

Anti-de Sitter-CFT correspondence in three-dimensional supergravity

Máximo Bañados,^{1,2} Karin Bautier,³ Olivier Coussaert,³ Marc Henneaux,^{2,3,4} and Miguel Ortiz⁵
¹*Departamento de Física Teórica, Universidad de Zaragoza, Ciudad Universitaria 50009, Zaragoza, Spain*

²*Centro de Estudios Científicos de Santiago, Casilla 16443, Santiago 9, Chile*

³*Physique Théorique et Mathématique, Université Libre de Bruxelles, Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium*

⁴*Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24, rue Lhomond, 75230 Paris Cedex 05, France*

⁵*Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom*

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Anti-de Sitter supergravity models are considered in three dimensions. Precise asymptotic conditions involving a chiral projection are given on the Rarita-Schwinger fields. Together with the known boundary conditions on the bosonic fields, these ensure that the asymptotic symmetry algebra is the superconformal algebra. The classical central charge is computed and found to be equal to the one of pure gravity. It is also indicated that the asymptotic degrees of freedom are described by 2D “induced supergravity” and that the boundary conditions “transmute” the non-vanishing components of the WZW supercurrent into the supercharges. [S0556-2821(98)01020-0]

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I. INTRODUCTION

It has been pointed out in [1] that the asymptotic symmetry group of anti-de Sitter gravity in three dimensions is the conformal group in two dimensions with a central charge $c = 3l/2G$. The emergence of the conformal group at infinity can be understood either in terms of the Penrose conformal treatment of infinity [2] or by working out explicitly the boundary conditions and solving the asymptotic Killing equations [1]. It is a purely asymptotic phenomenon, in the sense that the infinite-dimensional conformal group in two dimensions is not the isometry group of any 3D background geometry. This is one feature that makes the three-dimensional case particularly interesting and which actually allows for a nontrivial central charge in the dynamical realization of the asymptotic symmetry algebra [1].

Another interesting feature of three-dimensional gravitational theories is that they have no bulk degrees of freedom, so that the role of the boundary degrees of freedom in the adS-conformal-field-theory (CFT) correspondence [3–7] can be investigated more easily. That the boundary degrees of freedom may be quite significant has been stressed recently in [8]. The pure gravitational case has been analyzed in [9], where it was shown that the boundary dynamics at infinity is described by “induced 2D gravity” (Liouville theory) up to terms involving the zero modes and the holonomies that were not worked out.

The purpose of this paper is to extend the analysis of [1,9] to the supersymmetric context, which is known to play a central role in black hole physics. The new non-trivial ingredient to be fed in is the precise asymptotic behavior of the Rarita-Schwinger fields, which must be compatible with the symmetries. In particular, one must understand how the boundary conditions implement two-dimensional supersymmetry at infinity.

The supersymmetry properties of 3D black holes were investigated in [10], assuming the existence of asymptotic conditions on the Rarita-Schwinger fields satisfying the required properties. However, the asymptotic conditions in

question were not given. The main object of our paper is to fill this gap, which appears necessary since otherwise, the discussion of the asymptotic dynamics remains rather formal. We also verify that the given fall-off conditions reduce the theory to induced 2D supergravity. The symmetry algebra is the super-conformal algebra with unchanged central charge $c = 3l/2G$. The boundary conditions involve a chiral projection, in a way very similar to what has been discussed for Dirac fields in [11].

II. BOUNDARY CONDITIONS

AdS supergravity in three dimensional spacetime can be written as a Chern-Simons theory [12]. We shall adopt the Chern-Simons point of view from the outset and consider almost exclusively the (1,1) theory. The relevant group is $OSp(1|2) \times OSp(1|2)$. The (1,1)-supergravity action is

$$I[A, \psi; \tilde{A}, \tilde{\psi}] = I[A, \psi] - I[\tilde{A}, \tilde{\psi}] \quad (1)$$

where $I[A, \psi]$ and $I[\tilde{A}, \tilde{\psi}]$ are the Chern-Simons actions for the supergroup $OSp(1|2)$,

$$I[A, \psi] = \frac{k}{4\pi} \int \left[\text{Tr} \left(A dA + \frac{2}{3} A^3 \right) + i \bar{\psi} \wedge D\psi \right] \quad (2)$$

