Pointlike structure for super p-branes

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We present an efficient method to understand the p-brane dynamics in a unified framework. For this purpose, we reformulate the action for super p-branes in the form appropriate to incorporate the pointlike (parton) structure of higher dimensional p-branes and intend to interpret the p-brane dynamics as the collective dynamics of superparticles. In order to examine such a parton picture of super p-branes, we consider various superparticle configurations that can be reduced from super p-branes, especially a supermembrane, and study the partonic structure of classical p-brane solutions. [S0556-2821(98)10320-X]

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I. INTRODUCTION

The eleventh dimensions is the highest dimension in which supergravity theory can exist, with fields carrying spin $J\leqslant 2$ [1]. In fact, it is the only sensible supersymmetric theory in $d=11$ [2]. It has a membrane as a fundamental degree of freedom as well as gravitons [3,4], which may come from the massless excitations of a supermembrane. Recently, it was shown [5,6] that it is the low energy limit of the eleven dimensional M-theory. M theory is defined as the strong coupling limit of the type IIA string theory [5,6] and the double dimensional reduction of an eleven dimensional supermembrane action yields the Green-Schwarz action of the type IIA superstring [7]. These lead one to wonder whether a quantum supermembrane provides an intrinsic definition of M-theory. Moreover, it was shown [8] that the massless spectrum of a supermembrane in $d=11$ occurring in the sector of a completely collapsed membrane, i.e., superparticle, corresponds to the supergravity multiplet.

But the principal objection to this reasoning is that the spectrum is continuous [9,10], which would preclude a particle interpretation. It is known that, unlike string theory, membrane theory encounters new divergences coming from an infinite number of internal degrees of freedom. In order to make the supermembrane dynamics well-defined, we need to have some kind of regularization in a supersymmetric way. Such a regularized description, so-called matrix theory, is given in a light-cone gauge by $U(N)$ supersymmetric Yang-Mills quantum mechanics [9] and its underlying spacetime geometry is noncommutative at short distances. The classical spacetime geometry is a sensible concept only in a long distance regime. Thus, the spectrums of short distance physics may be dramatically changed [11] due to noncommutativity of spacetime.

The parton model of hadrons in the late 1960s was originally developed to describe the properties of high energy collisions of hadrons and later incorporated into the fabric of the quantum chromodynamics, generally accepted relativistic parton model of hadrons. In an infinite momentum frame in which partons are in extreme relativistic motion, the internal motions of the partons and the rate at which they interact with each other are slowed down (frozen) because of the relativistic time dilatation effect and the Fock space vacuum becomes extremely simple with the nonrelativistic nature of underlying dynamics [12]. The matrix regularization of supermembrane dynamics attempts to develop the theory based on the idea that the supermembrane is made of smaller entities, partons [9]. In addition, the Matrix theory for a complete nonperturbative formulation of M theory explicitly incorporates the parton picture in terms of D0-branes in infinite momentum frame [13,14]. When this is done, the spatial coordinates of the N D0-branes are represented by $N\times N$ Hermitian matrices.

The recent picture of M theory tells us that strings, membranes and other extended p-branes hold an equal rank as nonperturbative spectrums [14,15]. Recently, the ordinary string theory as a first quantized description was reformulated as the Matrix string theory, the Matrix theory compactified on a tiny circle, where it was shown that it provides a description of the Hilbert space of *second* quantized string theory [16]. In analogy with the quark picture that appeared to unify many ''fundamental'' hadrons, it may be reasonable to consider p-branes as the composites of smaller entities. It is thus desirable to reformulate in a unified framework the p-brane dynamics as the dynamics for possible constituents as the Matrix model for M theory [13].

In this paper, we construct the Barbour-Bertotti-Schild action [17,18] for super p-branes in order to incorporate the pointlike (parton) structure of higher dimensional p-branes and intend to interpret the p-brane dynamics as the collective dynamics of superparticles. In order to examine the parton picture of super p-branes, we consider various superparticle configurations that can be reduced from super p-branes, especially, supermembrane and study the partonic structure of classical p-brane solutions. Finally, we give some comments on the matrix formulation of the supermembrane from the viewpoint of composites of pointlike entities.

II. SUPER p-BRANE ACTION

For the purpose of illustrating the pointlike structure of p-branes, in this section, we will first consider the Green-Schwarz action [19] of a d-dimensional supermembrane and then super p-brane. The action for the supermembrane in flat superspace is [3,4]

$$I=-T_M\int d^3\xi\bigg\{\sqrt{-g(X,\theta)}+i\varepsilon^{ijk}\bigg(\frac{1}{2}\partial_i X^\mu \times(\partial_j X^\nu - i\bar{\theta}\Gamma^\nu\partial_j\theta)-\frac{1}{6}\bar{\theta}\Gamma^\mu\partial_i\theta\bar{\theta}\Gamma^\nu\partial_j\theta\bigg)\bar{\theta}\Gamma_{\mu\nu}\partial_k\theta\bigg\}, \tag{2.1}$$

where $X^\mu(\xi)$ and $\theta(\xi)$ denote the superspace coordinates of the membrane parametrized in terms of world volume parameters ξ^i $(i=0,1,2)$. Here T_M is a membrane tension proportional to l_P^{-3} and we will take the unit $T_M=1$. The metric $g_{ij}(X,\theta)$ is the induced metric on the world volume

$$g_{ij}=E_i^\mu E_j^\nu \eta_{\mu\nu}, \tag{2.2}$$

where E_i^μ are certain supervielbein components tangential to the world volume defined by

$$E_i^\mu=\partial_i X^\mu - i\bar{\theta}\Gamma^\mu \partial_i \theta \tag{2.3}$$

and $\eta_{\mu\nu}$ is the flat $d=11$ Minkowski metric. The action (2.1) is invariant under spacetime supersymmetric transformations

$$\delta X^\mu = i\bar{\epsilon}\Gamma^\mu\theta, \qquad \delta\theta=\epsilon. \tag{2.4}$$

Note that the above invariance is associated with the crucial gamma matrix identity

$$\bar{\psi}_{[1}\Gamma^\mu\psi_2\bar{\psi}_3\Gamma_{\mu\nu}\psi_{4]}=0 \tag{2.5}$$

only satisfied for $d=4,5,7$, and 11 [3,4].

