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Determination of the weak phase γ from color-allowed $B_u^{\pm}\to DK^{\pm}$ decays

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We show that it is possible to determine the weak phase $\gamma\equiv\arg(-V_{ub}^{*}V_{ud}/V_{cb}^{*}V_{cd})$ of the Cabibbo-Kobayashi-Maskawa flavor mixing matrix only from the measurement of the color-allowed $B_u^{\pm}\to DK^{\pm}$ decay rates. The uncertainty of this method, arising mainly from the factorization approximation for two tree-level spectator quark transitions, may be well controlled. [S0556-2821(98)04019-3]

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The first observation of the Cabibbo-suppressed but color-allowed transitions $B_u^+\to\bar{D}^0K^+$ and $B_u^-\to D^0K^-$, which involve the quark subprocesses $b\to c\bar{u}s$ and $\bar{b}\to\bar{c}u\bar{s}$, respectively, has been reported by the CLEO Collaboration [1]. These two decay modes, together with $B_u^+\to D^0K^+$, $B_u^+\to D_{1,2}K^+$ and their charge-conjugate counterparts,[1] can in principle be used to determine the weak phase $\gamma\equiv\arg(-V_{ub}^{*}V_{ud}/V_{cb}^{*}V_{cd})$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2,3].[2] In practice, however, such a method may have an essential problem, arising from the difficulty in measuring the color-suppressed transitions $B_u^+\to D^0K^+$ and $B_u^-\to\bar{D}^0K^-$. To get around this problem, Atwood, Dunietz and Soni have proposed the measurement of $B_u^+\to D^0K^+\to(K^-\pi^+)_{D^0}K^+$, $B_u^+\to\bar{D}^0K^+\to(K^-\pi^+)_{\bar{D}^0}K^+$, and their charge-conjugate channels for the extraction of γ [5]. Furthermore Soffer has shown how to remove some uncertainties from the methods mentioned above and increase their sensitivity by measuring CP-conserving phases at a τ-charm factory [6].

The interesting question remains of whether it is possible to constrain γ only from the observation of the color-allowed $B_u^{\pm}\to DK^{\pm}$ decay rates. Gronau has recently discussed this possibility by proposing a few promising measurables, such as the charge-averaged ratios for $B_u^{\pm}$ decays into D-meson CP and flavor states [7]:

$$R_i\equiv 2\frac{\Gamma(B_u^+\to D_iK^+)+\Gamma(B_u^-\to D_iK^-)}{\Gamma(B_u^+\to\bar{D}^0K^+)+\Gamma(B_u^-\to D^0K^-)} \qquad (1)$$

with $i=1$ or 2 [the factor 2 on the right-hand side of Eq. (1) is just taken to normalize R_i to a value close to 1]. It is easy to show that there exist two inequalities $\sin^2\gamma\leqslant R_{1,2}$, and one of the two ratios R_1 and R_2 must be smaller than one except that the value of γ happens to lie in a narrow band around $\pi/2$. Thus the measurement of $R_{1,2}$ may in most cases provide a useful constraint on γ.

The purpose of this paper is to show that γ can indeed be *determined* from R_1 and R_2 through the formula

$$\cos\gamma=\frac{\kappa}{2}\,\frac{R_1-R_2}{R_1+R_2-2}, \qquad (2)$$

where $\kappa\approx 0.077$ is a coefficient related to the ratio of the color-suppressed and color-allowed decay amplitudes

$$r\equiv\left|\frac{A(B_u^+\to D^0K^+)}{A(B_u^+\to\bar{D}^0K^+)}\right|=\left|\frac{A(B_u^-\to\bar{D}^0K^-)}{A(B_u^-\to D^0K^-)}\right|. \qquad (3)$$

The value of κ is obtained by means of the isospin symmetry and the factorization approximation. Since the factorization approximation made here involves only tree-level spectator quark diagrams, the uncertainty associated with κ may be well controlled. In fact, the reported branching ratio $\mathcal{B}(B_u^+\to\bar{D}^0K^+)=(0.257\pm 0.065\pm 0.032)\times 10^{-3}$ [1] is in good agreement with the prediction from the factorization scheme [8]. Therefore the measurement of the color-allowed $B_u^{\pm}\to DK^{\pm}$ decay rates should allow a determination of the weak phase γ from Eq. (2) to an acceptable degree of accuracy in the near future.