(with a similar expression for $I[\tilde{A}, \tilde{\psi}]$). Here $A = (1/2)A^a \gamma_a$ and the symbol Tr stands for the trace in the spinorial representation of $SO(2,1)$ generated by $J_a = (1/2)\gamma_a$ (our conventions are summarized in [13]). The constant k is related to the 3D Newton constant G and the anti-de Sitter radius l through $k = l/4G$.

Assuming that the topology of the three dimensional manifold M is $\Sigma \times \mathfrak{R}$, the action (2) can be recast in Hamiltonian form as

$$I = \int \left[-\frac{k \epsilon^{ij}}{4\pi} \left(\frac{\eta_{ab}}{2} A_i^a \dot{A}_j^b + i \bar{\psi}_i \dot{\psi}_j \right) - A_0^a \mathcal{G}_a - \bar{\psi}_0 \mathcal{S} \right] \quad (3)$$

where the constraints \mathcal{G}_a and \mathcal{S} are given by $\mathcal{G}_a \equiv -(k\epsilon^{ij}/8\pi)\eta_{ab}(F_{ij}^b - i\bar{\psi}_i\gamma^b\psi_j) = 0$, $\mathcal{S} \equiv -(ik/2\pi)\epsilon^{ij}D_i\psi_j = 0$ and satisfy the $OSp(1|2)$ algebra in the Poisson brackets $[A_i^a, A_j^b] = (4\pi/k)\eta^{ab}\epsilon_{ij}$, $\{\psi_i^\alpha, \psi_j^\beta\} = (2\pi i/k)\epsilon_{ij}(\gamma_0)^{\alpha\beta}$ which are derived from Eq. (3). The canonical generator of the gauge transformations (including the fermionic ones) is $G(\lambda^a) + S(\rho)$ with

$$G(\lambda^a) = \int_{\Sigma} \lambda^a \mathcal{G}_a + B, S(\rho) = \int_{\Sigma} \bar{\rho} \mathcal{S} + F. \quad (4)$$

The boundary terms B and F must be chosen so that the generators G and S have well-defined functional derivatives [14], and their precise form depends on the boundary conditions.

The boundary conditions at infinity on the bosonic fields have been given in [1] in the metric representation and they were reexpressed in the connection representation in [9]. (See [15,16] for a different approach to the problem of boundary conditions in the connection representation.) The bosonic boundary conditions must be supplemented by appropriate boundary conditions on the fermionic fields. We consider only one $OSp(1|2)$ copy, the other copy being treated similarly.

The searched-for boundary conditions can be determined by following the procedure of [17]: one starts with the known physical metrics that should be included in the theory—here, the black hole solutions [18]—and acts on them with the anti-de Sitter supergroup. This suggests to adopt for the Rarita-Schwinger fields the following boundary conditions (in the standard orthonormal frames):

$$\psi_t \sim r^{-1/2} [1 + \gamma_1] \chi(t, \phi), \quad (5)$$

$$\psi_\phi \sim r^{-1/2} [1 + \gamma_1] \chi(t, \phi), \quad (6)$$

$$\psi_r \sim r^{-5/2} [1 - \gamma_1] \chi_r(t, \phi). \quad (7)$$

Apart from an irrelevant replacement of γ_1 by $-\gamma_1$ (due to conventions), the boundary conditions differ from those of [17] [Eq. (V.I)] in two respects. First, they involve a slower rate of decrease at infinity (one less power of r). This was to be expected since we are one dimension lower and also holds for the bosonic fields [1]. Second, they have the same leading order for both ψ_t and ψ_ϕ . The equality of the leading orders of ψ_t and ψ_ϕ is consistent with the fact that the adS Killing spinors of (1,0) supergravity depend only on $t + \phi$ [10]. The boundary conditions are otherwise identical and in particular, they crucially involve a projection onto the eigenspaces of the radial γ -matrix, which makes the induced spinors chiral in two dimensions (recall that γ_1 appears as the “ γ_5 ”-matrix on the surface at infinity). A similar phenomenon is described in [11].