We would like to rewrite the first term represented as the Nambu-Goto type as the following Schild type action [18]:

$$-\int d^3\xi\sqrt{-g(X,\theta)}=\frac{1}{2}\int d^3\xi e\left(\frac{1}{3!e^2}(\varepsilon^{ijk}E_i^\mu E_j^\nu E_k^\rho)^2-1\right). \tag{2.6}$$

Using the equation of motion about the auxiliary field e, i.e., $e=\sqrt{-(1/3!)(\varepsilon^{ijk}E_i^\mu E_j^\nu E_k^\rho)^2}=\sqrt{-\det g_{ij}}$, it is easy to show that the original Nambu-Goto type action can be recovered.

We assume that the topology of the membrane is fixed to be $\Sigma\times\mathbf{R}$, with Σ a compact two manifold, so that the three coordinates of the world volume, ξ^i, are broken down into $\xi^0=\tau$ and $\xi^a=\sigma^a$, $a=1,2$.[1] We introduce a two dimensional induced metric on Σ defined by

$$q_{ab}=E_a^\mu E_b^\nu\eta_{\mu\nu}, \qquad \varepsilon^{ab}=-\varepsilon^{0ab}. \tag{2.7}$$

Note that

$$\varepsilon^{ab}\varepsilon^{cd}=q(q^{ac}q^{bd}-q^{bc}q^{ad}), \tag{2.8}$$

where q^{ab} is the inverse of q_{ab}, i.e., $q^{ac}q_{cb}=\delta_b^a$ and $q=\det q_{ab}$.

The action (2.6) can then be rewritten as the Barbour-Bertotti-Schild (BBS) type [17,18] appropriate to incorporating the partonic picture of supermembrane

$$I_{BBS}=\frac{1}{2}\int d\tau d^2\sigma\sqrt{q}[\tilde{e}^{-1}(\dot{X}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta})\times G_{\mu\nu}(\dot{X}^\nu - i\bar{\theta}\Gamma^\nu\dot{\theta})-\tilde{e}], \tag{2.9}$$

where dot means the derivative with respect to the time-like parameter τ and $\tilde{e}=e/\sqrt{q}$. Here we have introduced the ''manifold Poisson brackets'' (MPB) using the simplectic structure $\varepsilon^{ab}/\sqrt{q}$ on the two manifold Σ:

$$\langle f,g\rangle=\frac{\varepsilon^{ab}}{\sqrt{q}}\partial_a f\partial_b g \tag{2.10}$$

for $C^\infty(\Sigma)$ functions f and g. Note that this simplectic structure on Σ, which is the cotangent bundles of configuration space Σ, is uniquely defined by the metric and orientation of Σ [21]. Our definition of the MPB manifestly respects the full diffeomorphism group of Σ, $\mathrm{Diff}(\Sigma)$, and satisfies the Jacobi identity

$$\langle\langle f,g\rangle,h\rangle+\langle\langle g,h\rangle,f\rangle+\langle\langle h,f\rangle,g\rangle=0 \tag{2.11}$$

for $C^\infty(\Sigma)$ functions f, g, and h. The metric $G_{\mu\nu}$ on the configuration space of the embeddings $X^\mu(\sigma^a)$ and $\theta(\sigma^a)$ is given by[2]

$$G_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}, \tag{2.12}$$

where $h_{\mu\nu}$ is a useful quantity defined as

$$h_{\mu\nu}=\langle E_\mu,E^\rho\rangle\langle E_\rho,E_\nu\rangle=-q^{ab}E_{a\mu}E_{b\nu} \tag{2.13}$$

and used the abbreviated notation

$$\langle E_\mu,E_\nu\rangle=\langle X_\mu,X_\nu\rangle - i\bar{\theta}\Gamma_\mu\langle\theta,X_\nu\rangle + i\bar{\theta}\Gamma_\nu\langle\theta,X_\mu\rangle+\bar{\theta}\Gamma_\mu\langle\theta,\bar{\theta}\rangle\Gamma_\nu\theta. \tag{2.14}$$

The metric $h_{\mu\nu}$ and $G_{\mu\nu}$ *induced* by neighboring superparticles, by the definition of q_{ab}, satisfy the following identity, respectively,

$$h_{\mu\lambda}h^{\lambda\nu}=-h_\mu^\nu, \tag{2.15}$$

$$G_\mu^\lambda G_\lambda^\nu=G_\mu^\nu. \tag{2.16}$$

The Eq. (2.16) implies that the metric G_μ^ν, indeed, acts as a kind of projection operator in the target space $\mathbf{R}^{d-1,1}$. In addition we have the important identity related with $\mathrm{Diff}(\Sigma)$ constraints generating the reparametrization of the membrane surface

$$E_a^\mu G_{\mu\nu}=0, \tag{2.17}$$

which can be directly derived from the definition (2.12) of $G_{\mu\nu}$. From the Eq. (2.17), one can obtain the relation $q_{ab}=-E_a^\mu h_{\mu\nu}E_b^\nu$, which is consistent with the Eq. (2.13).

The action (2.9) is also invariant under the local reparametrization, $\tau\rightarrow f(\tau)$, provided that the auxiliary field $\tilde{e}$ (a sort

[1]The $2+1$ splitting corresponds to a gauge fixing to put shift vectors N^a of world volume metric to zero [20].

[2]The Lorentz indices such as μ and ν are raised and lowered by using the metric $\eta_{\mu\nu}$.

of ‘‘metric’’ along the particle worldline) transforms as $\tilde{e} \to \tilde{e}(df/d\tau)^{-1}$. This reflects that there is no intrinsic preferred time variable on the membrane. Note that, from the equation of motion with respect to the auxiliary field $\tilde{e}$,

$$\tilde{e} = \sqrt{-(\dot{X}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta})G_{\mu\nu}(\dot{X}^\nu - i\bar{\theta}\Gamma^\nu\dot{\theta})}, \tag{2.18}$$

we can easily recover the usual Barbour-Bertotti form for the membrane as well [22,23].

