | \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 | M \rangle_{nf}}{\langle P_1 P_2 | \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 | M \rangle_f}$$

$$\times \frac{\langle P_1 P_2 | \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 | M \rangle_f}{\langle P_1 P_2 | (\bar{q}_1 q_2)_{V-A} (\bar{q}_3 q_4)_{V-A} | M \rangle_f},$$

$$\varepsilon_8 = \frac{1}{2} \frac{\langle P_1 P_2 | \bar{q}_1 \lambda^a \Gamma q_2 \, \bar{q}_3 \lambda^a \Gamma' q_4 | M \rangle}{\langle P_1 P_2 | \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 | M \rangle_f}$$

$$\times \frac{\langle P_1 P_2 | \bar{q}_1 \Gamma q_2 \, \bar{q}_3 \Gamma' q_4 | M \rangle_f}{\langle P_1 P_2 | (\bar{q}_1 q_2)_{V-A} (\bar{q}_3 q_4)_{V-A} | M \rangle_f}. \quad (3.4)$$

However, the Fierz transformation of the $(V-A)(V+A)$ operators $O_{5,6,7,8}$ is quite different from that of $(V-A)(V-A)$ operators $O_{1,2,3,4}$ and $O_{9,10}$; that is,

$$(V-A)(V+A) \to -2(S-P)(S+P),$$
$$(V-A)(V-A) \to (V-A)(V-A). \quad (3.5)$$

Therefore, Γ and Γ' are the combinations of the Dirac matrices 1 and γ_5 for the Fierz transformation of $(V-A)(V+A)$ operators and the combinations of γ_μ and $\gamma_\mu \gamma_5$ for $(V-A)(V-A)$ operators. As a result, nonfactorizable effects in the matrix elements of $(V-A)(V+A)$ operators are *a priori* different from that of $(V-A)(V-A)$ operators, i.e., $\chi(V+A) \neq \chi(V-A)$. Since $1/N_c^{\text{eff}} = 1/N_c + \chi$ [cf. Eq. (1.6)], theoretically it is expected that

$$N_c^{\text{eff}}(V-A) \equiv (N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \approx (N_c^{\text{eff}})_3 \approx (N_c^{\text{eff}})_4$$
$$\approx (N_c^{\text{eff}})_9 \approx (N_c^{\text{eff}})_{10},$$

$$N_c^{\text{eff}}(V+A) \equiv (N_c^{\text{eff}})_5 \approx (N_c^{\text{eff}})_6 \approx (N_c^{\text{eff}})_7 \approx (N_c^{\text{eff}})_8, \quad (3.6)$$

and $N_c^{\text{eff}}(V+A) \neq N_c^{\text{eff}}(V-A)$ in general. In principle, N_c^{eff} can vary from channel to channel, as in the case of charm decay. However, in energetic two-body B decays, N_c^{eff} is expected to be process insensitive as supported by data [6]. As stressed in the Introduction, if N_c^{eff} is process independent, then we have a generalized factorization. Contrary to the naive one, the improved factorization does incorporate

nonfactorizable effects in a process-independent form. For example, $\chi_1 = \chi_2 = -\frac{1}{3}$ in the large-N_c approximation of factorization.

The unknown parameters $(N_c^{\text{eff}})_i$ in charmless B decays in principle can be determined if the decay rates are measured for a handful of decay modes with sufficient accuracy. Because of the limited data and limited significance available at present, we shall use Eqs. (3.6) and the experimental result for $N_c^{\text{eff}}(B \to D\pi)$ as a guidance to determine $(N_c^{\text{eff}})_i$. To begin with, we focus in this section on the decay modes

dominated by the spectator diagrams induced by the current-current operators O_1 and O_2. In particular, we would like to study these modes which are sensitive to the interference between external and internal W-emission amplitudes. The fact that $N_c^{\text{eff}} < 3.5$ ($N_c^{\text{eff}} > 3.5$) implies a positive (negative) a_2^{eff} and hence a constructive (destructive) interference will enable us to differentiate between them. Good examples are the class-III modes $B^{\pm} \to \omega\pi^{\pm}, \pi^0\pi^{\pm}, \eta\pi^{\pm}, \pi^0\rho^{\pm}, \ldots,$ etc.

We first consider the decay $B^- \to \omega\pi^-$. Under the generalized factorization, its decay amplitude is given by

$$A(B^- \to \omega\pi^-) = \frac{G_F}{\sqrt{2}} \Bigg\{ V_{ub} V_{ud}^* (a_1 X^{(B\omega,\pi)} + a_2 X^{(B\pi,\omega)} + 2a_1 X^{(B,\pi\omega)})$$

'

$$- V_{tb} V_{td}^* \Bigg[\left(a_4 + a_{10} - 2(a_6+a_8) \frac{m_\pi^2}{(m_b+m_u)(m_u+m_d)} \right) X^{(B\omega,\pi)}$$

$$+ \frac{1}{2}(4a_3 + 2a_4 + 4a_5 + a_7 + a_9 - a_{10}) X^{(B\pi,\omega)}$$

$$+ 2 \left(a_4 + a_{10} - 2(a_6+a_8) \frac{m_B^2}{(m_b+m_u)(m_u+m_d)} \right) X^{(B,\pi\omega)} \Bigg] \Bigg\}, \qquad (3.7)$$

where we have dropped the superscript "eff" for convenience, and the notation $X^{(B\omega,\pi)}$, for example, denotes the factorization amplitude with the π meson being factored out:

$$X^{(B\omega,\pi)} \equiv \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle \langle \omega | (\bar{u}b)_{V-A} | B^- \rangle,$$

$$X^{(B\pi,\omega)} \equiv \langle \omega | (\bar{u}u)_{V-A} | 0 \rangle \langle \pi^- | (\bar{d}b)_{V-A} | B^- \rangle,$$

$$X^{(B,\pi\omega)} \equiv \langle \pi^- \omega | (\bar{d}u)_{V-A} | 0 \rangle \langle 0 | (\bar{u}b)_{V-A} | B^- \rangle. \qquad (3.8)$$

Note that in the penguin amplitude, the term $X^{(B,\pi\omega)}$ arises from the spacelike penguin diagram. Using the following parametrization for decay constants and form factors,[4]

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[4]Once the one-body matrix elements are defined, one can apply heavy-quark symmetry to the two-body matrix elements for heavy-to-heavy transition to show that all the form factors defined in Eqs. (3.9) are positively defined at $q^2 \geq 0$ and that the relative signs between two-body and one-body matrix elements are fixed. In this way, we find that the vector form factor $V(q^2)$ defined by Wirbel, Stech, and Bauer [31] has a sign opposite to ours. Note that our convention is $\epsilon_{0123} = 1$.

$$\langle 0 | A_\mu | P(q) \rangle = i f_P q_\mu, \quad \langle 0 | V_\mu | V(p,\varepsilon) \rangle = f_V m_V \varepsilon_\mu,$$

$$\langle P'(p') | V_\mu | P(p) \rangle = \left(p_\mu + p'_\mu - \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu \right) F_1(q^2)$$

$$+ F_0(q^2) \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu,$$

$$\langle V(p',\varepsilon) | V_\mu | P(p) \rangle = \frac{2}{m_P + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta V(q^2),$$

$$\langle V(p',\varepsilon) | A_\mu | P(p) \rangle = i \Bigg[(m_P + m_V) \varepsilon_\mu A_1(q^2)$$

$$- \frac{\varepsilon \cdot p}{m_P + m_V} (p+p')_\mu A_2(q^2)$$

$$- 2m_V \frac{\varepsilon \cdot p}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] \Bigg],$$

$$\qquad (3.9)$$

where $q = p - p'$, $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$, and

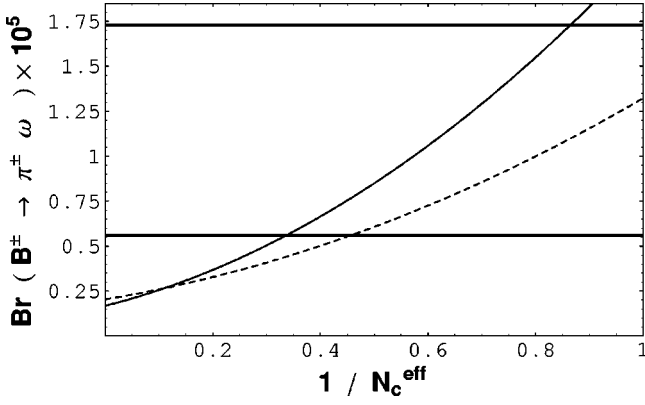$$A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2), \quad (3.10)$$

we obtain

FIG. 1. The branching ratio of $B^{\pm} \to \omega \pi^{\pm}$ vs $1/N_c^{\text{eff}}$. The solid and dashed curves are for $\eta = 0.34$, $\rho = 0.16$ and $\eta = 0.35$, $\rho = -0.12$, respectively. The solid thick lines are the CLEO measurements with one sigma errors.

$$X^{(B\omega,\pi)} = -2f_{\pi} m_{\omega} A_0^{B\omega}(m_{\pi}^2) \varepsilon \cdot p_B,$$

$$X^{(B\pi,\omega)} = -\sqrt{2} f_{\omega} m_{\omega} F_1^{B\pi}(m_{\omega}^2) \varepsilon \cdot p_B. \tag{3.11}$$

For the q^2 dependence of form factors in the region where q^2 is not too large, we shall use the pole dominance ansatz, namely,

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_*^2)^n}, \tag{3.12}$$

where m_* is the pole mass given in [7]. A direct calculation of $B \to P$ and $B \to V$ form factors at timelike momentum transfer is available in the relativistic light-front quark model [32] with the results that the q^2 dependence of the form factors A_0, F_1 is a dipole behavior (i.e., $n = 2$), while F_0 exhibits a monopole dependence ($n = 1$). The decay rate is then given by

$$\Gamma(B^- \to \pi^- \omega) = \frac{p_c}{8 \pi m_B^2} \left(\frac{(m_B^2 - m_{\pi}^2 - m_{\omega}^2)^2}{4 m_{\omega}^2} - m_{\pi}^2 \right)$$

$$\times \left| \frac{A(B^- \to \pi^- \omega)}{\varepsilon \cdot p_B} \right|^2, \tag{3.13}$$

where p_c is the c.m. momentum:

$$p_c = \frac{\sqrt{[m_B^2 - (m_{\omega} + m_{\pi})^2][m_B^2 - (m_{\omega} - m_{\pi})^2]}}{2 m_B}. \tag{3.14}$$

Since

$$V_{ub} V_{ud}^* = A\lambda^3(\rho - i\eta), \quad V_{cb} V_{cd}^* = -A\lambda^3,$$

$$V_{tb} V_{td}^* = A\lambda^3(1 - \rho + i\eta), \tag{3.15}$$

in terms of the Wolfenstein parametrization [29], are of the same order of magnitude, it is clear that $B^- \to \omega \pi^-$ is dominated by external and internal W emissions and that penguin
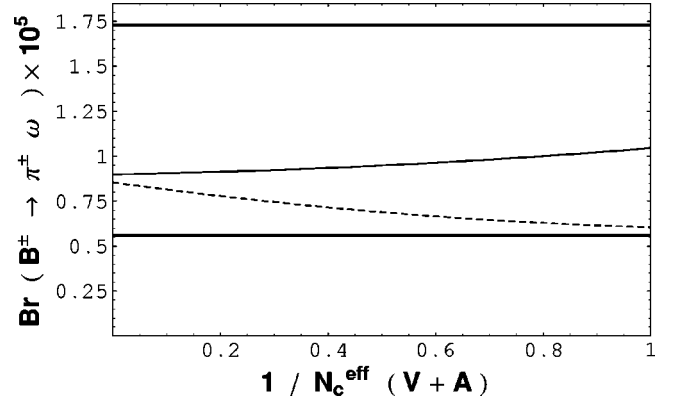

