Model-independent analysis of CERN LEP and SLAC SLD data on Z decays: Is the standard model confirmed?

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A model-independent analysis is performed on the CERN LEP and SLAC SLD data on Z decays. Using only very weak theoretical assumptions, the effective vector and axial-vector couplings of leptons, c quarks, and b quarks have been extracted. Although the lepton and c quark couplings agree well with standard model predictions, those of the b quark show deviations of more than three standard deviations. The effect is mainly in the right-handed b quark coupling, the left-handed coupling being consistent (at the 2σ level) with the standard model prediction. The probability that the observed deviations of all the measured effective couplings are statistical fluctuations from lepton universality and the standard model is estimated to be 0.9%. The estimated probability that the deviations in the leptonic and b quark couplings alone are a fluctuation is 0.18%. A thorough discussion is made of the internal consistency of the different measurements contributing to the average values A_l and A_b used to extract the b quark couplings, as well as possible sources of systematic error that may not, hitherto, have been taken into account. Excluding τ-polarization measurements, which show internal inconsistencies, from the averages increases the deviations of the extracted b quark couplings from the standard model predictions to the four-standard-deviation level. [S0556-2821(98)06119-0]

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I. INTRODUCTION

This study is based on a recent compilation [1] of experimental results on Z decays. The aim is to answer the following question: Are the data consistent with the predictions of the standard electroweak model (SM) [2]? To this end the analysis is carried out in three independent steps. In the first, the data are used to extract the effective vector ($\overline{v}$) and axial-vector ($\overline{a}$) coupling constants of the charged leptons (assuming lepton universality) and of the c and b quarks. This is done using only weak theoretical assumptions. The effective couplings are then compared with the predictions of the SM and the confidence levels (C.L.s) for consistency of the measurements with the SM are calculated. In the second step the experimental data contributing to deviations observed from the SM in the b quark couplings are critically examined. Issues addressed include the roles of statistical and systematic errors, as well as the internal consistency of physical parameters measured using different experimental methods. Finally, the observed deviations from the SM are assumed to represent real physical effects, whose interpretation is discussed. The three steps described above constitute the material of the following three sections. A summary and outlook are given in the final section.

II. EXTRACTION OF THE EFFECTIVE WEAK COUPLING CONSTANTS

It is convenient to define the following auxiliary quantities:[1]

$$\overline{r}_f \equiv \overline{v}_f / \overline{a}_f, \tag{2.1}$$

$$\overline{s}_f \equiv (\overline{a}_f)^2 + (\overline{v}_f)^2, \tag{2.2}$$

which may be simply derived from the measurements. The experimental errors on $\overline{r}_f$ and $\overline{s}_f$ (f here stands for lepton or quark) are, unlike those in $\overline{v}_f$ and $\overline{a}_f$, essentially uncorrelated, simplifying the calculation of the statistical significance of any deviations observed from the SM expectations. Throughout this section, the SM predictions quoted are those of the global SM fit with $m_t = 172$ GeV, $m_H = 149$ GeV reported in Ref. [1]. The effect of varying the Higgs boson mass, the only remaining unknown parameter of the SM, is discussed in Sec. IV below.

The quantities[2] $\overline{r}_f$ ($f = l,c,b$) and $\overline{s}_l$ may be directly obtained from the data without any additional assumptions concerning the poorly measured [3] couplings of the u, d, s quarks. A further assumption is, however, necessary in order to extract $\overline{s}_c$, $\overline{s}_b$ and hence the c and b quark couplings. In order to perform an analysis which is, as far as possible, ''model independent'' and to avoid the specific assumption of the validity of the SM, as used in the fits of Ref. [1], the weaker hypothesis of quark-lepton universality is made for the fermions e, μ, τ, u, d, s, c. That is, all these fermions are assumed to have the same effective weak mixing angle [1]. The derived values of $\overline{v}_c$ and $\overline{a}_c$ presented below are found to be, within errors, in good agreement with this hypothesis. Another possibility is to assume a value of $\alpha_s(M_Z)$ derived from non-electroweak-related measurements. In this case the c and b quark couplings may be derived from the ratios Γ_Q/Γ_l ($Q = c,b$) without any assumption concerning the couplings of the light quarks. There is now, however, the disadvantage that the extracted values of the electroweak

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[1]The fermion masses are set to zero in Eq. (2.2). Only for the b quark do the fermion mass terms give a non-negligible contribution.

[2]Unless otherwise stated, l is a generic lepton label and e-μ-τ universality is assumed.

TABLE I. Average values of electroweak observables used in the analysis [1]. SM denotes the standard model prediction for $m_t=172$ GeV, $m_H=149$ GeV [1].

Quantity	Measurement (total error)	SM	(Meas.−SM)/error
LEP			
$A_{FB}^{0,e}$	0.0160(24)	0.0159	0.04
$A_{FB}^{0,\mu}$	0.0162(13)	0.0159	0.04
$A_{FB}^{0,\tau}$	0.0201(18)	0.0159	2.3
Γ_l (MeV)	83.91(11)	83.96	−0.45
τ polarization			
A_e	0.1382(76)	0.1458	−1.0
A_τ	0.1401(67)	0.1458	−0.9
c and b quarks			
$A_{FB}^{0,c}$	0.0733(49)	0.0730	0.1
R_c	0.1715(56)	0.1723	−0.1
$A_{FB}^{0,b}$	0.0979(23)	0.1022	−1.8
R_b	0.2179(12)	0.2158	1.8
SLD			
A_e	0.1543(37)	0.1458	2.3
A_c	0.625(84)	0.667	−0.5
A_b	0.863(49)	0.935	−1.4
R_b	0.2149(38)	0.2158	−0.2

couplings are strongly correlated with the assumed $\alpha_s(M_Z)$ value.

At LEP, $\overline{r_f}$ is found from the measured, corrected, pole forward-backward charge asymmetries $A_{FB}^{0,f}$ [1] via the relations

$$A_{FB}^{0,f}=\frac{3}{4}A_eA_f, \tag{2.3}$$

$$A_f\equiv\frac{2\overline{v_f}\,\overline{a_f}}{(\overline{a_f})^2+(\overline{v_f})^2}=\frac{2\overline{r_f}}{1+\overline{r_f}^2}. \tag{2.4}$$

A_e and A_τ have also been measured at LEP via the angular dependence of the τ-polarization asymmetry:

$$\overline{P}_\tau(\cos\theta)=-\frac{A_\tau+A_eF(\theta)}{1+A_\tau A_eF(\theta)}, \tag{2.5}$$

where

$$F(\theta)\equiv 2\cos\theta/(1+\cos^2\theta)$$

TABLE II. LEP+SLD averages. The τ-polarization measurements are excluded from the averages quoted in the second row.

A_l	A_c	A_b	R_c	R_b
0.1501(24)	0.645(39)	0.869(22)	0.1715(56)	0.2176(11)
0.1533(27)	0.634(38)	0.853(22)		

TABLE III. Measured values of $\overline{r_f}=\overline{v_f}/\overline{a_f}$ compared to standard model predictions.

	$\overline{r_l}$	$\overline{r_c}$	$\overline{r_b}$
Measurement	0.07548(120)	0.366(29)	0.582(32)
SM	0.07332	0.383	0.689
(Meas.−SM)/error	−1.80	−0.59	−3.34
Meas. τ poln. out	0.07711(140)	0.357(29)	0.562(29)
(Meas.−SM)/error	−2.71	−0.90	−4.38

and θ is the angle between the incoming e^- and the outgoing τ^- in the τ-pair center-of-mass frame. At SLD, A_e is directly measured by the left-right beam-polarization asymmetry A_{LR}, while A_c and A_b are determined from the left-right–forward-backward asymmetries of tagged heavy quarks.

The separate LEP and SLD average values of the electroweak observables, which are directly sensitive to the effective couplings, are reported in Table I. The combined LEP and SLD averages of A_l, A_c, A_b, R_c, and R_b, where $R_Q=\Gamma_Q/\Gamma_{had}$ ($Q=c,b$), are reported in Table II.

It may be remarked that, while there is good agreement between the different values of A_l derived from $A_{FB}^{0,l}$ ($l=e,\mu,\tau$) using Eq. (2.3) and that derived from A_{LR} [weighted average 0.1533(27),[3] $\chi^2=3.85$ for three degrees of freedom (DOF), CL=28%], the values of both A_e and A_τ derived from the τ-polarization measurements are significantly lower. In fact, the average value of A_l from τ polarization lies 2.5σ below the average from $A_{FB}^{0,l}$, A_{LR}. Including or not including the τ-polarization data changes the LEP and SLD average value of A_l by more than one standard deviation (see the first column of Table II). Because of this possible inconsistency in the measured A_l values from different sources, the extraction of the coupling constants will be done throughout this paper using values of A_l that either include or exclude the τ-polarization results. Significant differences are found. Possible explanations for the apparent inconsistencies in the A_l measurements are discussed in the following section.

The values of $\overline{r_f}$ ($f=l,c,b$) derived from the measured values of A_l using Eq. (2.4) are presented in Table III. For the b quark, mass effects were taken into account by using the corrected form of Eq. (2.4):

$$A_b=\frac{2(\sqrt{1-4\mu_b})\overline{r_b}}{1-4\mu_b+(1+2\mu_b)\overline{r_b}^2}, \tag{2.6}$$

where $\mu_b=[\overline{m}_b(M_Z)/M_Z]^2\simeq 1.0\times10^{-3}$. The running b quark mass is taken as $\overline{m}_b(M_Z)=3.0$ GeV [4]. Agreement is seen with the SM at the 2σ level for $\overline{r_l}$, at $<1\sigma$ for $\overline{r_c}$, but only at the 3.3σ level for $\overline{r_b}$. A similar discrepancy for A_b was mentioned, but not discussed in terms of the b quark

[3]Throughout this paper total experimental errors are given in terms of the last significant figures: 0.1533(27) denotes 0.1533 ±0.0027.

TABLE IV. Measured values of $\overline{s}_f=\overline{a}_f^2(1-6\mu_f)+\overline{v}_f^2$ compared to standard model predictions.

	$\overline{s}_l$	$\overline{s}_c$	$\overline{s}_b$
Measurement	0.25244(33)	0.2877(95)	0.3676(24)
SM	0.25259	0.2880	0.3644
(Meas.−SM)/error	−0.45	−0.03	1.33

couplings, in Ref. [1]. If the τ-polarization measurements of A_l are excluded from the average, the discrepancy of $\overline{r}_l$ with the SM approaches[4] 3σ and that of $\overline{r}_b$ exceeds 4σ.

The quantity $\overline{s}_l$ is derived from the leptonic width Γ_l using the relation

$$\overline{s}_l=(\overline{a}_l)^2+(\overline{v}_l)^2=\frac{12\pi\Gamma_l}{\sqrt{2}G_\mu M_Z^3}\frac{1}{[1+3\alpha(M_Z)/4\pi]}. \quad (2.7)$$

The value obtained for $\overline{s}_l$, quoted in the first column of Table IV, uses the LEP average value of Γ_l from Table I together with $G_\mu=1.16639\times10^{-5}\,(\mathrm{GeV})^2$ [5], $M_Z=91.1863$ GeV, and $\alpha(M_Z)^{-1}=128.896$ [1]. Good agreement is found with the SM value. Solving Eqs. (2.1) and (2.2) for $\overline{a}_l$ and $\overline{v}_l$ yields the results presented in Table V. As in the calculation of all the other effective couplings, the signs of $\overline{a}_l$ and $\overline{v}_l$ are chosen to be the same as the SM predictions. The values of $\overline{a}_l$ and $\overline{v}_l$ are in good agreement with the LEP+SLD averages quoted in Ref. [1], taking into account the slightly different analysis procedures.[5] Both $\overline{a}_l$ and $\overline{v}_l$ are in agreement with the SM predictions.