Let us interpret the BBS action (2.9) as follows. The BBS action takes the form of superparticles with unit mass continuously distributed on the two manifold Σ moving in a background spacetime metric $G_{\mu\nu}$. We would like to interpret the supermembrane as the composite of the superparticles bound to each other by the surface tension and influenced by the *effective gravitational potential* $G_{\mu\nu}$, so, in this sense, the superparticles play a role of (classical) partons of supermembrane. Similarly, we may consider the supermembrane as the configuration of a fluid evolving from a fixed initial configuration. We can then consider the flow of a nonviscous compressible fluid on the region Σ moving along the timelike geodesic defined by the metric $G_{\mu\nu}$. Such a fluid is described by a curve $\tau \to g_\tau$, where the diffeomorphism g_τ is the map which carries every particle of the fluid from the place it was at time 0 to the place it is at time τ. From this picture, we see that the classical mass M of the static membrane is a sum of the mass of constituent superparticles: i.e.,

$$M = T_M \int_\Sigma d^2\sigma \sqrt{q}. \tag{2.19}$$

Recall that the Wess-Zumino term in Eq. (2.1) generates a femionic gauge symmetry, so-called κ-symmetry, which allows us to match the Bose and Fermi degrees of freedom. This fermionic gauge invariance of the supermembrane is only possible for specific number of spacetime dimensions, i.e., for $d=4,5,7,$ and 11. The Wess-Zumino action which is independent of the world volume metric is rewritten as [3,4]

$$I_{WZ} = -\frac{1}{6}\int d^3\xi \varepsilon^{ijk} E_i^A E_j^B E_k^C B_{CBA}, \tag{2.20}$$

where $E_i^A = (E_i^\mu, E_i^\alpha)$ with $E_i^\alpha = \partial_i \theta^\alpha$. The super 3-form B is such that $H = dB$, with all components of H vanishing except $H_{\mu\nu\alpha\beta} = -2i(\Gamma_{\mu\nu})_{\alpha\beta}$. The gamma matrix identity of Eq. (2.5) is nothing but the Bianchi identity $dH = d^2B = 0$ from which the brane scan comes in. Solving for B, one finds

$$\begin{aligned} &B_{\mu\nu\rho} = 0, && B_{\mu\nu\alpha} = i(\bar{\theta}\Gamma_{\mu\nu})_\alpha, \\ &B_{\mu\alpha\beta} = -(\bar{\theta}\Gamma_{\mu\nu})_{(\alpha}(\bar{\theta}\Gamma^\nu)_{\beta)}, && B_{\alpha\beta\gamma} = i(\bar{\theta}\Gamma_{\mu\nu})_{(\alpha}(\bar{\theta}\Gamma^\mu)_\beta(\bar{\theta}\Gamma^\nu)_{\gamma)}. \end{aligned} \tag{2.21}$$

Since the local κ-symmetry eliminates half of the θ fermionic modes, it is involved with some kind of the projection operator $\frac{1}{2}(1 \pm \Gamma)$, where the function Γ is defined by

$$\Gamma = \frac{1}{6e}\varepsilon^{ijk} E_i^\mu E_j^\nu E_k^\rho \Gamma_{\mu\nu\rho} = -\frac{1}{2\tilde{e}} E_0^\mu \langle E^\nu, E^\rho \rangle \Gamma_{\mu\nu\rho} \tag{2.22}$$

and satisfies the relation $\Gamma^2 = 1$ on shell.

In terms of the 2+1 splitting, the action (2.20) takes the form[3]

$$I_{WZ} = \frac{1}{2}\int d\tau d^2\sigma \sqrt{q} E_0^A \Pi_A, \tag{2.23}$$

where the ‘‘external’’ field Π_A is defined as follows:

$$\Pi_A = \langle E^B, E^C \rangle B_{CBA}. \tag{2.24}$$

Then the full BBS type action of the supermembrane takes the following form

$$\begin{aligned} I = \frac{1}{2}\int d\tau d^2\sigma \sqrt{q} [\tilde{e}^{-1}(\dot{X}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta}) \\ \times G_{\mu\nu}(\dot{X}^\nu - i\bar{\theta}\Gamma^\nu\dot{\theta}) - \tilde{e} + E_0^A \Pi_A]. \end{aligned} \tag{2.25}$$

Now the above supermembrane action can be interpreted as the collective dynamics of superparticles or the nonviscous charged fluid composed of the superparticles (which are charged with respect to the fermionic κ-transformation) under the influence of the ‘‘gravitational’’ field $G_{\mu\nu}$ and the ‘‘external’’ field Π_A. We have seen so far that the fields $G_{\mu\nu}$ and Π_A which couple to the superparticles are not arbitrary, but highly constrained by Diff(Σ) symmetry, supersymmetry, and κ-symmetry.

[3]We are now taking an analogy with electrodynamics, where the point particle with charge q is interacting with one-form potential A defined on the worldline Γ of the particle, i.e., $q\int_\Gamma A = q\int_\Gamma d\tau (dX^\mu/d\tau) A_\mu$, and electromagnetic one-form A should satisfy the Bianchi identity, $dF = d^2A = 0$.

Without doing any gauge fixing, we proceed directly to define the canonical momenta of the variables (X^μ,θ^α):

$$P_\mu=\delta I/\delta\dot X^\mu=\sqrt{q}\left\{\tilde e^{-1}G_{\mu\nu}(\dot X^\nu-i\bar\theta\Gamma^\nu\dot\theta)+\frac{1}{2}\Pi_\mu\right\},$$

$$\begin{aligned}P_\alpha&=-\delta I/\delta\dot\theta^\alpha=-i\sqrt{q}\left\{\tilde e^{-1}(\dot X^\mu-i\bar\theta\Gamma^\mu\dot\theta)G_{\mu\nu}(\bar\theta\Gamma^\nu)_\alpha+\frac{1}{2}\Pi^\mu(\bar\theta\Gamma_\mu)_\alpha-\frac{i}{2}\Pi_\alpha\right\}\\&=-iP^\mu(\bar\theta\Gamma_\mu)_\alpha-\frac{\sqrt{q}}{2}\Pi_\alpha.\end{aligned}\tag{2.26}$$

The phase space Poisson brackets of these canonical variables are the following:

$$\begin{aligned}\{X^\mu(\sigma),P_\nu(\sigma')\}_-&=\delta^\mu_\nu\delta^2(\sigma-\sigma'),\\\{\theta^\alpha(\sigma),P_\beta(\sigma')\}_+&=\delta^\alpha_\beta\delta^2(\sigma-\sigma'),\end{aligned}\tag{2.27}$$

where the graded Poisson bracket is defined by $\{A,B\}_\pm=\pm\{B,A\}_\pm$ and the brackets are evaluated at equal times.