III. ASYMPTOTIC SYMMETRIES

The algebra of coordinate and supersymmetry transformations that preserves the boundary conditions (“asymptotic symmetry algebra”) is the infinite-dimensional super-

Virasoro algebra. To discuss this issue, it is most convenient to work in the superconnection representation where the transformations take a simpler form.

Combining the above boundary conditions on the fermions with those of [1,9] on the bosons, one finds that the superconnection must satisfy

$$A_v = 0, \quad \psi_v = 0, \quad (8)$$

$$A_r = b^{-1} \partial_r b, \quad \psi_r = 0 \quad (9)$$

and

$$lA_u = b^{-1} \begin{pmatrix} 0 & L/k \\ 1 & 0 \end{pmatrix} b, \quad l\psi_u = b^{-1} \begin{pmatrix} Q/k \\ 0 \end{pmatrix} \quad (10)$$

asymptotically. Here, $u = t + l\phi$, $v = t - l\phi$ and $L = L(t, \phi)$ and $Q = Q(t, \phi)$ are arbitrary functions which will be shown to be equal to the generators of the super-*Virasoro* algebra. The group element $b(r)$ is equal to

$$b(r) = \begin{pmatrix} (r/l)^{1/2} & 0 \\ 0 & (r/l)^{-1/2} \end{pmatrix} \quad (11)$$

and satisfies $b\gamma_0 b = \gamma_0$. Note that $A_\phi = lA_u$, $\psi_\phi = l\psi_u$ since $A_v = 0$, $\psi_v = 0$. The other $OSp(1|2)$ field satisfies analogous boundary conditions, with u and v interchanged, and depends on two additional functions \tilde{L} and \tilde{Q} . For positive values of L_0 and \tilde{L}_0 the boundary conditions (10) represent a black hole. The black hole ground state ($M=0$) is obtained for $L = \tilde{L} = 0$. Anti-de Sitter space corresponds to $L/k = \tilde{L}/k = -1/4$ and is the only configuration for which the holonomies are trivial. [Note, however, that because A is written in the spinorial representation, the holonomy (in polar coordinates) is only trivial under a 4π rotation.]

The fact that the second component of ψ_u is zero just expresses the chirality condition on the fermion enforced by the boundary conditions (5) and (6). One may rewrite the asymptotic form (10) of the superconnection in terms of supermatrices as

$$\Gamma_u = b^{-1} \begin{pmatrix} 0 & L/k & Q/(\sqrt{2}k) \\ 1 & 0 & 0 \\ 0 & Q/(\sqrt{2}k) & 0 \end{pmatrix} b \quad (12)$$

where b is now the 3×3 supermatrix obtained by completing the above b by adding 0 in the fermionic positions and 1 in position (3,3). The advantage of the connection representation is that one can completely eliminate the r -dependence through the gauge transformation generated by b . After this gauge transformation has been performed, all the asymptotically relevant components of the fields occur at the same order $O(1)$. Furthermore, because r has dropped out, the analysis could be carried out in the same way at any finite value of r . A Virasoro algebra for all values of r has been investigated in [16].

The most general supergauge transformations that preserve the boundary conditions (8), (9) and (10) are characterized by gauge parameters (λ^a, ρ) that must satisfy, to leading order,

$$\lambda(u, r) = b^{-1} \eta(u) b, \quad \rho(r, u) = b^{-1} \varepsilon(u) \quad (13)$$

with

$$\eta^+ = \frac{\eta^- L}{k} - (1/2)(\eta^-)'' + \frac{iQ\varepsilon}{2k}, \quad (14)$$

$$\eta^1 = -(\eta^-)' \quad (15)$$

$$\varepsilon = \begin{pmatrix} -\varepsilon' + \eta^- Q/k \\ \varepsilon \end{pmatrix} \quad (16)$$

where ' denotes derivative with respect to the argument. We have expanded the algebra element η in the Cartan basis $\eta = \eta^1 J_1 + \eta^+ J_+ + \eta^- J_-$. Equations (14)–(16) imply that the full residual symmetry can be expressed in terms of two functions of the lightlike coordinate u , one bosonic (η^-) and one fermionic (ε).

