

Quantum creation of an open inflationary universe

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We discuss the dramatic difference between the description of the quantum creation of an open universe using the Hartle-Hawking wave function and the tunneling wave function. Recently Hawking and Turok have found that the Hartle-Hawking wave function leads to a universe with $\Omega=0.01$, which is much smaller than the observed value of Ω . Galaxies in such a universe would be 10^{10^8} light years away from each other, and so the universe would be practically structureless. We argue that the Hartle-Hawking wave function does not describe the probability of the creation of the universe. If one uses the tunneling wave function for the description of the creation of the universe, then in most inflationary models the universe should have $\Omega=1$, which agrees with the standard expectation that inflation makes the universe flat. The same result can be obtained in the theory of a self-reproducing inflationary universe, independently of the issue of initial conditions. However, there exist some models where Ω may take any value, from $\Omega>1$ to $\Omega\ll 1$.
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I. INTRODUCTION

Until very recently it was believed that the universe after inflation must become extremely flat, with $\Omega = 1 \pm 10^{-4}$. This implied that if observational data show that Ω differs from 1 by more than a fraction of a percent, inflationary theory should be ruled out. Of course, it was always possible to make inflation short and Ω different from 1 by fine-tuning the parameters, but in this case the problems of homogeneity and isotropy of the observable part of the universe would remain unsolved.

Fortunately, this problem was solved recently. The main idea is to use the well-known fact that the region of space created in the process of quantum tunneling tends to have spherically symmetric shape and a homogeneous interior, if the tunneling probability is suppressed strongly enough. Then such bubbles of the new phase tend to expand in a spherically symmetric fashion. Thus, if one could associate the whole visible part of the universe with an interior of one such region, one would solve the homogeneity and isotropy problems, and then all other problems would be solved by the subsequent relatively short stage of inflation.

For a closed universe the realization of this program could be relatively straightforward [1]. One should consider the process of the quantum creation of a closed inflationary universe from “nothing.” If the probability of such a process is exponentially suppressed (and this is indeed the case if inflation is possible only at an energy density much smaller than the Planck density [2–5]), then the universe created that way will be rather homogeneous from the very beginning. Typically it will grow exponentially large, and Ω will gradually approach the flat-space limit $\Omega=1$. However, there exist many inflationary models where the total duration of inflation cannot be longer than 60–70 e -foldings. In such models the present value of Ω can be noticeably higher than 1. These models have several potential drawbacks which will be discussed in the last section of this paper, but nevertheless creation of a closed inflationary universe, at least in principle, does not seem impossible.

The situation with an open universe is much more complicated. Indeed, an open universe is infinite, and it may seem impossible to create an infinite universe by a tunneling process. However, this is not the case: according to Coleman and De Luccia, any bubble formed in the process of the false vacuum decay looks from inside like an infinite open universe [6,7]. If this universe continues inflating inside the bubble, then we obtain an open inflationary universe.

Until a short while ago it was not quite clear whether it is possible to realize the one-bubble open universe scenario in a natural way. An important step in this direction was made when the first semirealistic models of open inflation were proposed [8]. These models were based on the investigation of chaotic inflation and tunneling in the theories of one scalar field ϕ . However, as was shown in [1], in the simplest versions of such theories with potentials of the type of $(m^2/2)\phi^2 - (\delta/3)\phi^3 + (\lambda/4)\phi^4$ the tunneling does not occur by bubble formation, but by jumping onto the top of the potential barrier described by the Hawking-Moss instanton [9]. This instanton was originally interpreted as describing homogeneous tunneling, but later it was found that this is not the case [10–14]. This process leads to the formation of inhomogeneous domains of a new phase, and the whole scenario fails.

This problem is in fact rather general; it appears not only in the models with the potential $(m^2/2)\phi^2 - (\delta/3)\phi^3 + (\lambda/4)\phi^4$. Indeed, Coleman–De Luccia instantons by their construction must be smaller than the size of the Euclidean continuation of de Sitter space, H^{-1} . Meanwhile, the typical size of a bubble is of the same order as the inverse mass of the field ϕ , which can be estimated as $1/\sqrt{V''(\phi)}$. This implies that these instantons can exist only if $V''(\phi) \gg H^2$ inside the bubble. This condition is incompatible with the assumption of Ref. [8], that inflation continues after the tunneling, which would require that $V''(\phi) \ll H^2$ inside the bubble.

In order to resolve this problem one is forced to “bend” the effective potentials in a rather specific way. The potential must be very flat everywhere except at one place where it should have a very deep minimum and a sharp maximum. In

addition, one should consider models where inflation inside the bubble begins not immediately after the tunneling, but much later. These requirements make the corresponding models of open inflation not only fine-tuned but also very complicated. No realistic versions of open inflation models of this type have been invented so far.

Fortunately, the same goal can be achieved if one considers models of two scalar fields [1]. The presence of two scalar fields allows one to obtain the required bending of the inflaton potential by simply changing the definition of the inflaton field in the process of inflation. The tunneling occurs with respect to a heavy field σ with a steep barrier in its potential, while after the tunneling the role of the inflaton is played by a light field ϕ , rolling along a flat direction “orthogonal” to the direction of quantum tunneling. Inflationary models of this type are quite simple, yet they have many interesting features. In these models the universe consists of infinitely many expanding bubbles immersed into an exponentially expanding false vacuum state. Each of these bubbles on the inside looks like an infinitely large open universe, but the values of Ω in these universes may take any value from 1 to 0.

Many versions of these two-field models have been considered in the recent literature; see, e.g., [1,15,16]. Some of them did not survive comparison with the observational data, and some of them are very fine-tuned, but in any case one can no longer claim that inflation and an open universe are incompatible. The simplest open inflationary model of this type describes two scalar fields with the effective potential

$$V(\phi, \sigma) = \frac{g^2}{2} \phi^2 \sigma^2 + V(\sigma), \quad (1)$$

where the effective potential for the field σ can be taken, e.g., in the following form: $V(\sigma) = (M^2/2)\sigma^2 - \alpha M\sigma^3 + (\lambda/4)\sigma^4 + V_0$ [1]. Here V_0 is a constant which is added to ensure that $V(\phi, \sigma) = 0$ at the absolute minimum of $V(\phi, \sigma)$. If the initial value of the field ϕ is sufficiently large, then the field σ is trapped at $\sigma = 0$. The field ϕ slowly drifts in different directions due to inflationary quantum fluctuations, and in the regions where it becomes smaller than a certain critical value ϕ_c , the phase transition to large σ becomes possible. Inside the bubbles of the field σ the field ϕ acquires nonvanishing mass squared $g^2\sigma^2$; it begins to slide towards $\phi = 0$ and yields the secondary stage of inflation. Depending on the initial value of the field ϕ , this stage may be either short, creating open universes with small Ω , or long, creating universes with $\Omega \approx 1$. If the probability of bubble production is very small, the vacuum state with $\sigma = 0$ will never completely decay, and the process of the creation of new bubbles will never end. This implies that in an eternally existing self-reproducing universe based on this scenario there will be infinitely many universes containing any particular value of Ω , from $\Omega = 0$ to $\Omega = 1$. Moreover, the effective value of Ω in this scenario may vary even within each of the bubbles [17].

An intriguing possibility which will be discussed in this paper is the quantum creation of an open universe from nothing. Until very recently such a process seemed impossible.

Indeed, in accordance with the investigation of inflationary universe creation performed in [2–5], the probability of the quantum creation of an inflationary universe is expected to be suppressed by $e^{-|2S|}$, where S is the value of Euclidean action on the trajectory describing the universe creation. For a closed universe with vacuum energy $V(\phi, \sigma)$ one has

$$P \sim e^{-2|S|} = \exp\left(-\frac{3M_p^4}{8V(\phi, \sigma)}\right). \quad (2)$$

One could expect that the action S on an instanton describing the creation of an infinitely large open universe must be infinitely large. Hence one would not expect that an open universe can be created unless it is topologically nontrivial and compact [18]. However, this problem disappears in the new class of open universe models considered above. The probability of the quantum creation of a closed inflationary universe is finite. After its creation it inflates and becomes flat and practically infinite. In the scenario described above, it unceasingly produces more and more bubbles, each of which represents a new infinite open universe. Thus, in this scenario one does not encounter any problems in creating an open universe from nothing. In fact one does not create a single open universe but infinitely many of them, with different values of Ω in each of the universes [19].

Recently the possibility of the quantum creation of an open universe was pursued even further in a paper by Hawking and Turok [20]. They argued that an open universe can be created from nothing even without passing through an intermediate stage of false vacuum inflation and subsequent tunneling. According to [20], this regime is possible in the theories of a single field ϕ with the simplest potentials of the chaotic inflation type [21]. This would be a very interesting and encouraging development. However, Hawking and Turok used the Hartle-Hawking wave function of the universe [22] to describe the probability of the creation of an open universe. As a result, they experienced severe problems usually associated with the description of the universe creation in the context of the Hartle-Hawking approach. Typical universes produced by the process described in [20] tend to be not only open, but entirely empty, $\Omega \rightarrow 0$. The only way to avoid this disastrous conclusion is to use the anthropic principle and argue that we live in a universe with small Ω simply because we cannot live in the universe with $\Omega = 0$. But even this does not help much. Estimates made in [20] show that the maximum of the probability to live in an open universe is sharply peaked at $\Omega = 0.01$, which does not agree with the observational data.

In this paper we will show that this result is practically model independent, and it appears solely due to the use of the Hartle-Hawking wave function. This wave function gives the probability of the universe creation of a very peculiar form,

$$P \sim e^{-2S} = \exp\left(\frac{3M_p^4}{8V(\phi)}\right), \quad (3)$$

which strongly disfavors inflation of any kind and suggests that it is much easier to create an infinite Minkowski space

rather than a Planckian-sized closed universe. The difference between Eqs. (2) and (3) appears due to the “wrong” sign of the gravitational action of the instanton describing creation of a de Sitter universe, $S = -3M_p^4/16V(\phi)$.

As was argued in [2,13,23], the Hartle-Hawking wave function does not describe the probability of the universe creation. Rather, it describes the probability of quantum fluctuations in a universe which has already been born. In particular, the probability distribution (3) implies that the universe in its ground state resides near the minimum of the effective potential, and the probability of its deviations from this state is exponentially small.

Meanwhile the essence of inflationary theory is that initially the universe could be very far from the minimum of $V(\phi)$. It takes a lot of time for the field to roll to this minimum, and during this time the universe becomes exponentially large. Thus, in our opinion, the tunneling wave function makes an attempt to describe the creation of an inflationary universe and inflationary theory tells us how the universe approached the minimum of $V(\phi)$, whereas the Hartle-Hawking wave function describes properties of the universe after it reaches its ground state, in the case where such a ground state exists.

To clarify this issue, in Sec. II of this paper we will recall the history of the debate related to the choice of the Hartle-Hawking versus the tunneling wave function. In Sec. III we will analyze this issue again, using the stochastic approach to inflation. This will allow one to have a better understanding of different approaches to the calculation of the most probable value of Ω in the context of quantum cosmology.

Then in Sec. IV we will discuss the properties of the Hawking-Turok instanton and the probability of an open universe creation. We will explain the origin of the result $\Omega \sim 0.01$, and show that this number practically does not depend on the choice of a particular inflationary model. We will also argue that if one applies the Hartle-Hawking approach to the creation of the universe, then this result will endanger all previous versions of the open universe scenario [7,8,1,15,16].

We will, however, show that if one uses the tunneling wave function of the universe for the description of the creation of the universe [2–5], a typical universe to be created in the simplest versions of the chaotic inflation scenario with polynomial potentials will have $\Omega = 1$, rather than $\Omega = 0.01$. This result is in agreement with the usual expectation that inflation typically leads to $\Omega = 1$. However, there exist several versions of the chaotic inflation scenario discussed in [1] and one recently proposed version of the hybrid inflation scenario in supergravity [24], where the typical duration of inflation is very small. In such models the most probable value of Ω can take any value between 1 and 0 depending on the parameters of the model, without any need to appeal to the anthropic principle. These models, however, have serious problems of their own, which require further investigation. If the mechanism which we will discuss is successful, we will have a new class of open inflationary models.

In Sec. V we will describe some problems with the more recent proposal of Hawking and Turok related to the theory

of the four form field strength [25]. We will argue that the unfortunate prediction $\Omega = 0.01$ appears in this theory as well.

Independently of the success or failure of the new class of models of open inflation, we will show that the use of the tunneling wave function for the description of the universe creation preserves the validity of the previous models of open inflation, proposed in [7,8,1,15,16]. Moreover, we will argue that the models of open inflation proposed in [7,8,1,15,16] remain valid independently of the choice of the wave function describing initial conditions if one takes into account the possibility of eternal inflation in these models.

II. WAVE FUNCTION OF THE UNIVERSE

A. Why do we need quantum cosmology?

The investigation of the wave function of the universe goes back to the fundamental papers by Wheeler and DeWitt [26]. However, for a long time it seemed almost meaningless to apply the notion of the wave function to the universe itself, since the universe is not a microscopic object. Only with the development of inflationary cosmology did it become clear that the whole universe could appear from a tiny part of space as small as the Planck length M_p^{-1} (at least in the chaotic inflation scenario [21]). Such a tiny region of space can appear as a result of quantum fluctuations of metric, which should be studied in the context of quantum cosmology. Later it was found that the global structure of the universe in the chaotic inflation scenario is determined not by classical physics, but by quantum processes [13].

Unfortunately, quantum cosmology is not a well developed science. This theory is based on the Wheeler-DeWitt equation, which is the Schrödinger equation for the wave function of the universe. This equation has many solutions, and at the present time the best method to specify preferable solutions of this equation, as well as to interpret them, is based on the Euclidean approach to quantum gravity. This method is very powerful, but some of its applications are not well justified. In some cases this method may give incorrect answers, but rather paradoxically sometimes these answers appear to be correct when applied to some other questions. Therefore it becomes necessary not only to solve the problem in the Euclidean approach, but also to check, using one’s best judgement, whether the solution is related to the original problem or to something else. An alternative approach is based on the use of stochastic methods in inflationary cosmology [10–13,23]. These methods allow one to understand such effects as the creation of inflationary density perturbations, the theory of tunneling, and even the theory of the self-reproduction of an inflationary universe. Both the Euclidean approach and stochastic approach to inflation have their limitations, and it is important to understand them.

B. Hawking-Moss tunneling

Before discussing the quantum creation of the universe, let us pause a little and study the problem of tunneling between two local minima of the effective potential $V(\phi)$ in

inflationary cosmology. As we will see, this subject is closely related to the issue of the quantum creation of the universe.