Let us collect the canonical constraints imposed on the phase space of the supermembrane [20]:

$$F_\alpha\equiv P_\alpha+iP^\mu(\bar\theta\Gamma_\mu)_\alpha+\frac{\sqrt{q}}{2}\Pi_\alpha\approx0,\tag{2.28}$$

$$\varphi_{ab}\equiv q_{ab}-\eta_{\mu\nu}E^\mu_aE^\nu_b\approx0,\tag{2.29}$$

$$\varphi_a\equiv E^\mu_a\left(P_\mu-\frac{\sqrt{q}}{2}\Pi_\mu\right)\approx0,\tag{2.30}$$

$$\varphi\equiv\frac{1}{2}\left(P_\mu-\frac{\sqrt{q}}{2}\Pi_\mu\right)G^{\mu\nu}\left(P_\nu-\frac{\sqrt{q}}{2}\Pi_\nu\right)+\frac{1}{2}q\approx0,\tag{2.31}$$

$$P\equiv\delta I/\delta\dot{\tilde e}\approx0,\tag{2.32}$$

$$P_{ab}\equiv\delta I/\delta\dot q_{ab}\approx0.\tag{2.33}$$

Note that all these constraints directly follow from the definition of the phase space variables. The constraints (2.28) and (2.31) come from the above definition of the conjugate momenta (P_μ,P_α), where the Eq. (2.18) is rendered into the form of the constraint (2.31). The constraint (2.29) is the definition of the induced metric on the membrane surface Σ and (2.30) is the Diff(Σ) constraint due to the relation (2.17). In fact, the constraints (2.29) can be understood as the secondary constraints of the second class constraints (2.33). On the other hand, the constraint (2.32) is the first class generating the reparametrization, $\tilde e\to\tilde e(df/d\tau)^{-1}$. Multiplying the constraints (2.28)–(2.33) with the Lagrange multipliers Σ^α, Λ^{ab}, Λ^a, Λ, λ, and λ^{ab}, respectively, and adding them to the Hamiltonian, we obtain the total Hamiltonian

$$\begin{aligned}H&=\int d^2\sigma\{(P_\mu\dot X^\mu+P_\alpha\dot\theta^\alpha-\mathcal{L})+\Sigma^\alpha F_\alpha+\Lambda^{ab}\varphi_{ab}+\Lambda^a\varphi_a+\Lambda\varphi+\lambda P+\lambda^{ab}P_{ab}\}\\&=\int d^2\sigma\left[\left(\frac{\tilde e}{\sqrt{q}}+\Lambda\right)\varphi+\Sigma^\alpha F_\alpha+\Lambda^{ab}\varphi_{ab}+\Lambda^a\varphi_a+\lambda P+\lambda^{ab}P_{ab}\right].\end{aligned}\tag{2.34}$$

In Ref. [20], Bergshoeff *et al.* analyzed the constraint structure of the eleven dimensional supermembrane and covariantly classified the constraint algebra. It was shown in Ref. [20] that Eqs. (2.29), (2.33), and $1/2(1-\Gamma)(F+4iP^{ab}E^\mu_a\Gamma_\mu\partial_b\theta)$ (which is an orthogonal part on the κ-transformation) are second class constraints.

It is generally possible that the Green-Schwarz action for any p-brane can be rewritten as the BBS action, which takes that of particles continuously distributed on a p-dimensional surface moving in a nontrivial external background. The Green-Schwarz action for super p-brane is [3,4]

$$I=-T_{p+1}\int d^{p+1}\xi\left\{\sqrt{-g(X,\theta)}+\frac{1}{(p+1)!}\varepsilon^{i_1i_2\cdots i_{p+1}}E^{A_1}_{i_1}E^{A_2}_{i_2}\cdots E^{A_{p+1}}_{i_{p+1}}B_{A_{p+1}\cdots A_2A_1}\right\},\tag{2.35}$$

where the superspace $(p+1)$-form B is the potential for a closed $(p+2)$-form $H=dB$. Possible super p-brane theories exist whenever there is a closed $(p+2)$-form in superspace.

As the case of supermembrane, we assume that the topology of the p-brane is fixed to be $\Sigma\times\mathbf{R}$, with Σ a compact p-dimensional manifold, so that the $(p+1)$ coordinates of the world volume, ξ^i, are split into $\xi^0=\tau$ and $\xi^a=\sigma^a$, $a=1,\cdots,p$. We introduce a p-dimensional induced metric on Σ defined by

$$q_{ab}=E^\mu_aE_{b\mu},\qquad\varepsilon^{a_1a_2\cdots a_p}=-\varepsilon^{0a_1a_2\cdots a_p}.\tag{2.36}$$

Then the following formula can be found

$$\varepsilon^{a_1a_2\cdots a_p}\varepsilon^{b_1b_2\cdots b_p}=q\det\begin{vmatrix}q^{a_1b_1}&q^{a_1b_2}&\cdots&q^{a_1b_p}\\q^{a_2b_1}&q^{a_2b_2}&\cdots&q^{a_2b_p}\\\vdots&\vdots&\ddots&\vdots\\q^{a_pb_1}&q^{a_pb_2}&\cdots&q^{a_pb_p}\end{vmatrix}\tag{2.37}$$

where q^{ab} is the inverse of q_{ab}, i.e., $q^{ac}q_{cb}=\delta^a_b$ and $q=\det q_{ab}$.