To derive the formula in Eq. (2) and calculate the parameter κ, we begin with the effective weak Hamiltonians responsible for $b\to c\bar{u}s$, $b\to u\bar{c}s$, and their charge-conjugate transitions

$$\mathcal{H}_{\rm eff}^{(1)}=\frac{G_F}{\sqrt{2}}V_{cb}V_{us}^{*}[c_1(\bar{s}u)_{V-A}(\bar{c}b)_{V-A}+c_2(\bar{c}u)_{V-A}(\bar{s}b)_{V-A}]+\text{H.c.},$$

$$\mathcal{H}_{\rm eff}^{(2)}=\frac{G_F}{\sqrt{2}}V_{ub}V_{cs}^{*}[c_1(\bar{s}c)_{V-A}(\bar{u}b)_{V-A}+c_2(\bar{u}c)_{V-A}(\bar{s}b)_{V-A}]+\text{H.c.}, \qquad (4)$$

where c_1 and c_2 are QCD correction coefficients. To calculate the hadronic matrix elements $\langle DK|\mathcal{H}_{\rm eff}|B\rangle$, one has to make some approximations. Here we use the factorization approximation, which factorizes each four-quark operator matrix element into a product of two current matrix elements. Wherever there is a color mismatch between the

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[1]where D_1 and D_2 denote the CP-even and CP-odd states of D^0 and $\bar{D}^0$ mesons, respectively.

[2]Possible new physics in D^0-$\bar{D}^0$ mixing could affect those methods proposed in Refs. [2,3]. See Ref. [4] for some detailed discussions.

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quark operator and the external state in this approach, a phenomenological parameter ξ is introduced and the factorized matrix element becomes related to one of the following two coefficients [9]:

$$a_1 \equiv c_1 + \xi c_2, \quad a_2 \equiv c_2 + \xi c_1. \tag{5}$$

The color-suppression factor ξ is naively expected to be 1/3, corresponding to exact vacuum saturation. Both a_1 and a_2 have been determined from experimental data [10]. The explicit knowledge of ξ, c_1 and c_2 will be irrelevant in our subsequent analysis.

When applying the effective Hamiltonians and the factorization approximation to $B_u^\pm \to DK^\pm$ decays, one has to take possible final-state interactions into account. Since both D and K are isospin 1/2 particles, the state DK can be either $I=0$ or $I=1$. An isospin analysis made by Deshpande and Dib [11] gives[3]

$$A(B_u^- \to \overline{D}^0 K^-) = \frac{G_F}{\sqrt{2}} (V_{ub} V_{cs}^*) \left(\frac{X}{2} e^{i\phi_0} + \frac{X}{2} e^{i\phi_1} \right),$$

$$A(B_u^- \to D^0 K^-) = \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) (X+Y) e^{i\phi_1}, \tag{6}$$

where ϕ_0 and ϕ_1 are the strong phase parameters, X and Y are the factorized hadronic matrix elements

$$X = a_2 \langle \overline{D}^0 | (\overline{u}c)_{V-A} | 0 \rangle \langle K^- | (\overline{s}b)_{V-A} | B_u^- \rangle,$$

$$Y = a_1 \langle K^- | (\overline{s}u)_{V-A} | 0 \rangle \langle D^0 | (\overline{c}b)_{V-A} | B_u^- \rangle. \tag{7}$$

Clearly the weak phase difference between $A(B_u^- \to \overline{D}^0 K^-)$ and $A(B_u^- \to D^0 K^-)$ amounts to $-\gamma$ to an excellent degree of accuracy in the standard model [4]. The strong phase difference between these two amplitudes, denoted by δ as the notation in Ref. [7], is equal to $(\phi_0 - \phi_1)/2$. By convention, we take $(\phi_1 - \phi_0) \in [-\pi, +\pi]$. Then $\cos\delta \geqslant 0$ holds.