The quantities $\overline{s}_Q$ $(Q=c,b)$, including quark mass effects, may be derived from the measured quantities R_Q via the relation

$$\overline{s}_Q=(\overline{a}_Q)^2(1-6\mu_Q)+(\overline{v}_Q)^2=\frac{R_Q S_Q}{(1-R_Q)C_Q^{\mathrm{QED}}C_Q^{\mathrm{QCD}}}, \quad (2.8)$$

where

$$S_Q\equiv\sum_{q\neq Q}[(\overline{a}_q)^2(1-6\mu_q)+(\overline{v}_q)^2]$$

and [6]

$$C_Q^i=1+\delta_Q^i-\langle\delta_{q\neq Q}^i\rangle(i=\mathrm{QED,QCD}),\quad \mu_q=0\quad \text{for } q\neq b,$$

$$\delta_q^{\mathrm{QED}}=\frac{3(e_q)^2}{4\pi}\,\alpha(M_Z),\quad \delta_{q\neq b}^{\mathrm{QCD}}=1.00a_s+1.42a_s^2,$$

$$\delta_b^{\mathrm{QCD}}=0.99a_s-1.55a_s^2.$$

q is a generic quark flavor index, e_q the quark electric charge in units of that of the positron, and $a_s\equiv\alpha_s(M_Z)/\pi$. $\langle X\rangle$ denotes the quark flavor average of X. As mentioned above, $\mu_b=1.0\times10^{-3}$, while, taking into account the present experimental error on R_c, μ_c is set to zero. The numerical values of the QED and QCD correction factors, with $\alpha_s(M_Z)=0.12$ and $\alpha(M_Z)^{-1}=128.9$, are presented in Table VI. The non-b-quark couplings in Eq. (2.8) are written, conventionally, as

$$\overline{a}_q=\sqrt{\rho_q}T_3^q, \quad (2.9)$$

$$\overline{v}_q=\sqrt{\rho_q}[T_3^q-2e_q(\overline{s_W^q})^2], \quad (2.10)$$

where, assuming non-b-quark lepton universality,[6]

$$\sqrt{\rho_q}=\sqrt{\rho_l}=2|\overline{a}_l|\quad (\text{all } q\neq b), \quad (2.11)$$

$$(\overline{s_W^q})^2=\frac{1}{4}(1-\overline{r}_l)\quad (\text{all } q\neq b), \quad (2.12)$$

and T_3^q is the third component of the weak isospin of the quark q. Substituting the measured values of $\overline{r}_l$, $\overline{a}_l$, from Tables III, V and of R_c, R_b from Table II leads to the values of $\overline{s}_c$, $\overline{s}_b$ reported in Table IV. Note that the values of $\overline{s}_b$ and, hence, $\overline{a}_b$ and $\overline{v}_b$ are extracted first. The latter are then substituted into Eq. (2.8) (taking into account their experimental errors) in order to find $\overline{s}_c$. In Table IV good agreement is seen between the measured values of $\overline{s}_l$ and $\overline{s}_c$ and the SM predictions. On the other hand, $\overline{s}_b$ lies 1.3σ above the prediction, a residual of the well-known ''R_b problem'' [1]. Solving Eqs. (2.1) and (2.8) then gives the effective coupling

TABLE V. Measured values of the effective electroweak coupling constants for the charged leptons. Dev(σ) =(meas.−SM)/error. The values given in the last two rows exclude τ-polarization data from the averages.

	Leptons		
	Meas.	SM	Dev(σ)
$\overline{a}_l$	−0.50101(33)	−0.50124	0.67
$\overline{v}_l$	−0.03782(68)	−0.03675	−1.57
$\overline{a}_l$	−0.50093(33)		0.91
$\overline{v}_l$	−0.03863(77)		−2.44

[4]As shown in Sec. IV below, the discrepancies with the SM predictions for the leptonic couplings, unlike those of the b quark, are reduced by assuming a smaller value of m_H and a larger value of m_t than that found in the global fit of Ref. [1].

[5]Reference [1] included small mass corrections in calculating $\overline{a}_l$ and $\overline{v}_l$ which are neglected here.

[6]Here the weak isospin symmetry of the SM is invoked to calculate the unobserved couplings. It is also assumed that the quantum corrections contained in ρ_q and $(\overline{s_W^q})^2$, though not necessarily those of the SM, are universal.

TABLE VI. QED and QCD correction factors for heavy quarks assuming $\alpha_s(M_Z)=0.12$ and $\alpha(M_Z)^{-1}=128.9$.

$C_c^{\rm QED}$	$C_b^{\rm QED}$	$C_c^{\rm QCD}$	$C_b^{\rm QCD}$
1.00046	0.99975	1.0012	0.9953

constants for the heavy quarks presented in Table VII. The values found, as well as the errors, agree well with those reported by Renton in a recent review [7]. The solutions for $\overline{a}_f$, $\overline{v}_f$ obtained from the essentially uncorrelated quantities $\overline{r}_f$ and $\overline{s}_f$ are shown graphically in Figs. 1a, 1b, 1c for $f=l$, c, b, respectively. The corresponding solutions when the τ-polarization measurements of A_l are excluded from the average are shown in Figs. 2a, 2b, 2c. It is clear from Figs. 1c, 2c that largest discrepancy with the SM is in the parameter $\overline{r}_b$ (completely determined by A_b) rather than in $\overline{s}_b$ (essentially determined by R_b). Indeed, if the SM value for the latter is used, instead of the measured one, to solve for $\overline{a}_b$ and $\overline{v}_b$, the discrepancies between the values found and the SM are almost unchanged.

Although the c quark couplings agree well with the SM and are also consistent with the quark-lepton universality hypothesis, both $\overline{a}_b$ and $\overline{v}_b$ differ from the SM values by more than three standard deviations. The errors in these quantities are, however, highly correlated. The statistical significance of these deviations is discussed in detail below.

It should be remarked that, although a particular value (0.12) of $\alpha_s(M_Z)$ has been assumed in order to extract the effective couplings of the heavy quarks, the sensitivity to the chosen value is very weak. Varying $\alpha_s(M_Z)$ over the range $0.1 < \alpha_s(M_Z) < 0.14$ leads variations of only $\simeq 3\times 10^{-4}$ in $\overline{a}_b$ and $\overline{v}_b$ to be compared with experimental errors $\simeq 1-4\times 10^{-2}$ (see Table VII).

A further constraint on the quark couplings is provided by the measurement of the mean quark forward-backward charge asymmetry:

$$\langle A_{\rm FB}^q\rangle = \frac{8A_l\Sigma_q\overline{v}_q\overline{a}_q}{\Sigma_q[(1-6\mu_q)(\overline{a}_q)^2+(\overline{v}_q)^2]}. \qquad (2.13)$$

All experimental analyses performed to date have assumed the correctness of the SM and have used measurements of $\langle A_{\rm FB}^q\rangle$ to determine a value of $\sin^2\theta_{\rm eff}^{\rm lept}$ [1]. Inserting the average value of the latter reported in Ref. [1] into the SM formula for $\langle A_{\rm FB}^q\rangle$ and propagating the error leads to the ''measured'' value:

$$\langle A_{\rm FB}^q\rangle = 0.1592(86).$$

As shown in Table VIII, this value is consistent with the SM prediction, with the ''model-independent'' prediction given by inserting the lepton and b quark couplings from Tables V and VII into Eq. (2.13) and assuming non-b-quark lepton universality for the u, d, s, c quarks, as well as the prediction when, in the latter case, the measured b quark couplings are replaced by the SM ones. With the present experimental errors, $\langle A_{\rm FB}^q\rangle$ is therefore insensitive to possible deviations of the b quark couplings from the SM, of the magnitude observed in the A_b measurements.

As mentioned earlier, in order to avoid having to introduce an accurate value of $\alpha_s(M_Z)$ as a correlated parameter in the extraction of the heavy quark effective couplings, the hypothesis of non-b-quark lepton universality was made in deriving the value of $\overline{s}_b$ from the measured quantity R_b. The consistency of this assumption may be checked by extracting $\alpha_s(M_Z)$ from the LEP average value of $R_l\equiv\Gamma_{\rm had}/\Gamma_l$ [1],

$$R_l = 20.778(29),$$

using the relation

$$R_l = 3\,\frac{\langle C_q^{\rm QED}\rangle\langle C_q^{\rm QCD}\rangle}{C_l^{\rm QED}}\,\frac{\Sigma_q\overline{s}_q}{\overline{s}_l}. \qquad (2.14)$$

The QED and QCD correction factors $\langle C_q^{\rm QED}\rangle$ and $\langle C_q^{\rm QCD}\rangle$ are averaged over all quark flavors. The QED correction factors are

$$\langle C_q^{\rm QED}\rangle = 1.00040, \quad C_l^{\rm QED} = 1.0019.$$

Inserting the measured values of $\overline{s}_b$ and $\overline{s}_l$ and using non-b-quark lepton universality to evaluate $\overline{s}_q$ $(q\neq b)$ gives, for the QCD correction factor:

$$\langle C_q^{\rm QCD}\rangle = 1.0394(21).$$

TABLE VII. Measured values of the effective electroweak coupling constants of c and b quarks. Dev(σ)=(meas.−SM)/error. The values given in the last two rows exclude τ-polarization data from the averages.

	c quark			b quark		
	Meas.	SM	Dev(σ)	Meas.	SM	Dev(σ)
$\overline{a}_f$	0.504(10)	0.501	0.30	−0.5252(75)	−0.4981	−3.61
$\overline{v}_f$	0.184(15)	0.192	−0.53	−0.3057(125)	−0.3434	3.18
$\overline{a}_f$	0.505(10)		0.40	−0.5298(70)		−4.53
$\overline{v}_f$	0.180(15)		−0.80	−0.2977(123)		3.72

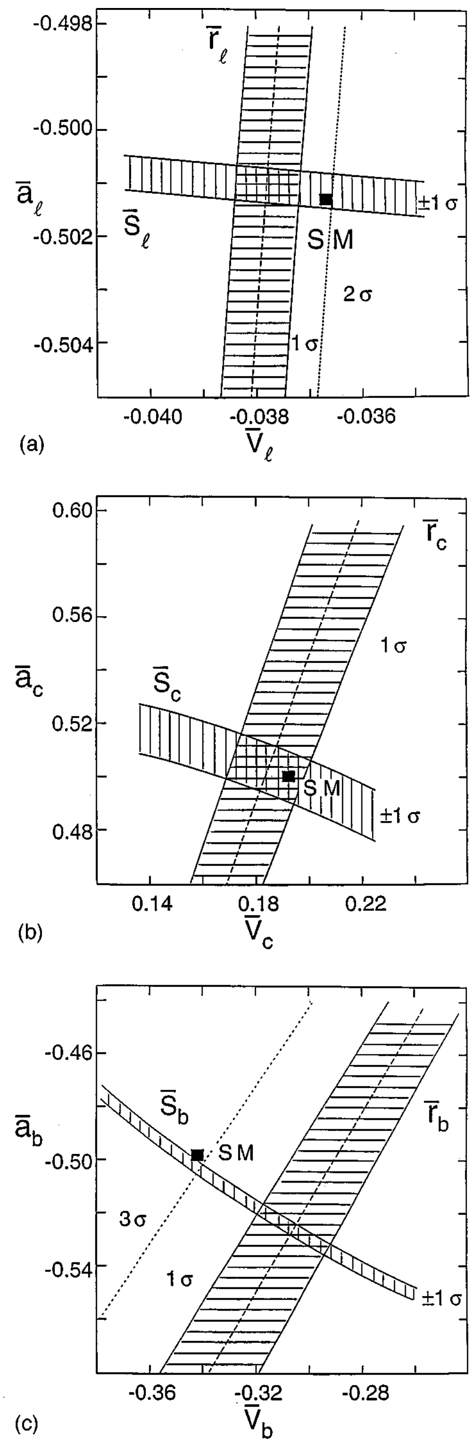

FIG. 1. Constraints on the effective couplings $\overline{a_f}$, $\overline{v_f}$ provided by the measurements of $\overline{r_f}$ and $\overline{s_f}$: (a) leptons, (b) c quarks, and (c) b quarks. The cross hatched areas show $\pm 1\sigma$ limits. The dotted lines in (a) [(c)] show 2σ [3σ] limits for $\overline{r_l}$,[$\overline{r_b}$]. SM is the standard model prediction for $m_t = 172$ GeV, $m_H = 149$ GeV.

