

Critical energy flux and mass in solvable theories of 2D dilaton gravity

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In this paper we address the issue of determining the semiclassical threshold for black hole formation in the context of a one-parameter family of theories which continuously interpolates between the RST and BPP models. We find that the results depend significantly on the initial static configuration of the spacetime geometry before the influx of matter is turned on. In some cases there is a critical energy density, given by the Hawking rate of evaporation, as well as a critical mass m_{cr} (eventually vanishing). In others there is neither m_{cr} nor a critical flux. [S0556-2821(98)07318-4]

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I. INTRODUCTION

Black holes are among the most fascinating and interesting objects in modern theoretical physics. Discovered in the context of general relativity, their understanding from the point of view of quantum theory is one of the essential ingredients in the search of a unified theory of all fundamental interactions.

Classically black holes are “simple” objects; i.e., as their name suggests they absorb any kind of matter, but since light itself gets trapped in their gravitational field, they are invisible to any external observer. This view has however been drastically modified by quantum considerations. The basic process can be understood, heuristically, by considering loops of virtual particles close to the event horizon; the gravitational field of the hole is capable of capturing one partner (provided its energy is negative), leaving the other free to reach infinity. Hawking [1] has shown, in fact, that they rather behave as hot bodies with temperature¹ $T_H = k_+/2\pi$, where k_+ is the surface gravity at the event horizon.

In this paper we will consider the aspect of the formation of black holes in a simplified context, namely two dimensional dilaton gravity. In the classical theory in 1 + 1 dimensions collapse of matter (in the form of conformally coupled scalar fields) always forms a stable black hole, no matter the amount of total incoming energy² M . The discovery of exactly solvable models at the semiclassical level [2], where the backreaction of the Hawking radiation on the background geometry can be analytically evaluated, has been very useful for understanding many features of quantum black hole physics.

In the present context we will consider a one-parameter

(a) family of models introduced in [3] given by the action $S = S_{cl} + S_q$, where

$$S_{cl} = \frac{1}{2\pi} \int d^2x \sqrt{-g} \times \left[e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\lambda^2] - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right] \quad (1.1)$$

and

$$S_q = \frac{1}{2\pi} \int d^2x \sqrt{-g} \times \left[-\frac{N}{48} R \square^{-1} R + \frac{N}{12} (1-2a)(\nabla\phi)^2 + (a-1)\phi R \right]. \quad (1.2)$$

For $a = 1/2$ we recover the Russo-Susskind-Thorlacius (RST) model [5] and when $a = 0$ the one given by Bose, Parker and Peleg (BPP) [6]. The classical limit of these theories, i.e. S_{cl} , is the Callan-Giddings-Harvey-Strominger (CGHS) model [7], which describes low-energy excitations along the infinite throat of extremal (magnetic) stringy black holes in four dimensions. Its general static solution is simply expressed in terms of a mass parameter M . When $M > 0$ it is a black hole and has the same causal structure as the Schwarzschild solution; the case $M = 0$ is the well-known linear dilaton vacuum and, finally, for $M < 0$ the spacetime geometry exhibits a naked timelike singularity.

In the semiclassical regime (which, we recall, makes sense as an approximation to the full quantum theory only for $N \rightarrow \infty$, $N e^{2\phi}$ fixed) it turns out that by requiring the absence of radiation at infinity Minkowski spacetime is no longer a solution of the equations of motion unless $a = 1/2$ (i.e. the RST model). For different values of a the “ground state” of the theory is a nonflat geometry asymptotically Minkowskian (as $e^{2\phi} \rightarrow 0$) and, in the strong coupling region, with generically a regular timelike boundary at a finite

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¹Here and throughout the paper we will consider units where $\hbar = G = c = 1$.²In four dimensions, however, there is a classical threshold for black hole formation; see [4].

proper distance from any other point (for $a=0$ it becomes, instead, an infinite throat and the spacetime is geodesically complete). These solutions also represent the end-point of the Hawking evaporation process.

There are however other solutions, obtained by imposing reflecting boundary conditions along some timelike surface in the strong coupling region, which can also be considered regular from the point of view of the semiclassical theory. We will use all such configurations as possible initial states for the gravitational collapse process that we will investigate. Starting with the simple case of an incoming shock-wave (Sec. IV), we will then consider a constant energy density flux (Sec. V) and show finally, in Sec. VI, that the results obtained have a rather general validity and apply for all types of collapsing null matter.