FIG. 2. Same as Fig. 1 except that the branching ratio is plotted against $1/N_c^{\text{eff}}(V+A)$ with $N_c^{\text{eff}}(V-A)$ being fixed at the value of 2.

contributions are suppressed by the smallness of the penguin coefficients. Neglecting the W-annihilation contribution denoted by $X^{(B,\pi\omega)}$, and using $f_{\pi} = 132$ MeV, $f_{\omega} = 195$ MeV for decay constants, $A_0^{B\omega}(0) = 0.28/\sqrt{2}, F_1^{B\pi}(0) = 0.33$ for form factors [31], and $\tau(B^{\pm}) = (1.67 \pm 0.04)$ ps [33] for the charged B lifetime, the branching ratio of $B^{\pm} \to \pi^{\pm} \omega$ averaged over CP-conjugate modes is shown in Fig. 1 where we have set $N_c^{\text{eff}}(V+A) = N_c^{\text{eff}}(V-A) = N_c^{\text{eff}}$ and plotted the branching ratio as a function of $1/N_c^{\text{eff}}$. We see that the branching ratio is sensitive to $1/N_c^{\text{eff}}$ and has the lowest value of order 2×10^{-6} at $N_c^{\text{eff}} = \infty$ and then increases with $1/N_c^{\text{eff}}$. Since, experimentally [16],[5]

$$\mathcal{B}(B^{\pm} \to \omega \pi^{\pm}) = (1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}, \tag{3.16}$$

it is evident that $1/N_c^{\text{eff}} > 0.35$ is preferred by the data. Because this decay is dominated by tree amplitudes, this in turn implies that

$$N_c^{\text{eff}}(V-A) < 2.9 \quad \text{from} \quad B^{\pm} \to \pi^{\pm} \omega. \tag{3.17}$$

With the value of $N_c^{\text{eff}}(V-A)$ being fixed to be 2, the branching ratio of $B^{\pm} \to \pi^{\pm} \omega$ is plotted in Fig. 2 as a function of $N_c^{\text{eff}}(V+A)$. We see that for positive ρ, which is preferred by the current analysis [30], the branching ratio is of order $(0.9-1.0) \times 10^{-5}$, very close to the central value of the measured one.

The fact that $N_c^{\text{eff}}(V-A) < 2.9$ in charmless two-body decays of the B meson is consistent with the nonfactorizable term extracted from $B \to (D,D^*)\pi, D\rho$ decays, namely, $\chi \sim 0.10$ or $N_c^{\text{eff}}(B \to D\pi) \approx 2$. Since the energy release in the energetic two-body decays $B \to \omega \pi$, $B \to D\pi$ is of the same order of magnitude, it is thus expected that $N_c^{\text{eff}}(V-A)|_{B \to \omega \pi} \approx 2$.

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[5]The significance of $B^{\pm} \to \omega \pi^{\pm}$ is reduced in the recent CLEO analysis and only an upper limit is quoted [34,19]: $\mathcal{B}(B^{\pm} \to \pi^{\pm} \omega) < 2.3 \times 10^{-5}$. Since $\mathcal{B}(B^{\pm} \to K^{\pm} \omega) = (1.5^{+0.7}_{-0.6} \pm 0.2) \times 10^{-5}$ and $\mathcal{B}(B^{\pm} \to h^{\pm} \omega) = (2.5^{+0.8}_{-0.7} \pm 0.3) \times 10^{-5}$ with $h = \pi, K$, the central value of $\mathcal{B}(B^{\pm} \to \pi^{\pm} \omega)$ remains about the same as in Eq. (3.16).
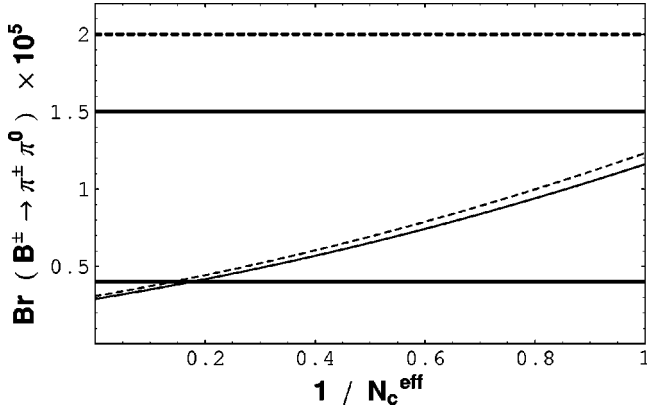
FIG. 3. Same as Fig. 1 except for $B^{\pm} \to \pi^{\pm} \pi^0$. The thick dotted line is the CLEO upper limit [see Eq. (3.20)].

The main uncertainty of the above analysis is the negligence of the spacelike penguin diagrams and W annihilations. It is common to argue that W annihilation is negligible due to helicity suppression, corresponding to form factor suppression at a large momentum transfer, $q^2 = m_B^2$ (for a recent study, see [35]). However, we see from Eq. (3.7) that the spacelike penguin contribution gains a large enhancement by a factor of $m_B^2 / [m_b(m_u + m_d)] \approx 670$. Therefore, there is no good reason to ignore the spacelike penguin effect [36] that has been largely overlooked in the literature. Unfortunately, we do not have a reliable method for estimating W-annihilation and hence spacelike penguin diagrams.

We next come to the decay $B^- \to \pi^- \pi^0$ which is quite clean and unique in the sense that this is the only two-body charmless B decay mode that does not receive any contributions from the QCD penguin operators. Under the generalized factorization approximation,

$$A(B^- \to \pi^- \pi^0) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^*(a_1 + a_2) i f_\pi$$

$$\times (m_B^2 - m_\pi^2) F_0^{B\pi^0}(m_\pi^2), \quad (3.18)$$

with $F_0^{B\pi^0} = F_0^{B\pi^\pm}/\sqrt{2}$, where we have neglected the very small electroweak penguin contributions. The decay rate is

$$\Gamma(B^- \to \pi^- \pi^0) = \frac{p_c}{8\pi m_B^2} |A(B^- \to \pi^- \pi^0)|^2. \quad (3.19)$$

Just like the decay $B^- \to \pi^- \omega$, the branching ratio of $B^- \to \pi^- \pi^0$ also increases with $1/N_c^{\mathrm{eff}}$ as shown in Fig. 3. The CLEO measurement is [17]

$$\mathcal{B}(B^{\pm} \to \pi^{\pm} \pi^0) = (0.9^{+0.6}_{-0.5}) \times 10^{-5} < 2.0 \times 10^{-5}. \quad (3.20)$$

However, the errors are so large that it is meaningless to put a sensible constraint on $N_c^{\mathrm{eff}}(V-A)$. Nevertheless, we see that in the range $0 \leq 1/N_c^{\mathrm{eff}} \leq 0.5$ [24], $N_c^{\mathrm{eff}}(V-A) \approx 2$ is favored.

In analogy to the decays $B \to D^{(*)}\pi(\rho)$, the interference effect of spectator amplitudes in class-III charmless B decay can be tested by measuring the ratios

$$R_1 \equiv 2\frac{\mathcal{B}(B^- \to \pi^- \pi^0)}{\mathcal{B}(\bar{B}^0 \to \pi^- \pi^+)}, \quad R_2 \equiv 2\frac{\mathcal{B}(B^- \to \rho^- \pi^0)}{\mathcal{B}(\bar{B}^0 \to \rho^- \pi^+)},$$

$$R_3 \equiv 2\frac{\mathcal{B}(B^- \to \pi^- \rho^0)}{\mathcal{B}(\bar{B}^0 \to \pi^- \rho^+)}. \quad (3.21)$$

Since penguin contributions are very small as we have checked numerically, to a good approximation we have

$$R_1 = \frac{\tau(B^-)}{\tau(B_d^0)}\left(1 + \frac{a_2}{a_1}\right)^2,$$

$$R_2 = \frac{\tau(B^-)}{\tau(B_d^0)}\left(1 + \frac{f_\pi}{f_\rho}\frac{A_0^{B\rho}(m_\pi^2)}{F_1^{B\pi}(m_\rho^2)}\frac{a_2}{a_1}\right)^2,$$

$$R_3 = \frac{\tau(B^-)}{\tau(B_d^0)}\left(1 + \frac{f_\rho}{f_\pi}\frac{F_1^{B\pi}(m_\rho^2)}{A_0^{B\rho}(m_\pi^2)}\frac{a_2}{a_1}\right)^2. \quad (3.22)$$

Evidently, the ratios R_i are greater (less) than unity when the interference is constructive (destructive). Numerically we find

$$R_1 = \begin{cases} 1.74, \\ 0.58, \end{cases} \quad R_2 = \begin{cases} 1.40, \\ 0.80, \end{cases}$$

$$R_3 = \begin{cases} 2.50 & \text{for } N_c^{\mathrm{eff}} = 2, \\ 0.26 & \text{for } N_c^{\mathrm{eff}} = \infty, \end{cases} \quad (3.23)$$

where use of $\tau(B_d^0) = (1.57 \pm 0.04)$ ps [33], $f_\rho = 216$ MeV, $A_0^{B\rho}(0) = 0.28$ [32] has been made. Hence, a measurement of R_i (in particular R_3), which has the advantage of being independent of the parameters ρ and η, will constitute a very useful test of the effective number of colors $N_c^{\mathrm{eff}}(V-A)$. The present experimental information on $\bar{B}^0 \to \pi^+ \pi^-$ is [17]

$$\mathcal{B}(\bar{B}^0 \to \pi^{\pm} \pi^{\mp}) = (0.7 \pm 0.4) \times 10^{-5} < 1.5 \times 10^{-5}. \quad (3.24)$$

As far as the experimental central value of R_1 is concerned, it appears that $1/N_c^{\mathrm{eff}} \sim 0.5$ is more favored than any other small values of $1/N_c^{\mathrm{eff}}$.

In short, using the central values of the branching ratios for class-III decay modes, $B \to \pi\omega$, $B \to \pi\pi$, we find that within the range $0 \leq 1/N_c^{\mathrm{eff}} \leq 0.5$, $N_c^{\mathrm{eff}}(V-A) \sim 2$ is certainly more preferred. Measurements of class-III decays are urgently needed in order to pin down the nonfactorizable effect in tree amplitudes. In particular, measurements of the inter-

ference effects in charged B decays $B^- \rightarrow \pi^-(\rho^-)\pi^0(\rho^0)$ will be very helpful in determining $N_c^{\mathrm{eff}}(V-A)$.

IV. NONFACTORIZABLE EFFECTS IN PENGUIN AMPLITUDES

In Sec. III we have shown that for spectator diagram amplitudes $N_c^{\mathrm{eff}}(V-A) \sim 2$ is preferred, as expected. However, the nonfactorizable effect in the penguin amplitude is not necessarily the same as that in the tree amplitude since the chiral structure and the Fierz transformation of the $(V-A)(V+A)$ four-quark operators $O_{5,6,7,8}$ are different from that of $(V-A)(V-A)$ operators; that is, $N_c^{\mathrm{eff}}(V+A)$ is *a priori* not the same as $N_c^{\mathrm{eff}}(V-A)$. By studying the penguin-dominated decays $B \rightarrow \phi K$ and $B \rightarrow \phi K^*$, we shall see that $N_c^{\mathrm{eff}}(V+A) \sim 2$ is ruled out by the current bound on $B^- \rightarrow K^- \phi$.