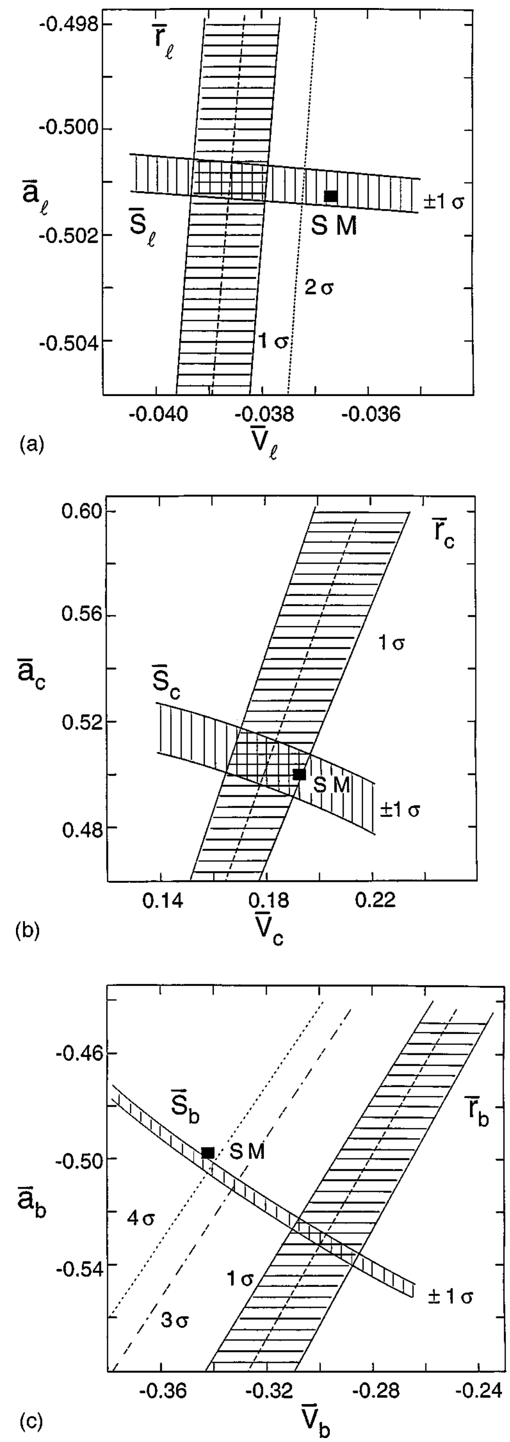

FIG. 2. As in Fig. 1, except that τ-polarization measurements are excluded from the LEP average value of A_l. The dotted line in (a) shows the 2σ limit for $\overline{r_l}$. The dash-dotted (dotted) line in (c) shows the 3σ (4σ) limit for $\overline{r_b}$.

Using the third-order perturbative QCD formula [8]

$$\langle C_q^{\mathrm{QCD}} \rangle = 1 + 1.06\,\frac{\alpha_s(M_Z)}{\pi} + 0.9\left(\frac{\alpha_s(M_Z)}{\pi}\right)^2 - 15\left(\frac{\alpha_s(M_Z)}{\pi}\right)^3 \quad (2.15)$$

gives

$$\alpha_s(M_Z) = 0.116^{+0.005}_{-0.007},$$

which may be compared to the global fit value of Ref. [1]:

$$\alpha_s(M_Z) = 0.120(3).$$

The good agreement of the model-independent analysis re-

TABLE VIII. Values of the mean quark charge asymmetry. ''MI pred.'' stands for model-independent prediction (see text). See also the text for the definition of ''measured.''

	''Measured''	SM pred.	MI pred.	MI pred. with SM b quark
$\langle A_{FB}^{q}\rangle$	0.1592(86)	0.1641	0.1639(28)	0.1692(28)
(''Meas.''−pred.)/error		−0.57	−0.52	−1.1

sult with the global world average value $\alpha_s(M_Z)=0.118(5)$, found in two recent reviews [9,10] of all published measurements of α_s, shows that an analysis assuming this value of $\alpha_s(M_Z)$, but without the assumption of non-b-quark lepton universality, would lead to essentially the same values of the b quark couplings as those reported in Table VII. In the fit used in Ref. [7], to determine the heavy quark effective couplings the constraint $\alpha_s(M_Z)=0.123(6)$ was imposed. As mentioned above, the fitted heavy quark couplings are very consistent with those found in the present analysis.

In order to correctly calculate the statistical significance of the deviations from the SM predictions of the effective couplings shown in Tables V and VII, it is necessary to take into account the correlations between the errors of the different quantities. To avoid the very large correlations between the errors on $\overline{a_f}$ and $\overline{v_f}$ (for the case of b quarks the correlation coefficient is −0.96), it is convenient to use, in calculating the χ^2, the equivalent quantities $\overline{r_f}$, $\overline{s_f}$ for which the errors are uncorrelated for a given fermion flavor f. Important correlations still exist, however, between the errors in $(\overline{r_l},\overline{r_c})$ and $(\overline{r_l},\overline{r_b})$ in the case that $\overline{r_c}$ and $\overline{r_b}$ are extracted from forward-backward asymmetries using Eqs. (2.3), (2.4), and (2.6). The correlation coefficient is

$$\mathcal{C}_{lQ}=-\frac{(1-\overline{r_l}^2)(1+\overline{r_Q}^2)}{(1+\overline{r_l}^2)(1-\overline{r_Q}^2)}\frac{\sigma_{\overline{r_l}}}{\overline{r_l}}\frac{\overline{r_Q}}{\sigma_{\overline{r_Q}}}\quad (Q=c,b). \quad (2.16)$$

Substituting the parameters from Table III gives

$$\mathcal{C}_{lc}=-0.29,\quad \mathcal{C}_{lb}=-0.52.$$

The results on the C.L.s for the agreement with the SM of different sets of effective weak coupling constants, parametrized in terms of $\overline{r_f}$ and $\overline{s_f}$, are collected in Table IX. These C.L.s assume perfect statistical consistency of the different measurements contributing to the averages. The entries in the first column of Table IX, giving the level of agreement of $(\overline{r_l},\overline{s_l})$ with the SM prediction, are simply calculated from the entries of Tables III and IV using a diagonal error matrix, since the errors in $\overline{r_l}$ and $\overline{s_l}$ are uncorrelated. Calculating separately the contributions to χ^2 from $\overline{r_l}$ and $\overline{r_b}$, where the latter is derived from the LEP $A_{FB}^{0,b}$ measurement and $\overline{r_b}$ derived via Eq. (2.6) directly from the SLD A_b measurement, gives the entries reported in the second column of Table IX. The C.L. for agreement with the SM prediction of 1.4% drops to only 0.06% if the τ-polarization measurements of A_l are excluded. The third column of Table IX results from adding to the χ^2 in the second column the (uncorrelated)[7] contributions of $\overline{s_l}$ and $\overline{s_b}$. In the fourth column of Table IX, the χ^2 and C.L.s of the variables $\overline{r_l}$, $\overline{r_b}$, and $\overline{r_c}$ taking into account the $\overline{r_l}$-$\overline{r_b}$ and $\overline{r_l}$-$\overline{r_c}$ correlations are given. In the last column of Table IX, the (uncorrelated) variables $\overline{s_l}$, $\overline{s_b}$, and $\overline{s_c}$ are added to those of the fourth column. Note that the number of degrees of freedom corresponding to the χ^2 values reported in the second, third, fourth, and fifth columns of Table IX is 3, 5, 5, and 8 respectively, since the $\overline{r_c}$ and $\overline{r_b}$ measurements derived from the SLD A_c, A_b determinations give separate, uncorrelated, contributions to the χ^2. As expected, the agreement with the SM improves as the number of degrees of freedom of the χ^2 increases (the more parameters considered, the more likely is a deviation associated with any one of the parameters to be consistent with a statistical fluctuation). However, there is still a factor of ≃10 difference between the C.L.s including (or excluding) the τ-polarization data. Taking into account the C.L. (8.4%) for self-consistency of the different A_l measurements, the probability[8] that all six effective couplings are consistent with lepton universality and the SM is 0.9%. The similar probability for the leptonic and b quark couplings alone is 0.18%. If the τ-polarization measurements are excluded, the latter probability drops to 0.018%.

It is important emphasize that a correct calculation of χ^2 and the associated confidence levels requires that all relevant correlations between errors be taken into account. If a χ^2 is calculated from ''raw'' experimental measurements, such as those presented in Table I, erroneous conclusions as to the consistency of the data with the SM will be drawn. Assuming non-b-quark lepton universality, the 14 measured electroweak observables presented in Table I depend on only four unknown parameters, the effective couplings of the leptons and of the b quarks. The ''raw'' χ^2 calculated from the ''pulls'' [1] in the last column of Table I is 21.3 for 14 DOF (C.L.=0.093) or, excluding the τ-polarization data 19.5 for 12 DOF (C.L.=0.077).[9] The strong sensitivity of the C.L. to inclusion or exclusion of the τ-polarization data is completely lost using the ''raw'' χ^2. In fact, the ≃2σ effects seen in the ''pulls'' of the observables $A_{FB}^{0,\tau}$, A_e (SLD), $A_{FB}^{0,b}$,

[7]Actually there is a weak correlation between $\overline{s_b}$ and $\overline{r_l}$ following from Eq. (2.8), where $\overline{r_l}$ is used to calculate S_Q. However, the correlation coefficient is only ≃0.08 and is neglected here.

[8]Here the term ''probability'' is used in the the usual sense of the fraction of all cases expected to have a C.L. less than the observed value. For independent χ^2 tests the probabilities are assumed to be uncorrelated.

[9]In Ref. [1], the SM prediction is obtained by fitting m_t, m_H and several other electroweak parameters to the observables of Table I as well as others which are not directly sensitive to the effective couplings (see Table 20 of Ref. [1]). Here, for comparison purposes, the values $m_t=172$ GeV, $m_H=149$ GeV are assumed so that the SM prediction has no free parameters.

TABLE IX. χ^2 and confidence levels for agreement with the SM ($m_t=172$ GeV, $m_H=149$ GeV) of different sets of electroweak observables sensitive to the effective couplings, assuming perfect statistical consistency of the LEP+SLD averages in Table II. The values given in the last two rows do not use τ-polarization measurements in the A_l average. See the text for the explanation of the number of degrees of freedom (DOF) in each case.

Observables	$\overline{r_l},\overline{s_l}$	$\overline{r_l},\overline{r_b}$	$\overline{r_l},\overline{s_l},\overline{r_b},\overline{s_b}$	$\overline{r_l},\overline{r_b},\overline{r_c}$	$\overline{r_l},\overline{s_l},\overline{r_b},\overline{s_b},\overline{r_c},\overline{s_c}$
DOF	2	3	5	5	8
χ^2	3.44	10.6	13.2	10.9	13.2
C.L. (%)	17.9	1.4	2.2	5.3	10.5
χ^2	7.55	17.2	19.4	17.4	19.6
C.L. (%)	2.3	0.064	0.16	0.38	1.2

and A_b (SLD) add, because of correlations, constructively in the parameter $\overline{r_b}$ to give the observed deviation from the SM of $>3\sigma$. First extracting the essential theoretical parameters (the effective couplings) and then comparing with the SM predictions using a χ^2 test is more sensitive to deviations of these parameters from the SM predictions than the ''raw'' χ^2. For the latter an inevitable statistical dilution occurs because of the large number of experimental observables (14) used as compared to only four effective coupling constants that determine the theoretical prediction. This dilution effect becomes even more marked when additional observables, not directly sensitive to the effective couplings, are added to the χ^2 as in the global fit of Ref. [1]. In fact, the five additional observables used in the fit contribute only 0.55 to the total χ^2 of 19.1, indicating an overestimation of the errors on these quantities[10] that reinforces the statistical dilution.