II. CGHS MODEL: CLASSICAL SOLUTIONS

In this section we will recall briefly the form of the classical solutions. The CGHS theory is given by the action S_{cl} of Eq. (1.1), where R is the 2D Ricci scalar, ϕ the dilaton field, λ^2 the cosmological constant and f_i represent N massless conformally coupled scalar fields.

Choosing the conformal frame $ds^2 = -e^{2\rho} dx^+ dx^-$ the equations of motion of this theory obtained by variation with respect to the metric are

$$e^{-2\phi}(4\partial_{\pm}\rho\partial_{\pm}\phi - 2\partial_{\pm}^2\phi) + \sum_{i=1}^N \frac{1}{2}\partial_{\pm}f_i\partial_{\pm}f_i = 0, \quad (2.1)$$

$$e^{-2\phi}(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2 e^{2\rho}) = 0. \quad (2.2)$$

Variation of the dilaton and the matter fields gives

$$-4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} = 0, \quad (2.3)$$

$$\partial_+\partial_-\phi = 0. \quad (2.4)$$

It is possible to fix the residual diffeomorphism invariance [i.e. the transformations $x^{\pm} \rightarrow x'^{\pm}(x^{\pm})$ that preserve the conformal frame] and impose the Kruskal gauge choice

$$\rho = \phi \quad (2.5)$$

for which the static solutions to the equations of motion take the simple form

$$e^{-2\phi} = e^{-2\rho} = \frac{M}{\lambda} - \lambda^2 x^+ x^-. \quad (2.6)$$

The parameter M is identified with the Arnowitt-Deser-Misner (ADM) mass.

The solutions with $M > 0$ represent black holes: they are characterized by a spacelike curvature singularity located at $x^+ x^- = M/\lambda^3$, event horizons at $x^{\pm} = 0$ and are asymptotically Minkowskian as $x^+ x^- \rightarrow \infty$.

The case $M = 0$ is the linear dilaton vacuum. This is easily seen by transforming to coordinates σ^{\pm} such that $\pm \lambda x^{\pm} = e^{\pm \lambda \sigma^{\pm}}$, where

$$ds^2 = -d\sigma^+ d\sigma^-, \quad \phi = -\lambda \sigma \quad (2.7)$$

and $\sigma \equiv (\sigma^+ - \sigma^-)/2$.

Finally, when $M < 0$ there is a timelike singularity at $x^+ x^- = -|M|/\lambda^3$. By cosmic censorship arguments this solution should be excluded from the physical spectrum. However, we will see in the next section that we can nonetheless introduce semiclassical configurations which reduce, in the classical limit, to these solutions. This simple fact will be important for the discussion of our results.

III. SEMICLASSICAL STATIC SOLUTIONS OF THE RST-BPP MODELS AND SPACETIME STRUCTURE

The solvability of the semiclassical theory $S_{cl} + S_q$, given in Eqs. (1.1) and (1.2), is essentially due to the fact that provided we perform the field redefinitions

$$\Omega = \frac{N}{12} a \phi + e^{-2\phi}, \quad (3.1)$$

$$\chi = \frac{N}{12} \rho + \frac{N}{12} (a-1) \phi + e^{-2\phi}, \quad (3.2)$$

and work in the conformal gauge, it is equivalent to a Liouville theory

$$S = \frac{1}{\pi} \int d^2x \left[\frac{12}{N} (-\partial_+ \chi \partial_- \chi + \partial_+ \Omega \partial_- \Omega) + \lambda^2 e^{24(\chi - \Omega)/N} + \frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_- f_i \right]. \quad (3.3)$$

The equations of motion of this theory take a very simple form

$$\partial_+ \partial_- (\chi - \Omega) = 0, \quad (3.4)$$

$$\partial_+ \partial_- \chi = -\lambda^2 e^{24(\chi - \Omega)/N}. \quad (3.5)$$

In addition to these equations the solutions to the equations of motion have also to satisfy the constraints (which are obtained by variation of the full covariant action with respect to $g^{\pm\pm}$)

$$\begin{aligned} \frac{N}{12} t_{\pm} = \frac{12}{N} (-\partial_{\pm} \chi \partial_{\pm} \chi + \partial_{\pm} \Omega \partial_{\pm} \Omega) \\ + \partial_{\pm}^2 \chi + \frac{1}{2} \sum_{i=1}^N \partial_{\pm} f_i \partial_{\pm} f_i, \end{aligned} \quad (3.6)$$