As a result of these formula, we have the BBS action for super p-brane,

$$I=\frac{1}{2}\int d\tau d^p\sigma\sqrt{q}[\tilde{e}^{-1}(\dot{X}^\mu-i\bar{\theta}\Gamma^\mu\dot{\theta})G_{\mu\nu}(\dot{X}^\nu-i\bar{\theta}\Gamma^\nu\dot{\theta}) -\tilde{e}+E_0^A\Pi_A], \tag{2.38}$$

where $\tilde{e}=e/\sqrt{q}$ and the ''external'' field Π_A is defined as follows:

$$\Pi_A=\frac{2}{p!}\langle E^{A_1},E^{A_2},\cdots,E^{A_p}\rangle B_{A_p,\cdots,A_2,A_1,A}. \tag{2.39}$$

Here we have introduced the ''manifold multiple bracket''[4] on the manifold Σ extending the previous MPB

$$\langle f_1,f_2,\cdots,f_p\rangle=\frac{1}{\sqrt{q}}\frac{\partial(f_1,f_2,\cdots,f_p)}{\partial(\sigma_1,\sigma_2,\cdots,\sigma_p)} \tag{2.40}$$

for $C^\infty(\Sigma)$ functions f_a. The metric $G_{\mu\nu}$ on the configuration space of the embeddings $X^\mu(\sigma^a)$ and $\theta(\sigma^a)$ is given by

$$G_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}, \tag{2.41}$$

where $h_{\mu\nu}$ is defined as

$$\begin{aligned} h_{\mu\nu}&=-(-)^{p(p-1)/2}\frac{1}{(p-1)!}\langle E_\mu,E^{\mu_1},\cdots,E^{\mu_{p-1}}\rangle \\ &\quad\times\langle E_{\mu_{p-1}},\cdots,E_{\mu_1},E_\nu\rangle \\ &=-q^{ab}E_{a\mu}E_{b\nu}. \end{aligned} \tag{2.42}$$

The similar formula for the metric $h_{\mu\nu}$ and $G_{\mu\nu}$ induced by neighboring superparticles also hold true for super p-branes

$$\begin{aligned} &h_{\mu\lambda}h^{\lambda\nu}=-h_\mu^\nu, \\ &trh^n=h_{\mu\nu}h_\rho^\nu\cdots h_\lambda^\sigma h^{\lambda\mu}=(-)^n\cdot p, \quad \forall n\geq 1, \\ &G_\mu^\lambda G_\lambda^\nu=G_\mu^\nu, \\ &trG^n=d-p, \qquad \forall n\geq 1. \end{aligned} \tag{2.43}$$

In the next section we will show that p-brane solutions always satisfy these relations.

We have exactly the same kind of identity as the supermembrane related with Diff(Σ) constraints generating the reparametrization of the p-brane surface

$$E_a^\mu G_{\mu\nu}=0. \tag{2.44}$$

From the Eq. (2.42), one can also obtain the relation $q_{ab}=-E_a^\mu h_{\mu\nu}E_b^\nu$.

Based on their equivalent canonical structure, it is apparent that the super p-brane ($p\geq 1$) action (2.38) will exhibit the same Hamiltonian structure as the supermembrane action (2.25).

III. PARTON CONFIGURATIONS OF SUPER *p*-BRANES

The parton picture in terms of superparticles is quite different from those of Matrix theory [13] and string bits model [27] where partons are described by a matrix transforming in the adjoint representation of some group G, mainly $SU(N)$ or $SO(N)$. Nevertheless, the formulation based on the idea that higher dimensional extended p-branes can be made of smaller entities, superparticles, is quite useful to understanding the dynamics of p-branes because the dynamics is conceptually simple and clear. In this section, we will try to understand the p-branes in viewpoint of composite of superparticles and study the parton configurations of p-brane solutions.

A. Superstring and superparticle from supermembrane

First, we consider a double dimensional reduction of eleven dimensional supermembrane, from which the type IIA superstring propagating in $d=10$ can be obtained, as shown by Duff *et al.* [7], and superparticle in $d=9$ by a further double dimensional reduction. In the present viewpoint, these solutions can be derived from the particular configurations of superparticles preserving the supersymmetry.

The type IIA superstring in $d=10$ considered by Duff *et al.* [7] is obtained by a compactification of both the world volume and the spacetime on the same circle, letting the membrane tension T_M tend to infinity, but the string tension $T_2=2\pi R_1 T_M$ maintain finite. This corresponds to the configuration of superparticles whose line mass density along the compactified circle tends to infinity, while the mass density along the extended string remains finite. This situation can be represented by the following ansatz:

$$X^{10}=\rho, \qquad \partial_\rho X^m=\partial_\rho\theta=0, \qquad m=0,1,\ldots,9. \tag{3.1}$$

Then the ''parton metric'' $h_{\mu\nu}$, $G_{\mu\nu}$, q_{ab}, and the external field $\Pi_A=(\Pi_m,\Pi_\alpha)$ induced by the superparticle configuration (3.1) can be found as

$$h_{\mu\nu}=\begin{pmatrix} g_{mn} & 0 \\ 0 & -1 \end{pmatrix}, \qquad G_{\mu\nu}=\begin{pmatrix} \eta_{mn}+g_{mn} & 0 \\ 0 & 0 \end{pmatrix},$$

[4]This multiple bracket was introduced a long time ago by Nambu [24] and the quantization for the generalized Hamiltonian dynamics was considered. And the basic principles of canonical formalism for the Nambu dynamics were presented by Takhtajan [25] and applied to the relativistic p-brane dynamics by Hoppe [26].

$$q_{ab}=\begin{pmatrix} E^m_\sigma E_{\sigma m} & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.2}$$

$$\Pi_m=-\frac{2}{\sqrt{q}}E^\alpha_\sigma B_{10m\alpha}=i\frac{2}{\sqrt{q}}\bar{\theta}\Gamma_{10}\Gamma_m\partial_\sigma\theta,$$

$$\Pi_\alpha=\frac{2}{\sqrt{q}}(E^m_\sigma B_{10m\alpha}+E^\beta_\sigma B_{10\beta\alpha})$$

$$=i\frac{2}{\sqrt{q}}(\bar{\theta}\Gamma_{10}\Gamma_m)_\alpha(\partial_\sigma X^m-i\bar{\theta}\Gamma^m\partial_\sigma\theta)$$

$$-\frac{1}{\sqrt{q}}[(\bar{\theta}\Gamma_{10}\Gamma_m)_\alpha\bar{\theta}\Gamma^m\partial_\sigma\theta$$

$$-(\bar{\theta}\Gamma_m)_\alpha\bar{\theta}\Gamma_{10}\Gamma^m\partial_\sigma\theta], \tag{3.3}$$

where η_{mn} is a ten dimensional Minkowski metric and the potentials $B_{10\alpha m}$ and $B_{10\beta\alpha}$ has been determined by Eq. (2.21). The reduced supervielbein E^A_σ and the metric g_{mn} are given by

$$E^A_\sigma=(E^m_\sigma,E^\alpha_\sigma)=(\partial_\sigma X^m-i\bar{\theta}\Gamma^m\partial_\sigma\theta,\ \partial_\sigma\theta^\alpha),$$

$$g_{mn}=-E_{\sigma m}E_{\sigma n}/q,\qquad q=E^l_\sigma E_{\sigma l}. \tag{3.4}$$

One can check that the solution (3.2) manifestly satisfies the identities (2.43), i.e., $trh^n=(-)^n\cdot 2$ and $trG^n=9$.