Consider a theory with an effective potential $V(\phi)$ which has a local minimum at ϕ_0 , a global minimum at ϕ_* , and a barrier separating these two minima, with the top of the barrier positioned at $\phi = \phi_1$. One of the first works on inflationary cosmology was the paper by Hawking and Moss [9] where they studied the possibility of tunneling from ϕ_0 to ϕ_* in the new inflationary universe scenario.

They have written equations of motion for the scalar field in an Euclidean space with the metric

$$ds^2 = d\tau^2 + a^2(\tau)(d\psi^2 + \sin^2\psi d\Omega_2^2). \quad (4)$$

The field ϕ and the radius a obey the field equations

$$\phi'' + 3\frac{a'}{a}\phi' = V_{,\phi}, \quad a'' = -\frac{8\pi G}{3}a(\phi'^2 + V), \quad (5)$$

where primes denote derivatives with respect to τ .

If the potential has an extremum at some particular value of the field ϕ , then the equation for the field ϕ is solved trivially by the field staying at this extremum. Then the equation for $a(\tau)$ has a simple solution $a(\tau) = H^{-1}\sin(H\tau)$, with $H^2 = 8\pi G V(\phi)/3 = 8\pi V(\phi)/3M_p^2$. This solution describes a sphere S^4 , the Euclidean version of de Sitter space. In this description τ plays the role of Euclidean time and $a(\tau)$ the role of the scale factor. One can try to interpret one half of this sphere as an instanton. The action on this instanton is negative:

$$S = \int d^4x \sqrt{-g} \left(-\frac{RM_p^2}{16\pi} + V(\phi) \right) = -\frac{3M_p^4}{16V(\phi)}. \quad (6)$$

It was argued in [9] that the probability of tunneling from ϕ_0 to the true vacuum ϕ_* is given by

$$P \sim \exp\left(\frac{3M_p^4}{8V(\phi_1)}\right) \exp\left(-\frac{3M_p^4}{8V(\phi_0)}\right). \quad (7)$$

The probability of tunneling, as usual, is suppressed by e^{-2S} [or by e^{-S} if by S we mean the result of integration over the whole sphere, $-3M_p^4/8V(\phi)$]. This is the standard result of the Euclidean theory of tunneling. Everything else about this result was rather mysterious.

First of all, instantons typically interpolate between the initial vacuum state and the final state. Here, however, the scalar field on the instanton solution was exactly constant. So why do we think that they describe tunneling from ϕ_0 if ϕ_0 never appears in the instanton solution?

A possible answer to this question can be given as follows. One can choose the coordinate system where the inflationary universe looks as a closed de Sitter space near the point of a maximal contraction, where its size becomes $H^{-1}(\phi_0)$; see region 1 in Fig. 1. Classically, such a universe at that moment begins expanding with the same value of the Hubble constant as before. However, since the total size of the universe at that moment is finite, it may also jump quan-

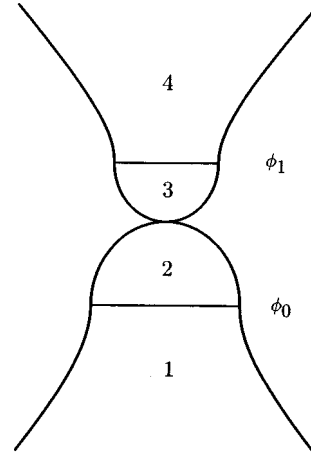


FIG. 1. A possible interpretation of the Hawking-Moss tunneling from ϕ_0 to ϕ_1 .

tum mechanically to a state with a different value of the field ϕ corresponding to a different extremum of the effective potential. One can represent this process by gluing two de Sitter instantons corresponding to two different values of the scalar field ϕ (ϕ_0 in region 2 and ϕ_1 in region 3 in Fig. 1) and by making analytical continuation to the Lorentzian regions 1 and 4.

This seems to be a plausible interpretation of the Hawking-Moss tunneling (see also [27]). But it certainly does not answer all questions. What will happen if we have several different local minima and maxima of $V(\phi)$? Why does the tunneling go to the top of the effective potential rather than to the absolute minimum of the effective potential, or to some other local maximum? Finally, if the instanton describes an exactly homogeneous scalar field ϕ , does it mean that the tunneling must simultaneously occur everywhere in an exponentially large inflationary universe? This does not seem plausible, but what else should we think about, if the field ϕ on the instanton solution is constant?

And, indeed, originally it was assumed that the tunneling described by this instanton must occur simultaneously in the whole universe. Then, in the second paper on this subject, Hawking and Moss said that their results were widely misunderstood, and that this instanton describes tunneling which is homogeneous only on the scale of horizon $\sim H^{-1}$ [28]. But how is it possible to describe inhomogeneous tunneling by a homogeneous instanton?

A part of the answer was given in Ref. [11]. We have found that if one deforms a little the Hawking-Moss instanton to make the field ϕ match ϕ_0 in some small region of the sphere, we will, strictly speaking, not get a solution, but an action on such a configuration can be made almost exactly coinciding with the Hawking-Moss action. Then such configurations can play the same role as instantons [29].

A full understanding of this issue was reached only after the development of the stochastic approach to inflation [10–13]. We will return to this question later.

C. Creation of the universe from nothing

Now we will discuss the problem of the universe creation. According to classical cosmology, the universe appeared

from the singularity in a state of infinite density. Of course, when the density was greater than the Planck density M_p^4 one could not trust the classical Einstein equations, but in many cases there is no demonstrated need to study the universe creation using the methods of quantum theory. For example, in the simplest versions of the chaotic inflation scenario [21], the process of inflation, at the classical level, could begin directly in the initial singularity. However, in certain models, such as the Starobinsky model [30] or the new inflationary universe scenario [31], inflation cannot start in a state of infinite density. In such cases one may speculate about the possibility that the inflationary universe appears due to quantum tunneling “from nothing.”

The first idea as to how one can describe the creation of an inflationary universe “from nothing” was given in 1981 by Zeldovich [32] in application to the Starobinsky model [30]. His idea was qualitatively correct, but he did not propose any quantitative description of this process. A very important step in this direction was made in 1982 by Vilenkin [33]. He suggested to calculate the Euclidean action on de Sitter space with the energy density $V(\phi)$, which coincides with the Hawking-Moss instanton with the action $S = -3M_p^4/16V(\phi)$. However, as we have seen, this instanton by itself does not tell us where the tunneling comes from. Vilenkin suggested to interpret this instanton as the tunneling trajectory describing the creation of the universe with the scale factor $a = H^{-1} = \sqrt{3M_p^2/8\pi V}$ from the state with $a = 0$. This would imply that the probability of the quantum creation of the universe is given by

$$P \propto \exp(-2S) = \exp\left(\frac{3M_p^4}{8V(\phi)}\right). \quad (8)$$

A year later this result received strong support when Hartle and Hawking reproduced it by a different though closely related method [22]. They argued that the wave function of the “ground state” of the universe with a scale factor a filled with a scalar field ϕ in the semiclassical approximation is given by

$$\Psi_0(a, \phi) \sim \exp[-S(a, \phi)]. \quad (9)$$

Here $S(a, \phi)$ is the Euclidean action corresponding to the Euclidean solutions of the Lagrange equation for $a(\tau)$ and $\phi(\tau)$ with the boundary conditions $a(0) = a$, $\phi(0) = \phi$. The reason for choosing this particular wave function was explained as follows. Let us consider the Green’s function of a particle which moves from point $(0, t')$ to point \mathbf{x}, t :

$$\begin{aligned} \langle \mathbf{x}, t | 0, t' \rangle &= \sum_n \Psi_n(\mathbf{x}) \Psi_n(0) \exp[iE_n(t-t')] \\ &= \int d\mathbf{x}(t) \exp[iS(\mathbf{x}(t))], \end{aligned} \quad (10)$$

where Ψ_n is a complete set of energy eigenstates corresponding to the energies $E_n \geq 0$.

To obtain an expression for the ground-state wave function $\Psi_0(\mathbf{x})$, one should make a rotation $t \rightarrow -i\tau$ and take the

limit as $\tau \rightarrow -\infty$. In the summation (10) only the term $n = 0$ with the lowest eigenvalue $E_0 = 0$ survives, and the integral transforms into $\int d\mathbf{x}(\tau) \exp[-S(\mathbf{x}(\tau))]$. This yields, in the semiclassical approximation,

$$\Psi_0(x) \sim \exp[-S(\mathbf{x})], \quad (11)$$

where the action is taken on the classical trajectory bringing the particle to the point \mathbf{x} . Hartle and Hawking have argued that the generalization of this result to the case of interest would yield Eq. (9).

The method described above is very powerful. For example, it provides the simplest way to find the wave function of the ground state of the harmonic oscillator in quantum mechanics. However, this wave function simply describes the probability of deviations of the harmonic oscillator from its equilibrium. It certainly does not describe the quantum creation of a harmonic oscillator. Similarly, if one applies this method to the hydrogen atom, one can obtain the wave function of an electron in the state with the lowest energy. Again, this result has no relation to the probability of the creation of an electron from nothing.

The gravitational action involved in Eq. (9) is the same action as before, corresponding to one-half of the Euclidean section S_4 of de Sitter space with $a(\tau) = H^{-1}(\phi) \cos H\tau$ ($0 \leq \tau \leq H^{-1}$). One can represent it in the following form:

$$\begin{aligned} S(a, \phi) &= -\frac{3\pi M_p^2}{4} \int d\eta \left[\left(\frac{da}{d\eta} \right)^2 - a^2 + \frac{8\pi V}{3M_p^2} a^4 \right] \\ &= -\frac{3M_p^4}{16V(\phi)}. \end{aligned} \quad (12)$$

Here η is the conformal time, $\eta = \int d\tau/a(\tau)$. Therefore, according to [22],

$$\Psi_0(a, \phi) \sim \exp[-S(a, \phi)] \sim \exp\left(\frac{3M_p^4}{16V(\phi)}\right). \quad (13)$$

By taking a square of this wave function one again obtains Eq. (8). The corresponding expression has a very sharp maximum as $V(\phi) \rightarrow 0$. This could suggest that the probability of finding the universe in a state with a large field ϕ and having a long stage of inflation should be strongly suppressed. But is it a correct interpretation of the Hartle-Hawking wave function? Just like in the examples with the harmonic oscillator and the hydrogen atom mentioned above, nothing in the “derivation” of the Hartle-Hawking wave function tells us that it describes the creation of the universe from nothing. The simplest way to interpret the Hartle-Hawking wave function in application to de Sitter space is as follows. At the classical level, de Sitter space has a definite speed of expansion, definite size of its throat H^{-1} , etc. At the quantum level, the de Sitter “trajectory” becomes wider because of quantum fluctuations. The Hartle-Hawking wave function of de Sitter space describes the probability of deviations of the metric of de Sitter space from its classical expectation value, which may occur due to the process shown

in Fig. 1. This is very much different from the probability of the spontaneous creation of the universe.

In fact, Eq. (8) from the very beginning did not seem to apply to the probability of the creation of the universe. The total energy of matter in a closed de Sitter space with $a(t) = H^{-1} \cosh Ht$ is greater than its minimal volume $\sim H^{-3}$ multiplied by $V(\phi)$, which gives the total energy of the universe $E \gtrsim M_p^3 / \sqrt{V}$. Thus the minimal value of the total energy of matter contained in a closed de Sitter universe *grows* when V decreases. For example, in order to create the universe at the Planck density $V \sim M_p^4$ one needs no more than the Planckian energy $M_p \sim 10^{-5}$ g. For the universe to appear at the grand unified theory (GUT) energy density $V \sim M_X^4$ one needs to create from nothing the universe with the total energy of matter of the order of $M_{\text{Schwarzenegger}} \sim 10^2$ kg, which is obviously much more difficult. Meanwhile, if one makes an attempt to use the Hartle-Hawking wave function for the description of the creation of the universe (which, as we believe, does not follow from its derivation), then Eq. (8) suggests that it should be much easier to create a huge universe with enormously large total mass rather than a small universe with Planckian mass. This seems very suspicious.

From uncertainty relations one can expect that the probability of a process of universe formation is not exponentially suppressed if it occurs within a time $\Delta t < E^{-1}$. This is quite possible if the effective potential is of the order of M_p^4 and $E \sim M_p^3 / \sqrt{V} \sim M_p$. In such a case one may envisage the process of the quantum creation of a universe of mass M_p within the Planck time M_p^{-1} . However, a universe of mass $E \gg M_p$ (which is the case for $V \ll M_p^4$) can be created only if the corresponding process lasts much shorter than the Planck time M_p^{-1} , which is hardly possible.

Another way to look at it is to calculate the total entropy \mathbf{S} of de Sitter space at the moment of its creation. It is equal to one quarter of the horizon area of de Sitter space (in Planck units), which gives $\mathbf{S} = 3M_p^4 / 8V(\phi)$. [Note its relation to the Euclidean action on the full de Sitter sphere $S = -3M_p^4 / 8V(\phi)$.] It seems natural to expect that the probability of the emergence of a complicated object of large entropy must be suppressed by a factor of $\exp(-\mathbf{S}) = \exp[-3M_p^4 / 8V(\phi)]$, which again brings us to Eq. (2); see [34]. Meanwhile the use of the Hartle-Hawking wave function for a description of the creation of the universe would indicate that it is much more probable to create a very large universe with a huge entropy rather than a small universe with entropy $O(1)$.

To avoid misunderstandings, one should note that the probability of fluctuations in a thermodynamical system is always *suppressed* by the factor $e^{\Delta \mathbf{S}}$, where $\Delta \mathbf{S}$ is the change of entropy between two different states of the system [35]. As we will see, this is exactly what happens during the tunneling between two different states of de Sitter space with two different values of $V(\phi)$. This is in perfect agreement with the prediction of the Hartle-Hawking wave function if one applies it not to the creation of the universe but to the probability of its change. However, now we are not talking about the probability of a change of the state of the system, but about the possibility of the creation of the whole system

together with a lot of information stored in it from nothing. We are not going to insist that this process be possible. In fact in the chaotic inflation scenario this assumption is not necessary because the universe formally can inflate even in a state with an indefinitely large density; so there is no need for any tunneling to take place. However, if creation from nothing is possible at all, then the tunneling wave function suggests that this process should be as unintrusive as possible, whereas the Hartle-Hawking approach implies that the greater the change, the easier it occurs. I leave it for the reader to decide whether this looks plausible.