The decay $B^- \rightarrow K^- \phi$ receives contributions from W-annihilation and penguin diagrams:

$$A(B^- \rightarrow K^- \phi) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_1 X^{(B,K\phi)} - V_{tb} V_{ts}^* \left[\left(a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \right) X^{(BK,\phi)} \right. \right.$$
$$\left. \left. + \left(a_4 + a_{10} - 2(a_6 + a_8) \frac{m_B^2}{(m_b + m_u)(m_s + m_u)} \right) X^{(B,K\phi)} \right] \right\}, \tag{4.1}$$

where

$$X^{(BK,\phi)} \equiv \langle \phi | (\bar{s}s)_{V-A} | 0 \rangle \langle K^- | (\bar{s}b)_{V-A} | B^- \rangle$$
$$= -2 f_\phi m_\phi F_1^{BK}(m_\phi^2)(\varepsilon \cdot p_B),$$

$$X^{(B,K\phi)} \equiv \langle \phi K^- | (\bar{s}u)_{V-A} | 0 \rangle \langle 0 | (\bar{u}b)_{V-A} | B^- \rangle. \tag{4.2}$$

Neglecting W-annihilation and spacelike penguin diagrams and using $f_\phi = 237$ MeV, $F_1^{BK}(0) = 0.34$ [32], we plot in Fig. 4 the branching ratio of $B^\pm \rightarrow \phi K^\pm$ against $1/N_c^{\mathrm{eff}}$ for two different cases: the dotted curve for the free parameter $N_c^{\mathrm{eff}}(V+A) = N_c^{\mathrm{eff}}(V-A) = N_c^{\mathrm{eff}}$ and the solid curve with $N_c^{\mathrm{eff}}(V-A)$ being fixed at the value of 2. In either case, it is clear that $N_c^{\mathrm{eff}}(V+A) = 2$ is evidently excluded from the present CLEO upper limit [34]

$$\mathcal{B}(B^\pm \rightarrow \phi K^\pm) < 0.5 \times 10^{-5}. \tag{4.3}$$

A similar observation was also made in [37]. The conclusion that $N_c^{\mathrm{eff}}(V+A) \neq 2$ will be further reinforced if the decay rate of $B^\pm \rightarrow \phi K^\pm$ is enhanced by the spacelike penguin diagrams. From Fig. 4 we also see that $1/N_c^{\mathrm{eff}}(V+A) < 0.23$ or $N_c^{\mathrm{eff}}(V+A) > 4.3$. Note that this constraint is subject to the corrections from spacelike penguin and W-annihilation contributions. At any rate, it is safe to conclude that

$$N_c^{\mathrm{eff}}(V+A) > N_c^{\mathrm{eff}}(V-A). \tag{4.4}$$

The branching ratio of $B \rightarrow \phi K^*$, the average of ϕK^{*-} and ϕK^{*0} modes, is also measured recently by CLEO with the result [34]

$$\mathcal{B}(B \rightarrow \phi K^*) \equiv \frac{1}{2} [\mathcal{B}(B^\pm \rightarrow \phi K^{*\pm}) + \mathcal{B}(B^0 \rightarrow \phi K^{*0})]$$
$$= (1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}. \tag{4.5}$$

As emphasized in the first footnote of Sec. I, the effective parameters a_i in general cannot be defined for the $B \rightarrow VV$ decay as its amplitude involves more than one Lorentz scalar. In the absence of information for the nonfactorizable contributions to various Lorentz scalars, we shall assume generalized factorization. Under this hypothesis, the decay amplitude of $B \rightarrow \phi K^*$ has a similar expression as that of $B \rightarrow \phi K$. Its decay rate is given by

$$\Gamma(B^- \rightarrow \phi K^{*-})$$
$$= \frac{p_c}{8 \pi m_B^2} \left| \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* f_\phi m_\phi (m_B + m_{K*}) \right.$$
$$\times A_1^{BK*}(m_\phi^2) \left(a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \right) \Bigg|^2$$
$$\times [(a - bx)^2 + 2(1 + c^2 y^2)], \tag{4.6}$$

with

$$x = A_2^{BK*}(m_\phi^2)/A_1^{BK*}(m_\phi^2), \quad y = V^{BK*}(m_\phi^2)/A_1^{BK*}(m_\phi^2),$$

$$a = \frac{m_B^2 - m_{K*}^2 - m_\phi^2}{2 m_{K*} m_\phi}, \qquad b = \frac{2 p_c^2 m_B^2}{m_{K*} m_\phi (m_B + m_{K*})^2},$$

$$c = \frac{2 p_c m_B}{(m_B + m_{K*})^2}, \tag{4.7}$$
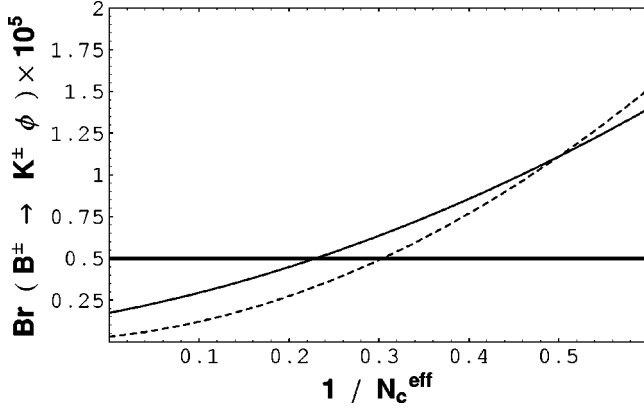
FIG. 4. The branching ratio of $B^{\pm} \to \phi K^{\pm}$ vs $1/N_c^{\text{eff}}$ for $\eta = 0.34$ and $\rho = 0.16$. The dotted curve is for $N_c^{\text{eff}}(V+A) = N_c^{\text{eff}}(V-A) = N_c^{\text{eff}}$ and the solid curve is the branching ratio against $1/N_c^{\text{eff}}(V+A)$ with $N_c^{\text{eff}}(V-A)$ being fixed at the value of 2. The solid thick line is the CLEO upper limit.

where we have neglected contributions proportional to $X^{(B,K^*\phi)}$.

We calculate the decay rates using two different sets of values for form factors:

$$A_1^{BK^*}(0) = 0.328, \quad A_2^{BK^*}(0) = 0.331, \quad V^{BK^*}(0) = 0.369 \tag{4.8}$$

from [7] and

$$A_1^{BK^*}(0) = 0.26, \quad A_2^{BK^*}(0) = 0.23, \quad V^{BK^*}(0) = 0.32 \tag{4.9}$$

from [32]. As for the q^2 dependence, light-front calculations indicate a dipole behavior for $V(q^2)$, $A_2(q^2)$ and a monopole dependence for $A_1(q^2)$ [32]. The result is shown in Fig. 5. It is interesting to note that the branching ratios are very insensitive to the choice of the values for form factors, Eqs. (4.8) or (4.9). We see that the allowed region is $0.7 \gtrsim 1/N_c^{\text{eff}}(V+A) \gtrsim 0.25$ or $4 \gtrsim N_c^{\text{eff}}(V+A) \gtrsim 1.4$, bearing in mind that this constraint is subject to the corrections from annihilation terms. This seems to be in contradiction to the constraint $N_c^{\text{eff}}(V+A) > 4.3$ derived from $B^{\pm} \to \phi K^{\pm}$. In fact, it is expected in the factorization approach that $\Gamma(B \to \phi K^*) \approx \Gamma(B \to \phi K)$ when the W-annihilation type of contribution is neglected. The current CLEO measurements (4.3) and (4.5) are obviously not consistent with the prediction based on factorization. One possibility is that generalized factorization is not applicable to $B \to VV$. Therefore, the discrepancy between $\mathcal{B}(B \to \phi K)$ and $\mathcal{B}(B \to \phi K^*)$ will measure the degree of deviation from the generalized factorization that has been applied to $B \to \phi K^*$. At any rate, in order to clarify this issue and to pin down the effective number of colors $N_c^{\text{eff}}(V+A)$, we need measurements of $B \to \phi K$ and $B \to \phi K^*$, especially the neutral modes, with sufficient accuracy.

Since CLEO has measured $B^- \to \pi^- \bar{K}^0$ and $\bar{B}^0 \to \pi^+ K^-$ [17], we have also studied these two decay modes. We found that for a fixed $N_c^{\text{eff}}(V-A) = 2$, the predicted branching ratios
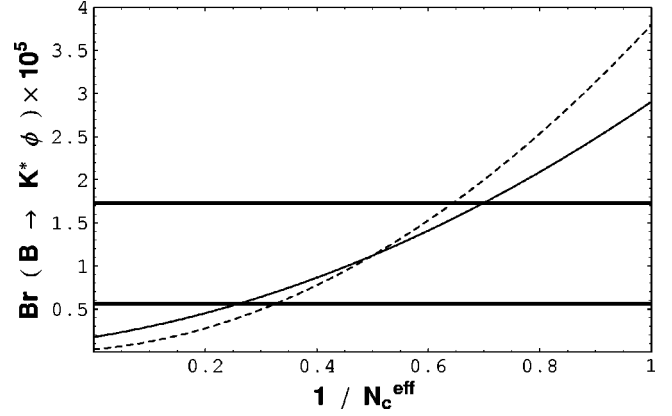

FIG. 5. Same as Fig. 1 except for $B \to \phi K^*$.

of $B \to \pi K$ are in agreement with the CLEO measurement within errors for all values of $1/N_c^{\text{eff}}(V+A)$. Hence, no useful constraint on $N_c^{\text{eff}}(V+A)$ can be derived from $B^- \to \pi^- \bar{K}^0$ and $\bar{B}^0 \to \pi^+ K^-$.

Since $N_c^{\text{eff}}(V+A) > N_c^{\text{eff}}(V-A)$, one may wonder if the leading $1/N_c$ expansion may happen to be applicable again to the matrix elements of $(V-A)(V+A)$ operators. We believe that $N_c^{\text{eff}}(V+A) = \infty$ is very unlikely for two reasons. First, it will predict a too small branching ratio of $B \to \phi K^*$ as shown in Fig. 5. Second, it implies a nonfactorizable term $\chi_B(V+A) \sim -\frac{1}{3}$, as in the charm case. Since the energy release in the energetic two-body decays of the B meson is much larger than that in charm decay, it is thus expected that

$$|\chi(D \to \bar{K}\pi)| > |\chi_B(V+A)| \sim |\chi(B \to D\pi)|. \tag{4.10}$$

Because $N_c^{\text{eff}}(V+A) \gtrsim 4.3$, it is then plausible to assume that $\chi_B(V+A) \sim -\chi(B \to D\pi) \approx -(0.10-0.12)$. Hence, $N_c^{\text{eff}}(V+A) \sim (4.3-4.9)$.

V. IMPLICATIONS ON CHARMLESS B DECAYS INTO η' AND η

When the preliminary CLEO measurement of $B^{\pm} \to \eta' K^{\pm}$ was reported last year [14],

$$\mathcal{B}(B^{\pm} \to \eta' K^{\pm}) = (7.8^{+2.7}_{-2.2} \pm 1.0) \times 10^{-5}, \tag{5.1}$$

it stimulated great interest in the community since early theoretical estimates of the $B^{\pm} \to \eta' K^{\pm}$ branching ratio [36,38,23] lie in the range of[6] $(1-2) \times 10^{-5}$. Since then, many theoretical studies and speculation have surged, as evidenced by the recent literature [24,27,39–49] that offers

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[6]The prediction $\mathcal{B}(B^{\pm} \to \eta' K^{\pm}) = 3.6 \times 10^{-5}$ given in [36] is too large by about a factor of 2 because the normalization constant of the η' wave function was not taken into account in the form factor $F_0^{B\eta'}$. This negligence was also erroneously made in some recent papers on $B \to \eta' K$. Note that all early calculations [36,38,23] did not take into account the anomaly contribution to the matrix element $\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle$ (see below).

various interpretations on the abnormally large branching ratios. It was soon realized [24,27] that the running strange quark mass at the scale $\mu = \mathcal{O}(m_b)$ and SU(3) breaking in the decay constants of the η_0 and η_8 will provide a large enhancement to the decay rate of $B \rightarrow \eta' K$ (for a review, see [50]). Unfortunately, as pointed out in [24], this enhancement is partially washed out by the anomaly contribution to the matrix element $\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle$, an effect overlooked previously. As a consequence, the branching ratio of $B \rightarrow \eta' K$ is of the order of $(2-3) \times 10^{-5}$ in the range $0 \leq 1/N_c^{\text{eff}} \leq 0.5$. The discrepancy between theory and current measurements [18],

$$\mathcal{B}(B^{\pm} \rightarrow \eta' K^{\pm}) = (6.5^{+1.5}_{-1.4} \pm 0.9) \times 10^{-5},$$

$$\mathcal{B}(B^0 \rightarrow \eta' K^0) = (4.7^{+2.7}_{-2.0} \pm 0.9) \times 10^{-5}, \qquad (5.2)$$

seems to call for some new mechanisms unique to η' production or even some new physics beyond the standard model.