III. DISCUSSION OF THE EXPERIMENTAL MEASUREMENTS OF ELECTROWEAK OBSERVABLES

In order to derive the average values of the electroweak observables presented in Tables I and II, many different experimental measurements were combined [1]. In the light of the apparent deviations seen from the SM predictions in the model-independent analysis described in the previous section, the first question that should be asked is whether the experimental measurements are reliable and consistent. An important general question is whether the uncertainty in the measured observable is dominated by statistical or systematic errors. Only in the former case can the error be interpreted, with confidence, in the statistical sense ($1\sigma \equiv 68\%$ C.L.) and the probabilistic meaning of the C.L. of a χ^2 test can be expected to be reliable. This is no longer the case if the systematic error is dominant. As there is no definite, agreed, procedure for assigning systematic errors, the meaning of the error can depend on psychological (or even sociological) factors. If the physicist is overconservative in assigning the error, real deviations from a theoretical expectation can be missed or, in the contrary case, spurious detector related effects wrongly interpreted as ''new physics.'' Some check on the degree of conservatism or, otherwise, of physicists is, however, provided if there are many repeated measurements of the same quantity, as provided, in the present case, by the different experiments at LEP. When the errors are dominated by systematic effects or contain a large systematic contribution, then comparing the weighted average error to that calculated by applying the central limit theorem to the different measurements of the same quantity gives an indication whether the systematic errors are over- or underestimated. Such a test only applies to errors which are uncorrelated for the different measurements of the same quantity. This test may be best applied when each experiment measures the same observable using a similar method. If consistency is found in this case, but inconsistencies are found when the same physical quantity is measured using different observables (for example, the quantity A_l may be measured using either forward-backward asymmetries for different dilepton final states or τ polarization), it is probable that there is an unknown source of correlated systematic error. As discussed further below, such an error can arise, for example, due to an inadequate treatment of QED radiative corrections.

The apparent deviations from the SM predictions of the b quark couplings seen in Table VII are due those observed in only two of the basic electroweak parameters A_b and A_l. Most of the current information on A_b is derived from the Z-peak $A_{\rm FB}^{0,b}$ measurements of the four LEP experiments. The error on this A_b measurement is roughly half that of the SLD A_b measurement. The individual measurements of the experiments [1] contributing to the LEP average $A_{\rm FB}^{0,b}$ measurement quoted in Table I are shown in Fig. 3, together with the weighted average value, its error, and the SM prediction. There is no hint of any badly understood systematic effect in the distribution of these measurements. Except for the ALEPH jet-charge measurement [11], all errors, even those using all LEP1 data, are statistically dominated. The weighted average value agrees well with the individual measurements ($\chi^2=5.9$ for 8 DOF, C.L.$=66\%$). The estimate of the error on the mean value, given by the sample variance of the seven most accurate determinations, using the central limit theorem is 0.00232, in excellent agreement with the weighted average error of the same data points, which is 0.00237. The situation is very different for the parameter A_l. Shown in Fig. 4 are the values of A_l derived from the LEP measurements of $A_{\rm FB}^{0,e}$, $A_{\rm FB}^{0,\mu}$, $A_{\rm FB}^{0,\tau}$, A_e, and A_τ from τ polar-

[10] A χ^2 of 0.55 for five degrees of freedom corresponds to a C.L. of 0.990.

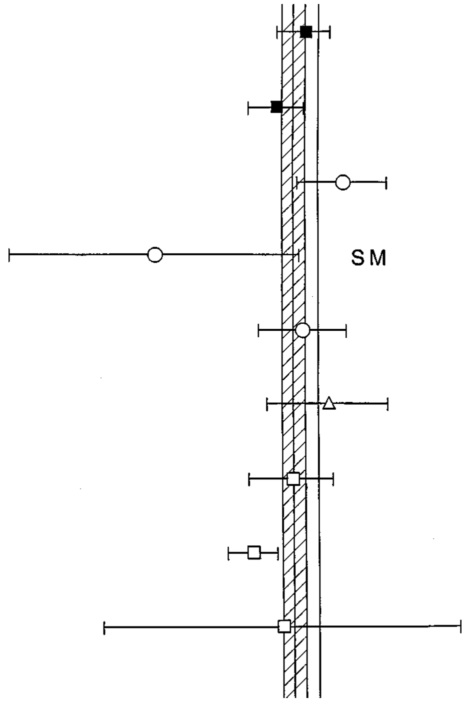

FIG. 3. LEP on-peak $A_{\rm FB}^{0,b}$ measurements. ALEPH solid squares; DELPHI, open circles; L3, open triangle; OPAL, open squares. The $\pm 1\sigma$ region around the weighted average value is indicated by the hatched band. The vertical line is the standard model prediction for $m_t = 172$ GeV, $m_H = 149$ GeV.

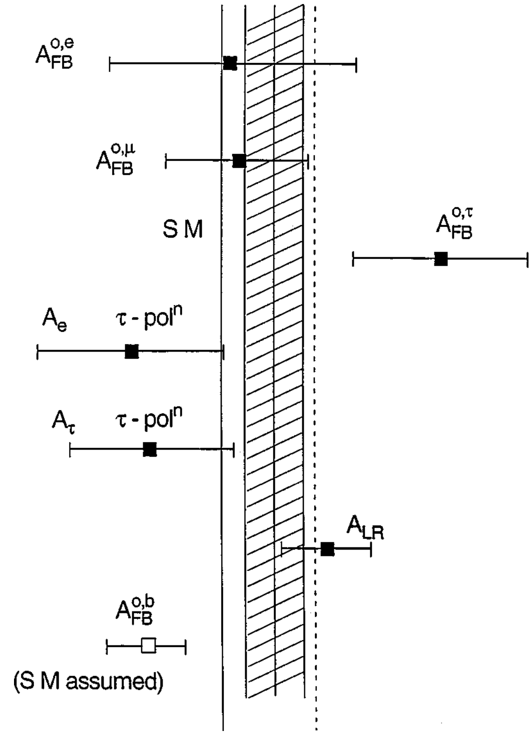

FIG. 4. LEP and SLD A_l measurements. The hatched band shows the $\pm 1\sigma$ region around the weighted average value. The weighted average value, excluding the τ-polarization measurements, is given by the dashed vertical line. The solid vertical line is the standard model prediction for $m_t = 172$ GeV, $m_H = 149$ GeV. The open square shows the value of A_l derived from the LEP average value of $A_{\rm FB}^{0,b}$ assuming the SM; this datum is not included in the weighted averages shown.

ization and A_e as measured by SLD using the left-right beam polarization asymmetry. Although the overall consistency of the individual measurements with the weighted average value seems acceptable ($\chi^2 = 9.7$ for 5 DOF, C.L.$=8.4\%$), the internal consistency of the various measurements is much worse. In particular, there are three, essentially independent,[11] $\simeq(2\text{–}3)\sigma$ deviations concerning τ-related measurements.

(i) $A_{\rm FB}^{0,\tau}$ is 1.8σ higher than the average of $A_{\rm FB}^{0,e}$ and $A_{\rm FB}^{0,\mu}$. Also, in all four LEP experiments (see Table 3 of Ref. [1]), $A_{\rm FB}^{0,\tau}$ is higher than $A_{\rm FB}^{0,e}$ or $A_{\rm FB}^{0,\mu}$. Assuming no systematic bias, the probability for this is 1 in 81.

(ii) The average value of A_l extracted from the τ-polarization data, 0.1393(50), lies 2.2σ below that, 0.1522(30), given by the weighted average of the non-τ measurements.

(iii) Assuming lepton universality, the LEP average value of $A_{\rm FB}^{0,\tau}$ gives, using Eq. (2.3), $A_l = 0.1649(71)$. This is 2.9σ higher than the mean A_l calculated from the τ-polarization measurements of A_e and A_τ.

These deviations cannot be explained by a breakdown of lepton universality for the τ. Using the τ-polarization A_e measurement to extract, using Eq. (2.3), A_τ from $A_{\rm FB}^{0,\tau}$ gives $A_\tau = 0.1968(204)$. This may be compared with the τ-polarization measurement: $A_\tau = 0.1401(67)$. There is a 2.6σ discrepancy. In the case of a breakdown of lepton universality, the two determinations of A_τ must give a consistent result that is significantly different from the measured A_e value. In fact, the values of A_e and A_τ found using τ polarization are consistent within 0.19σ.

Assuming Gaussian errors, the probability that *all* these τ-related apparent deviations from lepton universality are statistical fluctuations is 7.5×10^{-6}. This situation may be compared with that for the non-τ-related measurements of A_l derived from $A_{\rm FB}^{0,e}$, $A_{\rm FB}^{0,\mu}$, and $A_{\rm LR}$, which are, respectively, 0.147(11), 0.148(61), and 0.154(4). Labeling these measurements 1, 2, and 3, respectively, the deviations between the pairs 1-2, 1-3, and 2-3 are 0.08σ, 0.6σ, and 0.83σ. The measurements are perfectly consistent.

It may be remarked that the $A_{\rm FB}^{0,\tau}$ measurement, which occurs in the first of the three above-mentioned apparent deviations, has been included in the evaluation of the average values of A_l used in the model-independent analysis of the previous section. However, excluding this datum from the A_l average in addition to the τ-polarization data gives $A_l = 0.1516(29)$. Thus (see Table II) the deviations from the

[11]There is a weak correlation between two of the three effects, in that the $A_{\rm FB}^{0,e}$, $A_{\rm FB}^{0,\mu}$ measurements, used in the first consistency check, contribute also to the ''non-τ'' weighted average value of A_l used in the second. This can be avoided by comparing the τ-polarization value of A_l with the $A_{\rm LR}$ measurement. In this case an even larger discrepancy of 2.4σ is found.

TABLE X. Confidence levels for the consistency of LEP measurements of $A_{\rm FB}^{e,0}$, $A_{\rm FB}^{\mu,0}$, and $A_{\rm FB}^{\tau,0}$. STT, student's t test: EET, C.L. calculated from estimated experimental errors.

	τ-μ	τ-e	μ-e
C.L. (%) STT	2.7	6.0	68
C.L. (%) EET	8.0	17	94

SM predictions will lie between the ''τ-polarization-in'' and ''τ-polarization-out'' cases discussed above, if all τ-related measurements are excluded.

Also shown in Fig. 4 is the value of A_l derived from $A_{\rm FB}^{0,b}$, assuming the correctness of the SM. The value so obtained, 0.1396(33), differs from the weighted average of the purely leptonic measurements by 2.6σ, or by 3.2σ if the τ-polarization data are excluded. It is clear, from the analysis of the previous section, that these discrepancies are mainly due to the deviations of the b quark effective couplings from the SM predictions. The quantity $\sin^2\theta_{\rm eff}^{\rm lept}$ used in Ref. [1] is directly related to A_l via Eqs. (2.4), (2.12). The poor consistency of the different $\sin^2\theta_{\rm eff}^{\rm lept}$ determinations in Table 19 of Ref. [1] is largely due to the inclusion of values derived from $A_{\rm FB}^{0,b}$ and $\langle A_{\rm FB}^{q}\rangle$ assuming the correctness of the SM. The common origin, in the b quark couplings, of the poor agreement of the different $\sin^2\theta_{\rm eff}^{\rm lept}$ determinations and the 3σ deviation of the measured LEP-SLD average value of A_b from the SM prediction was not pointed out in Ref. [1].

Each type of observable contributing to the average values of A_l, $A_{\rm FB}^{0,l}$ $(l=e,\mu,\tau)$, A_e, and A_τ from τ polarization and A_e from $A_{\rm LR}$ is now discussed it turn, taking into account the internal consistency of measurements of the same quantity performed by different experiments, the relative importance of statistical and the estimated systematic errors, and possible systematic effects that may not have, so far, been taken into account.