One may wonder why the Hartle-Hawking wave function leads to rather counterintuitive predictions when applied to the probability of the creation of the universe? There is one obvious place where the derivation (or interpretation) of Eq. (8) could go wrong. The effective Lagrangian of the scale factor a in Eq. (12) has a wrong overall sign. Solutions of the Lagrange equations do not know anything about the sign of the Lagrangian; so we may simply change the sign before studying the tunneling. Only after switching the sign of the Lagrangian of the scale factor in Eq. (12) and representing the theory in a conventional form can we consider tunneling of the scale factor. But after changing the sign of the action, one obtains a different expression for the probability of quantum creation of the universe:

$$P \propto \exp(-2|S|) = \exp\left(-\frac{3M_p^4}{8V(\phi)}\right). \quad (14)$$

This equation predicts that a typical initial value of the field ϕ is given by $V(\phi) \sim M_p^4$ [if one does not speculate about the possibility that $V(\phi) \gg M_p^4$], which leads to a very long stage of inflation.

Originally I obtained this result by the method described above. However, because of the ambiguity of the notion of tunneling from the state $a=0$, one may try to look at the same subject from a different perspective, and reexamine the derivation of the Hartle-Hawking wave function. In this case the problem of the wrong sign of the Lagrangian appears again, though in a somewhat different form. Indeed, the total energy of a closed universe is zero, being a sum of the positive energy of matter and the negative energy of the scale factor a . Thus, the energy E_n of the scale factor is negative. If one makes the same Euclidean rotation as in Eq. (10), the contributions of all states with $n > 1$ will be greater than the contribution of the state with the lowest absolute value of energy; so such a rotation would not allow one to extract the wave function Ψ_0 as we did before. This is a simple mathematical fact, which means that the main argument used in [22] to justify their prescription of the quantization of the scale factor fails.

In order to suppress terms with large negative E_n and to obtain Ψ_0 from Eq. (10) one should rotate t not to $-i\tau$, but to $+i\tau$. This gives [2]

$$\Psi_0(a, \phi) \sim \exp[-|S(a, \phi)|] \sim \exp\left(-\frac{3M_p^4}{16V(\phi)}\right) \quad (15)$$

and

$$P(\phi) \sim |\Psi_0(a, \phi)|^2 \sim \exp\left(-\frac{3M_p^4}{8V(\phi)}\right). \quad (16)$$

Later this equation was also derived by Zeldovich and Starobinsky [3], Rubakov [4], and Vilenkin [5] using methods similar to the first method mentioned above (switching the sign of the Lagrangian). The corresponding wave function (15) was called ‘‘the tunneling wave function.’’ This wave function¹ is dramatically different from the Hartle-Hawking wave function [22], as well as from the Vilenkin’s wave function proposed few years earlier [33].

An obvious objection against this result is that it may be incorrect to use different ways of rotating t for the quantization of the scale factor and of the scalar field; see, e.g., [35]. If one makes the same rotation for the matter fields as the rotation which we proposed for the scale factor, then one may encounter catastrophic particle production and other equally unpleasant consequences. On the other hand, as we have seen, if one assumes without any proof that it is enough to make the standard Wick rotation to quantize the scale factor, one does not obtain the wave function of the ground state Ψ_0 , and one gets the counterintuitive result that large universes are created much easier than the small ones.

We believe that the problem here goes far beyond the issue of the Wick rotation. The idea that a consistent quantization of an unstable system of matter with positive energy density coupled to gravity with negative energy density can be accomplished by a proper choice of a complex contour of integration may be too optimistic. We know, for example, that despite many attempts to develop a Euclidean formulation of nonequilibrium quantum statistics or of the field theory in a nonstationary background, such a formulation still does not exist. It is quite clear from Eq. (10) that the $t \rightarrow -i\tau$ trick does not give us the ground-state wave function Ψ_0 if the spectrum E_n is not bounded from below. The absence of equilibrium, of any simple stationary ground state, seems to be a typical situation in quantum cosmology. A closely related instability is the basis of inflationary cosmology, where exponentially growing total energy of the scalar field appears as a result of pumping energy from the gravitational field, whereas the total energy of matter plus gravitational field remains zero.

Fortunately, in certain limiting cases this issue can be resolved in a relatively simple way. For example, at present the scale factor a is very big and it changes very slowly; so one can consider it as a classical background, and quantize only the usual matter fields with positive energy. In this case one should use the standard Wick rotation $t \rightarrow -i\tau$. On the other hand, in an inflationary universe the evolution of the scalar field is very slow; during typical time intervals $O(H^{-1})$, it behaves essentially as a classical field. Thus to a

good approximation one can describe the process of the creation of an inflationary universe filled with a homogeneous scalar field by the quantization of the scale factor a only and by the rotation $t \rightarrow i\tau$. When using the tunneling wave function, for example, for the description of particle creation in de Sitter space, instead of introducing a universal rule for the Wick rotation one should operate in a more delicate way, treating separately the scale factor and the particle excitations; see, e.g., [37].

Similarly, one should not use the Hartle-Hawking wave function for the description of the creation of an inflationary universe, but one can use it for the investigation of fluctuations of this background. These fluctuations are local, and often they appear simply as a result of quantum fluctuations of matter fields having positive energy. In particular, long-wavelength fluctuations of the scalar field ϕ in inflationary universe may change the local value of the energy density $V(\phi)$ inside domains of a size greater than the size of the event horizon H^{-1} . For a comoving observer, such a change looks like a homogeneous change of the scalar field ϕ and of the Hubble constant $H(\phi)$; so he might want to (erroneously) interpret it as a result of quantum fluctuations of the scale factor. These are local perturbations of the homogeneous classical background. These perturbations are produced by fields with positive energy. Therefore in all situations where the inflationary background changes slowly (and in this sense can be considered a ground state of the system) one can use the Hartle-Hawking wave function for the investigation of fluctuations of this background.

For example, the Hartle-Hawking wave function can be used for the description of black hole formation in a preexisting de Sitter background [38]. But this method should not be used for the description of the quantum creation of de Sitter space with a pair of black holes in it.

One can also obtain the amplitude of the density perturbation in an inflationary universe by a rather complicated method using the Hartle-Hawking wave function [39]. However, the same results for density perturbations can be obtained by assuming that the inflationary universe was created from nothing in accordance with the tunneling wave function, and then it expanded and produced perturbations in accordance to [40]. Moreover, as we already mentioned, in chaotic inflation there is no need to assume that any process of tunneling ever took place in the early universe. One may simply assume that the universe from the very beginning expanded classically, and then obtain the same results for the density perturbations using methods of Ref. [40].

The derivation of Eqs. (8),(16) and their interpretation is far from being rigorous, and therefore even now it remains a subject of debate. From time to time this issue attracts a lot of attention. For example, the famous proposal to solve the cosmological constant problem in the context of the baby universe theory, which was very popular ten years ago, was based entirely on the use of the wrong sign of the de Sitter action in the Hartle-Hawking approach to quantum gravity [41,42]. One of the main authors of this proposal, Coleman, emphasized: ‘‘The euclidean formulation of gravity is not a subject with firm foundations and clear rules of procedure; indeed it is more like a trackless swamp. I think that I have

¹In fact, the two different ‘‘derivations’’ of this wave function described above lead to two slightly different wave functions [36]. However, since the difference between these two versions of the tunneling wave function is exponentially small, we will neglect it in this paper.

threaded my way through it safely, but it is always possible that unknown to myself I am up to my neck in quicksand and sinking fast'' [42]. After two years of intensive investigation of this issue it became clear that the wrong sign of the Euclidean action can hardly provide a reliable explanation for the vanishing of the cosmological constant. Moreover, recent observational data indicate that the cosmological constant may not vanish after all.

To summarize, the derivation of the Hartle-Hawking wave function is rather ambiguous. Still, our main objection with respect to this wave function is related not to its derivation, but rather to its interpretation. The main purpose of the paper by Hartle and Hawking [22] was to find the wave function describing the least excited, stationary state of the gravitational system, which would be analogous to the ground state on the harmonic oscillator or of the hydrogen atom. And indeed it gives a nice description of quantum fluctuations near a de Sitter background, which in a certain sense is stationary. (There is a coordinate system where de Sitter space is static.) In such a situation one can consider matter fluctuations, and then find fluctuations of the scale factor induced by the fluctuations of matter. Then the problem of the negative energy of the scale factor does not arise, and one can use the Hartle-Hawking wave function to study fluctuations in the preexisting background. However, we do not see anything in the "derivation" of the Hartle-Hawking wave function which would indicate that it can be used for investigation of the probability of the quantum creation of the universe.

The tunneling wave function also has certain limitations, but it seems to have a better chance to describe the process of the quantum creation of the universe. In the subsequent discussion an exact form of this wave function will not be important for us. The only property of this wave function which we are going to use is that the quantum creation of the universe should not be strongly suppressed if it can be achieved by fluctuations of the metric on the Planck scale M_p^{-1} at the Planck density M_p^4 .

Since the debate concerning the wave function of the universe has continued for the last 15 years, it may be useful to look at it from a somewhat different perspective, which does not involve a discussion of the ambiguities of Euclidean quantum gravity. In the next section we will discuss the stochastic approach to quantum cosmology. Within this approach Eqs. (8) and (16) can be derived in a much more clear and rigorous way, but they will have a somewhat different interpretation.

III. WAVE FUNCTION OF THE UNIVERSE AND STOCHASTIC APPROACH TO INFLATION

In this section we will briefly describe the stochastic approach to inflation [10,13,23]. It is less ambitious, but also much less ambiguous than the approach based on the investigation of the wave function of the universe. One of the tools used in this approach is the probability distribution $P_c(\phi, t | \phi_0)$, which describes the probability of finding the field ϕ at a given point at a time t , under the condition that at the time $t=0$ the field ϕ at this point was equal to ϕ_0 .

The same function also describes the probability that the scalar field which at time t was equal to ϕ , at some earlier time $t=0$ was equal to ϕ_0 .

The probability distribution P_c is in fact the probability distribution per unit volume in *comoving coordinates* (hence the index c in P_c), which do not change during the expansion of the universe. By considering this probability distribution, we neglect the main source of the self-reproduction of inflationary domains, which is the exponential growth of their volume. Therefore, in addition to P_c , we introduced the probability distribution $P_p(\phi, \phi_0, t)$, which describes the probability to find a given field configuration in a unit physical volume [43,13].

Consider the simplest model of chaotic inflation based on the theory of a scalar field ϕ minimally coupled to gravity, with the effective potential $V(\phi)$. If the classical field ϕ is sufficiently homogeneous in some domain of the universe, then its behavior inside this domain is governed by the equation $3H\dot{\phi} = -dV/d\phi$, where $H^2 = 8\pi V(\phi)/3M_p^2$.

Inflation stretches all initial inhomogeneities. Therefore, if the evolution of the universe were governed solely by classical equations of motion, we would end up with an extremely smooth universe with no primordial fluctuations to initiate the growth of galaxies. Fortunately, new density perturbations are generated during inflation due to quantum effects. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially in the expanding universe. When the wavelength of any particular fluctuation becomes greater than H^{-1} , this fluctuation stops oscillating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ . The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more perturbations of the classical field with wavelengths greater than H^{-1} . The average amplitude of such perturbations generated during a time interval H^{-1} (in which the universe expands by a factor of e) is given by

$$|\delta\phi(x)| \approx \frac{H}{2\pi}. \quad (17)$$

The phase of each wave is random. Therefore, the sum of all waves at a given point fluctuates and experiences Brownian jumps in all directions in the field space.

One can describe the stochastic behavior of the inflaton field using diffusion equations for the probability distribution $P_c(\phi, t | \phi_0)$. The first equation is called the backward Kolmogorov equation,

$$\frac{\partial P_c(\phi, t | \phi_0)}{\partial t} = \frac{H^{3/2}(\phi_0)}{8\pi^2} \frac{\partial}{\partial \phi_0} \left(H^{3/2}(\phi_0) \frac{\partial P_c(\phi, t | \phi_0)}{\partial \phi_0} \right) - \frac{V'(\phi_0)}{3H(\phi_0)} \frac{\partial P_c(\phi, t | \phi_0)}{\partial \phi_0}. \quad (18)$$

In this equation one considers the value of the field ϕ at the time t as a constant, and finds the time dependence of the probability that this value was reached during the time t as a result of the diffusion of the scalar field from different possible initial values $\phi_0 \equiv \phi(0)$.

The second equation is the adjoint of the first one; it is called the forward Kolmogorov equation, or the Fokker-Planck equation [10]:

$$\frac{\partial P_c(\phi, t | \phi_0)}{\partial t} = \frac{\partial}{\partial \phi} \left(\frac{H^{3/2}(\phi)}{8\pi^2} \frac{\partial(H^{3/2}(\phi) P_c(\phi, t | \phi_0))}{\partial \phi} + \frac{V'(\phi)}{3H(\phi)} P_c(\phi, t | \phi_0) \right). \quad (19)$$

For notational simplicity we took $M_p = 1$ in these equations.

One may try to find a stationary solution of Eqs. (18), (19), assuming that $\partial P_c(\phi, t | \phi_0) / \partial t = 0$. The simplest stationary solution (subexponential factors being omitted) would be [10,44,23]

$$P_c(\phi, t | \phi_0) \sim N \exp\left(\frac{3M_p^4}{8V(\phi)}\right) \exp\left(-\frac{3M_p^4}{8V(\phi_0)}\right). \quad (20)$$

The first term in this expression is equal to the square of the Hartle-Hawking wave function of the universe (8), whereas the second one gives the square of the tunneling wave function (16); N is the overall normalization factor. This result was obtained without any ambiguous considerations based on the Euclidean approach to quantum cosmology.

This result has an obvious similarity with the Hawking-Moss expression for the probability of tunneling, Eq. (7). It provides a simple interpretation of the Hawking-Moss tunneling. During inflation, long-wavelength perturbations of the scalar field freeze on top of each other and form complicated configurations, which, however, look almost homogeneous on the horizon scale H^{-1} . If originally the whole universe was in a state ϕ_0 , the scalar field starts wandering around, and eventually it reaches the local maximum of the effective potential at $\phi = \phi_1$. The probability of this event (and the typical time that it takes) is suppressed by $\exp[3M_p^4/8V(\phi_1)]$. As soon as the field ϕ reaches the top of the effective potential, it may fall down to another minimum, because it looks nearly homogeneous on a scale of horizon, and gradients of the field ϕ are not strong enough to pull it back to ϕ_0 . This is not a homogeneous tunneling, but rather an inhomogeneous Brownian motion, which, however, looks homogeneous on the scale H^{-1} [13]. An important lesson is that when one finds an instanton in de Sitter space describing homogeneous tunneling, one should not jump to the conclusion that it really describes the creation of a homogeneous universe rather than an event which only looks homogeneous on a scale H^{-1} .