All the previous analyses of $B \rightarrow \eta' K$ in the literature are based on the assumption that $(N_c^{\text{eff}})_i$ are the same for $i = 1, 2, \ldots, 10$. In this section we will show that the fact that $N_c^{\text{eff}}(V+A)$ and $N_c^{\text{eff}}(V-A)$ are not the same and that they are subject to the constraints (3.17) and (4.10) will lead to a significant enhancement for the decay rate of $B \rightarrow \eta' K$ at small values of $1/N_c^{\text{eff}}$. Moreover, we shall see that the prediction of $\mathcal{B}(B \rightarrow \eta' K)$ is compatible with experiment. Especially, the measurement of $B^0 \rightarrow \eta' K^0$ is well explained, implying that no new mechanism in the standard model or new physics beyond the standard model is needed to account for the data.

To begin with, we write down the factorizable amplitude

$$
\begin{aligned}
A(B^- \rightarrow \eta' K^-) = \frac{G_F}{\sqrt{2}} \Bigg(& V_{ub} V_{us}^* (a_1 X^{(B\eta',K)} + a_2 X_u^{(BK,\eta')} + a_1 X^{(B,\eta'K)}) + V_{cb} V_{cs}^* a_2 X_c^{(BK,\eta')} \\
& - V_{tb} V_{ts}^* \Bigg\{ \left(a_4 + a_{10} + 2(a_6 + a_8) \frac{m_K^2}{(m_s + m_u)(m_b - m_u)} \right) X^{(B\eta',K)} \\
& + \left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 \right) X_u^{(BK,\eta')} + (a_3 - a_5 - a_7 + a_9) X_c^{(BK,\eta')} \\
& + \left(a_4 + a_{10} + 2(a_6 + a_8) \frac{m_B^2}{(m_s - m_u)(m_b + m_u)} \right) X^{(B,\eta'K)} \\
& + \left[a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8 \right) \frac{m_{\eta'}^2}{m_s(m_b - m_s)} \left(1 - \frac{f_{\eta'}^u}{f_{\eta'}^s} \right) \right] X_s^{(BK,\eta')} \Bigg\} \Bigg) \qquad (5.3)
\end{aligned}
$$

for $B^- \rightarrow \eta' K^-$ and

$$
\begin{aligned}
A(\bar{B}^0 \rightarrow \eta' \bar{K}^0) = \frac{G_F}{\sqrt{2}} \Bigg(& V_{ub} V_{us}^* a_2 X_u^{(BK,\eta')} + V_{cb} V_{cs}^* a_2 X_c^{(BK,\eta')} \\
& - V_{tb} V_{ts}^* \Bigg\{ \left(a_4 - \frac{1}{2}a_{10} + (2a_6 - a_8) \frac{m_K^2}{(m_s + m_d)(m_b - m_d)} \right) X^{(B\eta',K)} \\
& + \left(2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 \right) X_u^{(BK,\eta')} + (a_3 - a_5 - a_7 + a_9) X_c^{(BK,\eta')} \\
& + \left(a_4 + a_{10} + 2(a_6 + a_8) \frac{m_B^2}{(m_s - m_d)(m_b + m_d)} \right) X^{(B,\eta'K)} \\
& + \left[a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8 \right) \frac{m_{\eta'}^2}{m_s(m_b - m_s)} \left(1 - \frac{f_{\eta'}^u}{f_{\eta'}^s} \right) \right] X_s^{(BK,\eta')} \Bigg\} \Bigg) \qquad (5.4)
\end{aligned}
$$

for $\bar{B}^0 \rightarrow \eta' \bar{K}^0$, where

$$X^{(B\eta',K)} \equiv \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle \langle \eta' | (\bar{u}b)_{V-A} | B^- \rangle$$

$$= \langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \rangle \langle \eta' | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle$$

$$= if_K (m_B^2 - m_{\eta'}^2) F_0^{B\eta'}(m_K^2),$$

$$X_q^{(BK,\eta')} \equiv \langle \eta' | (\bar{q}q)_{V-A} | 0 \rangle \langle K^- | (\bar{s}b)_{V-A} | B^- \rangle$$

$$= \langle \eta' | (\bar{q}q)_{V-A} | 0 \rangle \langle \bar{K}^0 | (\bar{s}b)_{V-A} | \bar{B}^0 \rangle$$

$$= if_{\eta'}^q (m_B^2 - m_K^2) F_0^{BK}(m_{\eta'}^2),$$

$$X^{(B,\eta'K)} \equiv \langle \eta' K^- | (\bar{s}u)_{V-A} | 0 \rangle \langle 0 | (\bar{u}b)_{V-A} | B^- \rangle, \quad (5.5)$$

and use of the isospin relation $X_d^{(BK,\eta')} = X_u^{(BK,\eta')}$ has been made. For the amplitude of $B^- \rightarrow \eta' K^-$, the terms proportional to $X^{(B,\eta'K)}$ and $X_c^{(BK,\eta')}$ with penguin coefficients are often missed or not considered in previous analyses. Note that the neutral mode $\bar{B}^0 \rightarrow \eta' \bar{K}^0$ differs from the charged mode in that it does not receive contributions from external W-emission and W-annihilation diagrams. From the relevant quark mixing angles

$$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta), \quad V_{cb}V_{cs}^* = A\lambda^2 \left(1 - \frac{1}{2}\lambda^2\right),$$

$$V_{tb}V_{ts}^* = -A\lambda^2 + \frac{1}{2}A(1-2\rho)\lambda^4 + i\eta A\lambda^4, \quad (5.6)$$

it is clear that $B \rightarrow \eta' K$ decays are dominated by penguin diagrams.

The presence of the term $1 - (f_{\eta'}^u / f_{\eta'}^s)$ in Eqs. (5.3) and (5.4) is necessary and mandatory in order to ensure a correct chiral-limit behavior for the $(S-P)(S+P)$ matrix elements of the penguin operators $O_{5,6,7,8}$. In the chiral limit $m_u, m_d, m_s \rightarrow 0$, the ratio $m_K^2/(m_s + m_u) = m_\pi^2/(m_u + m_d)$ remains finite,[7] but this is no longer the case for $m_{\eta'}^2/m_s$ associated with the matrix element $\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle$ since the η' mass originates from the QCD anomaly and does not vanish in the chiral limit. As pointed out in [42,24], because of the presence of the anomaly in the equation of motion

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[7]For the annihilation term, the chiral-limit behavior of $[m_B^2/m_b(m_s - m_u)]X^{(B,\eta'K)}$ is supposed to be taken care of by the form factors in $X^{(B,\eta'K)}$.

$$\partial^\mu (\bar{s}\gamma_\mu \gamma_5 s) = 2m_s \bar{s} i \gamma_5 s + \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad (5.7)$$

it is erroneous to apply the relation

$$\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle = -i \frac{m_{\eta'}^2}{2m_s} f_{\eta'}^s, \quad (5.8)$$

as adopted previously in the literature, where $\langle 0 | \bar{q}\gamma_\mu \gamma_5 q | \eta' \rangle = if_{\eta'}^q p_\mu$. Neglecting the u and d quark masses in the equations of motion leads to [51]

$$\left\langle \eta' \left| \frac{\alpha_s}{4\pi} G\tilde{G} \right| 0 \right\rangle = f_{\eta'}^u m_{\eta'}^2, \quad (5.9)$$

and hence [42,24]

$$\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle = -i \frac{m_{\eta'}^2}{2m_s} (f_{\eta'}^s - f_{\eta'}^u). \quad (5.10)$$

It is easily seen that this matrix element has the correct chiral behavior. It should be stressed that in order to go the chiral-symmetry limit, one must consider both $m_s \rightarrow 0$ and $\theta \rightarrow 0$ together [52], where θ is the η-η' mixing angle to be defined below. Since $f_{\eta'}^u \sim \frac{1}{2} f_{\eta'}^s$ (see below) and the decay amplitude is dominated by $(S-P)(S+P)$ matrix elements, it is obvious that the decay rate of $B \rightarrow \eta' K$ is reduced considerably by the presence of the anomaly term in $\langle \eta' | \bar{s}\gamma_5 s | 0 \rangle$.

To determine the decay constant $f_{\eta'}^q$, we need to know the wave functions of the physical η' and η states which are related to that of the SU(3) singlet state η_0 and octet state η_8 by

$$\eta' = \eta_8 \sin\theta + \eta_0 \cos\theta, \quad \eta = \eta_8 \cos\theta - \eta_0 \sin\theta, \quad (5.11)$$

with $\theta \approx -20°$. When the η-η' mixing angle is $-19.5°$, the η' and η wave functions have simple expressions [36]

$$|\eta'\rangle = \frac{1}{\sqrt{6}} |\bar{u}u + \bar{d}d + 2\bar{s}s\rangle, \quad |\eta\rangle = \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d - \bar{s}s\rangle, \quad (5.12)$$

recalling that

$$|\eta_0\rangle = \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d + \bar{s}s\rangle, \quad |\eta_8\rangle = \frac{1}{\sqrt{6}} |\bar{u}u + \bar{d}d - 2\bar{s}s\rangle. \quad (5.13)$$

At this specific mixing angle, $f_{\eta'}^u = \frac{1}{2} f_{\eta'}^s$ in the SU(3) limit. Introducing the decay constants f_8 and f_0 by

$$\langle 0 | A_\mu^0 | \eta_0 \rangle = if_0 p_\mu, \quad \langle 0 | A_\mu^8 | \eta_8 \rangle = if_8 p_\mu, \quad (5.14)$$

$f_{\eta'}^u$ and $f_{\eta'}^s$ are then related to f_8 and f_0 by[8]

$$f_{\eta'}^u = \frac{f_8}{\sqrt{6}} \sin \theta + \frac{f_0}{\sqrt{3}} \cos \theta,$$

$$f_{\eta'}^s = -2\frac{f_8}{\sqrt{6}} \sin \theta + \frac{f_0}{\sqrt{3}} \cos \theta.$$

(5.15)

Likewise, for the η meson,

$$f_{\eta}^u = \frac{f_8}{\sqrt{6}} \cos \theta - \frac{f_0}{\sqrt{3}} \sin \theta, \quad f_{\eta}^s = -2\frac{f_8}{\sqrt{6}} \cos \theta - \frac{f_0}{\sqrt{3}} \sin \theta.$$

(5.16)