where $t_{\pm}(x^{\pm})$ are functions of their arguments and depend on boundary conditions such as the choice of the quantum state for the radiation fields. We can always choose the Kruskal gauge $\chi = \Omega$ (i.e. $\rho = \phi$) for which the general static solutions with no radiation at infinity in terms of the original fields read

$$e^{-2\phi} + \frac{Na}{12}\phi = -\lambda^2 x^+ x^- - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + C \quad (3.7)$$

and C is an integration constant. We can think of these solutions as being the ‘‘semiclassical versions’’ of those in Eq. (2.6).

It is easy to realize that the linear dilaton vacuum (2.7) is not included in these solutions unless $a = \frac{1}{2}$, in which case λC is identified with the ADM mass. For different values of a the only solutions that are completely regular are those for which

$$C = \hat{C} \equiv -\frac{N}{48} \left(1 - \ln \frac{N}{48}\right) + \frac{Na}{24} \left(1 - \ln \frac{Na}{24}\right). \quad (3.8)$$

The spacetime geometry for these cases is asymptotically Minkowskian as $x^+ x^- \rightarrow \infty$ (and $e^{2\phi} \rightarrow 0$). In the strong-coupling regime the critical line where $\Omega'(\phi) = 0$, i.e., $e^{-2\phi} = Na/24$, is generically at a finite distance (see [3]) except for $a=0$, where it takes the form of a semi-infinite throat (we refer to [6] for the details). This regular boundary can be considered on the same footing as the surface $r=0$ of 4D Minkowski spacetime.

Considering the case $C < \hat{C}$ one can show that we now have a timelike curvature singularity along the line

$$-\lambda^2 x^+ x^- - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + C = \frac{Na}{24} \left(1 - \ln \frac{Na}{24}\right). \quad (3.9)$$

When, instead, $C > \hat{C}$ the spacetime geometry presents light-like weakly coupled singularities at $x^\pm = 0$. It is however consistent in both these cases (see [6] for $a=0$) to impose reflecting boundary conditions on a suitable timelike hypersurface in order to avoid the region of strong coupling in the physical spacetime. The dynamical evolution of the boundaries for $C < \hat{C}$ has been considered in [8,9] for the RST model and in [10] for the BPP model.

The regularity of the solutions with $C = \hat{C}$ together with the fact that they represent the end-point of the Hawking evaporation of these models (see [3]) suggests that they can be considered the ground state of the theory. Any generic solution would therefore have ADM mass³ $\lambda(C - \hat{C})$.

IV. BLACK HOLE FORMATION WITH A SHOCK-WAVE

We begin our analysis of the dynamical solutions with infalling matter by recalling that the general solutions to the equations of motion (3.4), (3.5) and (3.6) take the form

$$e^{-2\phi} + \frac{Na}{12}\phi = -\lambda^2 x^+ \left(x^- + \frac{P(x^+)}{\lambda^2}\right) - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + \frac{M(x^+)}{\lambda} + C, \quad (4.1)$$

where $P(x^+) = \int dx^+ T_{++}^f$ and $M(x^+) = \lambda \int dx^+ x^+ T_{++}^f$ are, respectively, the Kruskal momentum and energy of the infalling matter (and $T_{++}^f = \frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_+ f_i$).

As a first simple example, let us consider the case of a shock-wave carrying an energy m and propagating along the null line $x^+ = x_0^+$, described by the energy-momentum tensor

$$T_{++}^f = \frac{m}{\lambda x_0^+} \delta(x^+ - x_0^+). \quad (4.2)$$

Consider our initial static configuration to be one of those studied in the previous section with generic C . The solution (4.1) is then

$$e^{-2\phi} + \frac{Na}{12}\phi = -\lambda^2 x^+ x^- - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) - \frac{m}{\lambda x_0^+} (x^+ - x_0^+) \theta(x^+ - x_0^+) + C. \quad (4.3)$$

The critical line at the future of the shock-wave is therefore given by

$$\alpha = -\lambda^2 x^+ (x^- + \Delta) - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + \frac{m}{\lambda} + C, \quad (4.4)$$

where we have defined $\Delta = m/\lambda^3 x_0^+$ and $\alpha = (Na/24)[1 - \ln(Na/24)]$.