Any three-dimensional gauge transformation whose parameters satisfy Eqs. (14)–(16) asymptotically is called ‘‘an asymptotic symmetry.’’ Two gauge transformations that tend to the same η^- and ε at infinity should be identified because they differ by a ‘‘proper gauge transformation,’’ which it is legitimate to quotient out [14,19] (as will be clear below, these transformations have in particular the same global charges). The resulting quotient superalgebra is the ‘‘asymptotic symmetry superalgebra.’’

If one computes the graded commutator of two Chern-Simons gauge transformations fulfilling the above asymptotic conditions and characterized by asymptotic parameters $(\eta_1^-, \varepsilon_1)$ and $(\eta_2^-, \varepsilon_2)$, one finds another such transformation with asymptotic gauge parameters related to $(\eta_1^-, \varepsilon_1)$ and $(\eta_2^-, \varepsilon_2)$ *exactly according to the graded commutation rules of the super-*Virasoro algebra**. Hence, after the quotient by the ideal of the proper gauge transformations (with $\eta^- = 0, \varepsilon = 0$) is taken, one is left with the super-*Virasoro algebra* as asymptotic symmetry superalgebra. This infinite-dimensional algebra contains *OSp(1|2)* as a subalgebra when the fermions are anti-periodic (Fourier modes 0 and ± 1 of η^- and modes $\pm \frac{1}{2}$ of ε). Note, in particular, that the Lie algebra commutator $[\lambda_1^a, \lambda_2^b]$ of two bosonic gauge transformations restricted by (13) and (14) reduces at infinity to the Lie bracket of the residual functions η_1^- and η_2^- viewed as vector fields on the circle. A similar statement holds for the fermionic sector.

IV. ADS CENTRAL CHARGE

We now turn to the discussion of the canonical realization of the asymptotic symmetry algebra. As is known, the bracket of the canonical generators of the asymptotic symmetries provides a projective representation of the algebra [1,20]. To determine the central charges, one must first work out the complete form of the generators (4). This is now

possible since the asymptotic form of both the fields and the symmetry transformations has been obtained. One finds that with the above asymptotic conditions, the boundary terms in the variation of the generators (4) cancel out if one takes

$$B = \frac{1}{2\pi} \int_{\partial\Sigma} \eta^- L, \quad F = \frac{-i}{2\pi} \int_{\partial\Sigma} \varepsilon Q \quad (17)$$

i.e., the surface terms are precisely L and Q (up to numerical factors). We have adjusted the constants in the charges so that these vanish for the zero mass black hole, which has $L = 0$. The surface terms (17) are of course equal to the surface terms that one would obtain through a more orthodox ‘‘non-Chern-Simons-based’’ approach (see [17] for the four-dimensional treatment). In particular, the bosonic piece B is equal to the charge (4.11) of [1] written in terms of the metric, while the fermionic surface term may be re-expressed as

$$F = \frac{ik}{2\pi} \int_{\partial\Sigma} \bar{\rho} \psi_\phi. \quad (18)$$

Because the components L and Q of the connection that remain at infinity enter the surface terms in the canonical generators of the asymptotic symmetries, it is useful to know how they transform under the asymptotic symmetry group. The transformation law for the superconnection $\delta\Gamma = D\Lambda$ yields

$$\delta L = (\eta^- L)' + (\eta^-)' L - \frac{k}{2} (\eta^-)''' + \left(\frac{iQ\varepsilon}{2} \right)' + iQ\varepsilon' \quad (19)$$

$$\delta Q = -k\varepsilon'' + L\varepsilon + (\eta^- Q)' + \frac{1}{2} (\eta^-)' Q. \quad (20)$$

Equations (19) and (20) indicate that L and Q form a super-*Virasoro algebra*. More importantly, the transformation laws (19) and (20) give also the central charge c , equal to $6k(c/12 = k/2)$.