The factorizable amplitude denoted by $X_c^{(BK, \eta')}$ involves a conversion of the $c\bar{c}$ pair into the η' via two gluon exchanges. Although the charm content of the η' is *a priori* expected to be small, its contribution is potentially important because the Cabibbo-Kobayashi-Maskawa (CKM) mixing angle $V_{cb}V_{cs}^*$ is of the same order of magnitude as that of the penguin amplitude [cf. Eq. (5.6)] and yet its effective coefficient a_2 is larger than the penguin coefficients by an order of magnitude. The decay constant $f_{\eta'}^c$, defined by $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'\rangle = if_{\eta'}^c q_\mu$, has been estimated to be $f_{\eta'}^c = (50-180)$ MeV, based on the operator product expansion (OPE), large-N_c approach and QCD low energy theorems [39]. It was claimed in [39] that $|f_{\eta'}^c| \sim 140$ MeV is needed in order to exhaust the CLEO observation of $B^\pm \to \eta' K^\pm$ and $B \to \eta' + X$ by the mechanism $b \to c\bar{c} + s \to \eta' + s$ via gluon exchanges. However, a large value of $f_{\eta'}^c$ seems to be

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[8]A two-mixing-angle parametrization of the η and η' wave functions, $\eta' = \eta_8 \sin\theta_8 + \eta_0 \cos\theta_0, \eta = \eta_8 \cos\theta_8 - \eta_0 \sin\theta_0$, is employed in [24] for the calculation of $B \to \eta'(\eta)K$. However, in the absence of mixing with other pseudoscalar mesons, this parametrization will destroy the orthogonality of the physical states η and η' if $\theta_0 \neq \theta_8$. Because of SU(3) breaking, the matrix elements $\langle 0|A_\mu^{0(8)}|\eta_{8(0)}\rangle$ do not vanish in general and they will induce a two-angle mixing among the decay constants:

$$f_{\eta'}^u = \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0,$$

$$f_{\eta'}^s = -2\frac{f_8}{\sqrt{6}} \sin\theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0.$$

Based on the ansatz that the decay constants in the quark flavor basis follow the pattern of particle state mixing, relations between θ_8, θ_0, and θ are derived in [53], where θ is the η-η' mixing angle introduced in Eqs. (5.11). It is found in [53] that phenomenologically $\theta_8 = -21.2°$, $\theta_0 = -9.2°$, and $\theta = -15.4°$. It must be accentuated that the two-mixing-angle formalism proposed in [54,53] applies to the decay constants of the η' and η rather than to their wave functions. Numerically, we find that the branching ratios shown in Table I (see below) calculated in one-angle and two-angle mixing schemes are different by at most 7%. In the present paper we shall employ the former scheme.

ruled out for several reasons [50]. For example, from the data of $J/\psi \to \eta_c \gamma$ and $J/\psi \to \eta' \gamma$, one can show that $|f_{\eta'}^c| \gtrsim 6$ MeV, where the lower bound corresponds to the nonrelativistic quark model estimate. Based on the $\eta\gamma$ and $\eta'\gamma$ transition form factor data, the range of allowed $f_{\eta'}^c$ was recently estimated to be -65 MeV $\leq f_{\eta'}^c \leq 15$ MeV [55]. A most recent reevaluation of $f_{\eta'}^c$ along the line of [39] yields $f_{\eta'}^c = -(12.3-18.4)$ MeV [56], which is in strong contradiction in magnitude and sign to the estimate of [39]. The sign of $f_{\eta'}^c$ can be fixed by using the QCD anomaly and is found to be negative [47] (see also [49,53,56]). In the presence of the charm content in the η_0, an additional mixing angle θ_c is needed to be introduced:

$$|\eta_0\rangle = \frac{1}{\sqrt{3}} \cos \theta_c |u\bar{u} + d\bar{d} + s\bar{s}\rangle + \sin \theta_c |c\bar{c}\rangle,$$

$$|\eta_c\rangle = -\frac{1}{\sqrt{3}} \sin \theta_c |u\bar{u} + d\bar{d} + s\bar{s}\rangle + \cos \theta_c |c\bar{c}\rangle.$$

(5.17)

Then $f_{\eta'}^c = \cos \theta \tan \theta_c f_{\eta_c}$ and $f_{\eta}^c = -\sin \theta \tan \theta_c f_{\eta_c}$, where the decay constant f_{η_c} can be extracted from $\eta_c \to \gamma\gamma$, and θ_c from $J/\psi \to \eta_c \gamma$ and $J/\psi \to \eta' \gamma$ [24]. In the present paper we shall use

$$f_{\eta'}^c = -6 \text{ MeV}, \quad f_{\eta}^c = -\tan \theta f_{\eta'}^c = -2.4 \text{ MeV},$$

(5.18)

for $\theta = -22°$ (see below), which are very close to the values

$$f_{\eta'}^c = -(6.3 \pm 0.6) \text{ MeV}, \quad f_{\eta}^c = -(2.4 \pm 0.2) \text{ MeV}$$

(5.19)

obtained in [53].

Using $F_0^{BK}(0) = 0.34$ [32], $\sqrt{3}F_0^{B\eta_0}(0) = 0.33$ for form factors,[9] $f_0 = f_8 = f_\pi$, $m_s(1 \text{ GeV}) = 150$ MeV, $\theta = -19.5°$, and Eq. (5.10) for the matrix element $\langle \eta'|\bar{s}\gamma_5 s|0\rangle$, we find that $\mathcal{B}(B \to \eta' K) = (0.9-1.0) \times 10^{-5}$ and it is insensitive to N_c^{eff} and the choice of Wolfenstein parameters ρ and η as long as $\sqrt{\rho^2 + \eta^2} \approx 0.37$, where $N_c^{\text{eff}} = N_c^{\text{eff}}(V+A) = N_c^{\text{eff}}(V-A)$. The discrepancy between theory and experiment can be greatly improved by the accumulation of several enhancements. First of all, the running quark masses appearing in the $(S-P)(S+P)$ matrix elements should be applied at the scale $\mu = \mathcal{O}(m_b)$ as given in Eq. (2.13) so that the $(S-P)(S+P)$ matrix element is enhanced due to the decrease

---

[9]The form factors $F_0^{B\eta'}(0) = 0.254$ and $F_0^{B\eta}(0) = 0.307$ given in [7] do not take into account the wave function normalization of the physical η' and η states. Since it is not clear to us what is the η-η' mixing angle employed in [7], we shall follow [24,27] to use the nonet symmetry relation $\sqrt{3}F_0^{B\eta_0}(0) = \sqrt{6}F_0^{B\eta_8}(0) = F_0^{B\pi^\pm}(0) \approx 0.33$ to obtain $F_0^{B\eta_0}$, $F_0^{B\eta_8}$ and hence the form factors $F_0^{B\eta'}$ as well as $F_0^{B\eta}$ for a given θ.
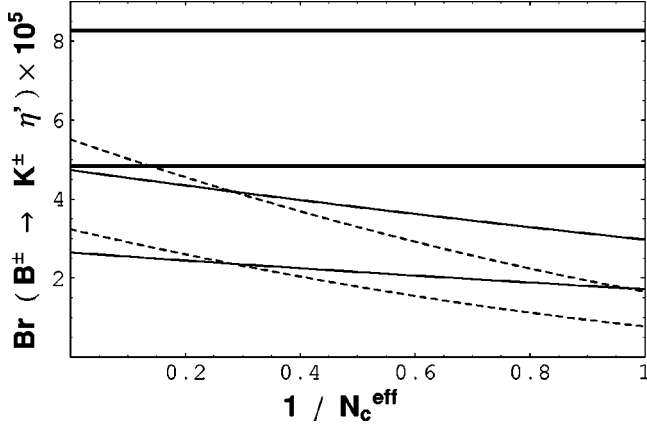
FIG. 6. The branching ratio of $B^{\pm} \to \eta' K^{\pm}$ as a function of $1/N_c^{\text{eff}}$ for $\eta = 0.34$ and $\rho = 0.16$. The charm content of the η' with $f_{\eta'}^c = -6$ MeV contributes to the solid curves, but not to the dotted curves. The lower set of solid and dotted curves takes into account the anomaly contribution to $\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$ [see Eq. (5.10)], whereas the upper set does not. The solid thick lines are the CLEO measurements with one sigma errors.

of $m_s(\mu)$ at $\mu = m_b$. (The sensitivity of the branching ratio to m_s was first noticed in [42].) Second, a recent analysis of the data of $\eta, \eta' \to \gamma\gamma$ and $\eta, \eta' \to \pi\pi\gamma$ yields [57]

$$\frac{f_8}{f_\pi} = 1.38 \pm 0.22, \quad \frac{f_0}{f_\pi} = 1.06 \pm 0.03, \quad \theta = -22.0° \pm 3.3°,$$

(5.20)

implying some SU(3) breaking in the decay constants. Applying the new values of the aforementioned parameters, the result for the branching ratio of $B^{\pm} \to \eta' K^{\pm}$ is shown in Fig. 6 vs $1/N_c^{\text{eff}}$ (see the lower set of solid and dotted curves). We find that $\mathcal{B}(B^{\pm} \to \eta' K^{\pm})$ is enhanced from $(0.9-1.0) \times 10^{-5}$ to $(2-3) \times 10^{-5}$. The latter result is in agreement with [24] (see the lower set of curves with negative $f_{\eta'}^c$ in Fig. 17 of [24]). The enhancement is due mainly to the running strange quark mass at $\mu = m_b$ and SU(3) breaking effects in the decay constants f_0 and f_8. From Fig. 6 we see that (i) in the absence of the anomaly contribution to $\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$, the branching ratios (the upper set of solid and dotted curves) will be further enhanced in a sizable way (of course, it is erroneous to neglect such an anomaly effect) and (ii) the contribution of $c\bar{c}$ conversion into the η' becomes destructive when $1/N_c^{\text{eff}} < 0.28$. This is understandable because a_2 becomes negative at small values of $1/N_c^{\text{eff}}$ so that the term $a_2 X_c^{(BK, \eta')}$ contributes in opposite sign to the penguin amplitudes. Therefore, the charm content of the η' is not welcome for explaining $\mathcal{B}(B \to \eta' K)$ at small $1/N_c^{\text{eff}}$.

Thus far it has been assumed in the analysis of $B \to \eta' K$ that the nonfactorizable effects lumped into a_i via $(N_c^{\text{eff}})_i$ are the same for $i = 1, 2, \ldots, 10$. However, we have pointed out in Sec. III that $N_c^{\text{eff}}(V-A)$ in hadronic charmless B decays is most likely very similar to that in $B \to D\pi$, namely, $N_c^{\text{eff}}(V-A) \sim N_c^{\text{eff}}(B \to D\pi) \approx 2$. In fact, we just showed that the charm content of the η' will make the discrepancy between
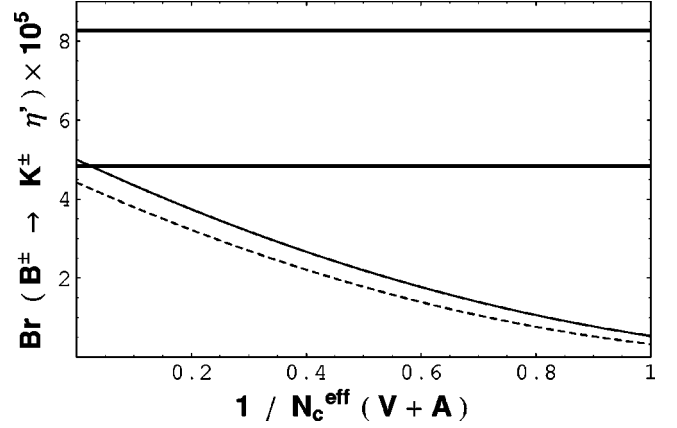

FIG. 7. Same as Fig. 6 except that the branching ratio is plotted against $1/N_c^{\text{eff}}(V+A)$ with $N_c^{\text{eff}}(V-A)$ being fixed at the value of 2. The anomaly contribution to $\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$ is included.