A. LEP leptonic forward-backward charge asymmetry measurements

The values of $A_{\rm FB}^{0,l}$ $(l=e,\mu,\tau)$ for the different LEP experiments are presented in Table 3 of Ref. [1]. A detailed breakdown of the systematic errors of the different data sets is found in Table 2 of the same paper. Although the individual experimental errors are usually statistics dominated, the statistical and systematic contributions to the error on the weighted average (see Table 20[12] of Ref. [1]) are almost equal. The most remarkable systematic feature of the $A_{\rm FB}^{0,l}$ measurements, as already mentioned above, is the relatively high value of $A_{\rm FB}^{0,\tau}$ found by all four experiments. Since each of the LEP experiments has comparable statistics in each channel, each can be considered to provide an independent estimate, with a similar weight, of $A_{\rm FB}^{0,l}$. Disregarding the estimated errors on each measurement, an unbiased statistical test of the consistency of the measured values of $A_{\rm FB}^{0,l}$ can be made using the student's t distribution. This is done by calculating the probability that, say, the $A_{\rm FB}^{0,\tau}$ and $A_{\rm FB}^{0,\mu}$ measurements of the different experiments are consistent with a common mean value [12]. The results of this comparison for the three possible pairings (τ,μ), (τ,e), and (μ,e) are presented in Table X. Also shown in this table are the C.L.s for consistency of the measurements, based on the total errors quoted in Table I. The systematically larger C.L.s found using these errors perhaps indicates that the point-to-point systematic errors tend to be overestimated. The good agreement between the μ-e measurements and the poor agreement between τ-μ and τ-e indicates the possible presence of a correlated systematic effect, not included in the present systematic error estimate, for the τ-pair channel.

An obvious candidate for such an effect is the QED radiative correction. On the Z peak this is large; the combined $O(\alpha)$ and $O(\alpha^2)$ corrections amount to $\simeq -110\%$ of the corrected pole asymmetry for muon pairs [13]. The systematic error in the weighted average value of $A_{\rm FB}^{0,l}$ is then $\simeq$4.4% of the radiative correction, and the observed deviation of the τ from the combined μ-e results is $\simeq$27% of it. It seems, however, unlikely that the error in the theoretical estimate of the QED radiative correction, essentially due to unobserved initial state radiation, and the associated virtual corrections could be large enough to explain the high value of the τ-pair asymmetry. One effect that can produce large changes in the forward-backward asymmetry is initial-final state interference in the case that hard cuts are applied to the radiated photons [14,15]. Indeed, for some decay channels, such as $\pi\nu$, $\pi\pi\pi\nu$, tight cuts are applied in some of the LEP analyses in order to cleanly separate them from channels containing extra π^0's. Such effects could be investigated by comparing the forward-backward asymmetries for different τ decay modes and different cuts on additional photons.

B. LEP τ-polarization measurements

Results of the measurements of the average τ polarization of the four LEP experiments [16,17,18,19], for each τ decay channel analyzed, are presented in Table XI. The first error shown in each measurement is statistical, the second one systematic. The weighted average values of A_τ and errors are given separately for each experiment and for each decay channel. The overall weighted average and its error are also given. The results shown in Table XI do not correspond exactly to those used in the LEP averages reported in Ref. [1], although there is a large overlap. The data sets chosen are those for which both statistical and systematic errors have been given for each decay channel.

It can be seen that, unlike for the forward-backward charge asymmetry measurements, the quoted statistical and systematic errors of individual experiments are comparable for almost every decay channel. The systematic error estimates can be tested by comparing the weighted errors, for each experiment or each decay channel, with the error estimate on the mean value given by the central limit theorem: $\sqrt{\Sigma(x-\bar{x})^2/n(n-1)}$. The latter error estimates are given in Table XI in square brackets next to the weighted

[12]Note there is a misprint in this table. The estimated systematic error in $A_{\rm FB}^{0,l}$ should presumably be 0.0007 not 0.007.

TABLE XI. LEP measurements of $A_\tau = -\langle \overline{P_\tau} \rangle$ The first error shown is statistical, the second systematic. For the weighted averages (WA) the weighted total error is given, where statistical and systematic errors are added in quadrature. Errors calculated from sample variances are shown in square brackets.

	ALEPH	DELPHI	L3	OPAL	WA
$e\nu\nu$	0.200(51,31)	0.179(52,67)	0.168(39,15)	0.161(33,29)	0.173(26) [16]
$\mu\nu\nu$	0.124(41,21)	0.097(38,22)	0.111(45,16)	0.138(33,22)	0.119(22) [11]
$\pi\nu$	0.142(20,11)	0.158(33,50)	0.135(21,17)	0.117(14,12)	0.130(12) [9]
$\rho\nu$	0.108(19,18)	0.199(46,39)	0.168(17,10)	0.116(13,11)	0.135(12) [22]
$a_1\nu$	0.135(35,20)	0.103(50,38)	-	0.151(37,31)	0.134(28) [14]
WA	0.132(15) [24]	0.137(26) [27]	0.154(14) [14]	0.123(11) [9]	0.1343(73)

averages. It can be seen that the level of agreement is better for experiments than for decay channels. For the $e\nu\nu$, $\mu\nu\nu$, $\pi\nu$, and $a_1\nu$ channels, there is an indication of an overestimation of the systematic errors, while for $\rho\nu$ the error calculated from the sample variance is much larger than the weighted average error. This is due to the wide spread of the measurements in this channel; the DELPHI and L3 values are much larger than those of ALEPH and OPAL.[13]

The common mean test using the student's t distribution, applied above to the forward-backward charge asymmetry measurements, has also been applied to the measurements of A_τ presented in Table XI. The results of the comparisons of different decay channels are presented in Table XII and of different experiments in Table XIII. In each case the C.L.s for consistency based on the total experimental errors are also given. The main systematic features of the A_τ measurements are the relatively high values found by all experiments for the $e\nu\nu$ channel and the high value of the L3 weighted average, as compared to those of the other three experiments. The latter effect is also seen in the τ-polarization measurements of A_e [1]. The systematically larger value for the $e\nu\nu$ channel is reflected in the poor confidence levels (0.35%–2.5%) of the student's t tests for the channels e-μ, e-π, and e-a_1 in Table XII. Relatively worse C.L.s are also seen for these channels in the test based on the total errors, but as expected from the larger experimentally assigned errors as compared to those calculated from the sample variance, higher absolute C.L.s (11%–31%) are found. The agreement between the two types of test is much better in Table XIII, from which one might be tempted to conclude that all four experiments give consistent results.[14]

As in the case of the τ forward-backward charge asymmetry, the radiative correction associated with final state radiation appears as an obvious candidate to explain both the systematic differences observed between different decay channels and those, mentioned above, between the τ-polarization and other measurements of A_l. The final state radiative corrections in the τ-polarization measurements are not only large for most decay channels, but depend strongly on the detection efficiency of the radiated photons and hence on the acceptance and resolution of the LEP detectors as well as the experimental cuts. The τ polarization is measured by fitting the energy spectra of τ decay products. The latter are directly effected by the rate and energy spectra of, and cuts applied to, the radiated photons. Any systematic errors in the treatment of final state radiation are thus directly correlated to systematic errors in the τ-polarization measurement. Consider, for example, the measurement of $\langle \overline{P_\tau} \rangle$ using the $\pi\nu$ decay channel on the Z peak. Measuring only the pion energy and neglecting that of the radiated photons has been estimated [20] to shift $\langle \overline{P_\tau} \rangle$ by $\simeq$20% of its value. This may be compared to the systematic error assigned to the LEP average value of $\langle \overline{P_\tau} \rangle$ of 3.3% of its value [1]. Since radiative corrections are not included in the systematic error estimate, a tacit assumption is thus made that they are known to much better than $\simeq$10% of their value.

All of the LEP experiments have used the same Monte Carlo program KORALZ [21], to correct for radiative effects in fitting the energy spectra of the τ decay products. No experimental checks (measurements of the rate and distributions of final state photons) have been published, and any systematic errors assigned by the experiments for radiative corrections have been very small in comparison to detector-related sources of systematic error. As for the case of the τ forward-backward asymmetry, it seems unlikely, however, that the approximations made in simulating the radiative corrections can account for the all the different values of A_τ found using different decay channels. For the decay mode of largest statistical power, $\tau \to \rho\nu$, where the sensitivity to the τ polarization is given by angular information from the ρ decay as well as the energy distributions of decay products, the sensitivity to radiative corrections is small. The entire radiative correction generated by KORALZ has been shown to generate a shift of only -0.011 in A_τ in this case.[15] For the

[13]The weighted average of DELPHI and L3 is $A_\tau = 0.171(19)$, that of ALEPH and OPAL $A_\tau = 0.113(15)$. The difference is 2.5 standard deviations.

[14]The student's t test is less well adapted to the comparison of the different LEP experiments since the different decay channels have very different statistical sensitivities, whereas each test datum is treated, in the test, on an equal footing. On the other hand, since each experiment has comparable statistics and sensitivity for a given decay channel, the student's t common mean test is well adapted to the channel-by-channel comparisons.

[15]W. Lohmann (private communication).

TABLE XII. Confidence levels for the consistency of LEP $\langle \overline{P_\tau} \rangle$ measurements using different τ decay channels. STT and EET are defined in Table X.

	e-μ	e-π	e-ρ	e-a_1	μ-π	μ-ρ	μ-a_1	π-ρ	π-a_1	ρ-a_1
C.L. (%) STT	0.3	1.5	36	2.5	15	35	9	68	45	95
C.L. (%) EET	11	13	19	31	67	51	66	74	79	97

case of the $\pi\nu$ decay channel, a large ''structure-dependent'' effect in the final state photon spectrum due to the decay $a_1 \to \pi\gamma$ is to be expected. This contribution and its interference with the bremsstrahlung amplitudes have been estimated [22] to change the yield of high energy photons with $x = 2E_\gamma/m_\tau > 0.6$ by a factor $\geqslant 2$. However, the radiative correction to A_τ is dominated by soft bremsstrahlung photons, and so no large corrections are to be expected from this effect. For the $e\nu\nu$, $\mu\nu\nu$ decays, KORALZ uses the exact $O(\alpha)$ matrix element. Particularly for the electron case where the radiative corrections are large, it is perhaps, however, of interest to investigate the effect of $O(\alpha^2)$ and higher order corrections.

Independently of any specific conjectures on possible sources of systematic effects, the systematic error of 0.0045 (3.3%) assigned to the LEP average value of A_τ in Ref. [1] would appear to be unduly optimistic, in view of the apparent inconsistencies between measurements from both different decay channels and different experiments, discussed above. The value of A_τ derived from the $e\nu\nu$ channel deviates from the overall weighted average by 8.6 times the above systematic error estimate and the L3 experiment by 4.4 times. The student's t common mean test shows that the uncorrelated systematic errors in the individual experiments are probably, on average, overestimated. Thus the real inconsistencies between experiments and decay channels are probably larger than those estimated from the assigned experimental errors. This argues even more strongly that the assigned systematic error on the average value of A_τ is too small.

To illustrate the effect of an underestimation of the systematic error in the τ-polarization measurements, consider the effect of doubling the systematic error in the LEP average value of A_τ and assigning the same systematic error to A_e measured from the τ-polarization asymmetry.[16]

This results in a weighted average value of A_l of 0.1518(26), which lies almost midway between the ''τ-polarization-in'' and ''τ-polarization-out'' values shown in Table II.

The conclusion of this discussion of the τ-polarization measurements is that there is evidence that the uncorrelated systematic errors of the different experiments for the same decay channels have been overestimated. On the other hand, correlated systematic errors between different decay channels seem to have been underestimated. The true world average value of A_l probably lies between the two values quoted in Table II.

C. SLD $A_{\rm LR}$ measurement

The discussion of this measurement can be very brief, as there is only one experiment and the present error is strongly statistics dominated [23]. The relative systematic error in $A^0_{\rm LR} = A_e$, which is almost completely determined by that in the SLC electron beam polarization, is only 0.7%, as compared to the relative statistical error of 2.8%. The accuracy of the beam polarization measurement has been checked at the 4% level by independent determinations using Compton and Møller polarimeters. For comparison, the difference between the LEP τ-polarization and SLD $A_{\rm LR}$ measurements of A_l amounts to 8%. The systematic error of the $A_{\rm LR}$ measurement would have to be wrong by an order of magnitude to account for this difference. Another very important advantage of the $A_{\rm LR}$ measurement is the low level of the QED radiative correction, of only $\simeq -3\times10^{-4}$ [24] as compared with -110% for $A^{0,l}_{\rm FB}$ or -8% to $+20\%$, depending on the decay channel and the cuts on the final state radiation [20], for $\langle \overline{P_\tau} \rangle$.