That is how the stochastic approach resolves all mysteries associated with the Hawking-Moss tunneling. I believe that it is a very important point which deserves a more detailed discussion. Consider for example the potential $V(\phi)$ shown in Fig. 2. There are five different de Sitter instantons with

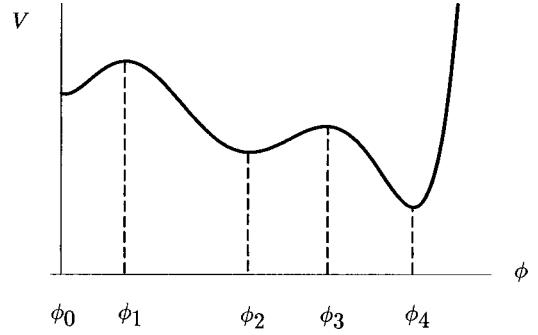


FIG. 2. Tunneling from the minimum at ϕ_0 occurs not to the points ϕ_2 or ϕ_3 , which, according to the naive estimates based on the instanton action, would be much more probable, but to the nearby maximum at ϕ_1 .

action $S = -3M_p^4/8V(\phi)$, corresponding to each of the five extrema of this effective potential. How one should interpret them? Do they describe tunneling between different minima, as suggested by Fig. 1, or creation from nothing, which can possibly be described by the upper half of Fig. 1?

There are two ways of interpreting instantons. The first one is to say that they *interpolate* between two different Lorentzian configurations, and describe the tunneling between them. Then one should specify initial and final states. This was the approach of Coleman and De Luccia [6]. The second one is to avoid any discussion of tunneling (and creation) but simply use instantons as a tool which allows one to calculate the wave function of the ground state. This was the approach of Hartle and Hawking.

As we already mentioned, using Hawking-Moss instantons as interpolating Euclidean solutions is difficult (because each of these instantons describes a constant field ϕ), but not impossible; see Fig. 1 and Ref. [11]. Suppose we study tunneling from ϕ_0 to ϕ_1 . Then the probability of tunneling is given by Eq. (20), where instead of ϕ one should use ϕ_1 . As one could expect, this result can be represented as $e^{\Delta S_{01}}$, where Δ is the change of entropy between the initial and the final states of the system, $\Delta S_{01} = 3M_p^4/8V(\phi_1) - 3M_p^4/8V(\phi_0) < 0$.

However, one could argue that it is much more probable to tunnel directly to ϕ_2 , or to ϕ_3 , or to ϕ_4 . Indeed, the Hawking-Moss instantons corresponding to each of these states do exist, and the absolute values of their actions are much greater than of the action corresponding to the tunneling to ϕ_1 . Naively, one would expect, for example, that the probability of tunneling from ϕ_0 to ϕ_3 would be given by $e^{\Delta S_{03}}$, where $\Delta S_{03} = 3M_p^4/8V(\phi_3) - 3M_p^4/8V(\phi_0) > 0$. Of course, a probability of tunneling greater than 1 does not seem to make much sense, but this is what we get when we uncritically use the Euclidean approach to tunneling. This is what one would expect in accordance with the argument of Ref. [35] implying that the universe should be created in a state with the greatest entropy, even if one encounters suspicious expressions like $e^{\Delta S}$ with $\Delta S < 0$.

From the point of view of the stochastic approach to inflation, the resolution of the paradox is pretty obvious and quite instructive. First of all, there is a subtle difference be-

tween the probability of tunneling and the probability to find the universe in a state with a given field ϕ . (This issue is directly related to the difference between the instantons interpolating between two different states, which describe tunneling, and the instantons used for the calculation of the wave function of the ground state.) Strictly speaking, Eq. (20) describes a stationary distribution of the probability to find a part of the universe in a state with a field ϕ . It does not necessarily describe the probability of tunneling (diffusion) to this state ϕ from the state ϕ_0 . These issues are related to each other, but only for $\phi \leq \phi_1$. According to [10,12,13], the typical time which is necessary for the field to move from the local minimum at ϕ_0 to any field $\phi \leq \phi_1$ by the process of diffusion is inversely proportional to $P_c(\phi)$. That is why the probability of jumping to the top of the barrier and rolling down to ϕ_2 is proportional to $P_c(\phi_1)$. However, the probability distribution $P_c(\phi)$, which is given by the square of the Hartle-Hawking wave function, has no direct relation to the probability of tunneling to $\phi > \phi_1$. Once the field ϕ rolled over the barrier, the probability of its subsequent rolling to ϕ_2 is neither suppressed nor enhanced by any additional factors.

Thus, despite expectations based on a naive interpretation of the Euclidean approach to tunneling, the universe does not jump to the state ϕ_2 , or ϕ_3 , or ϕ_4 with a probability greater than 1. If initially the main part of the universe was in a state ϕ_0 , then the process of diffusion gradually brings the scalar field ϕ in some parts of the universe to the nearby maximum of the effective potential at ϕ_1 . The probability of this event is *suppressed* by $e^{\Delta S_{01}} < 1$. Then the field ϕ falls to the minimum at ϕ_2 . Diffusion from ϕ_2 to ϕ_3 is also possible, and the probability to climb to ϕ_2 is *suppressed* by $e^{\Delta S_{23}} < 1$. As a result, the probability of diffusion (tunneling) from ϕ_0 to ϕ_4 is not enhanced by $e^{\Delta S_{04}} > 1$ or by $e^{\Delta S_{03}} > 1$, as one would naively expect, but is suppressed by $e^{\Delta S_{01}} e^{\Delta S_{23}} < 1$.

Suppose now that we were waiting for a very long time, so that the scalar field tunneled to its ground state ϕ_4 in the main part of the comoving volume. Then the probability distribution gradually reaches its stationary limit given by Eq. (20) provided that $V'' \ll H^2$ and inflation is possible all the way from ϕ_0 to ϕ_4 . This is exactly the result given by the Hartle-Hawking wave function: The probability to be in a stationary state with a small value of $V(\phi)$ is much greater than the probability to stay at large $V(\phi)$. But as we have seen, this result has no direct relation either to the probability of tunneling to the state near the absolute minimum of the effective potential or to creation of the universe in a state with a smallest possible vacuum energy density.

One could argue that Eq. (20) gives us much more than an interpretation of the Hawking-Moss tunneling. It appears to provide a direct confirmation and a simple physical interpretation of both the Hartle-Hawking wave function of the universe *and* the tunneling wave function. First of all, we see that the distribution of the probability to find the universe in a state with the field ϕ is proportional to $\exp[3M_p^4/8V(\phi)]$. Note that we are speaking here about the *state* of the universe rather than the probability of its *creation*. Meanwhile, the probability that the universe emerged from the state with

the field ϕ_0 is proportional to $\exp[-3M_p^4/8V(\phi_0)]$. Now we are speaking about the probability that a given part of the universe was created from the state with the field ϕ_0 , and the result coincides with our result for the probability of the quantum creation of the universe, Eq. (16).

This would be a great peaceful resolution of the conflict between the two wave functions. Unfortunately, the situation is even more complicated. In all realistic cosmological theories, in which $V(\phi) = 0$ at its minimum, the Hartle-Hawking distribution $\exp[3M_p^4/8V(\phi)]$ is not normalizable. The source of this difficulty can be easily understood: any stationary distribution may exist only due to the compensation of the classical flow of the field ϕ downwards to the minimum of $V(\phi)$ by the diffusion motion upwards. However, the diffusion of the field ϕ discussed above exists only during inflation. There is no diffusion upwards from the region near the minimum of the effective potential where inflation ends. Therefore the expression (20) is not a true solution of Eq. (19); all physically acceptable solutions for P_c are non-stationary (decaying) [43].

One can find, however, stationary solutions describing the probability distribution $P_p(\phi, t | \phi_0)$ introduced in [43]. This probability distribution takes into account different speeds of the exponential growth of the regions filled with different values of the field ϕ . The investigation of this question shows [23] that the relative fraction of the volume occupied by the field ϕ is described by a very complicated function which is completely different from the square of the Hartle-Hawking wave function. Meanwhile the relative fraction of the volume of the universe from which any given part of the universe could originate is given by a function, which in the limit $V(\phi_0) \ll M_p^4$ coincides with the square of the tunneling wave function. This is additional evidence indicating that if one wants to find out where our part of the universe came from, investigation of the tunneling wave function can be very useful. As we already mentioned, the Hartle-Hawking wave function can be very useful too if one applies it to the investigation of perturbations near a classical de Sitter background. One can use it for the investigation of density perturbations in an inflationary universe [39] and for the study of black hole formation in de Sitter space [38]. One can also use it for the investigation of tunneling from a quasistationary state in de Sitter space [9]. However, so far we did not find any evidence that the Hartle-Hawking wave function describes the probability of the quantum creation of the universe.

IV. QUANTUM CREATION OF OPEN UNIVERSES

A. Instantons describing the creation of open universes from nothing

We discussed the difference between the two types of wave functions at such length in order to put the results of Hawking and Turok into perspective. They have studied instanton solutions in theories with inflationary potentials such as ϕ^2 or ϕ^4 . Since in these theories the field ϕ moves, the corresponding instanton somewhat differs from the standard de Sitter instantons. In particular, it contains a singularity at

which the scalar field ϕ becomes infinitely large. However, this field only logarithmically grows near the singularity. At the same time the scale factor rapidly decreases, and the integral $-\pi^2 \int d\tau a^3(\tau) V(\phi)$ giving the action S converges. When tunneling occurs to the point ϕ where inflation is possible, the main contribution to the action is given by the nearly constant value of the field ϕ . As a result, the instanton action almost exactly coincides with the usual de Sitter action $-3M_p^4/16V(\phi)$ which we discussed before.² This, in fact, was the assumption which was made in [2] in the investigation of the probability of the quantum creation of an inflationary universe in the chaotic inflation scenario; so now this assumption is verified.

An important observation made in [20] was the possibility to make an analytical continuation of this instanton solution not only in the closed universe direction, but in the open universe direction as well. This is very interesting and non-trivial though not entirely new or unexpected because de Sitter space is known to have the amazing property of being simultaneously closed, open, and flat. A similar analytical continuation was employed in the paper by Coleman and De Luccia [6]. The possibility of an analytical continuation of the instanton containing a homogeneous scalar field to the open inflationary universe implies the possibility of the creation of an open inflationary universe containing a homogeneous field ϕ . This was the basis of recent models describing the quantum creation of an open universe [7,8,1,15,16].

The quantum creation of an open universe from nothing may seem to be entirely forbidden by the arguments contained in the previous section. Indeed, we are talking about the creation from nothing of a universe containing *infinite* energy. However, this may not be a real problem here. Let us remember that in the theory of the open universe creation by bubble formation [7,8,1] the universe inside the bubble looks finite from the point of view of an external observer, but it grows infinitely large in time. Its total energy grows because a false vacuum gives its energy to the expanding bubble wall. Thus, from the point of view of an inside observer, we have the instantaneous creation of an open universe with infinite total energy of matter. However, from the outside, the same process looks like a continuous and quite legitimate process of bubble growth and energy transfer from the surrounding de Sitter space.

Similarly, an open universe created by tunneling in the model of [20] does not appear alone, but as a part of a singular inflationary universe. At the moment when this complicated space-time emerges as the result of tunneling, the total volume of the part occupied by an open universe is vanishingly small, and it grows only gradually. However, just like in the growing bubble case described above, one can make a certain coordinate transformation, after which one

²The expression for the action given in Eq. (8) of Ref. [20] coincides with this expression, though it looks slightly different because the authors used a reduced Planck mass, which is smaller than the usual one by a factor of $(8\pi)^{-1/2}$.

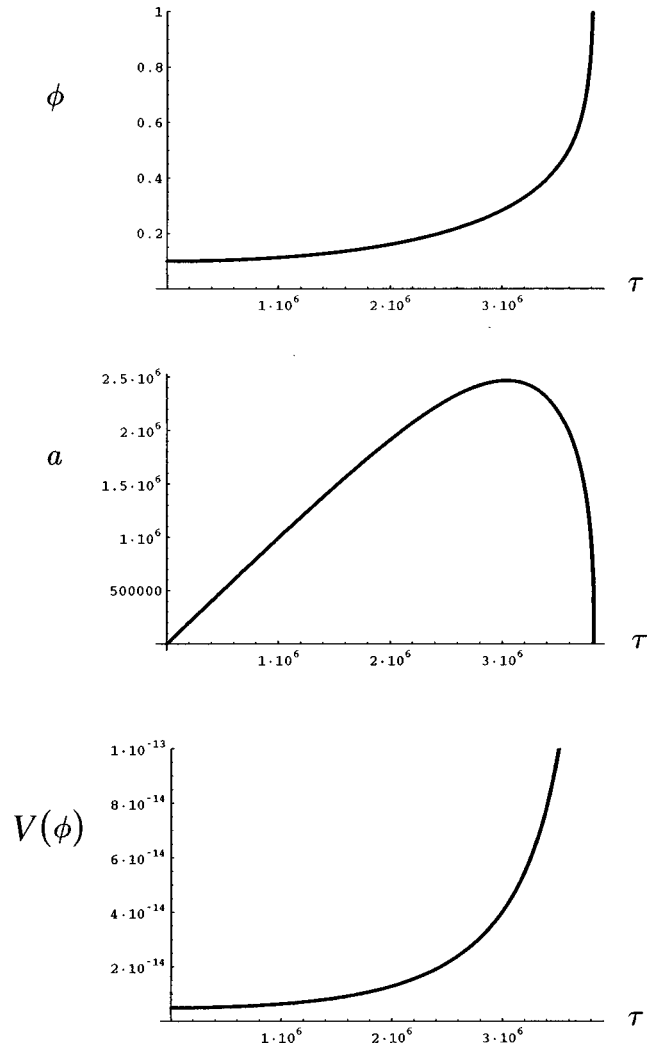


FIG. 3. Instanton in the theory $(m^2/2)\phi^2$ describing the creation of the universe with $\phi=0.1M_p$. In this case the scalar field rapidly changes in Euclidean space, and the universe does not inflate at all after the tunneling. If one considers greater values of ϕ at $\tau=0$, the scalar field becomes almost constant, but then it diverges logarithmically when τ approaches its maximal value.

may describe a part of the created space-time as an infinite open universe.

