Mass signature of supernova ν_μ and ν_τ neutrinos in the Sudbury Neutrino Observatory

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(Received 9 June 1998; published 5 October 1998)

Core-collapse supernovae emit of order 10^{58} neutrinos and antineutrinos of all flavors over several seconds, with average energies of 10–25 MeV. In the Sudbury Neutrino Observatory (SNO), which begins operation this year, neutrinos and antineutrinos of all flavors can be detected by reactions which break up the deuteron. For a future Galactic supernova at a distance of 10 kpc, several hundred events will be observed in SNO. The ν_μ and ν_τ neutrinos and antineutrinos are of particular interest, as a test of the supernova mechanism. In addition, it is possible to measure or limit their masses by their delay (determined from neutral-current events) relative to the $\bar{\nu}_e$ neutrinos (determined from charged-current events). Numerical results are presented for such a future supernova as seen in SNO. Under reasonable assumptions, and in the presence of the expected counting statistics, a ν_μ or ν_τ mass down to about 30 eV can be simply and robustly determined. If zero delay is measured, then the mass limit is *independent* of the distance D. At present, this seems to be the best possibility for direct determination of a ν_μ or ν_τ mass within the cosmologically interesting range. We also show how to separately study the supernova and neutrino physics, and how changes in the assumed supernova parameters would affect the mass sensitivity. [S0556-2821(98)00821-2]

PACS number(s): 14.60.Pq, 25.30.Pt, 95.55.Vj, 97.60.Bw

I. INTRODUCTION

As emphasized by Weinberg [1] and many others, whether or not neutrinos have masses or other properties beyond the standard model are questions which address some of the deepest issues in particle physics. Yet almost seventy years after they were proposed by Pauli, most of the properties of neutrinos are defined only by limits. In particular their masses, if any, are unknown. Results from several experiments strongly suggest that neutrino flavor mixing occurs in solar, atmospheric, and accelerator neutrinos, and proof of mixing would be a proof of mass. Direct searches for neutrino mass yield only the following limits: $m_{\bar{\nu}_e} \lesssim 5$ eV [2], $m_{\nu_\mu} < 170$ keV [3], and $m_{\nu_\tau} < 24$ MeV [3]. With current techniques, it will be very difficult to significantly improve these limits. In fact, the interesting mass scale is much lower than the latter two limits, and is given by the requirement that neutrinos do not overclose the universe (see [4] and references therein):

$$\sum_{i=1}^{3} m_{\nu_i} \lesssim 100 \ \ \mathrm{eV}. \tag{1}$$

Neutrino masses exceeding this bound are allowed for unstable neutrinos.

The most promising technique for direct determination of neutrino mass below the cosmological bound seems to be from time-of-flight measurements over astrophysical distances. With the present generation of detectors, neutrinos and antineutrinos of all flavors from a Galactic supernova will be readily detectable. Even a tiny mass will make the velocity slightly less than for a massless neutrino, and over the large distance to a supernova will cause a measurable delay in the arrival time. A neutrino with a mass m (in eV) and energy E (in MeV) will experience an energy-dependent delay (in s) relative to a massless neutrino in traveling over a distance D (in 10 kpc) of

$$\Delta t(E) = 0.515 \left(\frac{m}{E} \right)^2 D, \tag{2}$$

where only the lowest order in the small mass has been kept. For a ν_μ or ν_τ mass near the cosmological bound, the delay for a single neutrino is of order seconds. Because of the limit on the $\bar{\nu}_e$ mass, this delay can be measured from the arrival of the ν_μ and ν_τ events relative to the $\bar{\nu}_e$ events. With the statistical power of many events, it is possible to detect an average delay of order 0.1 s. Since one expects a type-II supernova about every 30 years in our Galaxy [5], and since supernova neutrino detectors are currently operating, there is a good chance that this technique can be used to dramatically improve the limits on the ν_μ and ν_τ masses.

In a previous paper [6], we considered a future supernova at 10 kpc (approximately the distance to the Galactic center) as seen by the SuperKamiokande (SK) detector. Such a supernova will cause about 710 neutral-current excitations of ^{16}O by ν_μ and ν_τ (and their antiparticles), followed by detectable gamma emission. In addition, about 8300 events are expected from $\bar{\nu}_e + p \to e^+ + n$. A ν_μ or ν_τ mass would cause a delay of the average arrival time of the neutral-current events as compared to the $\bar{\nu}_e$ events. We have shown how to test the statistical significance of the difference in average arrival times and how to extract the allowed neutrino mass range. Taking into account the finite statistics, we concluded that with this signal at SK, one can reach a mass sensitivity down to about 45 eV for the ν_μ or ν_τ mass.

In this paper, we consider the capabilities of the Sudbury Neutrino Observatory (SNO). For the same supernova, SNO

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will see in total about 400 events caused by the neutral-current breakup of deuterons by ν_μ and ν_τ (and their antiparticles). As in Ref. [6], the technique used is to compare the average arrival time of the (possibly massive) ν_μ and ν_τ events with the average arrival time of the $\overline{\nu}_e$ events. The sensitivity of SNO for this measurement has been estimated previously [7–11], with the claimed minimal detectable mass ranging from 10 eV to 200 eV. Here, we present a detailed calculation of the mass sensitivity of SNO, taking the finite statistics into account quantitatively. While the statistics are lower than for SK, the characteristic energy is lower (and so the delay is larger), leading to a sensitivity to a ν_μ or ν_τ mass down to about 30 eV.

The problem of ν_μ or ν_τ mass determination with supernova neutrinos has been discussed for other neutrino detection reactions and using different analysis techniques. Neutrino-electron scattering in water-Cerenkov detectors (e.g., SK) has been considered in Refs. [9–13]. There have also been proposals to use neutral-current excitation of various nuclei as a signal [14–17]. As shown below, some of the previous estimates seem to be too optimistic in that they assume a very sharp pulse of neutrinos in time, which makes any delay more apparent.

In Sec. II, we describe the details of the supernova model, the detector properties, and the neutrino detection signals. In Sec. III, we review our analysis technique and show our results for the sensitivity of SNO to small ν_μ or ν_τ masses. We also consider how the mass sensitivity would be modified if the actual supernova parameters differ from those assumed here. In Sec. IV, we discuss how the supernova parameters and neutrino properties can be separately extracted from the same data. In Sec. V, we summarize our results.

II. PRODUCTION AND DETECTION OF SUPERNOVA NEUTRINOS

A. Supernova neutrinos

When the core of a large star ($M \geqslant 8 M_\odot$) runs out of nuclear fuel, it collapses and forms a proto-neutron star with a central density well above the normal nuclear density (for a review of type-II supernova theory, see Ref. [18]). The total energy released in the collapse, i.e., the gravitational binding energy of the core ($E_B \sim G_N M_\odot^2 / R$ with $R \sim 10$ km), is about 3×10^{53} ergs; about 99% of that is carried away by neutrinos and antineutrinos, the particles with the longest mean free path. The proto-neutron star is dense enough that neutrinos diffuse outward over a time scale of several seconds, maintaining thermal equilibrium with the matter. When they are within about one mean free path of the edge, they escape freely, with a thermal spectrum characteristic of the surface of last scattering. The luminosities of the different neutrino flavors are approximately equal at all times; see, e.g., Ref. [19].

Those flavors which interact the most strongly with the matter will decouple at the largest radius and thus the lowest temperature. As explained in Ref. [6], the ν_μ and ν_τ neutrinos and their antiparticles, which we collectively call ν_x neutrinos, have a temperature of about 8 MeV (or $\langle E \rangle \simeq 25$ MeV). The $\overline{\nu}_e$ neutrinos have a temperature of about 5 MeV ($\langle E \rangle \simeq 16$ MeV), and the ν_e neutrinos have a temperature of about 3.5 MeV ($\langle E \rangle \simeq 11$ MeV); see, e.g., Refs. [19, 20]. These are the temperatures used in our analysis. While there is some variation between models in the actual values of the temperatures, all of the models have a temperature hierarchy as above. This is important for separating the ν_x neutrinos from the ν_e and $\overline{\nu}_e$ neutrinos. The energy distributions are taken here to be Fermi-Dirac distributions, characterized only by the temperatures given above. More elaborate models also introduce a chemical potential parameter to reduce the high-energy tail of the Fermi-Dirac distribution; the effect of this is considered below.