Let us first consider the case $C \leq \hat{C}$. The onset of the black hole phase is when this curve becomes light-like. We then expect an apparent horizon to form, thus shielding the singularity from the external observers, and, in the future asymptotic region, the evaporation to take place as has been shown in [3]. Once the singularity has become spacelike it is no longer possible to impose reflecting boundary conditions, which would then violate causality, and the spacetime becomes ‘‘truly’’ singular.

Differentiating Eq. (4.4) we get

$$\frac{dx^-}{dx^+} = \frac{\lambda^2 (x^- + \Delta) + N/48x^+}{-\lambda^2 x^+ - N/48x^-}. \quad (4.5)$$

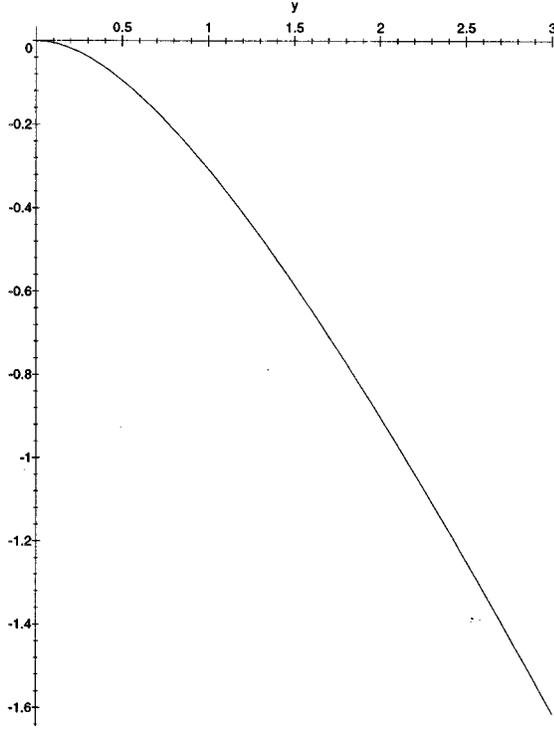
The critical condition

$$ds^2 = -e^{2\rho_{cr}} dx^+ dx^- = 0, \quad (4.6)$$

where ρ_{cr} is the value of ρ along the critical line, is then given, at $x^+ = x_0^+$, in terms of a critical mass m_{cr} :

$$\lambda^2 \left(x_0^- + \frac{m_{cr}}{\lambda^3 x_0^+}\right) + \frac{N}{48x_0^+} = 0 \quad (4.7)$$

³We note, although it could seem superfluous, that this definition of mass reduces to the classical ADM mass M in the classical limit.


FIG. 1. Graph of the function $f(y)$ (we chose $b=1$).

[here x_0^- is given by Eq. (4.4) at $x^+ = x_0^+$]. Combining Eqs. (4.4) and (4.7) we get

$$\alpha - C = \left(\frac{m_{cr}}{\lambda} + \frac{N}{48} \right) - \frac{N}{48} \ln \left(\frac{m_{cr}}{\lambda} + \frac{N}{48} \right). \quad (4.8)$$

In order to understand this equation better, let us first consider as our initial configuration the ground state solution $C = \hat{C} = (-N/48)[1 - \ln(N/48)] + \alpha$. This gives simply $m_{cr} = 0$, as already verified in the case of the RST model in [5].

To analyze the other cases let us write $C = \hat{C} + \beta$ and define, for simplicity, $y \equiv m_{cr}/\lambda$ (>0) and $b \equiv N/48$. Equation (4.8) can then be rewritten as

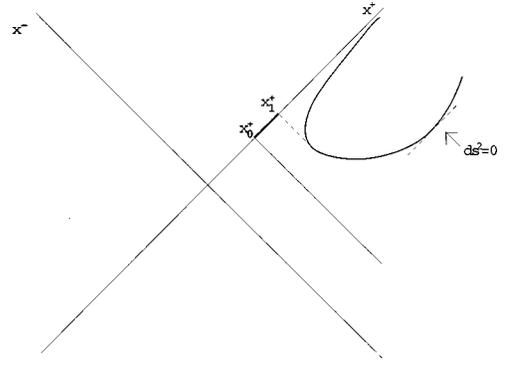
$$b \ln \left(1 + \frac{y}{b} \right) - y = \beta. \quad (4.9)$$

The graph of the function $f(y) = b \ln(1+y/b) - y$ is represented in Fig. 1.