This is an a very interesting possibility which deserves further investigation [45]. Several comments are in order here. First of all, instantons of this type describe tunneling not only to the part of the potential where inflation is possible, but to noninflationary parts as well. In this case the scalar field may rapidly change with the growth of the parameter τ , and the action is not given by $-3M_p^4/16V(\phi(0))$. Still the general tendency remains the same for all models we analyzed numerically: The smaller the potential $V(\phi)$ at $\tau=0$ [which corresponds to the initial value of $V(\phi)$ in the open universe], the greater the absolute value of the action.

As an example, Fig. 3 shows the behavior of $\phi(\tau)$, $a(\tau)$, and $V(\phi(\tau))$ for the instanton describing the universe creation in the theory $(m^2/2)\phi^2$. We consider the case when $\phi(0)=0.1$ in units of M_p . In this case the universe does not

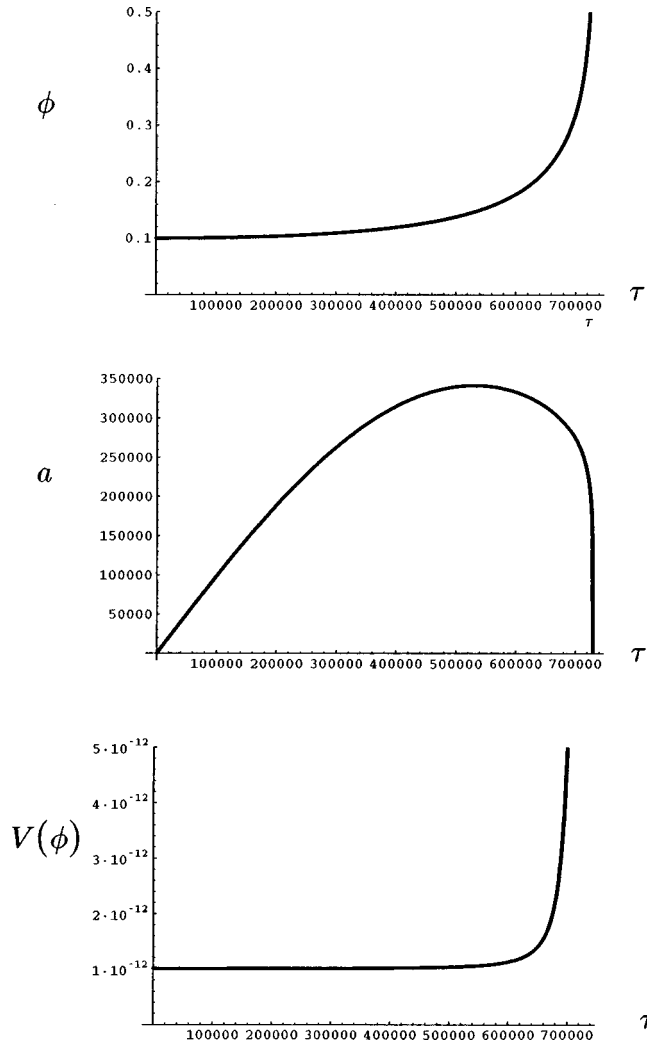


FIG. 4. Instanton in the hybrid inflation model based on supergravity, with $V(\phi) = M^4(1 - Q\phi^2 + Q^2\phi^4)e^{Q\phi^2}$. Everything is expressed in Planck units: $M \sim 10^{-3}$, $Q = 4\pi$; see Ref. [24].

inflate at all after the tunneling. Still the instanton does exist. Its action is given by $S = -\pi^2 \int d\tau a^3(\tau) V(\phi)$. For the realistic value $m \sim 10^{-6} M_p$ the action for this case shown in Fig. 3 is given by -6.55×10^{14} , which is greater by an order of magnitude than the absolute value of $-3M_p^4/16V(\phi(0))$. However, as soon as we consider tunneling to the inflationary part of the effective potential, the function $\phi(\tau)$ becomes nearly flat (until it blows up near the singularity), and the action practically coincides with the action on the usual de Sitter instanton with the constant energy density $V(\phi(0))$: $S = -3M_p^4/16V(\phi(0))$.

In Fig. 4 we show the instanton in the theory with the effective potential $V(\phi) = M^4(1 - Q\phi^2 + Q^2\phi^4)e^{Q\phi^2}$, with $M \sim 10^{-3}$, $Q = 4\pi$. This is the potential which (up to radiative corrections) appears in the hybrid inflation scenario in supergravity proposed in [24]; see also [46]. Note that the potential in this theory is extremely steep at $\phi > 0.3$. Therefore inflation is possible only for $\phi < 0.3$. Still the instanton solution does exist in this case as well.

In such theories, just like in theories where the effective potential is less steep, the field ϕ grows logarithmically near the singularity. If the effective potential depends on the field exponentially, the contribution to the action may blow up there. The integral still converges (or diverges only logarithmically) because of the sharp decrease of $a(\tau)$ near the singularity. It may still be necessary to make a cutoff of the integral, as soon as the sharply growing function $V(\phi)$ becomes greater than M_p^4 , and the semiclassical approximation breaks down.

Thus, now we have a candidate for a new mechanism of the creation of an open universe in inflationary cosmology. There are many questions associated with the new instantons. First of all, even though the singularity of the scalar field on these instantons is only logarithmic, the singularity of the energy density and of the curvature is a power law. If one takes such instantons into account, the corresponding method can no longer be called “the no-boundary proposal.” According to [47,48], the boundary terms give a contribution $-\pi M_p^2(a^3)'/4$ to the total action. We will not consider this correction here; it is relatively small if one considers creation of an inflationary universe. For example, the results of a numerical investigation performed in [48] show that in the theory $m^2\phi^2/2$ this correction, as compared to the action $-3M_p^4/16V(\phi)$, is suppressed by the factor $O(M_p/\phi)$ for $\phi \gg M_p$.

Another problem associated with the interpretation of the Hawking-Turok instanton as describing the creation of an open universe was given recently in [47], and was related to the singular nature of the instanton. While we tend to agree with the main conclusion of Ref. [47], we do not think that every instanton having a singularity is disallowed.

If one considers the Hawking-Turok instantons and cuts them at the time of the maximal expansion, $a = a_{\max}$, they will look almost exactly like the Hawking-Moss instantons. One may interpret them by saying that, just like in Fig. 1, they interpolate between two realizations of a closed de Sitter space. If one considers only the upper half of Fig. 1, starting from $a = 0$, and making the analytic continuation to de Sitter space at $a = a_{\max}$, then one may argue that the Hawking-Turok instanton interpolates between the state $a = 0$ (“nothing”) and de Sitter space. In such a case one may try to interpret this instanton as describing the creation of a closed universe from nothing. The results will not differ much if one calculate the action on the singular or on the nonsingular part of the instanton.

However, if one cuts the Hawking-Turok instanton not by the plane $a = a_{\max}$, but by the plane going through $a = 0$ and the singularity, as proposed in Ref. [20] in order to describe the creation of the open universe, it becomes much less obvious whether such an instanton interpolates between any two well-defined Lorentzian states or even between such states as the state with $a = 0$ and the singularity. This half-of-an-instanton seems to interpolate between half-of-nothing and half-of-singularity. Thus we are not quite sure that it really describes the quantum creation of an open universe. If the singularity is cut in half, and one needs to have a detailed knowledge of its structure to perform the analytical continu-

ation, the possibility to use such instantons for the description of tunneling becomes very suspicious. For a more detailed discussion of this issue see [48].

In addition, we have the general problem emphasized in the previous section. As we have found, only one of the instantons of the Hawking-Moss type really describes the tunneling (the instanton describing the tunneling from ϕ_0 to ϕ_1), whereas all other instantons are irrelevant even though they are perfectly nonsingular. It is very hard to find any reason to discard them within the Euclidean approach to tunneling, but the stochastic approach to inflation immediately explained which of them is relevant and what is its interpretation. We have found that it describes an inhomogeneous tunneling even though the instanton looks perfectly homogeneous.

We have a similar problem with respect to the Hawking-Turok instanton. So far we were unable to find any interpretation of the creation of a homogeneous open universe within the stochastic approach to inflation. The closest thing we were able to find was the nonperturbative effect of the creation of huge voids due to nonperturbative effects which might appear in a self-reproducing inflationary universe [49]. However, this effect appears only if one introduces some specific probability measure in inflationary cosmology, related to the probability distribution P_p mentioned in the previous section. Meanwhile the Hartle-Hawking wave function is related to the probability distribution P_c .

Despite all these problems, in what follows we will make an assumption that the Hawking-Turok instantons do describe the creation of a homogeneous open universe, and that the corresponding action with a good accuracy is given by $-3M_p^4/16V(\phi)$. We will study the consequences of this assumption if one interprets it using either the Hartle-Hawking or the tunneling wave function. But one should remember that the validity of this assumption made in [20] is less than obvious.

B. Open universes and the Hartle-Hawking wave function

Possible implications of the new class of instantons depend crucially on the choice of the wave function of the universe. Hawking and Turok suggested to use the Hartle-Hawking wave function, which implies that the probability of the quantum creation of an open universe with a field ϕ is given by Eq. (3):

$$P \sim e^{-2S} = \exp\left(\frac{3M_p^4}{8V(\phi)}\right). \quad (21)$$

This means that a typical open universe created by such a process would have the smallest possible value of the field ϕ ; i.e., the universe would tend to be created directly in the absolute minimum of the effective potential, which does not lead to any inflation whatsoever. As a result, such a universe at the present would be empty and would have $\Omega=0$. Of course we cannot live in a universe with $\Omega=0$; so we should discard the universes with too small Ω . Thus one may argue that the final probability distribution to live in a universe with a given value of Ω should be proportional to the prod-

uct of the probability of the creation of such a universe and the probability of galaxy formation there. An estimate of the most probable value of Ω which one can observe with an account taken of anthropic considerations can be made along the lines of [50]. This estimate has led the authors of [20] to the conclusion that Ω should be about 10^{-2} , which would be in disagreement with the observational data suggesting that $\Omega \gtrsim 0.3$.

Is there a chance that this disagreement might disappear after a more detailed investigation of the anthropic constraints on Ω ? After all, we are talking only about one order of magnitude; so is it perhaps possible to make things work? Let us consider this issue more carefully.

One can parametrize the present value of Ω for an open universe in the following way [20]:

$$\Omega \approx \frac{1}{1 + Ae^{-2N(\phi)}}, \quad (22)$$

where A is some factor depending on the efficiency of reheating and other details of the theory, and $N(\phi)$ is the number of e -foldings of inflation after the field ϕ begins rolling down.

Let us compare the probability P_ϕ for the universe to begin at some value of the scalar field ϕ and the corresponding probability to have a slightly greater field $\phi + \Delta\phi$:

$$\frac{P(\phi)}{P(\phi + \Delta\phi)} = \exp\left(\frac{3M_p^4 V' \Delta\phi}{8V^2(\phi)}\right). \quad (23)$$

Now one should take into account that $\Delta N = H\Delta t = H\Delta\phi/\dot{\phi} = 3H^2\Delta\phi/V'$, i.e., $\Delta\phi = \Delta N V' M_p^2/8\pi V$. Also, the amplitude of density perturbations, $\delta \sim M_p^3 V'/V^{3/2} \sim 10^{-5}$. Combining this all together and dropping factors $O(1)$, one has

$$\frac{P(\phi)}{P(\phi + \Delta\phi)} = \exp(10^{-1} \delta^{-2} \Delta N) \sim \exp(10^9 \Delta N). \quad (24)$$

A universe with $\Omega=0.3$ appears after the creation of the universe with $\phi_{0.3}$, where $0.3 = 1/(1 + Ae^{-2N(\phi_{0.3})})$. This gives $Ae^{-2N(\phi_{0.3})} \approx 2$. Meanwhile a universe with $\Omega=0.2$ appears if $Ae^{-2N(\phi_{0.2})} \approx 4$. Therefore $\Delta N = N(\phi_{0.3}) - N(\phi_{0.2}) \sim 0.5$. Thus the probability of the creation of a universe with $\Omega=0.2$ is approximately 10^{10^8} times greater than the probability of the creation of a universe with $\Omega=0.3$. Clearly, the probability of galaxy formation in these two cases cannot differ by a factor 10^{10^8} . This means that according to [20] it is entirely improbable to live in a universe with $\Omega=0.3$. Similarly, it seems entirely improbable to live in the universe with $\Omega=0.2$. One should consider absolutely extreme conditions in the universe in order to compensate for factors of the type of 10^{10^8} . Note that this conclusion is valid independently of the choice of the inflationary potential: The final result is determined by the amplitude of the density perturbations δ which is given by observations.

Since the probability of the universe formation grows roughly 10^{10^8} times when the number of e -foldings N decreases by $\Delta N = O(1)$, this growth can be compensated only by decreasing the probability of galaxy formation at small N (at small Ω). To compensate for the factor of $\sim 10^{10^8}$, the probability of galaxy formation must be smaller than 10^{-10^8} . In such a case we would not see any other galaxies around us; the next nearby galaxy in the Hartle-Hawking universe would be at a distance about $10^{100\,000\,000}$ light years away.

One could hope that in the worst case one can simply return to the old method of the creation of an open universe proposed in [7,8,1]. However, once the Hartle-Hawking approach is adopted, this does not seem possible either. The main difference between the previous mechanism of open universe formation and the new one was the existence of a deep local minimum of the effective potential at some $\phi = \tilde{\phi}$. In such a situation there exists the Coleman–De Luccia instanton, which describes the creation of an open universe immersed in a false vacuum with $\phi = \tilde{\phi}$. The minimal value of the scalar field ϕ on this instanton solution, ϕ_{\min} , should still be sufficiently large for the further 60 or 70 e -foldings of inflation to occur inside the bubble. This, if we would try to obtain an instanton solution for such theories just like we did for several other theories (see Figs. 1 and 2), we would begin our calculations at $\phi(0) = \phi_{\min}$, and we would see that the growing field ϕ stabilizes at $\phi = \tilde{\phi}$. However, if one starts the calculations at $\phi < \phi_{\min}$, the scalar field rolls over the local minimum of the effective potential (which looks like a local maximum from the point of view of equations of motion in Euclidean space) and continues its growth toward indefinitely large ϕ . Thus the Hawking-Turok instantons do exist even in theories where the effective potentials grow nonmonotonically at large ϕ . Therefore the conclusion concerning the tunneling to smallest possible values of $V(\phi)$ in the context of the Hartle-Hawking approach seems to be quite general.