While some numerical supernova models have temperatures decreasing with time, more recent models [19] have temperatures increasing with time. This is a consequence of the electron fraction and, hence, the opacities decreasing with time. The real temperature variation is probably not large (see, e.g., Ref. [19]). A well-motivated form for temperature variation may eventually be obtained from the supernova $\overline{\nu}_e$ data or from more-developed numerical models. The analysis of this paper could be easily modified to allow a varying temperature; until there is a compelling reason to use a particular form, we simply use constant temperatures.

The neutrino luminosity rises quickly over a time of order 0.1 s, and then falls over a time of order several seconds. The luminosity used here is composed of two pieces. The first gives a very short rise from zero to the full height over a time 0.09 s, using one side of a Gaussian with $\sigma = 0.03$ s. The rise is so fast that the details of its shape are irrelevant. The second piece is an exponential decay with time constant $\tau = 3$ s. The luminosity then has a width of 10 s or so, consistent with the SN 1987A observations. The detailed form of the neutrino luminosity is less important than the general shape features and their characteristic durations.

This description of a supernova is consistent with theoretical expectations, numerical supernova models, and the SN 1987A observations. With the next Galactic supernova, there will obviously be great improvements in the understanding of the supernova neutrinos. In Sec. IV, we discuss how to separately extract the supernova parameters and neutrino properties from the same data. Throughout the paper, we assume that the distance to the supernova is $D = 10$ kpc.

B. General form of the neutrino scattering rate

Here we briefly summarize the notation of Ref. [6]. Under the assumption that the neutrino energy spectra are time-independent, the double differential number distribution of neutrinos of a given flavor (one of $\nu_e, \overline{\nu}_e, \nu_\mu, \overline{\nu}_\mu, \nu_\tau, \overline{\nu}_\tau$) at the source can be written as

$$\frac{d^2 N_\nu}{dE\, dt_i} = f(E) \frac{L(t_i)}{\langle E \rangle}, \tag{3}$$

where E is the neutrino energy and t_i is the emission time. Here $f(E)$ is the normalized thermal spectrum, $L(t_i)$ is the the luminosity (energy flux per unit time), and $\langle E \rangle$ is the (time-independent) average neutrino energy. The double integral of this quantity is the total number N_ν of emitted neutrinos of that flavor. This form is convenient since we assume that the luminosities of the different flavors are approximately equal at every time t_i. As stated above, the energy spectrum $f(E)$ is assumed to be a Fermi-Dirac distribution, and the luminosity $L(t_i)$ is assumed to have a very sharp rise and an exponential decline.

The arrival time of a neutrino of mass m at the detector is $t=t_i+D+\Delta t(E)$, where D is the distance to the source, and the energy-dependent time delay is given by Eq. (2). For convenience, we drop the constant D. Then the double differential flux of neutrinos at the detector is given by

$$\frac{1}{4\pi D^2}\frac{d^2N_\nu}{dE\,dt}=\frac{1}{4\pi D^2}\int dt_i\frac{d^2N_\nu}{dE\,dt_i}\delta(t-t_i-\Delta t(E))$$
$$=\frac{1}{4\pi D^2}f(E)\frac{L(t-\Delta t(E))}{\langle E\rangle}. \tag{4}$$

Note that because of the mass effects, this is no longer the product of a function of energy alone and a function of time alone. The scattering rate for a given neutrino reaction in SNO is

$$\frac{dN_{sc}}{dt}=N_{D_2O}n\int dE\,\sigma(E)\frac{1}{4\pi D^2}\frac{d^2N_\nu}{dE\,dt}, \tag{5}$$

where N_{D_2O} is the number of heavy-water molecules in the detector, $\sigma(E)$ the cross section for a neutrino of energy E on the target particle, and n the number of targets per molecule for the given reaction. Using the results above, the scattering rate (in s^{-1}) can be written

$$\frac{dN_{sc}}{dt}=C\int dE\,f(E)\left(\frac{\sigma(E)}{10^{-42}\ \mathrm{cm}^2}\right)\left(\frac{L(t-\Delta t(E))}{E_B/6}\right), \tag{6}$$

$$C=8.28\left(\frac{E_B}{10^{53}\ \mathrm{ergs}}\right)\left(\frac{1\ \mathrm{MeV}}{T}\right)\left(\frac{10\ \mathrm{kpc}}{D}\right)^2\left(\frac{\mathrm{det.\ mass}}{1\ \mathrm{kton}}\right)n. \tag{7}$$

In the above, T is the spectrum temperature (where we assume $\langle E\rangle=3.15T$, as appropriate for a Fermi-Dirac spectrum), and $f(E)$ is in MeV^{-1}. For a light-water detector, the initial coefficient in C is 9.21 instead of 8.28. For equal luminosities in each flavor, the total binding energy released in a given flavor is $E_B/6$. Note that we ignore the prompt burst of ν_e neutrinos, since these carry only of order 1% of the total energy. When an integral over all arrival times is made, the luminosity term in Eq. (6) integrates to one, giving for the total number of scattering events:

$$N_{sc}=C\int dE\,f(E)\left(\frac{\sigma(E)}{10^{-42}\ \mathrm{cm}^2}\right). \tag{8}$$

For massless neutrinos, $\Delta t(E)=0$, and in Eq. (6) the luminosity can be taken outside of the integral, so that the time dependence of the scattering rate depends only on the time dependence of the luminosity.

C. The Sudbury Neutrino Observatory

When it begins operation this year, SNO will be the first deuterium-based detector for neutrino astrophysics. The SNO detector is described in Refs. [7,21]. Here we give a short summary of the properties relevant to our analysis. Deuterium is an excellent target for neutrinos since both the charged- and neutral-current cross sections are reasonably large. The active part of the detector is 1 kton of pure D_2O, separated by an acrylic vessel from 1.7 kton of light water. This entire volume is viewed by 10^4 phototubes which can see about 1.4 kton of the light water with good efficiency [22]. We assume that events in the D_2O can be separated from events in the H_2O.

The proposed threshold for solar neutrino studies is 5 MeV (the physics potential of the SNO solar neutrino studies is treated in Ref. [23] and references therein). At this energy, the contribution from the time-independent background is expected to be small [21]. For a Galactic supernova, one expects several hundred events over about 10 s, and a much higher background rate can be tolerated, allowing a lower threshold. For a threshold of 5 MeV, the solar neutrino rate of about 10^{-4} s^{-1} is a background for supernova neutrinos. Below 5 MeV, the background rate increases very steeply. From Ref. [21], we estimate that the threshold for the supernova analysis can be lowered by a few MeV while keeping the background contribution negligible. This would ensure that almost all low-energy gammas from neutron captures as well as electrons and positrons from charged-current events are detected.

Electrons and positrons will be detected by their Čerenkov radiation, and gammas via secondary electrons and positrons. It is not possible for SNO to distinguish between electron, positrons, and gammas of comparable energy. The only way to detect the neutral-current breakup of the deuteron is to detect the final neutron. There are three neutron detection techniques proposed for SNO: (n,γ) on deuterons in pure D_2O, giving a gamma of energy 6.25 MeV; (n,γ) on ^{35}Cl in D_2O with $MgCl_2$ salt added, giving a gamma cascade of energy 8.6 MeV; and direct n detection in ^3He proportional counter tubes suspended into the D_2O. If either of the latter two techniques are used, there will still be some contribution from neutron captures on deuterons. In this paper, as in all previous studies of the supernova capabilities of SNO, we assume that the neutron detection efficiency is nearly 100%. In Sec. III, we show that a reduced neutron detection efficiency has only a small effect on the mass sensitivity. It therefore makes very little difference which neutron detection technique or techniques are in place when the supernova occurs.