By using the general argument of [20,1], or by direct calculation, one finds that the Poisson brackets of the improved generators (4) are

$$[G(\lambda_1), G(\lambda_2)] = G([\lambda_1, \lambda_2]) - \frac{k}{4\pi} \int_{\partial\Sigma} \eta_1^- (\eta_2^-)''' \quad (21)$$

$$[G(\lambda), S(\rho)] = S(\lambda^a \gamma_{a\rho}) \quad (22)$$

$$\{S(\rho_1), S(\rho_2)\} = G(-i\bar{\rho}_1 \gamma^a \rho_2) + \frac{ik}{2\pi} \int_{\partial\Sigma} \varepsilon_1 (\varepsilon_2)'''. \quad (23)$$

The central term is just that of Eqs. (19), (20). The Chern-Simons formulation of adS supergravity provides a particularly efficient derivation of the adS central charge.

The above algebra involves both the proper gauge symmetries and the improper ones [14,19]. The proper gauge

symmetries have (weakly) vanishing Poisson brackets with all the other generators, i.e., form also an ideal in the Poisson sense. It follows that the generators of the asymptotic symmetries are “first class” and well defined in the reduced phase space obtained by quotientizing the proper gauge symmetries. Using standard terminology, they are “observables.” The Poisson bracket (in the reduced phase space) of these observables is the same as their Poisson bracket in the original phase space (see e.g. [21]). That the global charges at the boundary define observables has been particularly emphasized recently in [22]. The super-Virasoro algebra is therefore realized in the space of physical observables, where it is generated by L and Q (the constraints are zero in the reduced phase space). After Fourier transformation, the asymptotic superalgebra takes the familiar form (using quantum-mechanical notation and rescaling Q by $\sqrt{2}$)

$$[L_m, L_n] = (n - m)L_{n+m} + \frac{k}{2}n^3 \delta_{n+m,0} \quad (24)$$

$$[L_m, Q_n] = \left(\frac{1}{2}m - n\right) Q_{m+n} \quad (25)$$

$$\{Q_m, Q_n\} = 2L_{m+n} + 2km^2 \delta_{m+n,0} \quad (26)$$

with a central charge equal to $c = 6k$. A practical way to factor out the proper gauge symmetries is to fix the gauge in the bulk and use Dirac brackets [19]. In that case, the reduced brackets in the above algebra would appear as Dirac brackets.

Note that if we had imposed only the chirality boundary condition (8), as is usually done in the Chern-Simons \rightarrow chiral Wess-Zumino-Witten (WZW) reduction, we would have obtained a current (Kac-Moody) algebra rather than the super-Virasoro algebra. The key point leading to the super-Virasoro algebra is the presence of the extra boundary conditions (10) which “transmute” the residual gauge field components functions L and Q into super-Virasoro charges [23–27]. These extra boundary conditions express (with the other boundary conditions given above) adS asymptotics.

From the point of view of the chiral WZW theory, this transmutation can be seen explicitly as follows. Let affine $SL(2, \mathfrak{R})_k$ be generated by J^\pm, J^1 . Impose $J^- = 1$ and $J^1 = 0$ [see Eq. (10)]. These are second class constraints because their Poisson bracket is an invertible matrix. It follows that $J^+ = L$ satisfies, in the Dirac bracket, the Virasoro algebra with $c = 6k$. This argument is extended directly to the supergravity theory and will be given a dynamical interpretation in the next section.