In theories with two fields of the type of Eq. (1) the situation is even easier to analyze: One may consider the instanton with $\phi = 0$ describing the field σ climbing from the minimum of the effective potential to $\sigma \rightarrow \infty$. All results obtained in [20] and above apply to this case.

Thus, we expect that instantons of the type considered above should exist in the models considered in [7,8,1], and therefore all conclusions about the preferable creation of the universe with the smallest possible $V(\phi)$ and extremely small Ω should apply to such theories as well. This implies that the Hartle-Hawking approach makes it extremely difficult to propose any realistic open inflation model.

What happens if one makes a different analytical continuation and concentrates on the closed universe case instead? Similarly, the probability of the quantum creation of the universe grows 10^{10^8} times if the duration of inflation becomes one e -folding shorter. But a closed universe which inflates less than $N \sim 70$ (the exact number depends on the features of reheating) collapses in less than 10^{10} years, which makes the existence of life very problematic. Again, the only way to compensate for the factors $\sim 10^{10^8}$ pushing the probability

distribution toward small N (i.e., toward a premature death of the universe) is to assume that the probability of the existence of life near an “optimal” N , corresponding to the maximum of the total probability distribution, decreases by more than 10^{-10^8} when N decreases by 2. Thus, the no-boundary proposal based on the Hartle-Hawking wave function pushes us toward the region where the existence of life becomes nearly impossible. This does not mean that this proposal is incorrect. As we have already argued, it works perfectly well if one calculates the probability of events produced by usual quantum mechanical fluctuations having positive energy near an already existing cosmological background. However, we believe that it does not apply for the calculation of the probability of the formation of this background, which involves investigation of the fluctuations of the scale factor. If one does not make an attempt to extend the validity of the Hartle-Hawking wave function beyond a certain point, one does not face the consequences discussed above.

One possibility to resolve this problem is suggested by the form of the boundary terms found in [47,48,25]. For example, in chaotic inflation with $V(\phi) = (m^2/2)\phi^2$ the action with an account taken of boundary terms is given by [48]

$$S \approx -\frac{3M_p^4}{8V(\phi)} \left(1 - \frac{M_p}{2\phi}\right). \quad (25)$$

This equation shows that the action becomes minimal not at $\phi = 0$, but at $\phi \sim M_p$, which allows for a short stage of inflation. However, a numerical investigation of this question performed in [48] for several different versions of the chaotic inflation scenario has shown that this stage of inflation is extremely short, and the corresponding value of Ω would be exponentially small.

Hawking and Turok proposed to consider an inflationary model with a local maximum of the effective potential, such as $V(\phi) = \mu^4(1 - \cos(\phi/v))$ [25]. In this model the top of the effective potential corresponds to a local minimum of the action with an account taken of boundary terms. If one neglects the possibility of tunneling to small ϕ , the second best possibility is that the universe is created at the top of $V(\phi)$. But if the total duration of inflation is small (which is necessary to keep the universe open), then tunneling to the top is not allowed. Indeed, density perturbations produced during inflation are inversely proportional to V' ; so they are very large at the top of $V(\phi)$. A large amplitude of density perturbations on the horizon is anthropically forbidden; so it was argued in [25] that we should tunnel not to the top but to some point ϕ^* away from the top. This hopefully could give $\Omega \sim 0.3$ and $\delta\rho/\rho \sim 10^{-5}$ on the horizon, after a certain fine-tuning of the parameters of the model.

But suppose that we indeed have a model with parameters which make it possible. Then in the same model it is even more probable to tunnel directly to the top (because the action is smaller there), and then roll to ϕ^* from the top. We will still have $\delta\rho/\rho \sim 10^{-5}$ on the horizon, but in this case we will have $\Omega = 1$ because of the additional stage of inflation during the rolling from $\phi = 0$ to $\phi = \phi^*$. This is not

anthropically forbidden because $\delta\rho/\rho$ only very weakly depends on the length scale; it becomes much greater than 10^{-5} only at distances much greater than the size of the observable part of the universe. Thus it does not seem possible to get $\Omega < 1$ in this version of the scenario proposed in Ref. [25] even if one uses the Hartle-Hawking wave function and takes into account boundary terms.

Until now we tacitly assumed that the creation of the universe is a one-time event, and that it is correct to describe the total probability of forming a galaxy as a product of the probability of creating the universe with a given ϕ and the probability of forming a galaxy for a given $\Omega(\phi)$. This is a reasonable proposal in the minisuperspace approach to quantum cosmology, but it may fail if one takes into account the effect of the self-reproduction of the universe. Indeed, the probability of the creation of the universe with a large field ϕ is very small in the context of the Hartle-Hawking proposal. However, universes with large ϕ in the chaotic inflation scenario typically enter the stage of eternal self-reproduction, which leads to a permanent exponentially rapid growth of their total volume [43]. This process leads to the creation of an infinitely large number of galaxies. Then a typical galaxy will be produced not in the region suggested by the Hartle-Hawking probability distribution, but in the region where the scalar field ϕ was large enough for the process of the self-reproduction of the universe to begin.

One could object that if the Hartle-Hawking wave function correctly describes the creation of an open universe, then the universe has a very small energy density from the very beginning, and self-reproduction of the universe never happens. However, according to [23], if the universe is sufficiently large, the process of self-reproduction occurs even if the initial value of the field ϕ is so small that it can barely support inflation. Thus self-reproduction definitely occurs inside an infinite open inflationary universe. In such a case all negative (and positive) consequences of the description of quantum creation of the universe by the Hartle-Hawking wave function disappear, not because the consequences of the no-boundary proposal become different, but because the choice of initial conditions in quantum cosmology provided by the Hartle-Hawking (or tunneling) wave function becomes irrelevant for the description of the main part of our universe [44,23,51]. In the first universe produced by quantum creation from nothing one may have $\Omega \sim 0.01$, if it is described by the Hartle-Hawking wave function, or $\Omega = 1$, if it is described by the tunneling wave function. However, this universe will produce an infinite number of new inflationary universes. One may wonder, what is the most probable origin of a part of the universe of a given physical volume, which has density ρ at the time t , after the creation of the universe from nothing? The answer is that the relative fraction of the physical volume of a self-reproducing universe in a state with given properties (with given values of fields, with a given density of matter, etc.) does not depend on time t . The probability that a given part of the universe in this scenario originated from a state with a certain value of the scalar field ϕ is given by a function which is very similar to the square of the tunneling wave function [23].

C. Open universes and the tunneling wave function

Now let us see what happens if we use the results of Ref. [20], but interpret them from the point of view of the tunneling wave function. In this case, according to [2–5], the probability of the universe creation is proportional to $\exp(-3M_p^4/8V)$. Thus the universe tends to be born at the highest possible value of the effective potential V . In the simplest models with the effective potentials $(m^2/2)\phi^2$ or $(\lambda/4)\phi^4$ the total duration of inflation is so large that the resulting value of Ω becomes equal to 1 independently of the way the universe was born (i.e., whether it was closed or open from the very beginning). One may or may not like it, depending on one's beliefs concerning the total density of the universe at present, but at least this value is not as far away from recent observational results as the conclusion that we should live in a structureless universe with $\Omega = 0.01$.

On the other hand, now we have two classes of models where one can get $\Omega < 1$. The first class includes all models proposed in [7,8,1,15,16]. The universe may begin in a singularity, or it may appear due to the creation from nothing. The final result will be entirely insensitive to it. Indeed, as soon as inflation begins, in most versions of the of chaotic inflation scenario the universe enters the regime of eternal self-reproduction [13]. It produces an indefinitely large amount of space. For example, in the simplest model with the potential (1), eternal inflation may begin at very large σ and ϕ [1]. Then it produces exponentially large domains filled with all possible values of σ and ϕ . In particular, there will be domains trapped in the local minimum near $\sigma = 0$. These domains will continue to inflate eternally, like the de Sitter phase in the old inflation scenario, and they will continue producing open inflationary universes with all possible values of Ω . Thus, in this scenario a single act of creation of the universe produces not one but an infinite number of open universes.

One may wonder what is the most probable value of Ω in this scenario. At the moment we do not know a definite answer to this question. In all versions of eternal inflation theory we have to compare an infinite number of universes with different properties. As a result, the answer is ambiguous; it depends on the way one performs a cutoff and regularizes the infinities. For a discussion of different approaches to this question one may see, e.g., [1,23,52,53]; the problem is not settled yet. We do not even know whether it makes any sense to look for a definite answer. The reason is very simple [23,53]. Consider two *infinite* boxes, one with apples, another with oranges. One can pick one fruit from each box, an apple and an orange, then again an apple and an orange, and so on. This may give an idea that the number of apples is equal to the number of oranges. But one can equally well take each time one apple and two oranges, and conclude that the number of oranges is twice as large as the number of apples. The main problem here is that we are making an attempt to compare two infinities, and this gives an ambiguous result. Similarly, the total volume of a self-reproducing inflationary universe diverges in the future. When we make slices of the universe by hypersurfaces of constant time t , we are choosing one particular way of sorting out this infinite

volume. If one makes the slicing in a different way, the results will be different. The main statement, which does not depend on the choice of the probability measure, is that we have an infinite number of apples and oranges, and we have an infinite number of domains with various values of Ω . If we want to find in which of these universes we live, we should go and measure the value of Ω ; whichever we find will be ours.³

In addition to this class of theories, we may consider another class, which was introduced in [1] when we studied the possibility of the creation of a closed universe with Ω substantially greater than 1; see the Introduction. The main idea is to consider models where the self-reproduction of the universe is impossible and the total duration of inflation is very small. For example, one can consider a particular version of the chaotic inflation scenario with the effective potential

$$V(\phi) = \frac{m^2 \phi^2}{2} \exp\left(\frac{\phi}{CM_p}\right)^2. \quad (26)$$

Potentials of a similar type often appear in supergravity. In this theory inflation occurs only in the interval $M_p/2 \lesssim \phi \lesssim CM_p$. One may consider a model of tilted hybrid inflation proposed in [54] or a particular version of hybrid inflation in supergravity proposed in [24,46], where the effective potential at large ϕ (when logarithmic terms appearing due to quantum corrections become subdominant) looks as follows:

$$V(\phi) = M^4 \left(1 - Q \frac{\phi^2}{M_p^2} + Q^2 \frac{\phi^4}{M_p^4} \right) \exp\left(\frac{Q\phi^2}{M_p^2}\right). \quad (27)$$

Here $M \sim 10^{-3} M_p$, $Q = 4\pi$. As we have already mentioned in Sec. IV A, the effective potential in this theory is extremely steep at $\phi > 0.3$. Therefore inflation is possible only for $\phi < 0.3$. Still the instanton solution does exist, both for $\phi(0) < 0.3$ and for $\phi(0) > 0.3$. All coupling constants in this model are $O(10^{-1})$, and the total duration of inflation is $N \sim 10^2$. This makes it an interesting candidate for the open inflation model.

One may also consider models with the simple quadratic effective potential $(m^2/2)\phi^2$, but assume that the field ϕ has a nonminimal interaction with gravity of the form $-(\xi/2)R\phi^2$. In this case inflation becomes impossible for $\phi > M_p/\sqrt{8\pi\xi}$ [55,56]. In order to ensure that only a limited amount of inflation is possible for inflationary universes which can be produced during the process of the quantum creation of the universe in the theory $(m^2/2)\phi^2$, it is enough to assume that $M_p/\sqrt{8\pi\xi} < 3M_p$. This gives the condition $\xi > 1/72\pi \sim 4 \times 10^{-4}$.

If an open universe is created and it does not inflate much, then after inflation we have an open universe with $\Omega < 1$ in either of the models described above.

There are several different problems associated with this scenario. Consider for definiteness the model (26) and sup-

pose for a moment that tunneling may occur only to the region of small ϕ , where inflation is possible. Then, according to Eq. (2), the maximum of the probability of the creation of an inflationary universe appears near the upper range of values of the field ϕ for which inflation is possible, i.e., at $\phi_0 \sim CM_p$. The probability of such an event will be so strongly suppressed that the universe will be formed almost ideally homogeneous and spherically symmetric. As pointed out in [14], this solves the homogeneity, isotropy and horizon problems even before inflation really takes over. Then the size of the newly born universe in this model expands by the factor $\exp(2\pi\phi_0^2 M_p^{-2}) \sim \exp(2\pi C^2)$ during the stage of inflation [13]. If $C \gtrsim 3$, i.e., if $\phi_0 \gtrsim 3M_p \sim 3.6 \times 10^{19}$ GeV, the universe expands more than e^{60} times, and it becomes very flat. Meanwhile, for $C \ll 3$ the universe always remains ‘‘underinflated’’ and very curved. Its properties will depend on the way it was formed. If we make analytical continuation of the Hawking-Turok instanton in the usual way, it will describe the formation of a closed universe with $\Omega > 1$. On the other hand, the new analytical continuation proposed in [20] describes the creation of an open universe with $\Omega < 1$. In order to obtain Ω in the interval between 0.3 and 0.2 at the present time one should have the constant C fixed somewhere near $C = 3$ with an accuracy of a few percent. This is a fine-tuning, but not a terrible one.

However, in the above analysis we have assumed that tunneling may occur only to the inflationary part of the effective potential. Meanwhile we obtained instanton solutions which describe tunneling to noninflationary parts of the effective potential as well, with $\phi_0 \gg CM_p$. This may not look problematic. The field tunnels to the highest possible position with $V(\phi) \sim M_p^4$. Then after many oscillations the scalar field decreases to the region where inflation becomes possible; then the universe inflates a little, and we still get the universe with $\Omega < 1$. The problem is that if, as we expect, the probability to create a universe with a nearly Planckian density is not strongly suppressed, then at the moment of its creation the universe will not be very homogeneous. If the universe inflates a lot after its creation, these primordial inhomogeneities do not do us any harm. However, if inflation produces a universe with $\Omega < 1$, these inhomogeneities may cause significant anisotropy of the cosmic microwave background (CMB) radiation.

It may happen that this is not a real problem. A typical scale factor of an open universe at the moment of its creation in this scenario will be $O(M_p^{-1})$. Thus one may expect initial inhomogeneities to exist on this scale. Then the scale factor of the universe, as well as the wavelength of these perturbations, expands more slowly than the size of the horizon, $\sim t$, until the universe becomes inflationary. As a result, all initial inhomogeneities at the beginning of inflation have wavelengths much shorter than the horizon. Such perturbations rapidly decrease during inflation and become harmless. This may solve the homogeneity problem, but we believe that this issue requires a more detailed investigation.