D. Neutrino signals in SNO

Neutrinos and antineutrinos of all flavors cause the neutral-current breakup of the deuteron:

$$\nu+d\rightarrow\nu+p+n, \tag{9}$$

$$\bar{\nu}+d\rightarrow\bar{\nu}+p+n, \tag{10}$$

TABLE I. Calculated numbers of events expected in SNO for a supernova at 10 kpc. The other parameters (e.g., neutrino spectrum temperatures) are given in the text. In rows with two reactions listed, the number of events is the total for both. The notation ν indicates the sum of ν_e, ν_μ, and ν_τ, though they do not contribute equally to a given reaction, and X indicates either $n+{}^{15}\mathrm{O}$ or $p+{}^{15}\mathrm{N}$.

Events in 1 kton D_2O	
$\nu+d\to\nu+p+n$ $\bar{\nu}+d\to\bar{\nu}+p+n$	485
$\nu_e+d\to e^-+p+p$ $\bar{\nu}_e+d\to e^++n+n$	160
$\nu+{}^{16}\mathrm{O}\to\nu+\gamma+X$ $\bar{\nu}+{}^{16}\mathrm{O}\to\bar{\nu}+\gamma+X$	20
$\nu+{}^{16}\mathrm{O}\to\nu+n+{}^{15}\mathrm{O}$ $\bar{\nu}+{}^{16}\mathrm{O}\to\bar{\nu}+n+{}^{15}\mathrm{O}$	15
$\nu+e^-\to\nu+e^-$ $\bar{\nu}+e^-\to\bar{\nu}+e^-$	10
Events in 1.4 kton H_2O	
$\bar{\nu}_e+p\to e^++n$	365
$\nu+{}^{16}\mathrm{O}\to\nu+\gamma+X$ $\bar{\nu}+{}^{16}\mathrm{O}\to\bar{\nu}+\gamma+X$	30
$\nu+e^-\to\nu+e^-$ $\bar{\nu}+e^-\to\bar{\nu}+e^-$	15

with thresholds of 2.22 MeV. There are about 35 ν_e, 110 ν_μ, and 110 ν_τ events from the first reaction, and 50 $\bar{\nu}_e$, 90 $\bar{\nu}_\mu$, and 90 $\bar{\nu}_\tau$ events from the second reaction. Electron neutrinos and antineutrinos in addition cause the charged-current breakup of the deuteron:

$$\nu_e+d\to e^-+p+p, \tag{11}$$

$$\overline{\nu_e}+d\to e^++n+n, \tag{12}$$

with thresholds of 1.44 and 4.03 MeV, respectively. There are about 80 events each from these two reactions. The numbers of events are summarized in Table I. The cross sections for all these reactions have an energy dependence roughly of the form $\sigma(E)\sim(E-E_{\mathrm{th}})^2$. There have been many calculations of the neutrino-deuteron cross sections [24,25] and they are now rather well-determined. In this paper, we use the tabulated cross sections of Ref. [25]; the quoted uncertainty is 5% or less at the relevant energies. All of the outgoing particles in the deuteron breakup reactions are emitted approximately isotropically.

The neutral-current reactions are flavor-blind. However, since neutrinos and antineutrinos from a supernova have spectra with a hierarchy of temperatures, the energy dependence of the rates favors the higher-temperature flavors. In particular, most (about 82%) of the events will be from ν_x neutrinos. (Previous studies which indicated a percentage of about 90% used a higher ν_x temperature.)

In order to estimate the delay, Eq. (2) can be evaluated with a characteristic ν_x energy. However, one should not use the average energy $\langle E\rangle$, but rather a characteristic energy that takes into account the weighting with the cross section as well. For the neutral-current neutrino-deuteron reactions, this is shown later to be about 32 MeV. In addition, the fact that the neutrinos have a spectrum of energies means that different values of E contribute to the time delay, causing dispersion of the neutrino pulse as it travels from the supernova. It turns out that for the small masses we are primarily interested in, these dispersive effects are minimal.

Since the neutron capture time may be as large as several ms, and the event rates may be high, concerns were raised in Refs. [10, 11] that events would overlap in time and that it would therefore be difficult to distinguish charged-current and neutral-current events. Only during the first second or so are the rates likely to be high enough that this can occur. In the first second, there are about 150 neutral-current events and only about 45 charged-current events. Further, if the neutron mean free path is less than the diameter of the acrylic vessel, then it will be possible to use spatial information to distinguish events. In any case, the possible contamination of the neutral-current rate is very small, and the mass sensitivity will not be significantly affected.

For the solar neutrino studies, the electron from the charged-current ν_e reaction has a low energy and can be confused with a gamma from a neutron-capture event. Since the supernova ν_e energy is much higher, the electron energy in a charged-current reaction is high enough that only rarely can it be confused with a gamma. Thus neutrons can almost always be identified, either by direct detection with a proportional counter, or by the energy of the subsequent gamma. Positrons from charged-current events with $\overline{\nu_e}$ can be identified by their high energy and coincidence with two neutrons. The spectra of electron and positron energies will be broad, peaking at about 15 and 20 MeV, respectively.

So far, we have discussed only the neutrino-deuteron reactions. As noted in Ref. [26], the neutral-current excitation of ${}^{16}\mathrm{O}$ into the continuum, followed by neutron or proton emission, can also be detected. About 30% of the time, the A=15 nucleus is left in an excited state which decays by gamma emission, with gamma energies between 5 and 10 MeV. These gammas are detectable, as is the neutron. The remaining 70% of the time, the A=15 nucleus is left in the ground state. In these cases, only the final states with a neutron are detectable. All of the outgoing particles are emitted approximately isotropically. Including the ${}^{16}\mathrm{O}$ events only has a small effect on our final results.

There are also events from neutrino-electron scattering, which we ignore. These are forward-peaked, and we assume that they have been removed with an angular cut. A forward cone of half-angle 25 degrees would contain almost all neutrino-electron scattering events, while removing only 5% of the isotropic events.

III. SIGNATURE OF A SMALL NEUTRINO MASS

A. General description of the data

For a massless neutrino (ν_e or $\overline{\nu_e}$) the time dependence of the scattering rate is simply the time dependence of the luminosity of that flavor. For a possibly massive neutrino (ν_x), the time dependence of the scattering rate depends not only on the time dependence of the luminosity of that flavor, but also on the delaying effects of a mass. Since the luminosities of the different flavors are expected to be equal at all times, then if the ν_x is massive, we can compare the ν_x and ν_e scattering rates to search for a ν_x mass; see Eq. (6). In Ref. [11], it was proposed that the effects of a mass could be determined from the time dependence of the neutral-current rate divided by the total rate, which also has the effect of removing the the time dependence of the luminosity from the scattering rate. However, no quantitative technique for extracting the allowed mass range was given.

In order to implement the comparison of scattering rates, we define two rates: a Reference $R(t)$ containing only massless events, and a Signal $S(t)$ containing some fraction of possibly massive events, and some fraction of massless events (the possibly massive events cannot be completely separated from the massless events). We assume that only ν_τ is massive (this cannot be distinguished from the case that only ν_μ is massive). The analysis can easily be repeated for the case that both ν_μ and ν_τ are massive. Because of the quadratic dependence of the delay on the mass, this two-mass case will look like a one-mass case with the larger mass unless the two masses are nearly equal.

The Reference $R(t)$ can be formed in various ways. The largest sample of useful massless events will be the 7800 $\bar{\nu}_e$ events from SK with $E_{e^+}>10$ MeV. Below 10 MeV, there are gammas from the neutral-current excitation of ^{16}O which cannot be separated. We assume that SK and SNO will have synchronized clocks so that in principle, such a sharing of data will be possible. One can also use the 340 $\bar{\nu}_e$ events with $E_{e^+}>10$ MeV in the light water at SNO. Because of the smaller number of counts, the statistical error is larger and, hence, the mass sensitivity is slightly worse. This could be slightly improved by including the 160 charged-current events in the heavy water (a small fraction of the low-energy events would again have to be cut).