In Eqs. (24)–(26), the fermions can be periodic (index on Q integer-moded) or anti-periodic (index on Q half-integer moded). The form of the algebra is adapted to the periodic (“Ramond”) case, which has the zero mass black hole as the $L_0 = 0$ ground state [10]. The central charge vanishes for the sub-algebra generated by (L_0, Q_0) , which corresponds to the true symmetries of this background. The anti-de Sitter background has $M = -1$, i.e. $L_0 = -c/24$. It is the ground state of the anti-periodic (“Neveu-Schwarz”) sector [10]. If one shifts L_0 by $c/24$ so that L_0 vanishes on the anti-de Sitter

solution, one finds that the central charge vanishes for $(L_{\pm 1}, L_0, Q_{\pm 1/2})$, which are true $[OSp(1|2)]$ symmetries of the anti-de Sitter background).

What we have done for one $OSp(1|2)$ factor can be repeated for the other $OSp(1|2)$ factor. The corresponding spinor fields are projected on the other chirality and one finds another copy of the super-Virasoro algebra, this time depending on $v = t - l\phi$, with same central charge. The two super-Virasoro algebras give the conformal superalgebra.

V. DYNAMICS AT INFINITY

As anticipated above, the emergence of the super-conformal algebra at infinity with a non-vanishing central charge can be understood at the dynamical level, in the light of Polyakov’s discovery of the “hidden $SL(2, \mathfrak{R})$ symmetry” of 2D gravity [23–27]. The argument runs as follows [9]. As shown by [3,4], the Chern-Simons theory under the boundary condition (8) induces the chiral Wess-Zumino-Witten model at the boundary. The corresponding Kac-Moody currents are just the ϕ -components of the connection. Combining the two chiral WZW models of opposite chiralities obtained from each $SL(2, \mathfrak{R})$ -factor, one finds a non-chiral $SL(2, R)$ WZW theory (modulo zero modes and holonomies not discussed here because they affect neither the asymptotic symmetry nor the central charge). The constraints on the Kac-Moody currents arising from the anti-de Sitter asymptotics lead then to 2D-gravity [9].

In a similar way, the further constraint that the component of the Kac-Moody current along the fermionic generator f vanishes [see Eq. (10)] turns out to be precisely the constraint that reduces the WZW theory based on the supergroup $OSp(1|2)$ to chiral 2D supergravity [24,28–30]. Although the $OSp(1|2)$ -WZW theory is not superconformal, the resulting theory is. What happens is that the other component (along e) of the fermionic Kac-Moody supercurrent is “transmuted” into the super-Virasoro generator since its transformation law becomes Eq. (20) once the gauge parameters are restricted by Eqs. (14) and (16). From the WZW point of view, supersymmetry on the worldsheet arises therefore in a non-trivial way. It is rather interesting that these features are in fact all contained in the 3D boundary conditions expressing anti-de Sitter asymptotics, thus the adS/CFT correspondence is explicit in this context. Bringing in the other $OSp(1|2)$ factor leads to the non-chiral (1,1)-supergravity, which is described, in the 2D super-conformal gauge, by super-Liouville theory. We have explicitly checked, using the Gauss decomposition for $OSp(1|2)$ and following the same lines as in [9], that the 3D supergravity action (1) yields the super-Liouville action (up to zero modes and holonomies that we have not explicitly worked out).

VI. CONCLUSIONS

We have shown in this paper that the anti-de Sitter boundary conditions in (1,1) 3D-supergravity theory lead to an asymptotic symmetry algebra which is (twice) the super-Virasoro algebra with a central charge equal to $6k$. The precise boundary conditions given here on the spinors involve a

chiral projection and legitimate the assumptions of [10].