Note also that this problem appears only if we assume that tunneling to large values of $V(\phi)$ is possible. But what if the scalar field ϕ is only an effective degree of freedom describ-

³Inflationary theories of this type which allow a definite prediction for Ω are also possible [15].

ing, for example, the radius of compactification or a condensate of fermions? Then the effective potential may not be defined at $V(\phi) \sim M_p^4$, the tunneling to very large $V(\phi)$ becomes impossible, and the homogeneity problem may disappear.

One more thing which should be analyzed is the applicability of the simple rule $P \sim e^{-|S|}$ for the description of the universe creation in models with steep potentials. Indeed, as we emphasized, we expect this expression to be valid in situations when one can neglect the motion of the scalar field. In this case one can treat $V(\phi)$ as a cosmological constant and quantize only the scale factor. One may expect that this rule will remain approximately correct if the motion of the field ϕ is very slow. But if the effective potential is very steep, the field ϕ will move very fast. In such cases one should quantize simultaneously the scale factor a , which has negative energy, and the scalar field ϕ , which has positive energy. In this case the relation $P \sim e^{-|S|}$ must be considerably modified. One such example is pre-big-bang cosmology, where the action vanishes identically on equations of motion, whereas the entropy of an inflationary universe is exponentially large [34]. In our case there is an additional modification related to the boundary terms, which become very significant for tunneling to the steep parts of the effective potential. Indeed, the numerical investigation of this issue performed in [48] shows that in the regions where the effective potential is very steep the boundary terms may become so large that they may even change the sign of the action. This simply implies that the naive expression for the tunneling wave function obtained by modifying the sign of the action does not apply to such situations. However, this does not change our general qualitative conclusion that tunneling with the creation of the universe with $V(\phi) \sim M_p^4$ is not suppressed.

In addition to all the problems mentioned above, one should also make sure that the leading channel of the universe creation will produce topologically trivial open universes. First of all, tunneling may produce closed universes as well, with a similar probability [1]. This is not a real problem though, because if tunneling occurs to small ϕ , so that in an open universe case one obtains a universe with $\Omega \ll 1$, then in the closed universe case the same instanton will describe the universe with a very large Ω which collapses too early for any observers to appear there. But the creation of a closed universe is not the only competing process. There exist a variety of instantons describing Euclidean universes with a nonvanishing vacuum energy density V . The usual de Sitter instanton discussed above is just one of them. For example, the action on the Page instanton $P_2 + \bar{P}_2$ is $-9M_p^4/40V$, the action on the Fubini-Study instanton P_2 is $-9M_p^4/32V$, the action on the $S^2 \times S^2$ instanton is $-M_p^4/4V$ [57].⁴

⁴The values of the action we gave here are obtained by integration over the full Euclidean space; so they should be compared to the complete de Sitter action $-3M_p^4/8V$.

The most interesting of these solutions is the $S^2 \times S^2$ instanton. The absolute value of its action is smaller than that of the de Sitter instanton; so one may argue that it is easier to create an anisotropic Kantowski-Sachs universe rather than isotropic de Sitter space. Note that the resulting geometry is unstable with respect to the exponential growth of the radii of both spheres, and eventually this solution becomes locally indistinguishable from de Sitter space [58]. However, if tunneling occurs to small ϕ , the universe does not expand long enough to erase the large-scale anisotropy, which should therefore be detectable.

One should note that it is not quite correct to directly compare the action of the de Sitter instanton to the action of the $S^2 \times S^2$ instanton. Indeed, in theories where the effective potential sharply rises at large ϕ , the action describing tunneling to large ϕ is not given by simple expressions of the type of $-3M_p^4/16V$, but should be calculated anew for each particular configuration. If tunneling occurs near the Planck density, its probability is not expected to be exponentially suppressed for either of these instantons; so the probability of the creation of different spaces may be comparable, and then we may live in a universe with the simplest topological properties (if it is true) merely by chance.

It is also quite possible that tunneling may create spaces of a more complicated topology. The first attempt to study this possibility was made in [3]. It was found that the probability of tunneling to a flat exponentially expanding space with identified sides may not be suppressed at all unless one takes into account quantum corrections to the energy momentum tensor. This space has metric of a $3d$ -torus with identified sides:

$$ds^2 = dt^2 - [a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2], \quad (28)$$

with $x+L \equiv x$, $y+L \equiv y$, and $z+L \equiv z$. At large t this space locally looks like a de Sitter universe, but if the expansion is not long enough, then the universe will be noticeably anisotropic.

All these problems would not even arise in the standard situation when inflation lasts much more than 60 e -foldings, but if one adjusts the parameters of the model in such a way as to have inflation very short, the issue of global anisotropy and topology of the universe becomes quite important; see in this respect [59].

Another potential drawback of the new class of open inflation models is the unusual shape of the spectrum of density perturbations. By construction, inflation in these models begins at the point when the slope of the effective potential for the first time becomes not very steep, and the friction produced by the term $3H\dot{\phi}$ for the first time becomes sufficient to slow down the rolling of the field ϕ . But this automatically means that the amplitude of density perturbations produced at the beginning of inflation, which now corresponds to the scale of horizon, should be very small (blue spectrum); see, e.g., [24]. This may be a real problem for such models. Note, however, that this problem is somewhat opposite to the previously discussed problem of overproducing large-scale density perturbations created during the tunneling.

All these questions require a thorough investigation to make sure that the new models of open inflation discussed above can work. As we already emphasized in Sec. IV A, we are not sure that the Hawking-Turok instanton really describes the quantum creation of an open universe. It is important, however, that quite independently of these new possibilities, which may or may not prove to be realistic, the tunneling wave function allows us to have the usual inflationary models predicting $\Omega=1$, as well as the previously proposed class of models with $\Omega<1$ [7,8,1,15,16]. It seems much better than to have models predicting either $\Omega\gg 1$ for the closed universe case or $\Omega\sim 10^{-2}$ for the open inflationary universe.

V. MODELS WITH THE ANTISYMMETRIC TENSOR FIELD

In order to avoid the unfortunate consequence $\Omega\sim 10^{-2}$ of their original model, Hawking and Turok introduced recently a new class of models [25], where they added the four form field strength $F_{\mu\nu\rho\lambda}=\partial_{[\mu}A_{\nu\rho\lambda]}$. The Euclidean action for their model is

$$\begin{aligned} S_E = \int d^4x \sqrt{g} \left(-\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right. \\ \left. + \frac{1}{48} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} \right) + \frac{1}{8\pi G} \int d^3x \sqrt{h} K, \end{aligned} \quad (29)$$

The last term gives the boundary contribution, which is typically small when the tunneling occurs to the values of ϕ corresponding to a long stage of inflation [48,25].

The field F in four-dimensional space is not a real dynamical field. The Lagrange equation for F in the Euclidean regime has a solution $F^{\mu\nu\rho\lambda}=(c/\sqrt{g})\epsilon^{\mu\nu\rho\lambda}$ with c an arbitrary constant. In the Lorentzian regime this solution becomes $F^{\mu\nu\rho\lambda}=(ic/\sqrt{-g})\epsilon^{\mu\nu\rho\lambda}$. Its main role is to give a contribution to the effective cosmological constant, $V(\phi)\rightarrow V(\phi)-\frac{1}{48}F^2$, where $F^2\equiv F^{\alpha\beta\gamma\delta}F_{\alpha\beta\gamma\delta}$. The trick is to add simultaneously the vacuum energy $V_0=\frac{1}{48}F^2$. This operation leaves the original value of $V(\phi)$ intact, and thus it does not lead to any effects if one calculates the entropy of nearly de Sitter space:

$$\mathbf{S} = \frac{3M_p^4}{8V(\phi)}. \quad (30)$$

Here by $V(\phi)$ we mean the total energy density, including the energy of the scalar field $V(\phi)+V_0$ and the compensating F^2 contribution. For example, one can take $V(\phi)=m^2\phi^2/2+V_0-\frac{1}{48}F^2=m^2\phi^2/2$.

However, if instead one calculates the Euclidean action, which normally coincides with $-\mathbf{S}$ in inflationary cosmology, one gets a different result [60]. The action becomes a nontrivial function of $V(\phi)$ and F^2 . Neglecting the small boundary term, and integrating over the entire solution (which doubles the result), one gets [60,25]

$$S \approx -\frac{3M_p^4}{8V(\phi)^2} \left(V(\phi) - \frac{1}{24}F^2 \right). \quad (31)$$

This coincides with (minus) entropy $-\mathbf{S}$ for $F=0$. However, for $V_0=\frac{1}{48}F^2\neq 0$ one no longer has the maximum of the absolute value of the action S at $V(\phi)=0$. Instead of that, the maximum is reached at $V(\phi)\sim 4V_0$. By a proper choice of V_0 one can fine-tune the most probable initial value of ϕ (according to the Hartle-Hawking prescription) to be at any given place. In particular, one can have it at $\phi\sim 3M_p$, which would lead to about 60 e -folds of inflation. Thus, by choosing the proper value of the constant V_0 one can obtain any value of Ω , from 0 to 1.

A few comments are in order here. The main reason why the original idea of Hawking and Turok was so attractive is the postulated absence of any fine-tuning. Now this is no longer the case. Consider for example a realistic model of chaotic inflation with $V(\phi)=m^2\phi^2/2$, with $m\sim 10^{-6}M_p$. To obtain the most probable value of ϕ near $3M_p$ one would need to have $V_0\sim 10^{-12}$ in units of the Planckian energy density M_p^4 . This introduces a new extremely small parameter to the theory. The value of this parameter $V_0/M_p^4\sim 10^{-12}$ must be further fine-tuned with an accuracy of about 1% in order to get the desirable value of Ω . Then F^2 should be fine-tuned to cancel V_0 with an accuracy $10^{-123}M_p^4$, which is achieved in [25] by using anthropic considerations.

This mechanism can work only if F is imaginary in the Lorentzian regime. It is not quite clear therefore whether this model is realistic.

An additional complication appears if one remembers that now the entropy no longer coincides with the (minus) Euclidean action. Thus, one may wonder which of these functions should be maximized? The extremum of entropy \mathbf{S} , as before, appears at the point corresponding to the absolute minimum of $V(\phi)$, independently of the presence of the field F . Thus, the argument that one should maximize the entropy, given in the previous paper by Hawking and Turok [35], contradicts the proposal to maximize the Euclidean action.

One should note that Eq. (29) is not a unique way to write the action for the theory of the field F . One can add to the action the integral of a total derivative

$$S_{\text{extra}} = \frac{\alpha}{24} \int d^4x \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu\rho\lambda}, \quad (32)$$

as proposed by Aurelia, Nicolai, and Townsend [61]. Here α is an arbitrary constant. Since this is a total derivative, and it does not change the instanton solution, it does not modify the entropy, but it gives an extra contribution to the Euclidean action

$$\Delta S \sim -\frac{\alpha}{c} \frac{3M_p^4}{8V(\phi)^2} F^2.$$

For $\alpha=c$, this term cancels the F^2 term in Eq. (31) [60]. Thus, depending on α one gets different expressions for the Euclidean action, whereas the expression for the entropy is α

independent. This suggests that one should look for the extremum of the entropy rather than of the action.

One may try to resolve the ambiguity by applying the stochastic approach. In this case the presence of the F field will be entirely irrelevant as long as its contribution to the vacuum energy is canceled by V_0 . One obtains the same stationary probability distribution (20) determined by the exponent of the entropy e^S , independently of the existence of the field F . This means that the presence of the field F cannot change the prediction $\Omega=0.01$ based on the use of the Hartle-Hawking wave function.

In a new version of their paper [25] Hawking and Turok agreed with our conclusion. They noted that if one properly takes into account all boundary terms, an expression for the Euclidean action changes, and the disagreement between the calculation using the action and the entropy disappears [62,25]. This implies, just as we argued above, that the introduction of the field F in this model does not resolve the problem of having too small value of Ω .

A potentially interesting consequence of the introduction of the F field is the cosmological constant problem. In order to analyze it, in the new version of their paper [25] Hawking and Turok reverted the sign of the F^2 term in the action, to bring it closer to the Freund-Rubin work on supergravity compactification [63]. The exponent of the entropy, e^S , can be represented as

$$P \sim e^S = \exp\left(\frac{3M_p^4}{8[V(\phi) + V_0 + \rho_F]}\right), \quad (33)$$

where ρ_F is the (negative) energy density of the F field. [64]. If one interprets this result as the probability of the quantum creation of the universe, this may imply that the universe should be created in a state corresponding to the minimal value of the total energy density $V(\phi) + V_0 + \rho_F \ll M_p^4$ consistent with the subsequent emergence of life. The possibility of the creation of universes with different ρ_F then allows us to use the anthropic principle to make the observable value of the cosmological constant very small [25]. However, in this case one still has the problem of living in a structureless universe with $\Omega = 0.01$.

On the other hand, if one uses the tunneling wave function, one finds

$$P \sim \exp\left(-\frac{3M_p^4}{8[V(\phi) + V_0 + \rho_F]}\right). \quad (34)$$

This implies that the universe is created in a state with $V(\phi) + V_0 + \rho_F \sim M_p^4$. Note that the distribution of the probability of the creation of a universe in this scenario is practically flat with respect to ρ_F in an enormously wide interval $\Delta\rho_F \sim M_p^4$. Thus anthropic principle easily fixes $|V_0 + \rho_F| \lesssim 10^{-29} \text{ g/cm}^3$, which solves the cosmological constant problem. The initial value of $V(\phi)$ in this scenario is $O(M_p^4)$, which leads to a very long stage of inflation with $\Omega = 1$, or to $\Omega < 1$ in the models introduced in [7,8,1].

One should note, however, that the possibility to resolve the cosmological constant problem in realistic theories in-

volving the field F requires additional investigation. Indeed, the value of the (negative) energy density of this field in the models based on supergravity depends on the radius of compactification. In realistic models one expects $\rho_F \sim -M_p^4$. If, depending on compactification, ρ_F may take only a discrete set of values such that $\rho_F \sim -M_p^4$, the solution of the cosmological constant problem in this scenario would require that V_0 coincide with one of these values with an accuracy $10^{-123}M_p^4$. Thus the introduction of the antisymmetric tensor field F does not help to solve the problem of having too small Ω in the model of [25], and the possibility that it can help us to solve the cosmological constant problem also remains rather problematic.