The primary component of the Signal $S(t)$ is the 485 neutral-current events on deuterons. With the temperatures assumed here, these events are 18% ($\nu_e+\bar{\nu}_e$), 41% ($\nu_\mu+\bar{\nu}_\mu$), and 41% ($\nu_\tau+\bar{\nu}_\tau$). The flavors of the neutral-current events of course cannot be distinguished. Therefore, under our assumption that only ν_τ is massive, there is already some unavoidable dilution of the expected delaying effect of a mass. The Signal $S(t)$ should also contain all events below about 10 MeV which cannot otherwise be removed. There are 35 neutral-current events on ^{16}O in the heavy water. Because of the high threshold for this reaction, these events are 50% each of ($\nu_\mu+\bar{\nu}_\mu$), and ($\nu_\tau+\bar{\nu}_\tau$). Finally, low-energy charged-current events must also be included. Because of the high energies of supernova neutrinos, only a small number (about 15) of electrons must be included in the Signal $S(t)$. No positrons need be included since those events can be identified by their two accompanying neutrons. The inclusion of the ^{16}O and charged-current events only slightly changes the shape of $S(t)$, and barely changes the final mass limit. The total number of events in $S(t)$ is 535, of which still 41% are possibly massive.

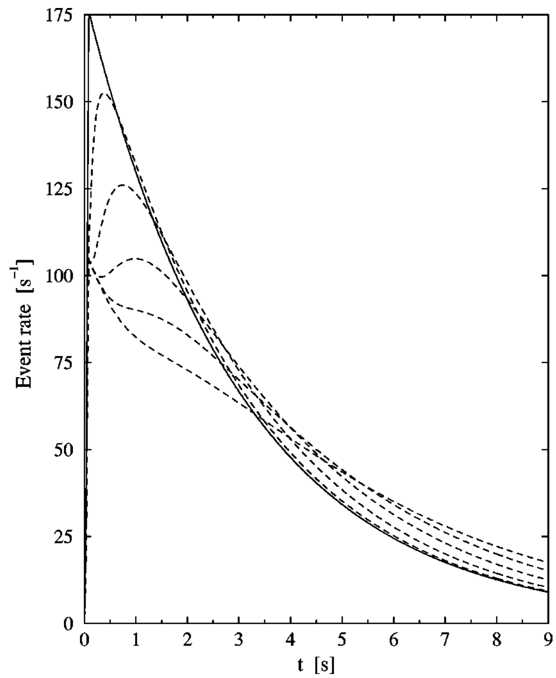

FIG. 1. The expected event rate for the Signal $S(t)$ at SNO in the absence of fluctuations for different ν_τ masses, as follows: solid line, 0 eV; dashed lines, in order of decreasing height: 20, 40, 60, 80, 100 eV. Of 535 total events, 100 are massless ($\nu_e+\bar{\nu}_e$), 217.5 are massless ($\nu_\mu+\bar{\nu}_\mu$), and 217.5 are massive ($\nu_\tau+\bar{\nu}_\tau$). These totals count events at all times; in the figure, only those with $t \leqslant 9$ s are shown.

If the Signal $S(t)$ contains some fraction of massive events, the shape of the scattering rate will be delayed and broadened. Relative to the Reference $R(t)$, there is a deficit of events at early times and an excess at late times. In the test for a mass, we test for this characteristic distortion in the shape. In Fig. 1, $S(t)$ is shown under different assumptions about the ν_τ mass. The shape of $R(t)$ is exactly that of $S(t)$ when $m_{\nu_\tau}=0$, though the number of events in $R(t)$ will be greater than in $S(t)$ if the SK $R(t)$ is used and comparable if the SNO $R(t)$ is used. For a very large ν_τ mass, the massless and massive components of $S(t)$ would completely separate in time. In Fig. 1, note that for $m=100$ eV, almost all of the massive events are delayed beyond 1 s.

The rates $R(t)$ and $S(t)$ will be measured with finite statistics, so it is possible for statistical fluctuations to obscure the effects of a mass when there is one, or to fake the effects when there is not. From Fig. 1, and Poisson statistics, one can easily get a rough idea of how finely the mass can be determined from the difference between $R(t)$ and $S(t)$. Note that if the SK $R(t)$ is used, the fluctuations in $R(t)$ when scaled down to the number of events in $S(t)$ will be small, and that if the SNO $R(t)$ is used, the fluctuations in $R(t)$ will be comparable to those in $S(t)$.

In this paper, we determine the mass sensitivity in the presence of the statistical fluctuations by Monte Carlo modeling. We use the Monte Carlo to generate representative statistical instances of the theoretical forms of $R(t)$ and $S(t)$, so that each run represents one supernova as seen in SNO. For $R(t)$, we pick a Poisson random number from a distribution with mean given by the expected number of events. This determines the number of events for a particular instance. We then use an acceptance-rejection method to sample the form of $R(t)$ until that number of events is obtained. This gives a statistical instance of $R(t)$, representative of what might be seen in a single experiment. A similar technique is used to generate an instance of $S(t)$. The massless and massive components of $S(t)$ are sampled separately, and then added together.

In Ref. [6], we considered two different tests of the shape distortion of $S(t)$ relative to $R(t)$. The first was a χ^2 test. A large value of χ^2 between statistical instances of $S(t)$ and $R(t)$ would indicate that they were likely to have been drawn from different distributions, and this would be taken as evidence of a ν_τ mass. However, this test is non-specific to testing for a mass, i.e., it is sensitive to any difference between $S(t)$ and $R(t)$. In addition, the χ^2 test requires binning in time, which washes out the effects of small masses. The second test, and the one advocated there, was a test of the average arrival time $\langle t\rangle$. Any massive component in $S(t)$ will always increase $\langle t\rangle$, up to statistical fluctuations. Besides being more directly related to the mass effect, the $\langle t\rangle$ technique is sensitive to somewhat smaller masses than the χ^2 technique, since no binning is required. In order that the results be believable, it is necessary that different reasonable statistical techniques yield consistent results. For this paper, we used both techniques and verified that they gave similar results. However, we present only the results of the $\langle t\rangle$ analysis.

B. $\langle t\rangle$ analysis

Given the Reference $R(t)$, the average arrival time is defined as

$$\langle t\rangle_R=\frac{\Sigma_k t_k}{\Sigma_k 1}=\frac{\int_0^{t_{max}} dt\, t R(t)}{\int_0^{t_{max}} dt R(t)}. \tag{13}$$

The summation form is used for real or simulated data sets, where the sum is over events (not time bins) in the Reference with $0\leqslant t\leqslant t_{max}$. The integral form would be used if the theoretical form for the rate were given. The starting time is assumed to be well-defined. With some 10^4 events expected in SK and SNO combined, and a risetime of order 0.1 s, this should not be a problem; the definition used here amounts to calling the starting point that time at which the $\overline{\nu}_e$ rate is about 1% of its peak rate. The choice of t_{max} is made as follows. The effect of the finite number of counts in $R(t)$ is to give $\langle t\rangle_R$ a statistical error:

$$\delta(\langle t\rangle_R)=\frac{\sqrt{\langle t^2\rangle_R-\langle t\rangle_R^2}}{\sqrt{N_R}}, \tag{14}$$

where both the width $\sqrt{\langle t^2\rangle_R-\langle t\rangle_R^2}$ and the number of events N_R depend on t_{max}. By choosing a moderate t_{max}, the width of $R(t)$ can be restricted. Such a choice will make the error on $\langle t\rangle_R$ as small as possible given that almost all of the events are to be included. Both the starting time and t_{max} were of course held constant over different Monte Carlo runs. For a purely exponential luminosity, and $t_{max}\to\infty$, $\langle t\rangle_R=\sqrt{\langle t^2\rangle_R-\langle t\rangle_R^2}=\tau$.