The appearance of the Virasoro algebra as the boundary symmetry algebra of anti-de Sitter space is purely kinematical in the sense that the only ingredients that enter the derivation of both the symmetry algebra and the central charge in Eq. (24) are (i) the asymptotic boundary conditions that dictate the approach to anti-de Sitter; and (ii) the fact that the surface terms at infinity in the Virasoro generators involve only the (bosonic) gravitational variables, i.e., the triads and the spin connection. Any theory with these features will have the same central charge in the commutator involving two L_n 's. In particular the extended (p,q) -supergravity models fulfill these properties and have as asymptotic symmetry algebra the relevant graded extension of the conformal group with Virasoro algebra satisfying Eq. (24) with same $c=3l/2G$. The triads, spin connection and spinor fields obey the same asymptotic conditions as above, while the $SO(N)$ -connection A_μ^{ij} fulfills $A_\nu^{ij}=0=A_r^{ij}$, $A_\mu^{ij}=T^{ij}(t,\phi)$. Of course, since the generators in these extended superconformal algebras contain, besides the Virasoro generators, only the N supercharges of conformal spin 3/2 and the $SO(N)$ -currents of conformal spin 1, with no generator of lower conformal spin, the supersymmetric extensions in question are those described in [31–33]. These algebra close quadratically in the $SO(N)$ -currents, except for $N=2$ and $N=4$ [with boundary conditions breaking $SO(4)$ to one of its $SU(2)$ subgroups], for which one recovers the linear al-

gebras of [34]. The Hamiltonian reduction of the corresponding WZW models has been analyzed in [35,36].

As observed in [37] the degeneracy of states for a conformal field theory with this central charge gives rise to, under appropriate conditions, exactly the Bekenstein-Hawking entropy for the 2+1 black hole (see also [38]). An earlier statistical description of the 2+1 black hole entropy was given by Carlip [39] in terms of horizon degrees of freedom. For further work in these directions, see [16,40–46]. In view of the relevance of the 2+1 black hole to higher dimensional ones [47,48], this question clearly deserves further study.

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- [12] A. Achúcarro and P.K. Townsend, *Phys. Lett. B* **180**, 89 (1986).
- [13] Our conventions for the spinors are the following. The spinors are real so that $\bar{\psi}=\psi^t\gamma_0$. The Dirac matrices are taken to be
- $$\gamma_0=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma_1=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_2=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
- and satisfy $[\gamma_a, \gamma_b]=2\epsilon_{abc}\gamma^c$, where we use $\epsilon^{012}=1$. The covariant derivative D in Eq. (2) acting on a spinor λ is defined as $D\lambda=d\lambda+\frac{1}{2}A^a\gamma_a\lambda$ and satisfies $D\wedge D\lambda=\frac{1}{2}F^a\gamma_a\lambda$ with $F^a=dA^a+(1/2)\epsilon_{bc}^aA^b\wedge A^c$. The generators of the superalgebra $OSp(1|2)$ are 3×3 supermatrices. The bosonic generators are obtained by augmenting the previous $J_a=(1/2)\gamma_a$ with one row and one column of zeros (they will still be denoted by J_a). The fermionic generators are
- $$e=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad f=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix},$$
- and one has $e^2=J_+$, $f^2=-J_-$ where $J_\pm=J_2\pm J_0$. Let ψ be a 2-component spinor and let Ψ be the fermionic supermatrix $\sqrt{2}\bar{\psi}=\psi_1e+\psi_2f$. Redefining the product of anticommuting numbers by inserting an i , $ab\equiv ia\cdot b$ [so that $(a\cdot b)^*=a^*\cdot b^*$], one gets $s\text{Tr}(\Psi\cdot\Xi)=i\bar{\psi}\xi$ and one may rewrite the action (2) in the manifest super-Chern-Simons form $(k/4\pi)\int s\text{Tr}(\Gamma\cdot d\Gamma+\frac{2}{3}\Gamma\cdot\Gamma\cdot\Gamma)$ with $\Gamma=A+\Psi$, where $s\text{Tr}$ is

- the supertrace. The supercurvature is $\mathcal{F}=d\Gamma+\Gamma\cdot\Gamma$, and the equations of motion are just $\mathcal{F}=0$. The gauge transformations are $\delta\Gamma=d\Lambda+\Gamma\cdot\Lambda-\Lambda\cdot\Gamma$ with $\Lambda\in osp(1|2)$.
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