To summarize the above detailed discussion of the different measurements contributing to the world average value of A_l used in the model-independent analysis of Sec. 2, although the two LEP measurements ($A^{0,l}_{\rm FB}$ and τ polarization) and the SLD $A_{\rm LR}$ measurement have, currently, similar statistical errors, the situation is very different with respect to systematic errors. Only for the $A_{\rm LR}$ measurement is the systematic error negligible. For $A^{0,l}_{\rm FB}$ and $\langle \overline{P_\tau} \rangle$ the estimated statistical and systematic errors are almost equal [1]. However, inconsistencies at the two to three standard deviation level in τ-related measurements of both $A^{0,l}_{\rm FB}$ and $\langle \overline{P_\tau} \rangle$ indicate that the true systematic error is probably considerably larger.

TABLE XIII. Confidence levels for the consistency of LEP $\langle \overline{P_\tau} \rangle$ measurements by different experiments. STT and EET are defined in Table X.

	A-D	A-L	A-O	D-L	D-O	L-O
C.L. (%) STT	85	31	15	31	31	64
C.L. (%) EET	87	29	63	57	63	9

[16] In Ref. [1] a systematic error of only 0.002 is assigned to A_e. Why this should be less than half of the already small error assigned to A_τ is not clear to this writer, taking into account the close similarity of the two measurements. Table 20 of Ref. [1] contains, however, the disclaimer that ''the determination of the systematic part of each error is approximate.''

TABLE XIV. Measured values of the quantum correction parameters $\Delta\rho_f$ and $\Delta\kappa_f$ compared to SM predictions. Dev(σ)=(expt.−SM)/error.

	Leptons		c quark		b quark	
	$\Delta\rho_l$	$\Delta\kappa_l$	$\Delta\rho_c$	$\Delta\kappa_c$	$\Delta\rho_b$	$\Delta\kappa_b$
Expt.	0.00404(133)	0.03445(134)	0.016(41)	0.064(49)	0.101(32)	0.403(107)
SM m_t=172 GeV						
m_H=149 GeV	0.00497	0.03686	0.005	0.037	−0.007	0.0436
Dev(σ)	−0.7	−1.8	0.27	0.55	3.38	3.36
SM m_t=180 GeV						
m_H=100 GeV	0.00563	0.03472	0.006	0.034	−0.008	0.0412
Dev(σ)	−1.2	−0.02	0.24	0.61	3.40	3.38

IV. PHYSICAL INTERPRETATION OF THE MEASURED EFFECTIVE WEAK COUPLING CONSTANTS

The test of the SM provided by measurements of Z decays at LEP and SLD is, essentially, that of the SM predictions for the quantum corrections, arising from massive virtual particle loops, to the Born level diagrams for $e^+e^-\to Z\to f\bar{f}$. These corrections may be conveniently expressed in terms of two parameters $\Delta\rho_f$ and $\Delta\kappa_f$ for each fermion flavor [25]. The parameters are given, in terms of the effective couplings, by the relations

$$\Delta\rho_f=-2(1-2|\overline{a_f}|), \tag{4.1}$$

$$\Delta\kappa_l=\frac{(1-\overline{r_l})}{4s_W^2}-1, \tag{4.2}$$

$$\Delta\kappa_c=3\,\frac{(1-\overline{r_c})}{8s_W^2}-1, \tag{4.3}$$

$$\Delta\kappa_b=3\,\frac{(1-\overline{r_b})}{4s_W^2}-1. \tag{4.4}$$

Here, following the usual on-shell definition [26],

$$\sin^2\theta_W=s_W^2=1-c_W^2\equiv 1-\frac{M_W^2}{M_Z^2}. \tag{4.5}$$

Since $\Delta\kappa_f$ is determined by $\overline{r_f}$, only the weak theoretical assumption of lepton universality is needed to extract it from the experimental measurements.

The SM predictions of Sec. II used the fixed values $m_t=172$ GeV, $m_H=149$ GeV found in the global fit of Ref. [1]. The effect on the SM prediction of varying m_t and m_H within the existing experimental bounds [27,28] is now considered. The leading dependence of $\Delta\rho_f$ on m_t and m_H is contained in the terms [25]

$$\Delta\rho_f^{\rm top}=\frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2}\,(1+\xi_f), \tag{4.6}$$

$$\Delta\rho_f^{\rm Higgs}=-\frac{\sqrt{2}G_\mu M_W^2}{8\pi^2}\tan^2\theta_W\left\{\frac{11}{3}\left[\ln\left(\frac{m_H}{M_W}\right)-\frac{5}{12}\right]\right\}, \tag{4.7}$$

where $\xi_f=0$ for $f\neq b$ and $-4/3$ for $f=b$. The quantum correction $\Delta\kappa_f$ is calculated using a parametrization[17] of the ZFITTER [29] prediction of the effective leptonic weak mixing angle:

$$(\overline{s_W^l})^2=0.233597-8.95\times10^{-8}m_t^2-3.86\times10^{-4}\ln m_t +5.43\times10^{-4}\ln m_H, \tag{4.8}$$

where m_t and m_H are in GeV units. $\Delta\kappa_f$ is related to $(\overline{s_W^l})^2=(1-\overline{r_l})/4$ by Eqs. (2.1), (2.9), (2.10), and (4.2)–(4.4). For the b quark there is an additional nonuniversal contribution

$$\Delta\kappa_b^{\rm top}=\frac{G_\mu m_t^2}{4\sqrt{2}\pi^2}. \tag{4.9}$$

The values of $\Delta\rho_f$, $\Delta\kappa_f$ for $f=l$, c, b, extracted from the measured effective couplings using Eqs. (4.1)–(4.5), are presented in Table XIV. Standard model predictions are shown for the cases $m_t=172$ GeV, $m_H=149$ GeV and for $m_t=180$ GeV, $m_H=100$ GeV. The latter choice gives a somewhat better description of the leptonic corrections. In Table XV corresponding results for $\Delta\rho_f$ and $\Delta\kappa_f$ are shown for the case when the τ-polarization measurements are excluded from the LEP average value of A_l.

Good agreement with the SM is seen for leptons and c quarks. For b quarks, however, the measured values of the quantum corrections are much larger than the SM predictions. For $\Delta\rho_b$ the measured value exceeds the SM prediction by a factor of 13–15 and is of opposite sign. The measured value of $\Delta\kappa_b$ has the same sign as the SM prediction, but is 9–11 times larger. Both effects are at the >3 standard deviation level, but they are highly correlated. The discrep-

[17]The relative accuracy of the formula (4.8) is about one per mille for the interesting range of values of m_t and m_H.

TABLE XV. Measured values of the quantum correction parameters $\Delta\rho_f$ and $\Delta\kappa_f$ compared to SM predictions. τ-polarization measurements are excluded from the A_l average. Dev(σ) = (expt. − SM)/error.

	Leptons		c quark		b quark	
	$\Delta\rho_l$	$\Delta\kappa_l$	$\Delta\rho_c$	$\Delta\kappa_c$	$\Delta\rho_b$	$\Delta\kappa_b$
Expt.	0.00372(133)	0.03260(157)	0.020(41)	0.079(49)	0.119(30)	0.470(97)
SM $m_t = 172$ GeV $m_H = 149$ GeV	0.00497	0.03686	0.005	0.037	−0.008	0.0439
Dev(σ)	−0.94	−2.7	0.37	0.86	4.23	4.39
SM $m_t = 180$ GeV $m_H = 100$ GeV	0.00563	0.03472	0.006	0.034	−0.008	0.0412
Dev(σ)	−1.4	−1.4	0.34	0.92	4.23	4.42

ancies seen are so large that the significance of the deviations shows almost no sensitivity to m_t and m_H.

It is also instructive to present the quantum corrections in terms of the ''epsilon parameters'' introduced by Altarelli *et al.* [30,31,32]. In terms of the variables used in the present paper to describe the effective couplings, these are defined as [30]

$$\epsilon_1 \equiv \Delta\rho_l = -2(1+2\overline{a_l}), \tag{4.10}$$

$$\epsilon_2 \equiv c_0^2\Delta\rho_l + \frac{s_0^2\Delta r_W}{(c_0^2-s_0^2)} - 2s_0^2\Delta k', \tag{4.11}$$

$$\epsilon_3 \equiv c_0^2\Delta\rho_l + (c_0^2-s_0^2)\Delta k', \tag{4.12}$$

here, $s_0^2 = 1-c_0^2$ and Δr_W are defined by the relations

$$\left(1-\frac{M_W^2}{M_Z^2}\right)\frac{M_W^2}{M_Z^2} = \frac{s_0^2c_0^2}{1-\Delta r_W} = \frac{\pi\alpha(M_Z)}{\sqrt{2}G_\mu M_Z^2(1-\Delta r_W)}$$

and

$$\Delta k' = \frac{(1-\overline{r_l})}{4s_0^2} - 1.$$

In Ref. [32] a fourth parameter ϵ_b was introduced. It may be defined in three distinct ways:

$$\epsilon_b(\overline{a_b}) \equiv \frac{\overline{a_b}}{\overline{a_l}} - 1, \tag{4.13}$$

$$\epsilon_b(\overline{r_b}) \equiv \frac{\overline{r_b} - \mathcal{R}_l}{1-\overline{r_b}}, \tag{4.14}$$

$$\epsilon_b(\overline{s_b}) \equiv \frac{\overline{s_b} - (\overline{a_l})^2(1-6\mu_b+\mathcal{R}_l^2)}{2(\overline{a_l})^2(1-6\mu_b+2\mathcal{R}_l)}, \tag{4.15}$$

where

$$\mathcal{R}_l = \frac{(2+\overline{r_l})}{3}.$$

In the SM, retaining only the leading terms $\simeq m_t^2$, the three definitions (4.13)–(4.15) are equivalent.[18] In previous phenomenological applications, however [32,33], only the third definition (4.15) based, via Eq. (2.8) on the measurement of R_b was used. The measured values of the six epsilon parameters defined above are presented in Table XVI, where they are compared with the SM predictions. As noted previously [33], the values of ϵ_1, ϵ_2, and ϵ_3 are in good agreement with the SM predictions. A small deviation is observed for $\epsilon_b(\overline{s_b})$, a residual of the much discussed [1] ''R_b problem.'' However, both $\epsilon_b(\overline{a_b})$ and $\epsilon_b(\overline{r_b})$ deviate from the SM prediction by about four standard deviations.[19] One may note the extreme sensitivity of the parameter $\epsilon_b(\overline{r_b})$ to the anomalous b coupling; the measured value is 39 times and 4.7σ larger[20] than the SM prediction. The SM predictions for these quantities are insensitive to m_H and are essentially given by the term $\simeq m_t^2$:

$$\epsilon_b = -\frac{2}{3}\,\Delta\rho^{\rm top} = -\frac{G_\mu m_t^2}{4\sqrt{2}\pi^2} = -0.0062 \quad (m_t = 172 \text{ GeV}). \tag{4.16}$$

Table XVII shows the epsilon parameters calculated excluding the τ-polarization measurements from the averages. It can be seen that the measured value of $\epsilon_b(\overline{r_b})$ exceeds the SM prediction by a factor of 44 and more than six standard deviations in this case. It may be remarked that, in this case, the leptonic parameter ϵ_3 also shows a deviation of (3–4)σ.

The conclusion to be drawn from Tables XVI and XVII is that the deviations observed for the b quark couplings, interpreted as a real physical effect, do not enter at all into the framework of the SM or any of its ''natural'' extensions. Supersymmetry, technicolor, anomalous $WW\gamma$ or WWZ cou-

[18]Modulo small b-mass-dependent corrections.

[19]Again, the errors in these quantities are highly correlated.