Given the Signal $S(t)$, the average arrival time is defined similarly as

$$\langle t\rangle_S=\frac{\Sigma_k t_k}{\Sigma_k 1}=\frac{\int_0^{t_{max}} dt\, t S(t)}{\int_0^{t_{max}} dt S(t)}, \tag{15}$$

where naturally the sums are now over events in the Signal. The widths of $R(t)$ and $S(t)$ are similar, each of order $\tau=3$ s [the mass increases the width of $S(t)$ only slightly for small masses.] If the SK $R(t)$ is used, then the statistical error on $\langle t\rangle_S$ is few times larger than that on $\langle t\rangle_R$ since there are several times fewer events. If the SNO $R(t)$ is used, then the statistical errors on $\langle t\rangle_S$ and $\langle t\rangle_R$ are comparable. Note that the errors on $\langle t\rangle_R$ and $\langle t\rangle_S$ are uncorrelated.

The signal of a mass is that the measured value of $\langle t\rangle_S-\langle t\rangle_R$ is greater than zero with statistical significance. From the Monte Carlo studies, $t_{max}=9$ s was found to be a very reasonable choice for the luminosity decay time $\tau=3$ s; about 95% of the data are then included while the width is somewhat reduced. The time-independent background events are negligible. For $t_{max}=9$ s, $\langle t\rangle_R=2.57$ s and $\sqrt{\langle t^2\rangle_R-\langle t\rangle_R^2}=2.12$ s. Near $t_{max}=9$ s, the significance of a delay, i.e., $\langle t\rangle_S-\langle t\rangle_R$ divided by its statistical error, is nearly maximal for the small masses we are considering. However, the results are not strongly dependent on the particular value of t_{max} used as long as it is reasonable (i.e., the vast majority of events are contained). Note that any shift in the starting time will cancel in the difference $\langle t\rangle_S-\langle t\rangle_R$.

Using the above procedure, we analyzed 10^4 simulated supernova data sets for a range of ν_τ masses. For each of them, $\langle t\rangle_S-\langle t\rangle_R$ was calculated and its value histogrammed. These histograms are shown in the upper panel of Fig. 2 for a few representative masses. (Note that the number of Monte Carlo runs only affects how smoothly these histograms are filled out, and not their width or placement.) These distributions are characterized by their central point and their width, using the 10%, 50% (equal to the average), and 90% confidence levels. That is, for each mass we determined the values of $\langle t\rangle_S-\langle t\rangle_R$ such that a given percentage of the Monte Carlo runs yielded a value of $\langle t\rangle_S-\langle t\rangle_R$ less than that value. With these three numbers, we can characterize the results of complete runs with many masses much more compactly, as shown in the lower panel of Fig. 2. Since the $\langle t\rangle_S-\langle t\rangle_R$ distributions are Gaussians, other confidence levels can easily be constructed. For convenience, the axes in the lower panel are inverted from how the plot was actually constructed. That is, given an experimentally determined value of $\langle t\rangle_S-\langle t\rangle_R$, one can read off the range of masses that would have been likely (at these confidence levels) to have given such a value of $\langle t\rangle_S-\langle t\rangle_R$ in one experiment. From the lower panel of Fig. 2, we see that SNO has a sensitivity to a

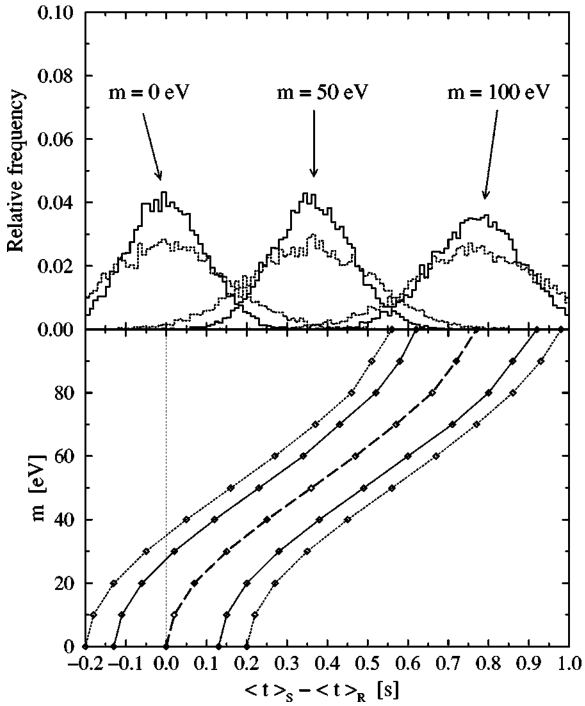

FIG. 2. The results of the $\langle t \rangle$ analysis for a massive ν_τ, using the Signal $S(t)$ from SNO defined in the text. In the upper panel, the relative frequencies of various $\langle t\rangle_S - \langle t\rangle_R$ values are shown for a few example masses. The solid line is for the results using the SK Reference $R(t)$ and the dotted line for the results using the SNO $R(t)$. In the lower panel, the range of masses corresponding to a given $\langle t\rangle_S - \langle t\rangle_R$ is shown. The dashed line is the 50% confidence level. The upper and lower solid lines are the 10% and 90% confidence levels, respectively, for the results with the SK $R(t)$. The dotted lines are the same for the results with the SNO $R(t)$. In this figure, $t_{max} = 9$ s and the time constant of the exponential luminosity is $\tau = 3$ s.

ν_τ mass down to about 30 eV (rounded from 27.5 eV) if the SK $R(t)$ is used, and down to about 35 eV if the SNO $R(t)$ is used.

We also investigated the dispersion of the event rate in time as a measure of the mass. A mass alone causes a delay, but a mass and an energy spectrum also cause dispersion. We defined the dispersion as the change in the width $\sqrt{\langle t^2\rangle_S - \langle t\rangle_S^2} - \sqrt{\langle t^2\rangle_R - \langle t\rangle_R^2}$, where all integrals are as above defined up to t_{max}. We found that the dispersion was not statistically significant until the mass was of order 80 eV or so; however, for such a large mass the statistical significance of the change in $\langle t\rangle$ cannot be missed. For these large masses, dispersion does increase the width of $S(t)$ and, hence, the error on $\langle t\rangle_S - \langle t\rangle_R$; this is just becoming visible in Fig. 2.

C. Sensitivity to the input parameters

Since the parameters governing the supernova neutrino emission are not perfectly known, it is worthwhile to examine the sensitivity of our conclusions to their assumed values. Most of this dependence can be obtained analytically by extending the time integration limit to $t_{max} \to \infty$. Note that the mass sensitivity will be slightly poorer than for $t_{max} = 9$ s used in the main analysis.

The characteristic delay is then

$$\langle t\rangle_S - \langle t\rangle_R \simeq \mathrm{frac}(m>0) \times 0.515 \left(\frac{m}{E_c}\right)^2 D, \tag{16}$$

where frac$(m>0)$ is the fraction (about 41%) of massive events in $S(t)$ and the units are as in Eq. (2). The characteristic energy E_c can be taken to be the peak of $f(E)\sigma(E)$, or more accurately from

$$\left\langle \left(\frac{m}{E_c}\right)^2 \right\rangle = \frac{\int dE f(E)\sigma(E)(m/E)^2}{\int dE f(E)\sigma(E)}. \tag{17}$$

The delay corresponding to E_c is the delay of the centroid of $S(t)$ relative to the centroid of $R(t)$, i.e., exactly $\langle t\rangle_S - \langle t\rangle_R$. For SNO, $E_c \simeq 32$ MeV. The statistical error on $\langle t\rangle_R$ is given by Eq. (14), and the statistical error on $\langle t\rangle_S$ is defined similarly.