As $C < \hat{C}$, i.e., $\beta < 0$, Eq. (4.9) has one solution $y_0 > 0$. As $\beta \ll 1$ we can expand the logarithm and find

$$m_{cr} \sim \lambda \sqrt{\frac{-2N(C - \hat{C})}{48}}. \quad (4.10)$$

The existence of a critical mass⁴ can be understood by considering the classical limit of the solutions (3.7) with $C < \hat{C}$, i.e. Eq. (2.6) with $M < 0$. Also in this case there is a


FIG. 2. Behavior of the singularity curve for $\beta > 0$ both for the shock-wave and $\epsilon < N/48$ cases.

critical mass for the formation of the black hole given by $|M|$. m_{cr} given by Eq. (4.9) can then be interpreted as the analogous of such a classical critical mass.

Turning now to the case $C > \hat{C}$, i.e., $\beta > 0$, we see that Eq. (4.9) has no solution. This is of no surprise, because the corresponding initial static solution is already a black hole in the classical limit. The singularity curve $x^- = 0$ persists until the time $x_1^+ = x_0^+[1 + (\lambda/m)\beta]$. At this point the singularity is given by the critical line (4.4), which again becomes light-like at the end-point of the evaporation process (Fig. 2).

V. CONSTANT ENERGY DENSITY FLUX

In this section a more general flux of matter will be considered, namely an influx of constant energy density $\lambda\epsilon$ starting at $x^+ = x_0^+$. In the Kruskal gauge it is described by the energy-momentum tensor

$$T_{++}^f(x^+) = \frac{\epsilon}{\lambda(x^+)^2} \theta(x^+ - x_0^+). \quad (5.1)$$

The solution to the equations of motion is, from Eq. (4.1),

$$e^{-2\phi} + \frac{Na}{12} \phi = -\lambda^2 x^+ \left(x^- + \frac{\epsilon}{\lambda^3 x_0^+} \right) - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + \frac{\epsilon}{\lambda} \left(1 + \ln \frac{x^+}{x_0^+} \right) + C. \quad (5.2)$$

The analysis of the formation of the black hole proceeds qualitatively as in the previous section. The critical line $e^{-2\phi} = Na/24$ is now described by the curve

$$\alpha = -\lambda^2 x^+ \left(x^- + \frac{\epsilon}{\lambda^3 x_0^+} \right) - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + \frac{\epsilon}{\lambda} \left(1 + \ln \frac{x^+}{x_0^+} \right) + C \quad (5.3)$$

and the critical condition $ds^2 = 0$ along this surface defines the relation

⁴See, for the BPP model, [10].

$$\lambda^2 \left(x^- + \frac{\epsilon}{\lambda^3 x_0^+} \right) + \left(\frac{N}{48} - \frac{\epsilon}{\lambda} \right) \frac{1}{x^+} = 0. \quad (5.4)$$

We can now rewrite Eq. (5.3) using Eq. (5.4) in the form

$$\alpha - C = \frac{N}{48} \left\{ 1 - \ln \left[\left(\frac{N}{48} - \frac{\epsilon}{\lambda} \right) + \frac{\epsilon x^+}{\lambda x_0^+} \right] \right\} + \frac{\epsilon}{\lambda} \ln \frac{x^+}{x_0^+}. \quad (5.5)$$

The case of the ground state solution $C = \hat{C}$ requires

$$\frac{\epsilon}{\lambda} = \frac{\epsilon_{cr}}{\lambda} \equiv \frac{N}{48}. \quad (5.6)$$

Provided we introduce the asymptotic Minkowskian null coordinate $\sigma^+ = (1/\lambda) \ln \lambda x^+$ we find that the threshold for black hole formation is given by an energy flux of the form

$$T_{\sigma^+ \sigma^+}^f = \frac{N \lambda^2}{48}. \quad (5.7)$$

This is nothing but the rate of evaporation of these two-dimensional black holes (see for instance [7]) and this result is quite plausible because we would not expect, on physical grounds, a black hole to form for subcritical fluxes because of the semiclassical Hawking effect. The same result was obtained in the RST model in [5].