In Ref. [7] the ratio r and the strong phase difference δ are formally taken as two independent parameters. In our factorization approach, however, r depends on δ through the relationship $r = \kappa \cos\delta$ with

$$\kappa = \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \left| \frac{X}{X+Y} \right|, \tag{8}$$

derived from Eqs. (3) and (6). The point is simply that the amplitude of $B_u^- \to \overline{D}^0 K^-$, after its CKM coefficient is factored out, contains two isospin components with equal magnitude but different phases. The result of $R_{1,2}$ obtained by Gronau [7] can be reproduced as follows:

$$R_{1,2} = 1 + r^2 \pm 2r \cos\delta \cos\gamma. \tag{9}$$

By use of $r = \kappa \cos\delta$, we arrive at

$$R_{1,2} = 1 + \kappa \cos^2\delta (\kappa \pm 2\cos\gamma). \tag{10}$$

It is easy to obtain two inequalities $\sin^2\gamma \leqslant R_{1,2}$ from either Eq. (9) or Eq. (10). Note that there exists a narrow band around $\gamma = \pi/2$, which makes both R_1 and R_2 equal to or larger than 1. With the help of Eq. (10), we find that the necessary condition for $R_{1,2} \geqslant 1$ is

$$-\frac{\kappa}{2} \leqslant \cos\gamma \leqslant +\frac{\kappa}{2}. \tag{11}$$

Beyond this band of γ, one of the two ratios R_1 and R_2 must be smaller than 1, thus one may get the constraint $\sin^2\gamma \leqslant R_1 < 1$ or $\sin^2\gamma \leqslant R_2 < 1$.

Note that the weak phase γ can indeed be determined from R_1 and R_2, provided the value of κ is known. From Eq. (10), we straightforwardly obtain

$$\cos\gamma = \frac{\kappa}{2} \frac{R_1 - R_2}{R_1 + R_2 - 2}. \tag{12}$$

This instructive result has been listed in Eq. (2). To evaluate κ, we express the hadronic matrix elements X and Y in terms of relevant decay constants and formfactors. The ratio X/Y turns out to be

$$\frac{X}{Y} = \frac{a_2}{a_1} \frac{m_B^2 - m_K^2}{m_B^2 - m_D^2} \frac{f_D}{f_K} \frac{F_0^{BK}(m_D^2)}{F_0^{BD}(m_K^2)}. \tag{13}$$

The main error bar of X/Y comes from the unknown decay constant f_D. Here we typically take $f_D = 220$ MeV. We also input $f_K = 160$ MeV, $a_2/a_1 = 0.25$ [10], $F_0^{BK}(0) = 0.38$, and $F_0^{BD}(0) = 0.69$ [9]. The uncertainties of the chosen values for $F_0^{BK}(0)$ and $F_0^{BD}(0)$ are expected to be within 15% [14]. By use of a simple monopole model for form factors [9], we obtain $F_0^{BK}(m_D^2) \approx 0.43$ and $F_0^{BD}(m_K^2) \approx 0.70$. Then we arrive at $X/Y \approx 0.24$. With the input $|(V_{ub}V_{cs}^*)/(V_{cb}V_{us}^*)| = 0.4$ [15], we finally get $\kappa \approx 0.077$. This implies that the value of $r = \kappa \cos\delta$ is smaller than the naive expectation $r \approx 0.1$, obtained in Ref. [7] with the assumption $\delta = 0$ and $X/(X+Y) \approx a_2/a_1$. Since the error associated with the CKM factor is only 25% or so [15] and those associated with the form factors can be partly cancelled in the ratio X/Y, it should be a conceivable argument that the realistic value of κ cannot be greater or smaller than our present result by a factor of 2. That is, $0.04 \leqslant \kappa \leqslant 0.15$ should be a sufficiently generous range of κ. Even if the uncertainty associated with the factorization approximation itself is taken into acount, the possibility of $\kappa \geqslant 0.2$ would remain extremely small.