Using the above, we can make a good estimate of the mass limit that SNO would set if the delay were measured to be zero. Ignoring the error on $\langle t\rangle_R$, and taking $t_{max} \to \infty$, we estimate the error to be $\tau/\sqrt{N_S} = 3/\sqrt{535} = 0.13$ s. The 10% and 90% confidence levels as used in Fig. 2 correspond to $\pm 1.3 \times \delta(\langle t\rangle_R - \langle t\rangle_S)$. For our estimate, this has magnitude 0.17 s, close to the result in Fig. 2 for the SK Reference. The mass limit that will be placed if no delay is seen (using $D = 1$, i.e., 10 kpc) is

$$m_{limit} = E_c \sqrt{\frac{1.3 \times \mathrm{error}}{\mathrm{frac}(m>0) \times 0.515 D}} \simeq 30 \ \ \mathrm{eV}, \tag{18}$$

in excellent agreement with the numerical result. If the neutron detection efficiency ϵ_n were not 100%, then N_S would be reduced by ϵ_n and the error increased by $1/\sqrt{\epsilon_n}$. Therefore, the mass limit would be increased only by $1/\sqrt[4]{\epsilon_n}$.

This formula can also be used to make an estimate of the mass limit that can be obtained with the forward-peaked neutrino-electron scattering at SK (which has the largest number of those events). Since $\sigma(E) \sim E$, $E_c \simeq \langle E\rangle \simeq 25$ MeV. The Signal $S(t)$ can be defined by the events in a forward cone of half-angle 25 degrees, and the Reference $R(t)$ defined by the events outside this cone. Assuming that all of the neutrino-electron scattering events are contained along with about 5% of the isotropic backgrounds, $N_S = 760$ and frac$(m>0) = 60/760 = 0.079$ [6]. The error is estimated to be $3/\sqrt{760} = 0.11$ s, and so $m_{limit} \simeq 50$ eV. This is smaller than the recent conclusion of Ref. [13], but the technique used here includes a much greater portion of the data.

Continuing to suppose that the mass is very small, and that SNO will simply place a limit, we can investigate the effects of varying the input parameters. We ignore dispersion and take the widths of $R(t)$ and $S(t)$ to be proportional to τ. If the cross section $\sigma(E)$ depends on energy as E^α ($\alpha \sim 2$ for $\nu + d$), then the characteristic energy $E_c \sim (2+\alpha)T$ and the thermally-averaged cross section is proportional to T^α, where T is the ν_x temperature. We take frac($m>0$) to be approximately constant and ignore the error on $\langle t\rangle_R$; this is only valid for small deviations from $T=8$ MeV. With these assumptions we can determine how the key parameters affect the result (for $\alpha=2$). The delay is

$$\langle t\rangle_S-\langle t\rangle_R\sim\left(\frac{m}{T}\right)^2 D. \tag{19}$$

The number of events is

$$N_S\sim\frac{1}{T}\frac{1}{D^2}T^2, \tag{20}$$

and so the error is

$$\delta(\langle t\rangle_S-\langle t\rangle_R)\sim\frac{\tau}{\sqrt{N_S}}\sim\frac{\tau D}{\sqrt{T}}. \tag{21}$$

Therefore, the significance (number of sigmas) of the delay is

$$\frac{\langle t\rangle_S-\langle t\rangle_R}{\delta(\langle t\rangle_S-\langle t\rangle_R)}\sim\left(\frac{m}{T}\right)^2\frac{\sqrt{T}}{\tau}. \tag{22}$$

Remarkably, this is *independent* of D. To place a mass limit at a given confidence level, the number of sigmas is fixed. Then the mass limit that can be obtained if no delay is observed has the following dependence:

$$m_{limit}\sim T^{3/4}\sqrt{\tau}, \tag{23}$$

also *independent* of D.

The D-independence can also be seen geometrically from Fig. 2. Under $D\rightarrow 2D$, for example, the delay and the error each become twice as large. However, the point at which the upper confidence level crosses the m-axis is unchanged. If a nonzero delay is measured, then the range of allowed masses does depend on D, with larger distances corresponding to smaller masses. The above arguments hold for the relevant range 1–30 kpc; for smaller D the detector may be overwhelmed and for larger D there are too few events and the derivation is not valid. Because of obscuration by dust, it may be difficult to measure the distance to a future Galactic supernova. It is therefore rather important that this does not affect the ability to place a limit on the ν_τ mass. That may not be true for other analysis techniques; for example, the estimates of Ref. [9] do depend on D. Note that a smaller distance would allow a better measurement of the temperatures and other supernova parameters.

Now consider the effect of a change in the temperature T_{ν_x} on the mass limit. As indicated, the dependence is weak, with a higher temperature making it slightly more difficult to limit the mass. The numerical results and the analytic estimate are shown in Fig. 3. Even under the large variation in T_{ν_x} of ± 2 MeV, the mass limit changes only by about ± 5 eV.

The effect of changing the luminosity decay time constant τ is even more straightforward and is obvious from Eq. (23).

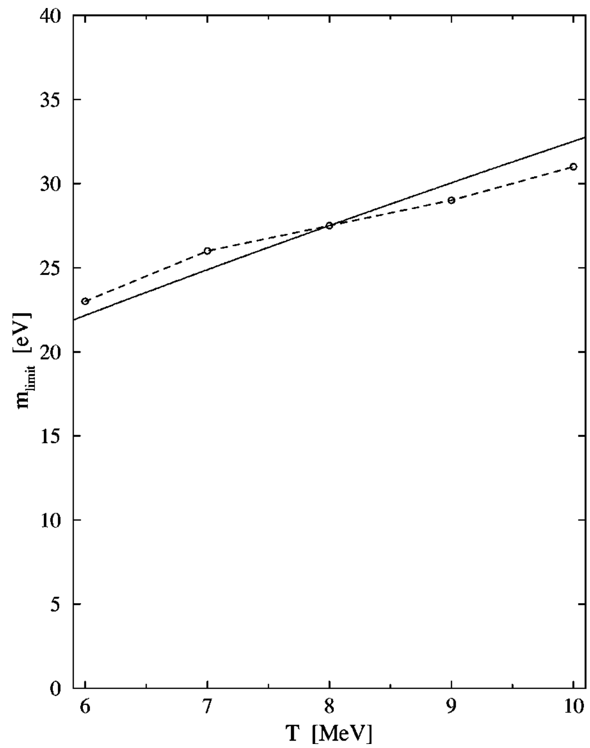

FIG. 3. The effect of a change in the ν_x temperature T on the mass limit that could be made using the SNO Signal and the SK Reference. The mass limit is defined as the mass at which the appropriate upper confidence level intersects the m-axis in Fig. 2. The solid line is the formula given in Eq. (23). The full numerical results are marked by circles and connected with a dashed line to guide the eye.

Thus, reducing that from $\tau=3$ s to $\tau=1$ s, as used in Ref. [10], would improve the limit at SNO (using the SK Ref) from about 30 eV to about 15 eV. Using $\tau=0.5$ s, as in Ref. [16], or $\tau=0.3$ s, as in Ref. [17], would improve the limit to about 10 eV. These values of τ were estimated from appropriate figures in these references, and are only approximations of the $L(t)$ time scale. The numerical results and the analytic estimate are shown in Fig. 4. Using an unrealistically sharp neutrino pulse makes the quoted mass limit very small.

We also considered the effect of a chemical potential in the ν_x spectrum, which would reduce the high-energy tail. We took $\mu=3T$, and then chose $T=6.31$ MeV to keep the average energy the same as for the $\mu=0$, $T=8$ MeV case. Because the neutrino-deuteron cross section only depends quadratically on energy, the effect is small. About 10% fewer events are obtained, but with a delay about 10% larger. These lead to a change of order a few percent in the mass sensitivity. Because of their steeper energy dependence [6], the $\nu_x+{}^{16}$O events would be more affected by a chemical potential.