To analyze Eq. (5.5) for other values of C we introduce, in order to simplify the expression, the quantities $x \equiv x^+/x_0^+ (> 1)$, ϵ instead of ϵ/λ , $b \equiv N/48$ and $\beta = C - \hat{C}$. We then rewrite Eq. (5.5) as

$$b \ln \left[\frac{\epsilon}{b} x + \left(1 - \frac{\epsilon}{b} \right) \right] - \epsilon \ln x = \beta. \quad (5.8)$$

On the basis of the results for the case $C = \hat{C}$ ($\beta = 0$) we will consider the ‘‘subcritical’’ $\epsilon < b$ and ‘‘supercritical’’ $\epsilon > b$ fluxes separately.

For $\epsilon < b$ the graph of the function $g(x) = b \ln[(\epsilon/b)x + (1 - \epsilon/b)] - \epsilon \ln x$ is represented in Fig. 3. We see that as $\beta < 0$, Eq. (5.8) is never satisfied, which means that with this subcritical flux the black hole is never formed, in complete analogy with the case $\beta = 0$. Turning to $\beta > 0$ we find a rather surprising result: for any values of $\epsilon < N/48$ Hawking radiation is always produced. The singularity curve $x^- = 0$ transforms into a space-like curve at the time given by the condition

$$x_1^+ - x_0^+ \ln \frac{x_1^+}{x_0^+} = x_0^+ \left(1 + \frac{\lambda}{\epsilon} \beta \right) \quad (5.9)$$

and finally it turns out to be light-like at the end-point of Hawking evaporation $x_2^+ \equiv x_0^+ g^{-1}(\beta)$ (see again Fig. 2).

The supercritical flux $\epsilon > b$ gives the function $g(x)$ in Fig. 4. We see clearly that as $\beta < 0$ the black hole forms at the time $x_0^+ g^{-1}(\beta) (> x_0^+)$. In this case there is a critical mass given by $\epsilon \ln g^{-1}(\beta)$. This happened also in the shock-wave scenario analyzed in the previous section and the possible

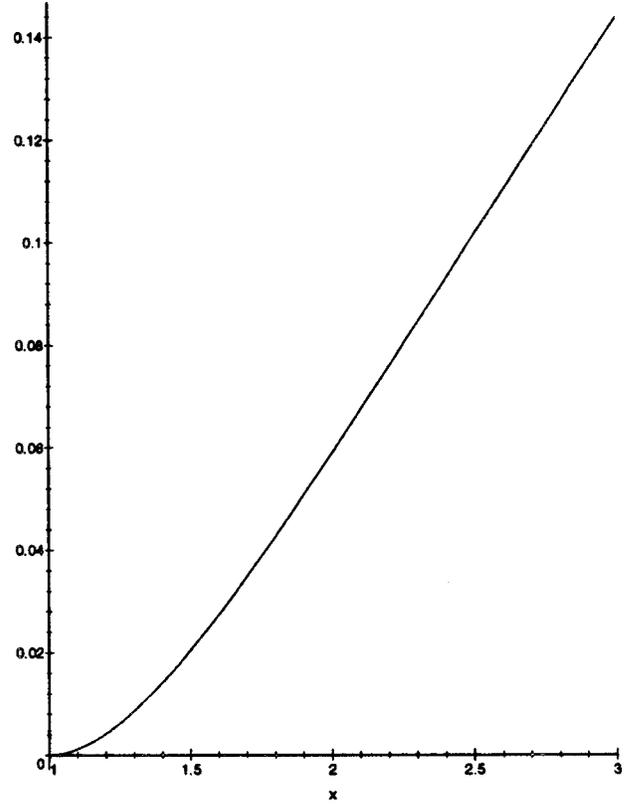


FIG. 3. $g(x) = b \ln[\epsilon/bx + (1 - \epsilon/b)] - \epsilon \ln x$ for $b=1$ and $\epsilon=1/2$ (subcritical flux).

physical interpretation is therefore the same. On the other hand, for $\beta > 0$ the equation $g(x) = \beta$ has no real solution; i.e., the black hole starts to radiate but never disappears.