theory and experiment even worse at small values of $1/N_c^{\text{eff}}$ if $N_c^{\text{eff}}(V-A)$ is the same as $N_c^{\text{eff}}(V+A)$. Setting $N_c^{\text{eff}}(V-A) = 2$, we find that (see Figs. 7 and 8) the decay rates of $B \to \eta' K$ are considerably enhanced especially at small $1/N_c^{\text{eff}}(V+A)$. That is, $\mathcal{B}(B^{\pm} \to \eta' K^{\pm})$ at $1/N_c^{\text{eff}}(V+A) \leq 0.2$ is enhanced from $(2.5-3) \times 10^{-5}$ to $(3.7-5) \times 10^{-5}$. First, the η' charm content contribution $a_2 X_c^{(BK, \eta')}$ now always contributes in the right direction to the decay rate irrespective of the value of $N_c^{\text{eff}}(V+A)$. Second, the interference in the spectator amplitudes of $B^{\pm} \to \eta' K^{\pm}$ is constructive. Third, the term proportional to

$$2(a_3 - a_5) X_u^{(BK, \eta')} + (a_3 + a_4 + a_5) X_s^{(BK, \eta')} \quad (5.21)$$

in Eqs. (5.3) and (5.4) is enhanced when $(N_c^{\text{eff}})_3 = (N_c^{\text{eff}})_4 = 2$. It is evident from Fig. 8 that the measurement of $\bar{B}^0 \to \eta' \bar{K}^0$ is well explained in the present framework based on the standard model within the allowed range $1/N_c^{\text{eff}}(V+A) \leq 0.23$ extracted from $B^{\pm} \to \phi K^{\pm}$. Contrary to some early claims, we see that it is not necessary to invoke some new mechanisms, say, the SU(3)-singlet contribution S' [43], to
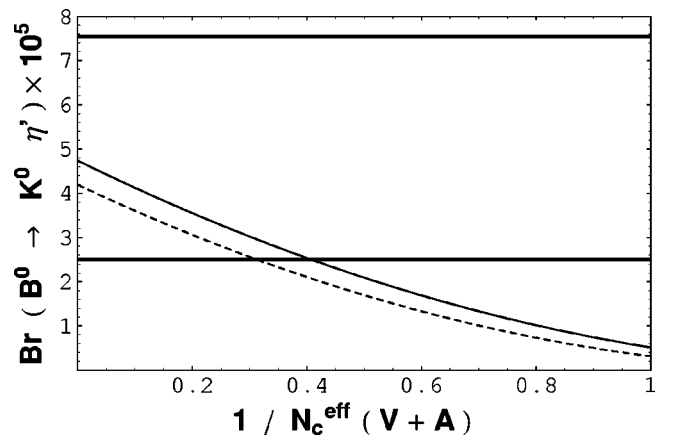

FIG. 8. Same as Fig. 7 except for $B^0 \to \eta' K^0$.

explain the data. The agreement with experiment provides another strong support for $N_c^{\mathrm{eff}}(V-A)\sim 2$ and for the relation $N_c^{\mathrm{eff}}(V+A)>N_c^{\mathrm{eff}}(V-A)$. As for the decay $B^{\pm}\to\eta'K^{\pm}$, the predicted branching ratio, say, 4×10^{-5}, at our preferred value $N_c^{\mathrm{eff}}(V+A)\sim 5$ (see Table I) is compatible with the data, though it is on the lower side. For a slightly enhanced decay constant $f_{\eta'}^c\approx -15$ MeV, as implied by a recent theoretical estimate [56], we obtain $\mathcal{B}(B\to\eta'K)=(4.6-5.9)\times 10^{-5}$ at $1/N_c^{\mathrm{eff}}(V+A)\lesssim 0.2$, which agrees with experiment very nicely. Note that the CLEO data of $B^{\pm}\to\eta'K^{\pm}$ and $B^0\to\eta'K$ are in good agreement within one sigma error [see Eq. (5.2)], though the charged mode is more

reliable. It is conceivable that when errors are improved and refined, the two values will converge eventually.

We have also studied the decays $B\to\eta K,\eta'K^*,\eta K^*$. The decay amplitude of $B\to\eta K$ is the same as $B\to\eta'K$ except for a trivial replacement of the index η' by η. As a general rule, the factorizable amplitude of $B\to\eta^{(')}K^*$ can be obtained from the $B\to\eta^{(')}K$ one by (i) replacing the term $m_P^2/[(m_1+m_2)(m_3-m_4)]$ by $-m_P^2/[(m_1+m_2)(m_3+m_4)]$ and the index K by K^* and (ii) discarding the $(S-P)(S+P)$ contribution associated with $X^{(B\eta^{(')},K^*)}$. For example, the decay amplitude of $B^-\to\eta'K^{*-}$ can be easily read from Eq. (5.3) to be

$$
\begin{aligned}
A(B^-\to\eta'K^{*-})=\frac{G_F}{\sqrt{2}}\Bigg(&V_{ub}V_{us}^*(a_1X^{(B\eta',K^*)}+a_2X_u^{(BK^*,\eta')}+a_1X^{(B,\eta'K^*)})+V_{cb}V_{cs}^*a_2X_c^{(BK^*,\eta')} \\
&-V_{tb}V_{ts}^*\Bigg\{(a_4+a_{10})X^{(B\eta',K^*)}+\Bigg[a_3+a_4-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9-\frac{1}{2}a_{10} \\
&-\left(a_6-\frac{1}{2}a_8\right)\frac{m_{\eta'}^2}{m_s(m_b+m_s)}\left(1-\frac{f_{\eta'}^u}{f_{\eta'}^s}\right)\Bigg]X_s^{(BK^*,\eta')} \\
&+\left(2a_3-2a_5-\frac{1}{2}a_7+\frac{1}{2}a_9\right)X_u^{(BK^*,\eta')}+(a_3-a_5-a_7+a_9)X_c^{(BK^*,\eta')} \\
&+\left(a_4+a_{10}-2(a_6+a_8)\frac{m_B^2}{(m_s+m_u)(m_b+m_u)}\right)X^{(B,\eta'K^*)}\Bigg\}\Bigg),
\end{aligned}
\tag{5.22}
$$

with

$$
\begin{aligned}
X^{(B\eta',K^*)}&\equiv\langle K^{*-}|(\bar{s}u)_{V-A}|0\rangle\langle\eta'|(\bar{u}b)_{V-A}|B^-\rangle \\
&=-2f_{K^*}m_{K^*}F_1^{B\eta'}(m_{K^*}^2)(\varepsilon\cdot p_B),
\end{aligned}
$$

$$
\begin{aligned}
X_q^{(BK^*,\eta')}&\equiv\langle\eta'|(\bar{q}q)_{V-A}|0\rangle\langle K^{*-}|(\bar{s}b)_{V-A}|B^-\rangle \\
&=-2f_{\eta'}^q m_{K^*}A_0^{BK^*}(m_{\eta'}^2)(\varepsilon\cdot p_B),
\end{aligned}
$$

$$
X^{(B,\eta'K^*)}\equiv\langle\eta'K^{*-}|(\bar{s}u)_{V-A}|0\rangle\langle 0|(\bar{u}b)_{V-A}|B^-\rangle.
\tag{5.23}
$$

From Table I we see that the electroweak penguin diagram is generally small due to the smallness of its Wilson coefficients, but it does play an essential role in the decays $B^{\pm}\to\eta K^{\pm}$ and $B^0\to\eta K^0$. It is interesting to note that the branching ratios of $B\to\eta^{(')}K^{(*)}$ are all less than 1×10^{-5} except for $B\to\eta'K$, which has a very large branching ratio, of order $(4-6)\times 10^{-5}$. It has been argued in [39] that $\mathcal{B}(B$

$\to\eta'K^*)$ is about twice larger than that of $B\to\eta'K$, which is certainly not the case in our calculation. The ratios of various decay rates are predicted to be

$$
\frac{\mathcal{B}(B\to\eta'K)}{\mathcal{B}(B\to\eta K)}=\begin{cases}72, \\ 296,\end{cases}\quad\frac{\mathcal{B}(B\to\eta'K^*)}{\mathcal{B}(B\to\eta K^*)}=\begin{cases}0.06 & \text{charged }B, \\ 0.02 & \text{neutral }B,\end{cases}
\tag{5.24}
$$

for positive ρ. The destructive (constructive) interference between the terms $X^{(B\eta^{(')},K)}$ and $a_6X_s^{(BK,\eta^{(')})}$ explains the ratio $\mathcal{B}(B\to\eta'K)/\mathcal{B}(B\to\eta K)$: $X_s^{(BK,\eta')}$ has a sign opposite to $X_s^{(BK,\eta)}$ as one can easily see from the wave functions of the η and η', Eq. (5.11). Since the sign of $a_6X_s^{(BK^*,\eta^{(')})}$ is flipped in $B\to\eta^{(')}K^*$ decays, the interference effect becomes the other way around: constructive in $B\to\eta K^*$ and destructive in $B\to\eta'K^*$.

To discuss the decays $B\to\eta^{(')}\pi(\rho)$, we consider $B^-\to\eta'\pi^-$ as an illustration. Its decay amplitude is

$$A(B^- \to \eta'\pi^-) = \frac{G_F}{\sqrt{2}}\left(V_{ub}V_{ud}^*(a_1 X^{(B\eta',\pi)} + a_2 X_u^{(B\pi,\eta')} + 2a_1 X^{(B,\eta'\pi)}) + V_{cb}V_{cd}^* a_2 X_c^{(B\pi,\eta')} \right.$$

$$-V_{tb}V_{td}^*\left\{ \left(a_4 + a_{10} + 2(a_6+a_8)\frac{m_\pi^2}{(m_d+m_u)(m_b-m_u)} \right) X^{(B\eta',\pi)} \right.$$

$$+ \left(a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 \right) X_s^{(B\pi,\eta')} + (a_3 - a_5 - a_7 + a_9)X_c^{(B\pi,\eta')}$$

$$+ \left(2a_4 + 2a_{10} + 4(a_6+a_8)\frac{m_B^2}{(m_d-m_u)(m_b+m_u)} \right) X^{(B,\eta'\pi)}$$

$$+ \left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) \right.$$

$$\left. \left. \left. + \left(a_6 - \frac{1}{2}a_8 \right)\frac{m_{\eta'}^2}{m_s(m_b-m_d)}\left(\frac{f_{\eta'}^s}{f_{\eta'}^u} - 1 \right) r_{\eta'} \right] X_u^{(B\pi,\eta')} \right\} \right), \tag{5.25}$$

where

$$r_{\eta'} = \frac{\sqrt{2f_0^2 - f_8^2}}{\sqrt{2f_8^2 - f_0^2}}\frac{\cos\theta + \frac{1}{\sqrt{2}}\sin\theta}{\cos\theta - \sqrt{2}\sin\theta} \tag{5.26}$$

and

$$X^{(B\eta',\pi)} \equiv \langle \pi^-|(\bar{d}u)_{V-A}|0\rangle\langle \eta'|(\bar{u}b)_{V-A}|B^-\rangle$$

$$= if_\pi(m_B^2 - m_{\eta'}^2)F_0^{B\eta'}(m_\pi^2),$$

$$X_q^{(B\pi,\eta')} \equiv \langle \eta'|(\bar{q}q)_{V-A}|0\rangle\langle \pi^-|(\bar{d}b)_{V-A}|B^-\rangle$$

$$= if_{\eta'}^q(m_B^2 - m_\pi^2)F_0^{B\pi}(m_{\eta'}^2),$$

$$X^{(B,\eta'\pi)} \equiv \langle \eta'\pi^-|(\bar{d}u)_{V-A}|0\rangle\langle 0|(\bar{u}b)_{V-A}|B^-\rangle. \tag{5.27}$$

TABLE I. Branching ratios averaged over CP-conjugate modes for charmless B decays to the η' and η, where ''Tree'' refers to branching ratios from tree diagrams only, ''Tree+QCD'' from tree and QCD penguin diagrams, and ''Full'' denotes full contributions from tree, QCD, and electroweak (EW) penguin diagrams in conjunction with contributions from the process $c\bar{c} \to \eta_0$. Predictions are for $k^2 = m_b^2/2$, $\eta = 0.35$, $\rho = -0.12$ (the first number in parentheses) and $\eta = 0.34$, $\rho = 0.16$ (the second number in parentheses). The decay constants $f_{\eta'}^c = -6$ MeV and $f_\eta^c = -2.4$ MeV are used. The effective number of colors is taken to be $N_c^{\text{eff}}(V-A) = 2$ and $N_c^{\text{eff}}(V+A) = 5$. The running quark masses at $\mu = m_b$ are given by Eqs. (2.13).