In order to recast the Green-Schwarz action for the type IIA superstring [19], the 32 components of Majorana spinor θ can be split into two Majorana-Weyl spinors in terms of the ten dimensional chiral matrix Γ_{10}

$$\theta_\pm=\frac{1}{2}(\mathbf{1}\pm\Gamma_{10})\theta.$$

Using these results, we can obtain the BBS action for the type IIA superstring[5]

$$I=\frac{T_2}{2}\int d\tau d\sigma\sqrt{q}[\tilde{e}^{-1}(\dot{X}^m-i\bar{\theta}\Gamma^m\dot{\theta})$$

$$\times G_{mn}(\dot{X}^n-i\bar{\theta}\Gamma^n\dot{\theta})-\tilde{e}+E^A_0\Pi_A], \tag{3.5}$$

where $G_{mn}=\eta_{mn}+g_{mn}$ and $\theta=(\theta_+,\theta_-)$.

From the double dimensional reduction (3.1), the Hamiltonian formulation of the superstring can be also derived from the Eqs. (2.26)–(2.34) and the constraint structure of the superstring is the same as that of the supermembrane [28]. Note that the string action (3.5) can be rewritten as the superconformally invariant theory through the Polyakov action even though the membrane action we started from cannot.

Consider a further double dimensional reduction of the superstring constructed by the Kaluza-Klein truncation (3.1) of the supermembrane [29]. The string is then wrapping around another circle of radius R_2. Thus the membrane has a toroidal topology embedded in a spacetime $\mathbf{R}^9\times S^1\times S^1$. Choosing the $S^1\times S^1$ to be in the X^{10} and X^9 directions and letting the string tension T_2 tend to infinity, but the static membrane mass (2.19), $M=(2\pi R_2)(2\pi R_1)T_M$, maintain finite,[6] the classical solution of this configuration can be taken as the following form

$$X^{10}=\rho,\quad X^9=\sigma,\quad \partial_a X^m=\partial_a\theta=0,$$

$$a\in(\sigma,\rho)\quad\text{and}\quad m=0,1,\cdots,8. \tag{3.6}$$

This configuration corresponds to a supermembrane that has completely collapsed to a point. In fact, we find the parton metric

$$h_{\mu\nu}=\begin{pmatrix} g_{mn}=0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},\ G_{\mu\nu}=\begin{pmatrix} \eta_{mn} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$q_{ab}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.7}$$

$$\Pi_m=\Pi_\alpha=0. \tag{3.8}$$

For these the supermembrane action reduces to that of superparticle with mass M and propagating in $d=9$ with $\mathcal{N}=2$ supersymmetries[7]

[5]It can be easily shown that the other $2+1$ splitting from the supermembrane action (2.1), $\xi^2=\rho$, $\xi^a=(\tau,\sigma)$, $a=0,1$, directly gives the Nambu-Goto action of superstring. In this case, the analogue of the Eq. (2.25) is involved with the derivative with respect to ρ instead of τ. Thus, it is sufficient that we consider only terms involved with G_{1010} and Π_{10}.

[6]The classical mass of the toroidal solution considered here is nonvanishing. This mass can be interpreted as the winding energy of the membrane wrapping around the toroidal surface or that of the string wrapping around the X^9-circle. For this reason, the mass is essentially quantized.

[7]A representation of the Γ-matrices appropriate to the $11=9+2$ split that we are making is:

$$\Gamma^m=\gamma^m\otimes\sigma^3,\qquad m=0,1,\cdots,8,$$

$$\Gamma^{8+a}=\mathbf{1}_{16}\otimes\sigma^a,\qquad a=1,2,$$

where the 9-dimensional γ-matrices satisfy

$$\{\gamma^m,\gamma^n\}=2\eta^{mn}.$$

$$I=\frac{M}{2}\int d\tau[\tilde{e}^{-1}(\dot{X}^m-i\overline{\theta^A}\gamma^m\dot{\theta}^A)\eta_{mn}\times(\dot{X}^n-i\overline{\theta^B}\gamma^n\dot{\theta}^B)-\tilde{e}], \tag{3.9}$$

where $\theta^A=(\theta_+,\theta_-)$ are two 16 component Majorana spinors in $d=9$. For the case of the superparticle, the ''external fields'' $h_{\mu\nu}$ and Π_A disappear in the action. As a well-known fact, in the case of point particles, there is no need for Wess-Zumino term to realize the κ-symmetry [30] as illustrated in Eq. (3.8).

Using the above results, the Hamiltonian formulation for the superparticle [30,32] can be also derived from the Eqs. (2.26)–(2.34) where the nontrivial constraints come from Eqs. (2.28), (2.31), and (2.32), the other constraints identically (or strongly) vanish.

B. Pulsating spherical membrane

Consider a periodic pulsating membrane, originally described by Collins and Tucker [31], where a spherical membrane contracts to a point and expands again with the opposite orientation. This solution was recently reconsidered in the Matrix theory context [33], where it was argued that, upon gravitational collapse, the spherical membrane has a possibility to form a Schwarzschild black hole and then decay quantum mechanically via Hawking radiation. As a simple good example of this formalism, the dynamics of a spherical membrane can be described by the $SU(N)$ Yang-Mills quantum mechanics in a light-cone gauge [9,10,33].