VI. DISCUSSION AND CONCLUSIONS

We could ask, at this point, whether the results obtained in the last two sections are only specific to the types of infalling matter considered. We can show quite easily that they have instead a rather general validity. In the general case, for infalling fluxes of matter switched on at $x^+ = x_0^+$, the critical line is given by

$$\alpha = -\lambda^2 x^+ \left(x^- + \frac{P(x^+)}{\lambda^2} \right) - \frac{N}{48} \ln(-\lambda^2 x^+ x^-) + \frac{M(x^+)}{\lambda} + C. \quad (6.1)$$

The time at which the singularity becomes null is related to the critical Kruskal momentum $P_{cr}(x^+)$ through the equation

$$\lambda^2 \left(x^- + \frac{P_{cr}(x^+)}{\lambda^2} \right) + \frac{N}{48 x^+} = 0. \quad (6.2)$$

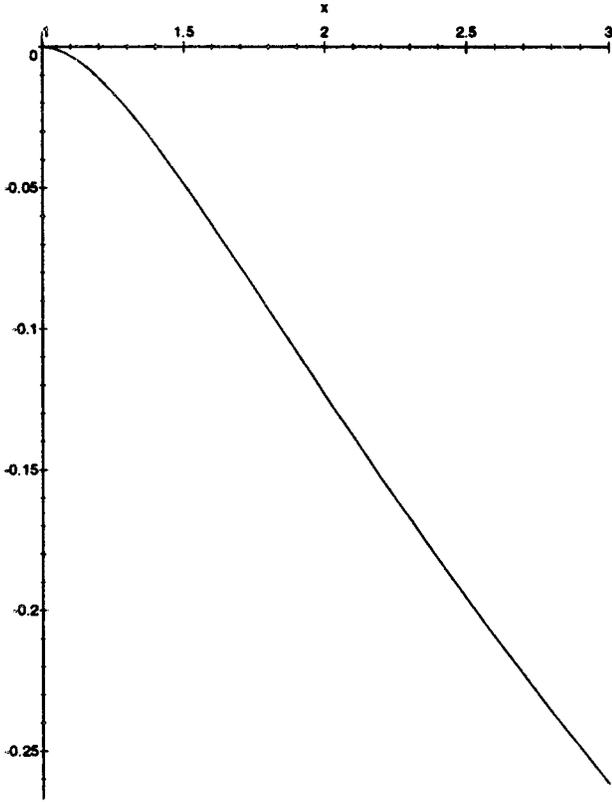


FIG. 4. $g(x)$ as in Fig. 3, but with $b=1$ and $\epsilon=3/2$ (supercritical flux).

Combining the previous two equations and considering the quantities β and b defined in the last section we obtain

$$\beta = b \ln \left[1 + \frac{x^+ P_{cr}(x^+)}{b} \right] - \frac{M_{cr}(x^+)}{\lambda}. \quad (6.3)$$

The function $h(x^+) = b \ln[1 + x^+ P_{cr}(x^+)/b] - M_{cr}(x^+)/\lambda$ is the analogue of $f(y)$ and $g(x)$ considered in Secs. IV and V.

Starting from $h(x_0^+) = 0$ the behavior of this function for $x^+ > x_0^+$ is essentially given by its first derivative

$$h'(x^+) = \frac{P_{cr}(x^+)(1 - x^{+2} T_{++}^{cr}/b)}{1 + x^+ P_{cr}(x^+)/b}. \quad (6.4)$$

Provided that $P_{cr}(x^+) > 0$ (which is always true for classical matter) we easily see that $h'(x^+) < 0$ for $T_{++}^{cr} > N/48x^{+2}$ and $h'(x^+) > 0$ as $T_{++}^{cr} < N/48x^{+2}$. The qualitative behavior of the function $h(x)$ is therefore the same as in Figs. 3 and 4.

We can now summarize the results of our investigation as follows. We have considered initial static geometries parametrized by the continuous parameter C . As $C \leq \hat{C}$, where \hat{C} denotes the ground state solution, there is essentially a threshold on the energy density of the incoming radiation $\epsilon_{cr} = N\lambda^2/48$ given by the Hawking rate of evaporation. For $\epsilon < \epsilon_{cr}$, in fact, it is not possible to form the black hole and as $\epsilon > \epsilon_{cr}$ there is, in addition, also a critical mass (vanishing when $C = \hat{C}$). When $C > \hat{C}$ the static semiclassical solution can be interpreted as a sort of ‘‘black hole’’ in an (unstable) equilibrium state. By sending in a small amount of energy one induces the evaporation process, irrespective of the incoming density flux ϵ and with no critical mass. This is in contrast with the thermal equilibrium black hole solutions which maintain equilibrium even in the presence of incoming matter.

We would like to mention that it could be of interest to study the critical behavior for black hole formation in other solvable models of 2D dilaton gravity with a different thermodynamic [11]. This will be considered in a future publication.

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