Decay	Tree	Tree+QCD	Tree+QCD+EW	Full	Expt. [18]
$B^\pm \to \eta'K^\pm$	1.48×10^{-7}	$(3.56,3.33)\times10^{-5}$	$(3.42,3.20)\times10^{-5}$	$(3.99,3.74)\times10^{-5}$	$(6.5^{+1.5}_{-1.4}\pm0.9)\times10^{-5}$
$B^\pm \to \eta K^\pm$	4.18×10^{-7}	$(0.59,1.27)\times10^{-6}$	$(3.91,7.10)\times10^{-7}$	$(3.88,5.17)\times10^{-7}$	$<1.4\times10^{-5}$
$B^\pm \to \eta'K^{*\pm}$	2.44×10^{-7}	$(3.66,4.00)\times10^{-7}$	$(3.54,4.62)\times10^{-7}$	$(5.73,3.53)\times10^{-7}$	$<13\times10^{-5}$
$B^\pm \to \eta K^{*\pm}$	5.98×10^{-7}	$(6.42,4.09)\times10^{-6}$	$(8.30,5.58)\times10^{-6}$	$(9.22,6.32)\times10^{-6}$	$<3.0\times10^{-5}$
$B^\pm \to \eta'\pi^\pm$	2.13×10^{-6}	$(1.47,2.53)\times10^{-6}$	$(1.49,2.51)\times10^{-6}$	$(1.52,2.75)\times10^{-6}$	$<3.1\times10^{-5}$
$B^\pm \to \eta\pi^\pm$	6.06×10^{-6}	$(4.16,7.11)\times10^{-6}$	$(4.11,7.22)\times10^{-6}$	$(4.14,7.38)\times10^{-6}$	$<1.5\times10^{-5}$
$B^\pm \to \eta'\rho^\pm$	4.44×10^{-6}	$(3.93,4.69)\times10^{-6}$	$(3.94,4.68)\times10^{-6}$	$(3.87,4.88)\times10^{-6}$	$<4.7\times10^{-5}$
$B^\pm \to \eta\rho^\pm$	1.08×10^{-5}	$(0.98,1.13)\times10^{-5}$	$(0.95,1.14)\times10^{-5}$	$(0.95,1.15)\times10^{-5}$	$<3.2\times10^{-5}$
$B_d \to \eta'K^0$	5.38×10^{-9}	$(3.20,3.23)\times10^{-5}$	$(3.00,3.03)\times10^{-5}$	$(3.52,3.55)\times10^{-5}$	$(4.7^{+2.7}_{-2.0}\pm0.9)\times10^{-5}$
$B_d \to \eta K^0$	2.05×10^{-8}	$(3.99,5.54)\times10^{-7}$	$(1.62,2.57)\times10^{-7}$	$(0.64,1.20)\times10^{-7}$	$<3.3\times10^{-5}$
$B_d \to \eta'K^{*0}$	4.49×10^{-9}	$(1.33,3.29)\times10^{-7}$	$(1.46,4.56)\times10^{-7}$	$(2.40,0.87)\times10^{-7}$	$<3.9\times10^{-5}$
$B_d \to \eta K^{*0}$	1.75×10^{-8}	$(5.19,3.70)\times10^{-6}$	$(6.99,4.69)\times10^{-6}$	$(7.85,5.40)\times10^{-6}$	$<3.0\times10^{-5}$
$B_d \to \eta'\pi^0$	2.14×10^{-10}	$(1.75,1.10)\times10^{-7}$	$(1.34,0.85)\times10^{-7}$	$(1.87,1.27)\times10^{-7}$	$<1.1\times10^{-5}$
$B_d \to \eta\pi^0$	1.01×10^{-8}	$(3.99,2.97)\times10^{-7}$	$(3.77,2.83)\times10^{-7}$	$(4.09,3.11)\times10^{-7}$	$<0.8\times10^{-5}$
$B_d \to \eta'\rho^0$	1.34×10^{-8}	$(3.43,1.81)\times10^{-8}$	$(2.85,1.65)\times10^{-8}$	$(2.15,1.83)\times10^{-8}$	$<2.3\times10^{-5}$
$B_d \to \eta\rho^0$	1.99×10^{-8}	$(4.27,9.07)\times10^{-8}$	$(3.14,5.84)\times10^{-8}$	$(3.11,5.36)\times10^{-8}$	$<1.3\times10^{-5}$

In deriving Eq. (5.25) we have applied the matrix elements[10]

$$\langle \eta' | \bar{u} \gamma_5 u | 0 \rangle = \langle \eta' | \bar{d} \gamma_5 d | 0 \rangle = r_{\eta'} \langle \eta' | \bar{s} \gamma_5 s | 0 \rangle, \tag{5.28}$$

with $r_{\eta'}$ being given by Eq. (5.26).

Since $V_{ub} V_{ud}^*$, $V_{cb} V_{cd}^*$, $V_{tb} V_{td}^*$ are all comparable in magnitude [cf. Eq. (3.15)] and since the Wilson coefficients of penguin operators are rather small, it is expected that $B \to \eta^{(')} \pi, \eta^{(')} \rho$ are dominated by spectator diagrams.[11] From Table I we see that this is indeed the case except for the decay modes $B^0 \to \eta^{(')} \pi^0$ which are penguin dominated. To compute the decay rate of $B \to \eta \pi(\rho)$ we have applied the matrix element $\langle \eta | \bar{u} \gamma_5 u | 0 \rangle = r_\eta \langle \eta | \bar{s} \gamma_5 s | 0 \rangle$ with

$$r_\eta = -\frac{1}{2} \frac{\sqrt{2 f_0^2 - f_8^2}}{\sqrt{2 f_8^2 - f_0^2}} \frac{\cos \theta - \sqrt{2} \sin \theta}{\cos \theta + \frac{1}{\sqrt{2}} \sin \theta}. \tag{5.29}$$

The mechanism of $c\bar{c} \to \eta_0$ is less significant in $B \to \eta^{(')} \pi(\rho)$ decays because it does not gain advantage from the quark mixing angle as in the case of $B \to \eta^{(')} K(K^*)$. We see from Table I the minor role played by the charm content

of the η' except for the decay $B^0 \to \eta' \pi^0$. In general, the decay rates of $B \to \eta^{(')} \pi(\rho)$ are not sensitive to the values of $N_c^{\text{eff}}(V-A)$ and $N_c^{\text{eff}}(V+A)$ and do not vary significantly from channel to channel:

$$\mathcal{B}[B^\pm \to \eta^{(')} \pi(\rho)] \sim (3-10) \times 10^{-6},$$

$$\mathcal{B}[\bar{B}^0 \to \eta^{(')} \pi(\rho)] \sim (0.2-4) \times 10^{-7}. \tag{5.30}$$

It is interesting to note that $\mathcal{B}[B \to \eta \pi(\rho)] > \mathcal{B}[B \to \eta' \pi(\rho)]$.

VI. DIFFICULTIES WITH $B^- \to K^- \omega$

Up to now we have shown that CLEO results on hadronic charmless B decays can be satisfactorily explained provided that $N_c^{\text{eff}}(V-A) \approx 2$ and $N_c^{\text{eff}}(V+A) \gtrsim \mathcal{O}(4)$. However, there is one CLEO measurement, namely, the decay $B^\pm \to \omega K^\pm$, which is beyond our explanation and hence may impose a potentially serious difficulty. In this section we will first explore the problem and then proceed to suggest some possible solutions.

The decay amplitude of $B^- \to \omega K^-$ is very similar to $B^- \to \omega \pi^-$ and has the expression

$$A(B^- \to \omega K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* (a_1 X^{(B\omega, K)} + a_2 X^{(BK, \omega)} + a_1 X^{(B, K\omega)}) \right.$$

$$- V_{tb} V_{ts}^* \left[\left(a_4 + a_{10} - 2(a_6 + a_8) \frac{m_K^2}{(m_b + m_u)(m_s + m_u)} \right) X^{(B\omega, K)} \right.$$

$$+ \frac{1}{2} (4a_3 + 4a_5 + a_7 + a_9) X^{(BK, \omega)}$$

$$\left. \left. + \left(a_4 + a_{10} - 2(a_6 + a_8) \frac{m_B^2}{(m_b + m_u)(m_s + m_u)} \right) X^{(B, K\omega)} \right] \right\}, \tag{6.1}$$

where

---

[10]The matrix element $\langle \eta' | \bar{u} \gamma_5 u | 0 \rangle$ can be obtained from [53] and it is slightly different from the corresponding one in [52,51]:

$$\langle \eta' | \bar{u} \gamma_5 u | 0 \rangle = \frac{f_8 \cos \theta + \frac{1}{\sqrt{2}} f_0 \sin \theta}{f_8 \cos \theta - \sqrt{2} f_0 \sin \theta} \langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$$

$$= -\frac{1}{2} \frac{f_{\eta'}^s}{f_{\eta'}^u} \langle \eta' | \bar{s} \gamma_5 s | 0 \rangle.$$

[11]The branching ratios of $B \to \eta^{(')} \pi, \eta^{(')} \rho$ are largely overestimated in [40,50] as the incorrect matrix element $\langle \eta' | \bar{u} \gamma_5 u | 0 \rangle = -i m_{\eta'}^2 f_{\eta'}^u / (2 m_u)$ is applied there.

$$X^{(B\omega, K)} \equiv \langle K^- | (\bar{s} u)_{V-A} | 0 \rangle \langle \omega | (\bar{u} b)_{V-A} | B^- \rangle$$

$$= -2 f_K m_\omega A_0^{B\omega}(m_K^2)(\varepsilon \cdot p_B),$$

$$X_u^{(BK, \omega)} \equiv \langle \omega | (\bar{u} u)_{V-A} | 0 \rangle \langle K^- | (\bar{s} b)_{V-A} | B^- \rangle$$

$$= -\sqrt{2} f_\omega m_\omega F_1^{BK}(m_\omega^2)(\varepsilon \cdot p_B),$$

$$X^{(B, \omega K)} \equiv \langle \omega K^- | (\bar{s} u)_{V-A} | 0 \rangle \langle 0 | (\bar{u} b)_{V-A} | B^- \rangle. \tag{6.2}$$

We see from Fig. 9 that the calculated branching ratio using $N_c^{\text{eff}}(V-A) = 2$, $F_1^{BK}(0) = 0.34$, and $A_0^{B\omega}(0) = 0.28/\sqrt{2}$ [7] is too small compared to experiment [34]:

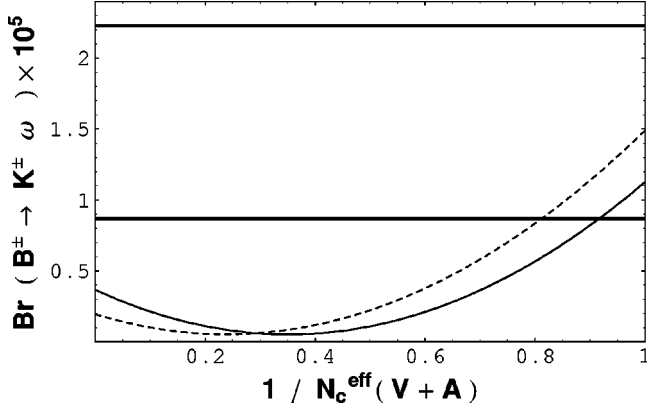$$\mathcal{B}(B^\pm \to \omega K^\pm) = (1.5^{+0.7}_{-0.6} \pm 0.2) \times 10^{-5}. \tag{6.3}$$

FIG. 9. The branching ratio of $B^{\pm} \to \omega K^{\pm}$ vs $1/N_c^{\text{eff}}(V+A)$ with $N_c^{\text{eff}}(V-A)$ being fixed at the value of 2. The solid and dashed curves are for $\eta = 0.34$, $\rho = 0.16$ and $\eta = 0.35$, $\rho = -0.12$, respectively. The solid thick lines are the CLEO measurements with one sigma errors.