VI. CONCLUSIONS

Prior to the invention of the inflationary universe scenario it seemed that quantum cosmology was very important for understanding the underlying principles of the theory of the evolution of the universe, but it may not have any observational consequences. During the last 15 years quantum cosmology has become a more established science, which allows us to make testable observational predictions.

As we have seen, both the Hartle-Hawking and the tunneling wave function of the universe can describe the creation of an open inflationary universe. This is a very interesting possibility in view of the recent tendency to claim that the observations favor a smaller value of Ω .

However, different versions of quantum cosmology predict completely different values of Ω . The Hartle-Hawking wave function predicts that if the universe is closed, then $\Omega \gg 1$, and if it is open, one has $\Omega \sim 10^{-2}$. This is experimentally unacceptable. In this paper we confirmed that this result is practically model independent if galaxy formation occurs due to adiabatic density perturbations produced during inflation. One may try to avoid this conclusion by appealing to some unspecified versions of string theory or M-theory where the situation might be better [20,35]. But in the absence of any realization of this idea one may conclude that at the present time the Hartle-Hawking wave function, *if used to calculate the probability of the quantum creation of the universe*, is in a direct contradiction with observational data.

Is it really possible to rule out the Hartle-Hawking wave function on the basis of these results? Perhaps such a conclusion would be premature. The main argument which pushed the most probable value of Ω toward 10^{-2} was based on the equation for adiabatic density perturbations in a theory of a single scalar field, Eq. (24). This conclusion can change if adiabatic perturbations are very small, and perturbations responsible for galaxy formation are isocurvature, or if they are produced by topological defects. For example, one may imagine that the phase transition which leads to the formation of topological defects occurs during the last stages of chaotic inflation; see, e.g., [65]. Then defect production is a threshold effect, which occurs only if the universe is formed with a sufficiently large scalar field ϕ . In such a situation the Hartle-Hawking wave function will suggest that the scalar field should be as small as possible, but still large enough for the phase transition to take place, because density

perturbations would be too small in a universe without strings. Then the unfortunate prediction $\Omega = 10^{-2}$ may disappear, but it will be replaced by the fine-tuning of the moment of the onset of the phase transition. Also, the possibility to produce the large scale structure of the universe using isothermal perturbations or topological defects is currently out of favor; so we are not sure whether one should consider it seriously.

In our opinion, the whole problem appears here because one tries to apply the Hartle-Hawking wave function for the investigation of the probability of the creation of the universe. Our analysis of this issue contained in Secs. II and III suggests that it should not be used for that purpose. In particular, we have seen that the stochastic approach to inflation unambiguously produces the same probability distribution as the Hartle-Hawking wave function; see Eq. (20). This equation has a simple interpretation: the Hartle-Hawking wave function (in agreement with its derivation in [22]) describes the probability distribution to find the field ϕ in a stationary state (if this state exists) *after* the field relaxes towards the minimum of the effective potential. This wave function does not describe the creation of the universe, inflation, and the process of relaxation toward this ground state, which is the main subject of our investigation.

If one uses the tunneling wave function for the description of initial conditions in the universe, then in most inflationary models the universe should have $\Omega = 1$, which agrees with the standard expectation that inflation makes the universe flat. This result is not sensitive at all to the exact features of the tunneling wave function and, in fact, to the very use of the tunneling wave function. The only thing which we need to assume is that there is no exponential suppression of the quantum creation of a very small universe as compared to the probability of the creation of a very large universe [13].

Moreover, according to the theory of a self-reproducing inflationary universe, which applies to most versions of chaotic inflation [43], one can avoid making even this assumption. The theory of a self-reproducing universe asserts that initial conditions are nearly irrelevant for the description of the properties of the main part of the universe [44,23]. In most models of that type one has $\Omega = 1$ after inflation.

There exists a new potentially interesting class of models

where the creation of an open universe described by the tunneling wave function may be possible. A thorough investigation is needed in order to verify whether this possibility is realistic or not. There are many reasons to be sceptical about it; see Sec. IV C and also [47,48]. It is important, however, that independently of this possibility we still have the class of models proposed in [7,8,1], which does not seem to work in the context of the Hartle-Hawking proposal, but which is quite compatible with the tunneling wave function of the universe, as well as with the theory of a self-reproducing inflationary universe.

Investigation of quantum cosmology in application to open universe creation is very difficult. Much work was to be done in order to investigate the new possibilities which we now have. However, one should not underestimate recent progress. Until very recently, we did not have *any* consistent cosmological models describing a homogeneous open universe. Even though the open universe model did exist from the point of view of mathematics, it simply did not appear to make any sense to assume that all parts of an infinite universe can be created simultaneously and have the same value of energy density everywhere.

That is why it is very encouraging that during the last few years we have found several different mechanisms of the creation of an open universe. All of these mechanisms require the universe to be inflationary. It is still true that inflationary models describing the universe with $\Omega = 1$ are much simpler than the models with $\Omega \neq 1$. Hopefully, the universe will appear to be flat, and we will never need to use any of the models of open inflation. But if we find out that nature has chosen to build the universe in a way which does not look particularly natural, this may give us a rare opportunity to reexamine some of our ideas and to learn more about quantum cosmology.

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- [1] A.D. Linde, Phys. Lett. B **351**, 99 (1995); A. D. Linde and A. Mezhlumian, Phys. Rev. D **52**, 6789 (1995).
 - [2] A.D. Linde, Sov. Phys. JETP **60**, 211 (1984); Lett. Nuovo Cimento **39**, 401 (1984).
 - [3] Ya.B. Zeldovich and A.A. Starobinsky, Sov. Astron. Lett. **10**, 135 (1984).
 - [4] V.A. Rubakov, Phys. Lett. **148B**, 280 (1984).
 - [5] A. Vilenkin, Phys. Rev. D **30**, 509 (1984).
 - [6] S. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
 - [7] J.R. Gott, Nature (London) **295**, 304 (1982); J.R. Gott and T.S. Statler, Phys. Lett. **136B**, 157 (1984).
 - [8] M. Bucher, A.S. Goldhaber, and N. Turok, Phys. Rev. D **52**, 3314 (1995); K. Yamamoto, M. Sasaki, and T. Tanaka, Astrophys. J. **455**, 412 (1995).
 - [9] S.W. Hawking and I.G. Moss, Phys. Lett. **110B**, 35 (1982).
 - [10] A.A. Starobinsky, in *Fundamental Interactions* (MGPI, Moscow, 1984), p. 55.
 - [11] A.S. Goncharov and A.D. Linde, Sov. J. Part. Nucl. **17**, 369 (1986).
 - [12] A.A. Starobinsky, in *Current Topics in Field Theory, Quantum Gravity and Strings*, Lecture Notes in Physics Vol 206, edited by H.J. de Vega and N. Sanchez (Springer, Heidelberg, 1986), p. 107.
 - [13] A.D. Linde, *Particle Physics and Inflationary Cosmology*

- (Harwood, Chur, Switzerland, 1990).
- [14] A.D. Linde, Nucl. Phys. **B372**, 421 (1992).
- [15] L. Amendola, C. Baccigalupi, and F. Occhionero, Phys. Rev. D **54**, 4760 (1996); A.M. Green and A.R. Liddle, *ibid.* **55**, 609 (1997).
- [16] J. García-Bellido, A. Linde, Phys. Rev. D **55**, 7480 (1997).
- [17] J. García-Bellido, J. Garriga, and X. Montes, Phys. Rev. D **57**, 4669 (1998).
- [18] S.S. Costa and H.V. Fagundes, gr-qc/9801066.
- [19] A.D. Linde, “Recent Progress in Inflationary Cosmology,” in Proceedings of the Erice School “Highlights: 50 Years Later,” 1997 (unpublished).
- [20] S.W. Hawking and N. Turok, Phys. Lett. B **425**, 25 (1998).
- [21] A.D. Linde, Phys. Lett. **129B**, 177 (1983).
- [22] J.B. Hartle and S.W. Hawking, Phys. Rev. D **28**, 2960 (1983).
- [23] A.D. Linde, D.A. Linde, and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994).
- [24] A.D. Linde and A. Riotto, Phys. Rev. D **56**, 1841 (1997).
- [25] S.W. Hawking and N. Turok, Phys. Lett. B **432**, 271 (1998).
- [26] J.A. Wheeler, in *Relativity, Groups and Topology*, edited by C.M. DeWitt and J.A. Wheeler (Benjamin, New York, 1968); B.S. DeWitt, Phys. Rev. **160**, 1113 (1967).
- [27] R. Bousso and A. Chamblin, gr-qc/9803047.
- [28] S.W. Hawking and I.G. Moss, Nucl. Phys. **B224**, 180 (1983).
- [29] A.D. Linde, Nucl. Phys. **B216**, 421 (1983).
- [30] A.A. Starobinsky, JETP Lett. **30**, 682 (1979); Phys. Lett. **91B**, 99 (1980).
- [31] A.D. Linde, Phys. Lett. **108B**, 389 (1982); **114B**, 431 (1982); **116B**, 335, 340 (1982); A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
- [32] Ya. B. Zeldovich, Sov. Astron. Lett. **7**, 322 (1981); L.P. Grishchuk and Ya. B. Zeldovich, in *Quantum Structure of Space-Time*, edited by M. Duff and C. J. Isham (Cambridge University Press, Cambridge, England, 1983), p. 353.
- [33] A. Vilenkin, Phys. Lett. **117B**, 25 (1982).
- [34] N. Kaloper, A. Linde, and R. Bousso, hep-th/9801073.
- [35] S.W. Hawking and N. Turok, “Comment on ‘Quantum Creation of an Open Universe,’ by Andrei Linde,” gr-qc/9802062.
- [36] A. Vilenkin, Phys. Rev. D **58**, 067301 (1998).
- [37] J. Garriga and A. Vilenkin, Phys. Rev. D **56**, 2464 (1997).
- [38] R. Bousso and S.W. Hawking, Phys. Rev. D **54**, 6312 (1996).
- [39] J.J. Halliwell and S.W. Hawking, Phys. Rev. D **31**, 1777 (1985).
- [40] V.F. Mukhanov and G.V. Chibisov, JETP Lett. **33**, 532 (1981); V.F. Mukhanov and G.V. Chibisov, Sov. Phys. JETP **56**, 258 (1982); S.W. Hawking, Phys. Lett. **115B**, 295 (1982); A.A. Starobinsky, *ibid.* **117B**, 175 (1982); A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); J. Bardeen, P.J. Steinhardt, and M. Turner, Phys. Rev. D **28**, 679 (1983); V.F. Mukhanov, JETP Lett. **41**, 493 (1985).
- [41] S. Hawking, Phys. Lett. **134B**, 403 (1984).
- [42] S. Coleman, Nucl. Phys. **B307**, 867 (1988); **B310**, 643 (1989); T. Banks, *ibid.* **B309**, 493 (1988); S. Giddings and A. Strominger, *ibid.* **B307**, 854 (1988); **B306**, 890 (1988); G.V. Lavrelashvili, V.A. Rubakov, and P.G. Tinyakov, JETP Lett. **46**, 167 (1987); Nucl. Phys. **B299**, 757 (1988).
- [43] A.D. Linde, Phys. Lett. B **175**, 395 (1986); Phys. Scr. **T15**, 169 (1987); Phys. Today **40** (9), 61 (1987).
- [44] M. Mijić, Phys. Rev. D **42**, 2469 (1990); Int. J. Mod. Phys. A **6**, 2685 (1991).
- [45] In a paper which was published in Daily Telegraph (March 14, 1998), Neil Turok described the significance of their discovery by the following words: “. . . if a divine being wanted to create a universe, the simplest way to do it would be to use our instanton.” To see a photo of this titanic instanton through the Golden Gate bridge, go to <http://physics.stanford.edu/~linde>
- [46] C. Panagiotakopoulos, Phys. Rev. D **55**, R7335 (1997); Phys. Lett. B **402**, 257 (1997).
- [47] A. Vilenkin, Phys. Rev. D **57**, 7069 (1998).
- [48] R. Bousso and A.D. Linde, Phys. Rev. D (to be published) gr-qc/9803068.
- [49] A.D. Linde, D.A. Linde, and A. Mezhlumian, Phys. Lett. B **345**, 203 (1995); Phys. Rev. D **54**, 2504 (1996).
- [50] G. Efstathiou, Mon. Not. R. Astron. Soc. **274L**, 73 (1995).
- [51] D.N. Page, Phys. Rev. D **56**, 2065 (1997).
- [52] A. Vilenkin, Phys. Rev. D **52**, 3365 (1995); A. Vilenkin and S. Winitzki, *ibid.* **55**, 548 (1997).
- [53] J. García-Bellido and A.D. Linde, Phys. Rev. D **51**, 429 (1995); A.D. Linde and A. Mezhlumian, *ibid.* **53**, 4267 (1996).
- [54] J. García-Bellido and A.D. Linde, Phys. Rev. B **398**, 18 (1997).
- [55] T. Futamase and K. Maeda, Phys. Rev. D **39**, 399 (1989); A.L. Berkin and K. Maeda, *ibid.* **44**, 1691 (1991).
- [56] J. García-Bellido and A.D. Linde, Phys. Rev. D **51**, 429 (1995).
- [57] T. Eguchi, P. Gilkey, and A. Hanson, Phys. Rep. **66**, 213 (1980).
- [58] L.A. Kofman and V. Sahni, Phys. Lett. A **117**, 275 (1986).
- [59] Ya.B. Zeldovich, Comments Astrophys. Space Phys. **5**, 169 (1973); D.D. Sokolov and V.F. Shwartsman, Zh. Eksp. Teor. Fiz. **66**, 412 (1974); A.A. Starobinsky, JETP Lett. **57**, 622 (1993); N.J. Cornish, D.N. Spergel, and G.D. Starkman, Phys. Rev. Lett. **77**, 215 (1996); J.J. Levin, J.D. Barrow, E.F. Bunn, and J. Silk, *ibid.* **79**, 974 (1997).
- [60] M. Duff, Phys. Lett. B **226**, 36 (1989).
- [61] A. Aurelia, H. Nicolai, and P.K. Townsend, Nucl. Phys. **B176**, 509 (1980).
- [62] M.J. Duncan L.G. Jensen, Nucl. Phys. **B336**, 100 (1990).
- [63] P. Freund and M. Rubin, Phys. Lett. **97B**, 233 (1980).
- [64] S.W. Hawking, Phys. Lett. B **175**, 395 (1984).
- [65] L.A. Kofman and A.D. Linde, Nucl. Phys. **B282**, 555 (1987).