A significant deviation of the strong phase difference δ from zero, implying significant rescattering effects of DK states, may reduce the magnitude of r further. Considering

[3]The factorized matrix element

$$\langle \overline{D}^0 K^- | (\overline{s}c)_{V-A} | 0 \rangle \langle 0 | (\overline{u}b)_{V-A} | B_u^- \rangle,$$

corresponding to an annihilation process, is expected to be form factor suppressed [12,13] and has been neglected here.

DK scattering via a t-channel exchange of Regge trajectories, Deshpande and Dib have made an estimation of the strong phase difference $(\phi_1-\phi_0)$ [11]. They obtained $\tan(\phi_1-\phi_0)\approx-0.14$, equivalent to $\delta\approx4°$. This value leads to $\cos\delta\approx1$ as an excellent approximation. Nevertheless, one should take such a result more qualitative rather than quantitative, as the method of Regge scattering itself involves large uncertainties. It has been argued that rescattering effects in decays of a B meson into lighter hadrons might not be as small as commonly imagined [16].

Taking $\kappa\approx0.077$, we obtain the narrow band of γ from Eq. (11): $87.8°\leqslant\gamma\leqslant92.2°$. In comparison, analyses of current data show that $\gamma\sim65°$ with a generous range $30°\leqslant\gamma\leqslant150°$ [17]. For illustration we plot R_1 and R_2 as functions of δ in Fig. 1, where three different values of γ have been typically taken. One can see that $R_1>1$ and $R_2<1$ for cases (a) and (b); but both $R_1>1$ and $R_2>1$ for case (c), as γ is within the narrow band mentioned above. It remains unclear that to what extent the constraint $\sin^2\gamma\leqslant R_1<1$ or $\sin^2\gamma\leqslant R_2<1$ will work.

The idea of extracting the weak phase γ from $R_{1,2}$ through Eq. (2) or Eq. (12), however, is valid for all allowed values of γ. The feasibility of this method depends on a reliable determination of the coefficient κ. This should be available in the near future, after f_D is measured from experiments (or calculated from lattice QCD) and the CKM factor $|V_{ub}/V_{cb}|$ is more accurately extracted from charmless B decays. At least, our present approach can be complementary to that proposed recently [7] and those suggested previously [2,3,5]. It may also confront the nearest data on $B_u^\pm\to DK^\pm$ and give a ballpark number to be expected for γ, before the delicate determination of γ becomes available in experiments.

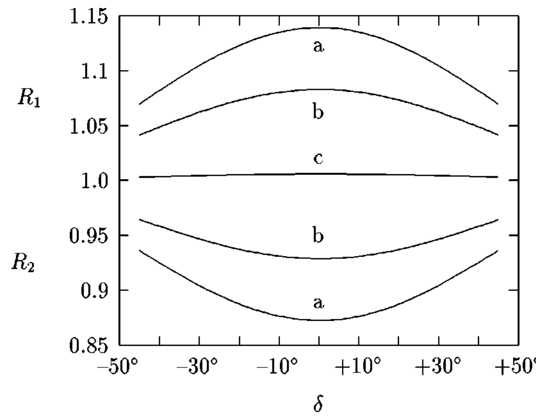

FIG. 1. Illustrative plot for $R_{1,2}$, where $\kappa=0.077$ and $\gamma=$(a) 30°, (b) 60°, and (c) 90°.

In conclusion, we have shown that it is possible to determine the weak angle γ only from the measurement of the color-allowed $B_u^\pm\to DK^\pm$ decay rates. Such measurements are expected to be carried out at both e^+e^- B-meson factories and high-luminosity hadron machines in the coming years.

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