IV. SEPARATE EXTRACTION OF SUPERNOVA AND NEUTRINO PARAMETERS

The observation of the neutrino signal of a future Galactic supernova will be extremely significant test of the physics

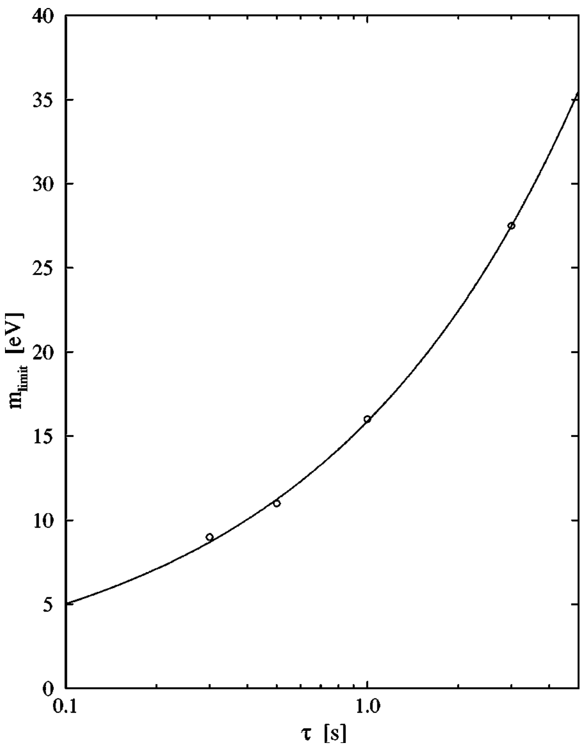

FIG. 4. The effect of a change in the luminosity decay constant τ on the mass limit that could be made using the SNO Signal and the SK Reference. The mass limit is defined as the mass at which the appropriate upper confidence level intersects the m-axis in Fig. 2. The solid line is the formula given in Eq. (23). The full numerical results are marked by circles, at the points $\tau = 3$ s (this paper), $\tau = 1$ s (Ref. [10]), $\tau = 0.5$ s (Ref. [16]), and $\tau = 0.3$ s (Ref. [17]); see the text for explanation.

involved. It will allow, among other things, determination of the imprecisely-known supernova neutrino emission parameters. In addition, we hope to be able to use the same data to determine or constrain neutrino properties. In this section, we discuss how both of these goals can be achieved simultaneously. Throughout, we use standard values for all parameters to numerically evaluate the expected precision.

The measured neutrino signal can be used to determine the temperatures (and possibly the chemical potentials) of ν_e, $\bar{\nu}_e$, and ν_x. Ideally, we would like to know these parameters as a function of time. In addition, the data can be used to determine the apparent source strength E_B/D^2 and the luminosity time profile. Since neutrino oscillations could change the neutrino flavor between emission and detection, their presence would make this task more difficult. Thus, we consider first the simpler situation without neutrino oscillations.

A. Without neutrino oscillations

The ν_e and $\bar{\nu}_e$ neutrinos will be detected by the charged-current reactions in which the energy of the outgoing electron or positron will be measured. Neglecting recoil corrections and detector resolution, the relation $E_\nu = E_e + E_{th}$ allows one to relate the measured electron or positron spectrum to the weighted neutrino spectrum $f(E_\nu)\sigma(E_\nu)$ which appears in Eq. (8). Since the charged-current cross sections are well-known, the actual neutrino spectrum $f(E_\nu)$ can be obtained. If this has the expected thermal shape, then the corresponding temperatures T_{ν_e} and $T_{\bar{\nu}_e}$ (and possibly chemical potentials) can be extracted. The recoil corrections and detector resolution can of course be taken into account by fitting the measured electron or positron spectrum with an appropriately convolved trial neutrino spectrum. Note that these temperature determinations depend only on the shape of the electron or positron spectrum. The normalization (the total number of events) can be used to determine the apparent source strength E_B/D^2.

There are 7800 $\bar{\nu}_e + p \rightarrow e^+ + n$ events with $E_{e^+} > 10$ MeV expected in SK, so that the average positron energy can be determined to about 1%. Following the procedure above, one should be able to determine the temperature $T_{\bar{\nu}_e}$ to a comparable precision. In this case, one can also check for the presence of a chemical potential and for time variation of $T_{\bar{\nu}_e}$. The situation with ν_e is more difficult, since there are only 80 events expected for the reaction $\nu_e + d \rightarrow e^- + p + p$ in SNO. (Note that ν_e has no charged-current interaction with protons, that SNO is much smaller than SK, and that $T_{\nu_e} < T_{\bar{\nu}_e}$.) In this case, the average electron energy can be determined to about 10%; as above, therefore a similar precision for the temperature T_{ν_e}. These two measurements thus allow one to test $T_{\nu_e} < T_{\bar{\nu}_e}$, as expected for the supernova temperature hierarchy. In addition, the two independent measurements of E_B/D^2 can be tested for consistency with each other and with possible determinations unrelated to neutrinos. If E_B/D^2 is assumed to be known, then T_{ν_e} and $T_{\bar{\nu}_e}$ can also be determined from the total numbers of charged-current events in SNO, as in Ref. [27].

These measurements can also be compared with other available, but lower-statistics data. For $T_{\bar{\nu}_e}$, other hydrogen-containing detectors (MACRO, SNO, Borexino, and Kamland) will have events from $\bar{\nu}_e + p \rightarrow e^+ + n$. For T_{ν_e}, there are small numbers of events expected from charged-current reactions on ^{12}C in Borexino and Kamland, and ^{16}O in SK. These consistency checks will be useful because the different reactions may have different systematic errors.

The ν_x neutrino can be detected only by the neutral current reactions. Since the outgoing neutrino carries an unknown amount of energy, it is not possible to determine the neutrino spectrum as above. Therefore, T_{ν_x} can only be extracted from the observed total number of events, and must rely on the measured value of E_B/D^2 and the assumption of luminosity equipartition. For the neutral-current reactions, the rate is independent of the neutrino flavor. If the supernova temperature hierarchy holds, then the neutral-current events will be dominated by the ν_x neutrinos. However, one cannot directly associate a flavor with the extracted T_{ν_x}.

The Signal at SK is expected to contain 710 events from $\nu_x + ^{16}O$. Low-energy charged current events from $\bar{\nu}_e + p \rightarrow e^+ + n$ can be confused with the neutral-current events; these add 530 events to the Signal. The latter events can be subtracted from the total number of events observed in the Signal, since their expected number can be calculated using the parameters measured from the higher-energy $\bar{\nu}_e$ data. For the SK Signal, since the dependence of the cross section on energy is stronger, the 4% precision on the number of events would translate to a 1% precision on T_{ν_x}. However, uncertainties on the cross section itself should also be considered. In principle, the neutrino-electron scattering events at SK could also be used to measure T_{ν_x}; however, the temperature dependence of the number of events is extremely weak. The Signal at SNO is expected to contain 400 events from $\nu_x + d$, and 135 events from the sum of neutral-current $\nu_e + \bar{\nu}_e$ reactions on deuterons, neutral-current excitation of ^{16}O, and low-energy ν_e charged-current reactions on deuterons. Again, the numbers expected for the latter can be calculated and subtracted from the total number of events observed in the Signal. The thermally-averaged cross section for $\nu_x + d$ depends roughly on $T^2_{\nu_x}$, so by Eq. (8) the number of events is roughly proportional to T_{ν_x}. Ignoring all other uncertainties, this would therefore allow a determination of T_{ν_x} to about 5%. There may also be T_{ν_x} measurements from $\nu_x + {}^{12}$C at Borexino and Kamland, particularly with the excitation of the 15.11 MeV state in ^{12}C. If each neutral-current reaction were thoroughly understood, it would be possible to use their different thresholds and energy dependences to map out the ν_x spectrum. At the very least, it should be possible to extract a good value of T_{ν_x} and to make important consistency tests.

Therefore, all three temperatures can be extracted with reasonable precision and the supernova temperature hierarchy $T_{\nu_e} < T_{\bar{\nu}_e} < T_{\nu_x}$ experimentally verified. As for the time variation of the temperatures, it is likely that only the $\bar{\nu}_e$ data will have sufficient statistics. Those data, after correcting for any temperature variation, will give the time profile of the luminosity $L(t)$ and the characteristic time scale τ.