[20]The errors in this quantity, determined essentially by those on A_b, are skewed and non-Gaussian. The average error is quoted in Tables XVI and XVII. The confidence level of the deviation of $\epsilon_b(\overline{r_b})$ from the SM, assuming Gaussian errors for A_b, is in fact almost the same as that of the latter, about one per mille.

TABLE XVI. Measured values of the epsilon parameters of Refs. [30–32] compared to SM predictions. Dev(σ)=(expt.−SM)/error.

	ϵ_1	ϵ_2	ϵ_3	$\epsilon_b(\overline{s_b})$	$\epsilon_b(\overline{a_b})$	$\epsilon_b(\overline{r_b})$
Expt.	0.00404(133)	−0.0073(8)	0.0031(8)	−0.0017(18)	0.048(15)	−0.263(57)
SM m_t=172 GeV						
m_H=149 GeV	0.00497	−0.0076	0.0051	−0.0045	−0.0060	−0.0068
Dev(σ)	−0.7	0.38	−2.5	1.6	3.6	−4.5
SM m_t=180 GeV						
m_H=100 GeV	0.00563	−0.0062	0.0045	−0.0046	−0.0068	−0.0055
Dev(σ)	−1.2	−1.4	−1.8	1.6	3.7	−4.5

plings, and new U(1) gauge bosons are all expected, via vacuum polarization effects in the gauge boson propagators, to produce deviations from the SM predictions for ϵ_1, ϵ_2, or ϵ_3 [30,31,32]. These parameters are much more sensitive to any flavor-independent modifications of the couplings, due to anomalous vacuum polarization effects, because of the high precision of the purely leptonic measurements. Although the ϵ_3 parameter shows a quite large deviation from the SM prediction in the case that the τ-polarization measurements are excluded, the most important apparently anomalous effect occurs in the quantum corrections to the b quark couplings that disagree, by an order of magnitude, with the expectations of the SM.

A clue as to the origin of the anomalous b quark couplings is provided by considering the right- and left-handed effective couplings $\overline{g_b^R}$, $\overline{g_b^L}$ related to $\overline{a_b}$ and $\overline{v_b}$ by the relations

$$\overline{g_b^R}=\frac{1}{2}(\overline{v_b}-\overline{a_b})=-\sqrt{\rho_b}e_b(\overline{s_W^b})^2, \tag{4.17}$$

$$\overline{g_b^L}=\frac{1}{2}(\overline{v_b}+\overline{a_b})=\sqrt{\rho_b}[T_3^b-e_b(\overline{s_W^b})^2]. \tag{4.18}$$

From the measured values of $\overline{a_b}$ and $\overline{v_b}$ presented in Table VII, the following values of the left- and right-handed effective couplings of the b quarks are found:

$$\overline{g_b^L}=-0.4155(30), \quad \overline{g_b^R}=0.1098(101),$$

which may be compared with the SM predictions of:

$$\overline{g_b^L}=-0.4208, \quad \overline{g_b^R}=0.0774.$$

The value of $\overline{g_b^L}$ is quite consistent with the SM prediction (for m_t=172 GeV, m_H=149 GeV) (a 1.3%, 1.8σ deviation), whereas the discrepancy for $\overline{g_b^R}$ is much larger (a 42%, 3.2σ deviation). Excluding the τ-polarization measurements gives the results

$$\overline{g_b^L}=-0.4138(29), \quad \overline{g_b^R}=0.1160(90).$$

The deviations from the SM prediction are 1.7% and 2.4σ for $\overline{g_b^L}$ and 50% and 4.3σ for $\overline{g_b^R}$. One may remark that the weak isospin of the SM affects only $\overline{g_b^L}$, not $\overline{g_b^R}$, and so it is possible that the SM does correctly describe $\overline{g_b^L}$, but that there is a new, anomalous, right-handed coupling for the b quark.

The right- and left-handed effective couplings of the s, d quarks have recently been measured by the OPAL Collaboration [34] with the results

$$\overline{g_{d,s}^L}=-0.44^{+0.13}_{-0.09}, \quad \overline{g_{d,s}^R}=0.13^{+0.15}_{-0.17},$$

to be compared with the SM predictions −0.424 and 0.077, respectively. These measurements are in good agreement with both the SM predictions and the measured b quark couplings given above.

Limits can also be set on possible anomalous couplings of the other ''d-type'' quarks, d, s, by comparing the measured values of $\langle A_{FB}^q\rangle$ and Γ_{had} with the predictions of a model in which the d and s quarks are assumed to have the same effective coupling constants as those measured for the b quarks. The prediction of this model for $\langle A_{FB}^q\rangle$ is 0.1600(72),

TABLE XVII. Measured values of the epsilon parameters of Refs. [30–32] compared to SM predictions. τ-polarization measurements are excluded from the A_l average. Dev(σ)=(expt.−SM)/error.

	ϵ_1	ϵ_2	ϵ_3	$\epsilon_b(\overline{s_b})$	$\epsilon_b(\overline{a_b})$	$\epsilon_b(\overline{r_b})$
Expt.	0.00372(133)	−0.0068(8)	0.0021(8)	−0.0018(18)	0.058(14)	−0.298(47)
SM m_t=172 GeV						
m_H=149 GeV	0.00497	−0.0076	0.0051	−0.0045	−0.0060	−0.0068
Dev(σ)	−0.94	1.0	−3.8	1.5	4.6	−6.2
SM m_t=180 GeV						
m_H=100 GeV	0.00563	−0.0062	0.0045	−0.0046	−0.0068	−0.0055
Dev(σ)	−1.4	−0.75	−3.0	1.6	4.6	−6.2

which is consistent with the ''measured'' value (see Sec. II above and Table VIII) of 0.1592(86) at the 0.68σ level. No useful constraint on possible anomalous couplings of the d and s quarks, of a size similar to those observed for the b quark, is therefore obtained using the measured value of $\langle A_{\mathrm{FB}}^{q}\rangle$ with the present experimental errors. A more favorable case is Γ_{had}. Using the world average value of $\alpha_s(M_Z)$ of 0.118(5) [9,10] in Eq. (2.15) to calculate the QCD correction and with $C_q^{\mathrm{QED}}=1.00040$, the predicted value of Γ_{had} in the model with a universal right-handed anomaly for down-type quarks is 1.7249(46) GeV. This differs by 3.6σ from the LEP average measurement [1] of 1.7436(25) GeV. Thus the model is essentially excluded by the measurement of Γ_{had}. It is interesting to note that the precise measurement of Γ_{had} currently gives a much more stringent constraint on possible anomalous couplings of the d and s quarks than the direct measurement of their left- and right-handed couplings cited above [34].

The values of the left- and right-handed couplings of the c quarks derived from the measured values of $\overline{a_c}$ and $\overline{v_c}$ given in the first two rows of Table VII are

$$\overline{g_c^L}=0.3440(92),\quad \overline{g_c^R}=-0.1600(70),$$

in very good agreement with the SM predictions for $m_t=172$ GeV, $m_H=149$ GeV of

$$\overline{g_c^L}=0.3465,\quad \overline{g_c^R}=-0.1545.$$

The $\pm 2\sigma$ limits for deviations of $\overline{g_c^R}$ from the SM prediction extends from -0.174 to -0.146. Thus, at 95% C.L., any anomalous right-handed couplings of the c quark lie between -6% and $+13\%$ of the SM prediction.

V. SUMMARY AND OUTLOOK

The analysis presented in this paper has been performed in such a way as to separate, as far as possible, the actual results of experimental measurements, expressed in terms of the effective weak coupling constants of the leptons, c quarks, and b quarks, from the comparison of these results with the SM. The quantities $\overline{v_l}$, $\overline{a_l}$, and $\overline{r_Q}=\overline{v_Q}/\overline{a_Q}$ ($Q=c,b$) were extracted assuming only lepton universality. A further theoretical assumption is needed to extract $\overline{v_Q}$ and $\overline{a_Q}$ separately. The hypothesis of non-b-quark lepton universality was chosen. It was shown that the alternative of assuming $\alpha_s(M_Z)$ to be known[21] would give essentially the same results for the b quark couplings, if the world average value of $\alpha_s(M_Z)=0.118(5)$ [9,10] is used. Thus the actual non-b-quark couplings are consistent with the universality hypothesis.

The measured effective couplings of the leptons and c quarks are in good agreement with the SM predictions for a values of m_t and m_H well consistent with existing experimental limits. However, the b quark couplings deviate from the SM predictions by more than three standard deviations. Taking carefully into account all relevant error correlations, the probability that all six effective couplings are consistent with lepton universality and the SM is found to be 0.9%. This number is the product of the C.L. for consistency with lepton universality, given by the χ^2 of the different A_l when compared with their weighted average (8.4%) and the C.L. of the SM comparison using the average value from Table IX (10.5%). The probability that the leptonic and b quark couplings are consistent with lepton universality and the SM is similarly calculated to be 0.18%. The latter probability drops to only 0.018% if the τ-polarization measurements are excluded in calculating the average value of A_l.

In Sec. III the possibility is discussed that systematic effects, beyond those taken into account in the present analyses and correlated between different experiments, exist in the τ-polarization measurements.

In Sec. IV the measured effective couplings are analyzed in terms of the one-loop quantum correction parameters $\Delta\rho_f$ and $\Delta\kappa_f$. These quantities are found to be in agreement with the SM predictions for the leptons and c quarks, but to be an order of magnitude larger than these predictions for the b quarks. An analysis in terms of the ''epsilon'' parameters [30,31,32] ϵ_1, ϵ_2, ϵ_3, $\epsilon_b(\overline{s_b})$, $\epsilon_b(\overline{a_b})$, and $\epsilon_b(\overline{r_b})$ shows reasonable agreement with the SM for the first four parameters, but $\simeq(4\text{–}6)\sigma$ deviations for the last two. The agreement found between experiment and the SM for ϵ_1, ϵ_2, and ϵ_3 demonstrates that there is no experimental evidence for contributions from natural extensions of the SM such as supersymmetry, technicolor, anomalous $WW\gamma$ or WWZ couplings, or new U(1) gauge bosons, which are all expected to produce deviations from the SM model predictions for one or more of these parameters [31]. It may be remarked, however, that if the τ-polarization measurements are excluded, the ϵ_3 parameter differs from the SM prediction by three standard deviations, even for a Higgs boson mass ($m_H=100$ GeV) close to the current experimental lower limit [28] of about 70 GeV. This additional potential problem for the SM merits further investigation, but is beyond the scope of the present paper.

A simple picture emerges in terms of the right- and left-handed couplings of the b quark. While the latter agrees at the 2σ level or better with the SM prediction, the former shows a (42–50)% and $(3.2\text{–}4.3)\sigma$ deviation. The larger deviations occur when τ-polarization measurements are excluded. Thus the most significant deviation from the SM predictions of the effective couplings is found in the right-handed b quark weak coupling constant.

The large deviations from the SM found here for the b quark couplings confirm results presented in a previous paper [7]. The paper contains, however, neither any discussion of the overall statistical significance of the deviations nor any physical interpretation of them. Other recent reviews [33,35,36] did not extract the heavy quark couplings from the measurements. They discussed rather the high precision purely leptonic data and the quantities R_c and R_b all of which agree well with SM predictions. Of the three parameters $\epsilon_b(\overline{s_b})$, $\epsilon_b(\overline{a_b})$, and $\epsilon_b(\overline{r_b})$ defined in Ref. [32], only

[21]This was the procedure adopted in Ref. [7], where where heavy quark effective couplings consistent with those extracted in the present paper were found.

TABLE XVIII. A scenario for the ''final values'' and errors of A_l and $\overline{r_b}$ at the end of the LEP+SLD experimental program. $A_l^{\rm SLD}=0.1547(19)$ and $A_b^{\rm SLD}=0.897(27)$ are assumed, and $A_l^{\rm SLD}$ varies by $-2\sigma_0$ to $+2\sigma_0$ from the central value, where σ_0 is its error. For $\overline{r_b}$, the number in square brackets denotes (value−SM)/error, where $\overline{r_b}^{\rm SM}=0.689$.