First, we introduce a parametrization of a unit sphere by coordinates $\sigma^a=(x,\theta)$ with $-1\leqslant x\leqslant 1$ and $0\leqslant\theta\leqslant 2\pi$. The embedding Cartesian coordinates on the sphere

$$x_1=x, \qquad x_2=\sqrt{1-x^2}\sin\theta, \qquad x_3=\sqrt{1-x^2}\cos\theta \tag{3.10}$$

obey the $SU(2)$ algebra

$$\{x_i,x_j\}=\varepsilon_{ijk}x_k, \qquad i,j,k=1,2,3. \tag{3.11}$$

The pulsating spherical membrane is described by setting

$$X_0(\tau,\sigma^a)=t(\tau), \quad X_i(\tau,\sigma^a)=r(\tau)x_i(\sigma^a),$$
$$X_4=\cdots=X_{d-1}=0, \quad \theta=0. \tag{3.12}$$

Using the result (3.11), the parton metric can be found as

$$G_{ij}=\begin{pmatrix} x^2 & x\sqrt{1-x^2}\sin\theta & x\sqrt{1-x^2}\cos\theta \\ x\sqrt{1-x^2}\sin\theta & (1-x^2)\sin^2\theta & (1-x^2)\sin\theta\cos\theta \\ x\sqrt{1-x^2}\cos\theta & (1-x^2)\sin\theta\cos\theta & (1-x^2)\cos^2\theta \end{pmatrix}, \tag{3.13}$$

$$q_{ab}=\begin{pmatrix} \frac{r^2}{1-x^2} & 0 \\ 0 & r^2(1-x^2) \end{pmatrix}, \qquad q=r^4,$$

where we have explicitly presented only the non-flat spatial components of $G_{\mu\nu}$. The membrane action (2.25) can then be reduced to the following simple form:

$$I=\frac{4\pi}{2}\int d\tau\, r^2[\tilde{e}^{-1}(-\dot{t}^2+\dot{r}^2)-\tilde{e}]$$
$$=-4\pi\int d\tau\, r^2\sqrt{\dot{t}^2-\dot{r}^2}, \tag{3.14}$$

where we find that $h_{\mu\nu}$ gives no contribution in the action (3.14) due to the relation $x_i^2=1$. The equation of motion coming from the variation δt is given by

$$\partial_\tau\left(\frac{r^2\,\dot{t}}{\sqrt{\dot{t}^2-\dot{r}^2}}\right)=0. \tag{3.15}$$

The action (3.14) still has the reparametrization symmetry; $\tau\rightarrow f(\tau)$. Using this freedom, let us choose a synchronous gauge

$$t=\tau. \tag{3.16}$$

Then the solution takes the form of energy conservation

$$\dot{r}^2+\frac{r^4}{r_0^4}=1, \tag{3.17}$$

where r_0 is the radial position at $\tau=0$. The bosonic partons perform a pulsating motion by the attractive r^4 potential [31]. Note that the potential proportional to r^4 comes from the time-dependent effective mass of parton, Eq. (2.19), due to the tension of the membrane.

It is easy to check that the equation of motion with respect to r is consistent with Eq. (3.17). Thus the dynamics of the spherical membrane is fully determined by Eq. (3.17) which can be solved in terms of elliptic functions [34].

C. Hoppe-Nicolai solution

We will consider more general solutions presented by Hoppe and Nicolai [35] and, in the Matrix theory context, by Hoppe and Rey [36], which describes pulsating and rigidly rotating classical surfaces (of arbitrary dimension) embedded into Euclidean spheres.

We take a natural ansatz corresponding to the simple motions of pulsation [described by a radial function $r(\tau)$] and rotation [described by a time-dependent real orthogonal matrix $D(\tau)$],

$$X_0(\tau,\Omega)=t(\tau), \qquad \mathbf{X}(\tau,\Omega)=r(\tau)D(\tau)\mathbf{m}(\Omega), \tag{3.18}$$

where $\Omega=(\sigma^1,\cdots,\sigma^p)$ stands for the world volume parameters of a p-dimensional surface and $\mathbf{m}(\Omega)$ is a unit vector

$$\mathbf{m}^2(\Omega)=1. \tag{3.19}$$

Then $\mathbf{X}(\tau,\Omega)$ has the simple interpretation of a rotating p-dimensional surface embedded in a sphere S^{d-2} of time dependent radius $r(\tau)$.

In this case, the parton metric has the form

$$q_{ab}=r^2\partial_a\mathbf{m}\cdot\partial_b\mathbf{m}\equiv r^2\tilde{q}_{ab}, \qquad q=r^{2p}\tilde{q}, \qquad a,b=1,\cdots,p,$$

$$h_{\mu\nu}=[D\tilde{h}D^T]_{ij}, \qquad \tilde{h}_{ij}\equiv-\tilde{q}^{ab}\partial_a m_i\partial_b m_j, \qquad i,j=1,\cdots,d-1, \tag{3.20}$$

where $\tilde{q}^{ab}$ is the inverse of $\tilde{q}_{ab}$ and $\tilde{q}=\det\tilde{q}_{ab}$. Taking the rotation matrix as

$$D(\tau)=\exp[\varphi(\tau)A], \tag{3.21}$$

where the matrix A is antisymmetric, the Hoppe-Nicolai solution corresponds to the ansatz choosing $\mathbf{m}(\Omega)$ to be [35]

$$A^2\mathbf{m}(\Omega)=-\mathbf{1}\cdot\mathbf{m}(\Omega), \tag{3.22}$$

$$\partial_a\mathbf{m}^TA\mathbf{m}=0. \tag{3.23}$$

The above equations can be satisfied by choosing

$$\mathbf{m}=(n_1,n_2,\cdots,n_k,0,\cdots,0), \qquad p+1\leqslant k\leqslant\frac{d-1}{2}, \tag{3.24}$$

and

$$A=\begin{pmatrix} 0 & -\mathbf{1} & 0\\ \mathbf{1} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{3.25}$$

where $\mathbf{1}$ is the $k\times k$ unit matrix. Then the p-brane action for the pulsating and rotating surfaces also takes the simple form

$$I=\frac{A_p}{2}\int d\tau\, r^p[\tilde{e}^{-1}(-\dot{t}^2+\dot{r}^2+r^2\dot{\varphi}^2)-\tilde{e}]$$

$$=-A_p\int d\tau\, r^p\sqrt{\dot{t}^2-\dot{r}^2-r^2\dot{\varphi}^2}, \tag{3.26}$$

where $A_p=\int_\Sigma d^p\sigma\sqrt{\tilde{q}}$ is the area of p-dimensional surface embedded in a $(k-1)$-dimensional unit sphere. In deriving Eq. (3.26), the terms involved with $h_{\mu\nu}$ identically vanish due to the Eq. (3.19), Eq. (3.23), and the orthonormality relation $D^T(\tau)D(\tau)=\mathbf{1}$.