Mass effects will not influence the temperature determination, provided the mass is not too large, since a delay changes only the time structure of the rate $S(t)$, not its normalization nor the time-integrated energy spectra. However, if $m_{\nu_x} \sim 1$ keV or larger, some low-energy events will be lost in the time-independent background due to their large delay, and the extracted temperatures would be affected. The considerations in this section, combined with the result above that the mass limit is only weakly dependent on T_{ν_x}, shows that the temperatures and the ν_x mass can be separately and robustly determined from the data.

B. With neutrino oscillations

Assuming that the presence of oscillations does not substantially change the supernova dynamics, it is relatively straightforward to separately search for oscillations and a mass delay. The effects of supernova neutrino oscillations (without a mass delay) were considered for SNO in Ref. [28]. Since the numbers of ν_μ, $\bar{\nu}_\mu$, ν_τ, and $\bar{\nu}_\tau$ emitted are expected to be the same, mixing among them have no effect on the number of ν_x events. What propagates are the mass eigenstates; the flavor content of the heavy mass eigenstate at the detector is irrelevant, and so oscillations among the ν_x neutrinos do not affect the delay.

Now consider mixing between either ν_e and ν_τ (or ν_μ) or $\bar{\nu}_e$ and $\bar{\nu}_\tau$ (or $\bar{\nu}_\mu$). We assume that the ν_τ mass is large, which makes δm^2 large. Large-angle, large-δm^2 mixing is ruled out by reactor and accelerator experiments. Small-angle, large-δm^2 mixing is allowed; however, the effects are minimal unless there is an Mikheyev-Smirnov-Wolfenstein (MSW) enhancement. For a normal mass hierarchy, this can only happen for mixing between ν_e and ν_τ, and at a very high density, of order 10^{12} g/cm^3, which does occur in supernovae. If there are such oscillations, then some high-T ν_τ neutrinos will become high-T ν_e neutrinos. As noted, the spectrum of electrons from $\nu_e + d$ can be related to the spectrum of ν_e neutrinos. For no oscillations, that would be a thermal spectrum with $T_{\nu_e} = 3.5$ MeV; for complete oscillations, that would be a thermal spectrum with $T_{\nu_e} = 8$ MeV (and a much greater number of events). For partial mixing, there would be two peaks. The mixing parameters and temperatures can thus be extracted from the measured electron spectrum. One should note that the charged-current reactions $\nu_e + {}^{12}$C at Borexino and Kamland and $\nu_e + {}^{16}$O at SK would have large numbers of events if such oscillations occur. If there is an inverted mass hierarchy, then mixing between $\bar{\nu}_e$ and $\bar{\nu}_x$ could have an MSW enhancement. Then similar considerations to those above could be used to examine the positron spectrum from $\bar{\nu}_e$ reactions. It would in fact be somewhat easier, due to the larger numbers of events. In either of these cases, extracting the allowed ν_τ mass range from the measured value of $\langle t \rangle_S - \langle t \rangle_R$ requires some care. A given delay could be caused by a ν_τ mass and no mixing or by a larger mass and partial mixing. Ideally, the mixing parameters will be known from the considerations above, so that the range of possible masses can be reasonably restricted.

V. CONCLUSIONS AND DISCUSSION

One of the key points of our technique is that the abundant $\bar{\nu}_e$ events can be used to calibrate the neutrino luminosity of the supernova and to define a clock by which to measure the delay of the ν_x neutrinos. The internal calibration substantially reduces the model dependence of our results and allows us to be sensitive to rather small masses. Our calculations indicate that a significant delay can be seen for $m = 30$ eV with the SNO data, corresponding to a delay in the average arrival time of about 0.15 s. Even though the duration of the pulse is expected to be of order 10 s, such a small average delay can be seen because several hundred events are expected. Without such a clock, one cannot determine a mass limit with the $\langle t \rangle_S - \langle t \rangle_R$ technique advocated here, since the absolute delay is unknown. Instead, one would have to constrain the mass from the observed dispersion of the events; only for a mass of $m = 150$ eV or greater would the pulse become significantly broader than expected from theory.

Moreover, the technique used here allows accurate analytic estimates of the results, so that it is easy to see how the conclusions would change if different input parameters were used. If the ν_τ mass is very small, then the most probable measured delay is $\langle t\rangle_S - \langle t\rangle_R = 0$. In that case, one can only place a limit m_{limit} on the ν_τ mass. We have shown that this limit is only weakly dependent on the ν_x temperature T or the presence of a chemical potential in the thermal spectrum, and is *independent* of the distance D for the reasonable range of distances for a Galactic supernova. The weak dependence of the results on T_{ν_x} means that allowing a time variation of T_{ν_x} would not significantly affect our conclusions. The value of m_{limit} is sensitive to the time scale over which the luminosity decreases. If a very small value for the time scale is assumed, as is sometimes the case in the literature, then one can obtain apparent sensitivity to a very small ν_τ mass. However, such short time scales are unreasonable, given the observed $\sim$10 s duration of the SN 1987A pulse.

The supernova parameters are not yet well-known. However, the sensitivity with which the neutrino properties can be determined using the data from a future Galactic supernova depends upon the supernova parameters assumed. We have shown how the supernova and neutrino physics can be separately studied using the same data, so that the extracted neutrino parameters can be determined in an almost model-independent way.

We assumed that only ν_τ is massive (note that this cannot be distinguished from the case that only ν_μ is massive). If no delay is seen, and nothing further is known, then the mass limit obtained would in fact apply to both the ν_μ and ν_τ masses. If both are massive, then because of the quadratic dependence of the delay on the mass, the one-mass case is recovered with the larger mass unless the masses are similar. If both are massive and have the same mass, the delay would be increased by a factor 2, while the error would be unchanged. This would decrease the mass limit by a factor $\sqrt{2}$ to about 20 eV.

If the ν_τ mass is very large, then the pulse will be so delayed and broadened that it will eventually disappear below the time-independent background (which for this purpose includes solar neutrinos). Assuming that the background rate with a lowered threshold is about 10^{-3} s^{-1} (about 10 times the solar neutrino rate), the maximal mass which can be seen in SNO is about $m \sim 10$ keV, similar to SK [6]. Above this mass, the neutrino would be likely to decay over the distance to the supernova, as pointed out in Ref. [29].

Our previous paper [6] showed that using the neutral-current excitation of ^{16}O in SK gives a mass sensitivity down to about 45 eV. In SK, one can also use the neutrino-electron scattering data, and as noted above, that should have a sensitivity down to about 50 eV. If no delay is seen, then SNO will place the best limit, about 30 eV, using neutral-current events on deuterons and calibrating the $\bar{\nu}_e$ data from SK. In that case, the two limits from SK using neutral-current excitation of ^{16}O and neutrino-electron scattering can only confirm the result. For those scattering rates, a mass of 30 eV is insignificant, and may as well be zero, so that there is nothing to be gained by combining those Signals with that from neutrino-deuteron breakup at SNO. On the other hand, if a significant delay is seen, then there will be three independent determinations of the allowed mass range.

In conclusion, we have presented a general method, including a thorough statistical analysis, of extracting information about the possible ν_τ and ν_μ masses from the future detection of a Galactic supernova neutrino burst by the Sudbury Neutrino Observatory. When such an event in fact occurs, the existing mass limits will be vastly improved and will approach, or cross over, the cosmological bound.

ACKNOWLEDGMENTS

This work was supported in part by the US Department of Energy under Grant No. DE-FG03-88ER-40397. J.F.B. was supported by the Sherman Fairchild Foundation. We thank P. J. Doe, K. T. Lesko, A. B. McDonald, and E. B. Norman of the SNO Collaboration for discussions about the detector properties, and Y.-Z. Qian for discussions about supernova neutrino oscillations.

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