In fact, all the region of $1/N_c^{\text{eff}}(V+A) < 0.9$ is excluded. Nevertheless, if $N_c^{\text{eff}}(V-A)$ is taken to be the same as $N_c^{\text{eff}}(V+A)$, then a rather small value of $1/N_c^{\text{eff}} < 0.05$ is experimentally allowed [24,27] (see Fig. 10). In other words, N_c^{eff} is preferred to be very large in $B^{\pm} \to \omega K^{\pm}$. In our opinion, however, a very large value of $N_c^{\text{eff}}(V-A)$ is rather unlikely for several reasons: (i) A small $N_c^{\text{eff}}(V-A) \approx 2$ is favored in other charmless B decays: $B \to \pi\pi, \pi\omega$ and $B \to \eta' K$. (ii) It will lead to a too large nonfactorizable term, which is not consistent with the small nonfactorizable effect observed in the spectator amplitudes of $B \to D\pi$ and the picture that the nonperturbative feature of nonfactorizable effects is loose in the energetic two-body decays of the B meson, as we have elaborated before (see the end of Sec. IV). It thus appears to us that the observed large decays rate of $B^{\pm} \to \omega K^{\pm}$ is attributed to other mechanisms rather than to a very large value of N_c^{eff}.

So far we have neglected three effects in the consideration of $B^{\pm} \to \omega K^{\pm}$: W annihilation, spacelike penguin diagrams, and final-state interactions (FSIs); all of them are difficult to estimate. In order to understand why $\mathcal{B}(B^{\pm} \to \omega\pi^{\pm}) \lesssim \mathcal{B}(B^{\pm} \to \omega K^{\pm})$ experimentally, we need a mechanism which will only enhance the latter. It appears that FSIs may play this role. Since $B^- \to \omega K^-$ involves only a single isospin amplitude, inelastic scattering will be the dominant effect of FSIs. For example, $b \to c\bar{c}s$ and $b \to u\bar{u}s$ modes can mix with each other so that the decay $B^- \to \omega K^-$ arises either from $b \to c\bar{c}s$ or indirectly through $B^- \to D^0 D_s^{*-}$ or $D^{*0} D_s^-$ (via $b \to c\bar{c}s$) with a rescattering $D^0 D_s^{*-}$ (or $D^{*0} D_s^-$) $\to \omega K^-$. For the decay $B^- \to \omega\pi^-$, the inelastic scattering $B^- \to \{DD^*\} \to \omega\pi^-$ is Cabibbo suppressed. Therefore, it is possible that $B^- \to \omega K^-$ receives large FSIs from inelastic scattering but $B^- \to \omega\pi^-$ does not. Since $B^0 \to \omega K^0$ does not receive contributions from W annihilation, its measurement can be used to test the relative strength between FSIs and annihilation terms. If the branching ratios of $B^0 \to \omega K^0$ and $B^{\pm} \to \omega K^{\pm}$ are close, this will imply the importance of FSIs.
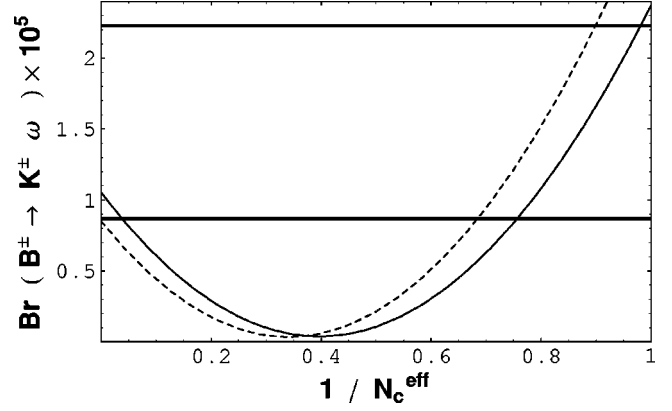

FIG. 10. Same as Fig. 9 except that $N_c^{\text{eff}}(V+A) = N_c^{\text{eff}}(V-A) = N_c^{\text{eff}}$.

VII. DISCUSSIONS AND CONCLUSIONS

For a given effective weak Hamiltonian, there are two important issues in the study of the hadronic matrix elements for nonleptonic decays of heavy mesons: one is the renormalization scale and scheme dependence of the matrix element, and the other is the nonfactorizable effect. For the former, we have emphasized that it is important to first evaluate the vertex and penguin corrections to the matrix element of four-quark operators at the scale μ so that $\langle O(\mu) \rangle = g(\mu) \langle O \rangle_{\text{tree}}$ and then apply factorization or any model calculation to $\langle O \rangle_{\text{tree}}$. The resulting effective coefficients $c_i^{\text{eff}} = c_i(\mu) g(\mu)$ are renormalization-scale and -scheme independent. We pointed out that while $c_{1,2}^{\text{eff}} \approx c_{1,2}^{\text{LO}}(\mu)$ at $\mu = m_b(m_b)$ for current-current operators, the real parts of c_{3-6}^{eff} are about one and half times larger than the leading-order penguin Wilson coefficients. This means that to describe the hadronic charmless B decays dominated by penguin diagrams, it is necessary and inevitable to take into account the penguin corrections to the four-quark operators.

Nonfactorizable effects in hadronic matrix elements of $B \to PP, VP$ decays can be parameterized in terms of the effective number of colors, N_c^{eff}, in the so-called generalized factorization scheme; the deviation of $1/N_c^{\text{eff}}$ from $1/N_c$ ($N_c = 3$) characterizes the nonfactorizable effect. We show that, contrary to the common assumption, $N_c^{\text{eff}}(V+A)$ induced by the $(V-A)(V+A)$ operators $O_{5,6,7,8}$ are theoretically and experimentally different from $N_c^{\text{eff}}(V-A)$ generated by the $(V-A)(V-A)$ operators. The CLEO data of $B^{\pm} \to \omega\pi^{\pm}$ available last year clearly indicate that $N_c^{\text{eff}}(V-A)$ is favored to be small, $N_c^{\text{eff}}(V-A) < 2.9$. This is consistent with the observation that $N_c^{\text{eff}}(V-A) \approx 2$ in $B \to D\pi$ decays. Unfortunately, the significance of $B^{\pm} \to \omega\pi^{\pm}$ is reduced in the recent CLEO analysis and only an upper limit is quoted. Therefore, a measurement of its branching ratio is urgently needed. In analogy to the class-III $B \to D\pi$ decays, the interference effect of spectator amplitudes in charged B decays $B^- \to \pi^-\pi^0, \rho^-\pi^0, \pi^-\rho^0$ is sensitive to $N_c^{\text{eff}}(V-A)$; measurements of them [see Eqs. (3.23)] will be very useful to pin down the value of $N_c^{\text{eff}}(V-A)$.

Contrary to the nonfactorizable effects in spectator-

dominated rare B decays, we found that $N_c^{\text{eff}}(V+A)$ extracted from the penguin-dominated decay $B^{\pm} \to \phi K^{\pm}$ is larger than $N_c^{\text{eff}}(V-A)$. This means that nonfactorizable effects in tree and penguin amplitudes behave differently. It turns out this observation is the key element for understanding the CLEO measurement of $B \to \eta' K$. In the conventional way of treating $N_c^{\text{eff}}(V+A)$ and $N_c^{\text{eff}}(V-A)$ in the same manner, the branching ratio of $B^{\pm} \to \eta' K^{\pm}$ after including the anomaly effect in the matrix element $\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle$ is naively only of the order of 1×10^{-5}. The running strange quark mass at $\mu = m_b$ and SU(3) breaking in the decay constants f_8 and f_0 will enhance $\mathcal{B}(B \to \eta' K)$ to the order of $(2-3) \times 10^{-5}$ with $f_{\eta'}^c = -6$ MeV. This is still lower than the central value of the CLEO measurements. Also, the charm content of the η' is not welcome for explaining the decay rate of $B \to \eta' K$ at small values of $1/N_c^{\text{eff}}$. We showed that the fact that $N_c^{\text{eff}}(V+A) > N_c^{\text{eff}}(V-A) \approx 2$ will substantially enhance the branching ratio of $B^{\pm} \to \eta' K^{\pm}$ to $(3.7-5) \times 10^{-5}$ at $1/N_c^{\text{eff}}(V+A) \lesssim 0.2$. Unlike the previous analysis, the small charm content of the η' is now always in the right direction for enhancement irrespective of the values of $1/N_c^{\text{eff}}(V+A)$. The predicted branching ratio of $B^0 \to \eta' K^0$ is in good agreement with experiment and the calculation of $B^{\pm} \to \eta' K^{\pm}$ is compatible with the data. For a slightly enhanced $f_{\eta'}^c \approx -15$ MeV, as implied by a recent theoretical estimate, we found that the agreement of the predicted branching ratio for $B \to \eta' K$ with experiment is very impressive. It is thus important to pin down the decay constant $f_{\eta'}^c$, recalling that the commonly used value $|f_{\eta'}^c| = 6$ MeV is extracted from experiment within the nonrelativistic quark model framework. We conclude that no new mechanism in the standard model or new physics beyond the standard model is needed to explain $B \to \eta' K$. We have also analyzed charmless B decays into the η' and η in some detail. The branching ratios of the spectator-dominated decays $B \to \eta^{(')} \pi, \eta^{(')} \rho$ were largely overestimated in the previous analysis because the matrix element $\langle \eta^{(')} | \bar{u} \gamma_5 u | 0 \rangle$ was not evaluated correctly before.

Although the CLEO measurements of hadronic charmless B decays are satisfactorily explained in the present framework, we found that it is difficult to understand the experimental observation that $\Gamma(B^{\pm} \to \omega \pi^{\pm}) \lesssim \Gamma(B^{\pm} \to \omega K^{\pm})$. The calculated branching ratio of $B^{\pm} \to \omega K^{\pm}$ is too small compared to experiment. We conjecture that final-state interactions via inelastic scattering may contribute in a sizable way to $B^{\pm} \to \omega K^{\pm}$, but are negligible for $B^{\pm} \to \omega \pi^{\pm}$ due to the Cabibbo-angle suppression. Clearly this decay mode deserves further serious investigation and a measurement of the neutral decay mode $B^0 \to \omega K^0$ will be very useful to clarify the issue.

Under the factorization hypothesis, the decays $B \to \phi K$ and $B \to \phi K^*$ should have almost the same branching ratios, a prediction not borne out by current data. Therefore, it is crucial to measure the charged and neutral decay modes of $B \to \phi(K, K^*)$ in order to see if the generalized factorization approach is applicable to $B \to \phi K^*$ decay.

To conclude, based on the available CLEO data on hadronic charmless two-body decays of the B meson, we have shown that the nonfactorizable effect induced by the $(V-A)(V+A)$ operators is different from that generated by the $(V-A)(V-A)$ operators. This is the key element for explaining the CLEO measurement of $B \to \eta' K$.

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