	''Final values'' of A_l			''Final values'' of $\overline{r_b}$		
$A_l^{\rm SLD}\pm x,\ x=$	$-2\sigma_0$	0	$+2\sigma_0$	$-2\sigma_0$	0	$+2\sigma_0$
LEP+SLD	0.1501(16)	0.1525(16)	0.1553(16)	0.604(26)	0.592(25)	0.579(23)
Full average				[−3.3]	[−3.9]	[−4.8]
τ poln.	0.1513(17)	0.1545(17)	0.1577(17)	0.598(26)	0.583(24)	0.568(23)
Excluded				[−3.5]	[−4.4]	[−5.3]

$\epsilon_b(\overline{s_b})$, derived from R_b, was considered in Ref. [33]. As also only the ALEPH R_b measurement [37] (which is in almost perfect agreement with the SM) was used, the anomalies in the b quark couplings were undetected. As shown in Tables XVI and XVII it is the other two ϵ_b parameters that are most sensitive to the apparent deviations from the SM.

The most important remaining question is whether the observed deviation in the right-handed b quark coupling is likely to be confirmed or excluded by currently planned measurements of electroweak observables. The effect observed in the effective couplings, interpreted as a statistical fluctuation, is $\simeq(3-4)\sigma$. A similar sized deviation for R_b was reported in 1995 by the Electroweak Working Group [38]. A year later, as a result of a better systematic understanding of both the R_b and the correlated R_c measurements, as well as much improved statistical errors, the deviation of the average R_b measurement was reduced to below two standard deviations [1]. Can the presently observed anomaly in the b quark right-handed coupling be expected to meet the same fate as that of R_b? Arguments will now be given that this is unlikely, given the current status of the analysis of the LEP+SLD electroweak data. If there is some large, as yet unknown, systematic effect in the LEP $A_{\rm FB}^{0,b}$ and/or the SLD A_b measurements similar to those uncovered during 1996 in the R_b and R_c measurements, the anomaly might disappear (or become larger). This hypothetical question will not be further addressed here.

The discussion concentrates on the observed (and possible future) deviations from the SM of $\overline{r_b}$, which is extracted from measurements using only the weak theoretical assumption of lepton universality. As can be seen in Figs. 1c, 2c, this quantity shows much larger deviations from the SM predictions than $\overline{s_b}$. The measurements of several of the relevant electroweak observables $A_{\rm FB}^{0,l}$, $A_{\rm LR}$, $A_{\rm FB}^{0,b}$, and A_b, have recently been improved as compared to numbers quoted in Table I [39]. The updated value of $\overline{r_b}$ following the same analysis procedure as described in Sec. II above is 0.591(30). This is very consistent with the value reported in Table III. The discrepancy with the SM is slightly reduced from 3.34 to 3.26 standard deviations. The forseen improvements in the precision of measurements of electroweak observables have been discussed in detail in the review of Renton [7]. Only very modest improvements are to be expected from completing the analysis of the existing and final LEP1 data. Their impact on the $\overline{r_b}$ measurement is expected to be essentially negligible. As shown in Sec. II and III above, this is not true; however, of possible systematic effects, witness the sensitivity of the C.L.s in Table IX and the $\Delta\rho$, $\Delta\kappa$, and epsilon parameters in Tables XIV–XVII to the inclusion or exclusion of the τ-polarization data. A better systematic understanding of the latter could have a large effect on the LEP average value of A_l. Ultimately, however, the LEP+SLD average value of A_l is expected to be dominated by the improved precision of the ongoing $A_{\rm LR}$ measurement. According to Ref. [7], the errors on the existing SLD measurements of $A_{\rm LR}$ and A_b [39] should be reduced, by the end of the SLD experimental program, by 41% and 45%, respectively. The most significant improvement is expected to be in the accuracy of the LEP+SLD average value of A_l due to the $A_{\rm LR}$ measurement. The error of the latest measurement, $A_l^{\rm SLD}=0.1547(32)$ [39], is expected to be reduced to 0.0019. This is significantly smaller than the error, 0.0033, on the LEP average value of A_l, which will, in the absence of new, presently unknown, systematic corrections, change little in the future. The improvement on the most recent SLD measurement of A_b, $A_b^{\rm SLD}=0.897(47)$ [39], where the error is expected to be reduced to 0.0027, is also large. However, the LEP average value of A_b, derived from $A_{\rm FB}^{0,b}$ is already more precise: $A_b^{\rm LEP}=0.861(21)$. To study the possible impact of the new SLD measurements on the expected future value of $\overline{r_b}$ the value of $A_l^{\rm SLD}$ is allowed to vary by $\pm2\sigma_0$ (where $\sigma_0=0.0019$ is the final expected error) about the latest measured value. The weighted average of $A_l^{\rm SLD}$ and the latest LEP measurement are calculated (including, or not, the τ-polarization data) to estimate the likely range of the ''final'' LEP+SLD average value of A_l. For each A_l value, A_b is extracted from the LEP average $A_{\rm FB}^{0,b}$ using Eq. (2.3) to yield ''final'' values of $A_b^{\rm LEP}$. The weighted average of $A_b^{\rm LEP}$ and $A_b^{\rm SLD}$ is then made, assuming for the latter the latest value given above and the expected final error of 0.0027. In the last step, for each ''final'' A_b value $\overline{r_b}$ is extracted using Eq. (2.6) and compared with the SM prediction. The results of this exercise are presented in Table XVIII. This purely statistical study shows that the deviations in $\overline{r_b}$ are unlikely to drop below 3σ or to become more significant than 5 σ. Larger deviations are still seen when the τ-polarization data are excluded, but the effect is less than shown in Table III ($\simeq0.5\sigma$ instead of $\simeq1\sigma$) due to the much higher statistical weight expected from the final SLD A_l measurement. Unlike in the case of the R_b anomaly in 1995, most of the foreseeable LEP+SLD data contributing to the measurement of $\overline{r_b}$ have already been analyzed. The predictions presented in Table XVIII show that only small changes are to be expected in the statistical significance of the observed anomaly in the b quark couplings.

The most important message of this paper for future electroweak analyses is that the step of extracting the important physical quantities from the experimental measurements should be clearly separated from the comparison of these quantities with theoretical predictions. This was done routinely in the past for the leptonic weak coupling constants [1]. A similar procedure should be followed, in the future, also for the heavy quark couplings. Only in this way can the physical origins of apparent deviations from theory be precisely located, and the C.L.s for agreement of the measurements with the theory be easily and correctly calculated. The second point (exemplified here by the discussion of the LEP τ-polarization measurements) is to check carefully the internal consistency of different experimental measurements of the same physical quantity before they are averaged. This can give indications of hitherto unconsidered systematic effects. In any case, the C.L. for the internal consistency of different measurements used to calculate an average physical quantity should be multiplied by that given by the χ^2 of the theory-experiment comparison based on the averaged quantities, in order to give a more meaningful overall probability for the theory-experiment comparison.

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[1] The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group, and the SLD Heavy Flavour Group, Report No. CERN-PPE/96-183, 1996.

[2] S. L. Glashow, Nucl. Phys. **22**, 579 (1961); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Proceedings of the 8th Nobel Symposium*, edited by N. Svartholm (Almqvist & Wiksells, Stockholm, 1968), p. 367.

[3] Measurements of forward-backward asymmetries for light quarks have been made, but the current precision is rather poor. See DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **67**, 1 (1995); OPAL Collaboration, K. Ackerstaff *et al.*, *ibid.* **76**, 387 (1997).

[4] G. Rodrigo, Nucl. Phys. B (Proc. Suppl.) **54A**, 60 (1997).

[5] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[6] K. G. Chetyrkin, J. H. Kühn, and A. Kwiatowski, in ''Report of the Working Group on Precision Calculations for the *Z*-Resonance,'' edited by D. Bardin, W. Hollik, and G. Passarino, Report No. CERN 95-03, 1995, p. 175.

[7] P. B. Renton, Int. J. Mod. Phys. A **12**, 4109 (1997).

[8] T. Hebbeker *et al.*, Phys. Lett. B **331**, 165 (1994).

[9] M. Schmelling, in Proceedings of the 28th International Conference on High Energy Physics, Warsaw, 1996, edited by Z. Ajduk and A. K. Wroblewski (unpublished).

[10] P. N. Burrows, presented at the 3rd International Symposium on Radiative Corrections, Cracow, Poland, 1996, Report No. SLAC-PUB-7293, MIT-LNS-96-214, hep-ex/9612007.

[11] ALEPH Collaboration, D. Buskulic *et al.*, Phys. Lett. B **335**, 99 (1994); ALEPH Collaboration ''An upgraded measurement of $A_{FB}^{b\bar{b}}$ from the charge asymmetry in lifetime tagged *Z*-decays,'' presented at ICHEP96 Warsaw, 1996, Report No. PA10-018.

[12] See, for example, R. J. Barlow, *Statistics* (Wiley, New York, 1989), p. 156.

[13] W. Beenakker *et al.*, Phys. Lett. B **251**, 299 (1990).

[14] S. Jadach and Z. Wąs, Phys. Lett. B **219**, 103 (1989).

[15] DELPHI Collaboration, P. Abreu *et al.*, Report No. CERN-PPE/94-44.

[16] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C **69**, 183 (1996).

[17] DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **67**, 183 (1995); ''An upgraded measurement of tau polarisation,'' Report No. DELPHI 96-114 CONF 42; presented at ICHEP96, Warsaw, 1996, Report No. PA7-008.

[18] L3 Collaboration, Phys. Lett. B **341**, 245 (1994); ''A Preliminary Update of A_τ and A_e Using 1994 Data,'' presented at ICHEP96, Warsaw, 1996, Report No. PA7-056.

[19] OPAL Collaboration, G. Alexander *et al.*, Z. Phys. C **72**, 365 (1996).

[20] S. Jadach and Z. Wąs, in ''Z Physics at LEP1,'' edited by G. Altarelli, Report No. CERN 89-08, p. 235.

[21] S. Jadach, B. F. L. Ward, and Z. Wąs, Comput. Phys. Commun. **66**, 276 (1991); **79**, 503 (1994).

[22] R. Decker and M. Finkemeier, Phys. Rev. D **48**, 4203 (1993).

[23] SLD Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **78**, 2075 (1997).

[24] SLD Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **70**, 2515 (1993).

[25] See Ref. [20], p. 39, 65. Notice the following misprints: In Eq. (35), p. 65, $3/s_W^2-2\rightarrow 3/s_W^2-4$ and an overall minus sign on the right side. Equation (36), p. 65, $10/3\rightarrow 11/3$.

[26] A. Sirlin, Phys. Rev. D **22**, 971 (1980), W. J. Marciano and A. Sirlin, *ibid.* **22**, 2695 (1980); **29**, 75 (1984).

[27] See Refs. [62–64] of Ref. [1].

[28] ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. B **412**, 173 (1997); DELPHI Collaboration, B. Abreu *et al.*, Eur. Phys. J. C **2**, 1 (1998). L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. B **411**, 373 (1997); OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. C **1**, 425 (1998).

[29] D. Bardin *et al.* FORTRAN package ZFITTER, Report No. CERN-TH 6443/92.

[30] G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991).

[31] G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. **B376**, 3 (1992).
[32] G. Altarelli, R. Barbieri, and F. Caravaglios, Nucl. Phys. **B405**, 3 (1993).
[33] G. Altarelli, ''Status of Precision Tests of the Standard Model,'' Report No. CERN-TH/91-265, 1996.
[34] See the second reference in [3].
[35] W. de Boer *et al.*, Z. Phys. C **75**, 627 (1997).
[36] K. Hagiwara, D. Haidt, and S. Matsumoto, Eur. Phys. J. C **2**, 95 (1998).
[37] ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. B **401**, 163 (1997).
[38] The LEP-SLD Electroweak Working Group (see Ref. [1]), Report No. CERN PPE/95-172, 1995.
[39] The LEP-SLD Electroweak Working Group (see Ref. [1]), Report No. LEPEWWG/97-01, 1997.