Notice that the variation of A_p, together with the constraint $\mathbf{n}^2=1$, leads to the requirement that $\mathbf{n}$ describes a minimal surface in S^{k-1} [35]:

$$\nabla^2\mathbf{n}(\Omega)=-p\,\mathbf{n}(\Omega), \tag{3.27}$$

where $\nabla^2=(1/\sqrt{\tilde{q}})\,\partial_a\sqrt{\tilde{q}}\tilde{q}^{ab}\partial_b$.

The equations of motion obtained by the variations δt and $\delta\varphi$, respectively, are given by

$$\partial_\tau\left(\frac{r^p\,\dot{t}}{\sqrt{\dot{t}^2-\dot{r}^2-r^2\dot{\varphi}^2}}\right)=0,$$

$$\partial_\tau\left(\frac{r^{p+2}\,\dot{\varphi}}{\sqrt{\dot{t}^2-\dot{r}^2-r^2\dot{\varphi}^2}}\right)=0. \tag{3.28}$$

Taking into account τ-reparametrization symmetry in the action (3.26), the above equations of motion take the form of the energy and the angular momentum conservation, respectively,

$$\dot{r}^2+r^2\dot{\varphi}^2+\alpha^2r^{2p}=1,$$

$$r^2\dot{\varphi}=\text{constant}\equiv L, \tag{3.29}$$

where $\alpha=\sqrt{1-L^2/r_0^2}/r_0^p$ and r_0 is the radial position at $\tau=0$. From Eq. (3.29), it follows that

$$\dot{r}^2+\frac{L^2}{r^2}+\alpha^2r^{2p}=1, \tag{3.30}$$

which is compatible with the equation of motion determined by variation δr. For the case of $L=0$ and $p=2$, the Eq. (3.30) equals to the Eq. (3.17) for the spherical membrane. In the case of $L=0$, there is no need to put the restriction on $\mathbf{m}(\Omega)$ such as Eqs. (3.22) and (3.23).

The other solutions of Eq. (3.30) are obtained by straightforward integration

$$\tau=\frac{1}{2}\int\frac{dz}{\sqrt{z-\alpha^2z^{p+1}-L^2}}, \tag{3.31}$$

where $z=r^2$. For $p=1$,

$$r(\tau)=(1/\sqrt{2}\alpha)\sqrt{1+2\alpha a\sin(2\alpha\tau+\theta_0)},$$

where $a=\sqrt{1/4\alpha^2-L^2}$. For $p=2,3$, the solutions can be also solved by elliptic functions [34]. They describe the motion of partons pulsating by r^{2p} potential with angular momentum L. Note that, for $L\neq0$ and finite energy, the pulsating and rotating *p*-branes need not collapse to a point, that is, there is a nonzero minimum radius r_{min} determined by Eq. (3.30).

IV. DISCUSSION

The aim of this paper is to understand *p*-brane dynamics in terms of superparticles. Although the parton picture in terms of superparticles is quite different from those of Matrix theory and string bits model, we have found that the super *p*-brane dynamics can be understood by the collective dynamics of superparticles in a unified framework. Here we summarize our formulation and add some comments on the matrix formulation of the supermembrane.

The Matrix formulation of supermembrane was constructed according to the following scheme. In light-cone gauge, the residual reparametrization symmetry reduces to an area preserving diffeomorphism, SDiff(Σ). According to the relation between the representation of $s$diff(Σ) and the $N\rightarrow\infty$ limit of some Lie algebra [37], the light-cone supermembrane is mapped to a *physically equivalent* system with the corresponding gauge symmetry. Here, *physically equivalent* means that the physical degrees of freedom and their Hilbert space structure exactly match with each other. Interestingly, such a system exists and is given by a supersymmetric Yang-Mills quantum mechanics [9]. When this is done, the embedding coordinates $X^{\mu}(\tau,\sigma^a)$ and $\theta(\tau,\sigma^a)$ are mapped to matrices $X^{\mu}_{IJ}(\tau)$ and $\theta_{IJ}(\tau)$ transforming in the adjoint representation of the Lie group G. The Σ-dependences of X and θ are transformed to matrix degrees of freedom. That is, the matrix coordinates X and θ are the collective variables describing the many pointlike parton degrees of freedom. The important point is that the matrix regularization of membrane dynamics is performed in a supersymmetric way.

It is the recent picture of Matrix theory [13,14] that some spectrums of M-theory in infinite momentum frame can be understood as the collective excitations of D0-particles whose dynamics is given by a matrix quantum mechanics. In the BBS action of supermembrane in terms of superparticles, the Σ-dependences are collected into the form of the ''effective potentials,'' $G_{\mu\nu}$ and Π_A, between superparticles and summed over all constituent superparticles. The Diff(Σ) symmetry restricts the form of the effective potentials. In other words, they should be given by the Diff(Σ)-invariants such as the MPB. Moreover, the κ-symmetry determines the ''gauge potentials'' Π_A coupled to the superparticles. These potentials determine an effective background about superparticle dynamics. Speculatively, the full matrix formulation of supermembrane may reduce to a problem to encode the effective background geometry determined by the potentials $G_{\mu\nu}$ and Π_A into the collective (matrix) coordinates of superparticles.

The possibility of a covariant (in the sense of the target space) matrix formulation rests on whether or not we can find a *physically equivalent* system with supersymmetric matrix regularization that the dynamical degrees of freedom and their Hilbert space structure exactly match with each other. As pointed out by Smolin [22], the only SDiff(Σ) is linearly realized by the Poisson algebra (2.10), which is mapped to the Lie algebra of a gauge group in light-cone gauge. The area non-preserving part, Diff(Σ)/SDiff(Σ), is non-linearly realized by the Poisson algebra. If we want to have a covariant matrix formulation of membrane, we should find a matrix realization (regularization) of the full Diff(Σ) [22,23]. It is desirable in the practical sense that the matrix formulation would provide the linear realizations on the Diff(Σ), κ-symmetry and supersymmetry. Unfortunately, it seems that there is no definite recipe for the above issues at the moment.

We think that, if the full matrix formulation of supermembrane should be incorporated with all the recent pictures appeared in the nonperturbative string theory and M-theory [14,15], e.g., noncommutative spacetime geometry, holographic principle, and *p*-brane democracy, it will need a fundamental unit defining spacetime quanta, bits of information, and partons of *p*-brane. We hope, in this sense, that the reformulation of *p*-brane dynamics by smaller entities presented in this paper will be helpful to understanding the nonperturbative dynamics of the supermembrane.

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